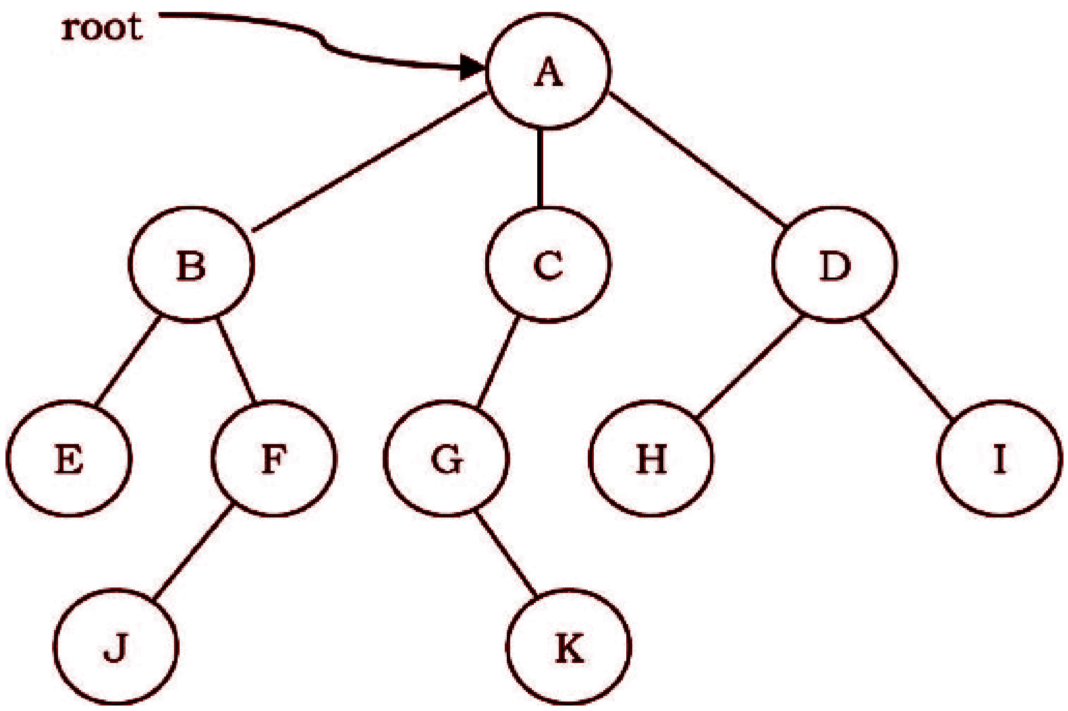
Binary Trees- 1

What is A Tree?

- A tree is a data structure similar to a linked list but instead of each node pointing simply to the next node in a linear fashion, each node points to several nodes.
- A tree is an example of a non- linear data structure.
- A tree structure is a way of representing the hierarchical nature of a structure in a graphical form.

Terminology Of Trees

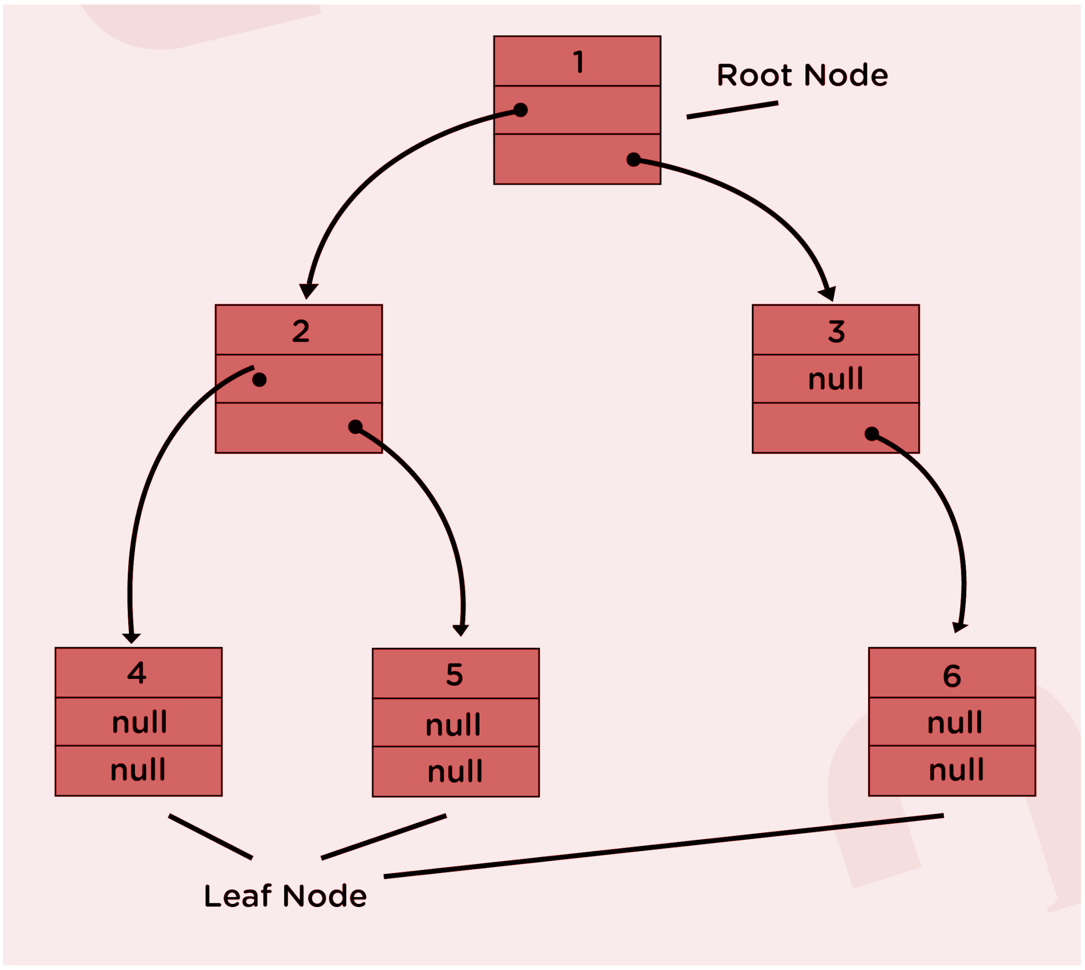


- The root of a tree is the node with no parents. There can be at most one root node in a tree **(node A in the above example)**.

- An **edge** refers to the link from a parent to a child **(all links in the figure)**.
- A node with no children is called a **leaf node** **(E, J, K, H, and I)**.
- The children nodes of the same parent are called **siblings** **(B, C, D are siblings of parent A and E, F are siblings of parent B)**.
- The set of all nodes at a given depth is called the **level** of the tree **(B, C, and D are the same level)**. The root node is at level zero.
- The **depth** of a node is the length of the path from the root to the node **(depth of G is 2, *A -> C –> G*)**.
- The **height** of a node is the length of the path from that node to the deepest node.
- The **height** of a tree is the length of the path from the root to the deepest node in the tree.
- A (rooted) tree with only one node (the root) has a height of zero.

Binary Trees

- A generic tree with at most two child nodes for each parent node is known as a binary tree.
- A binary tree is made of nodes that constitute a **left** pointer, a **right** pointer, and a data element. The **root** pointer is the topmost node in the tree.
- The left and right pointers recursively point to smaller **subtrees** on either side.
- An empty tree is also a valid binary tree.
- *A formal definition is:* A **binary tree** is either empty (represented by a None pointer), or is made of a single node, where the left and right pointers (recursive definition ahead) each point to a **binary tree**.

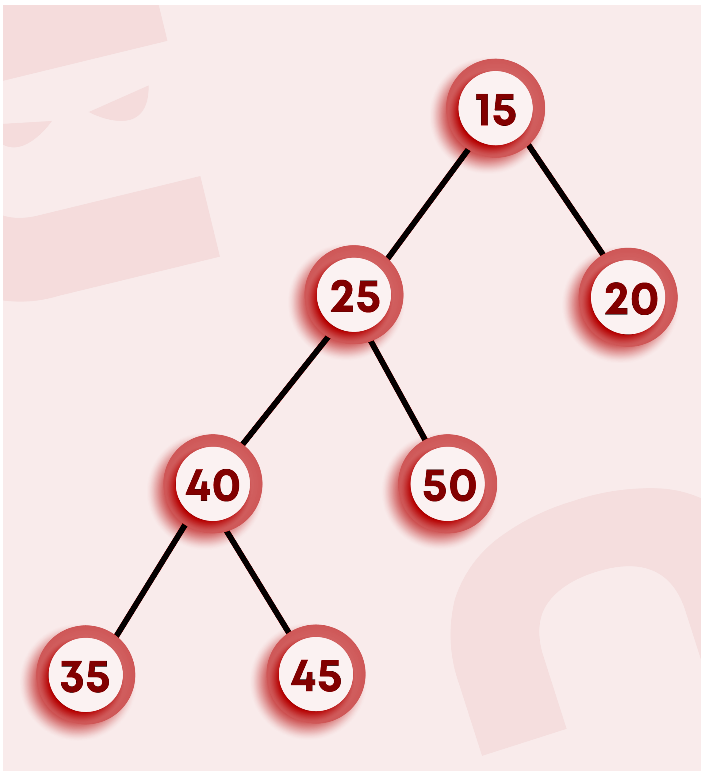


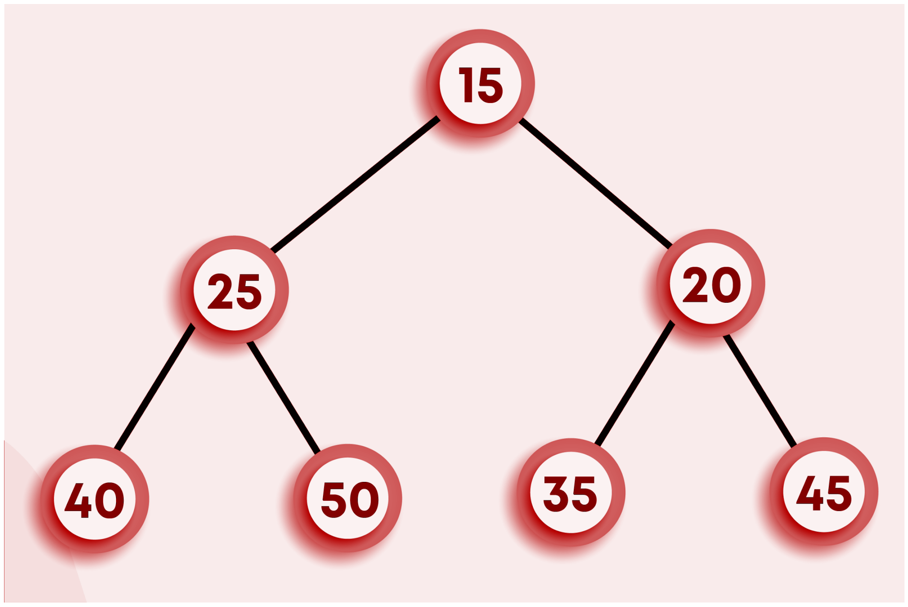
Types of binary trees:

Full binary trees: A binary tree in which every node has 0 or 2 children is termed as a full binary tree.

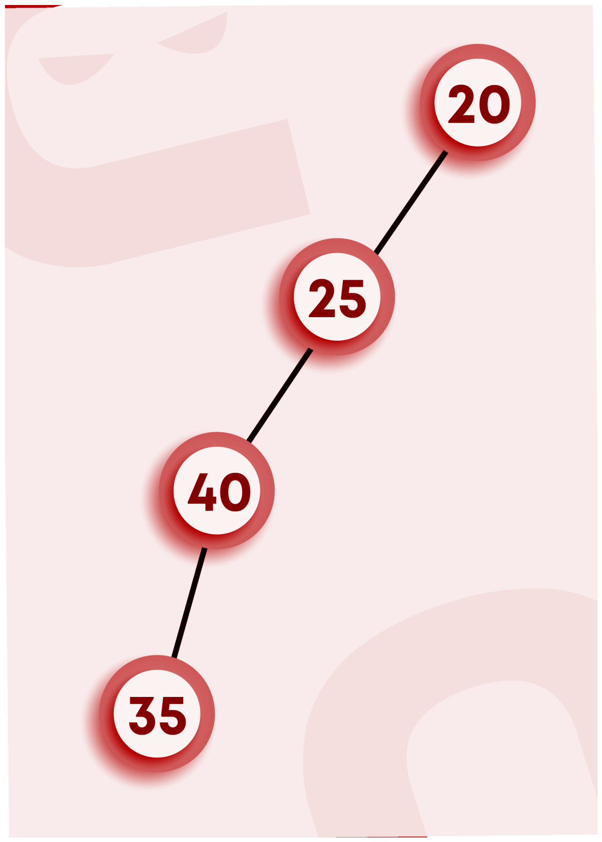
Complete binary tree: A complete binary tree has all the levels filled except for the last level, which has all its nodes as much as to the left.

Perfect binary tree: A binary tree is termed perfect when all its internal nodes have two children along with the leaf nodes that are at the same level.



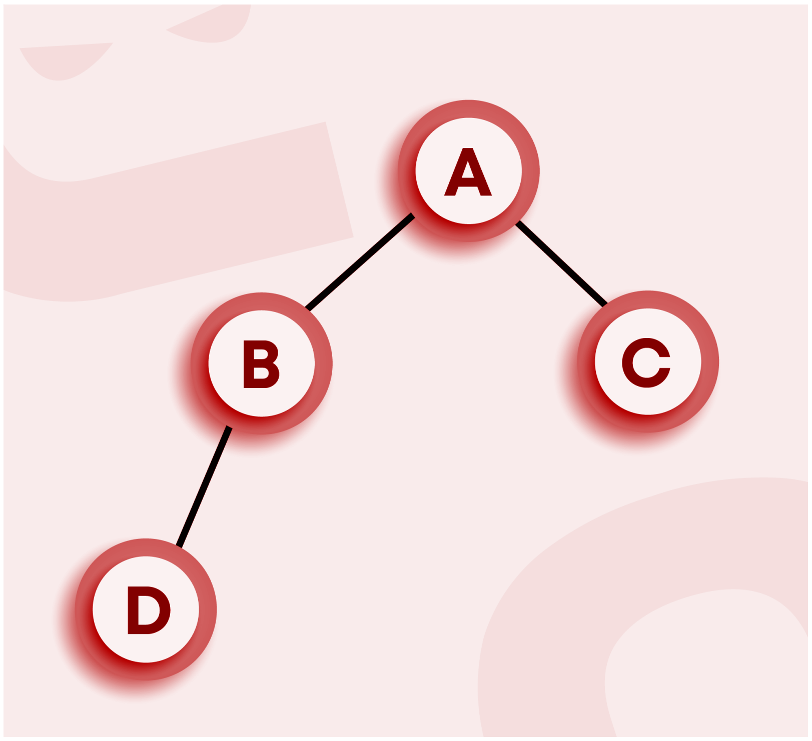


A degenerate tree: In a degenerate tree, each internal node has only one child.



The tree shown above is degenerate. These trees are very similar to linked-lists.

Balanced binary tree: A binary tree in which the difference between the depth of the two subtrees of every node is at most one is called a balanced binary tree.



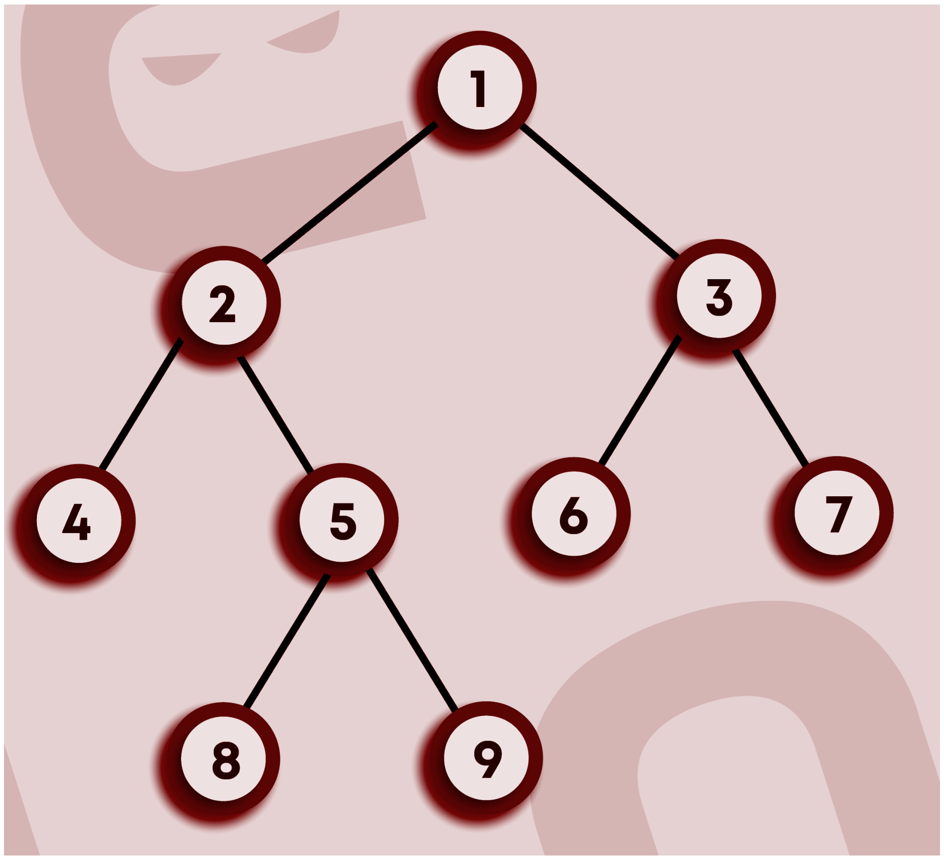
Binary tree representation:

Binary trees can be represented in two ways:

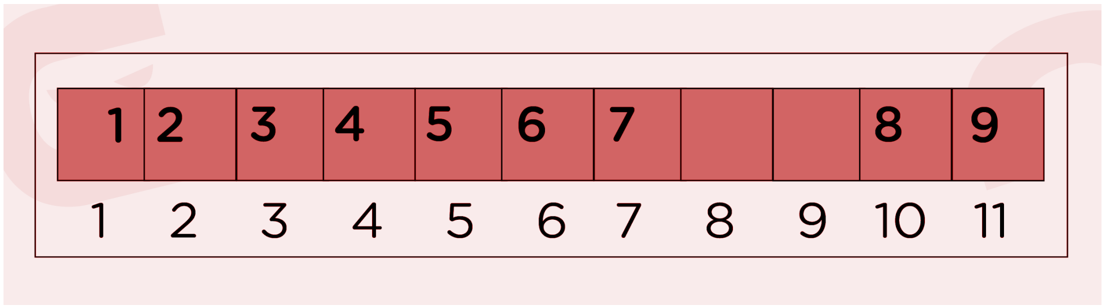
Sequential representation

- This is the most straightforward technique to store a tree data structure. An array is used to store the tree nodes.
- The number of nodes in a tree defines the size of the array.
- The root node of the tree is held at the first index in the array.
- In general, if a node is stored at the **i**$^{\text{th}}$ location, then its **left** and **right** child are kept at **(2i)**$^{\text{th}}$ and **(2i+1)**$^{\text{th}}$ locations in the array, respectively.

Consider the following binary tree:



The array representation of the above binary tree is as follows:



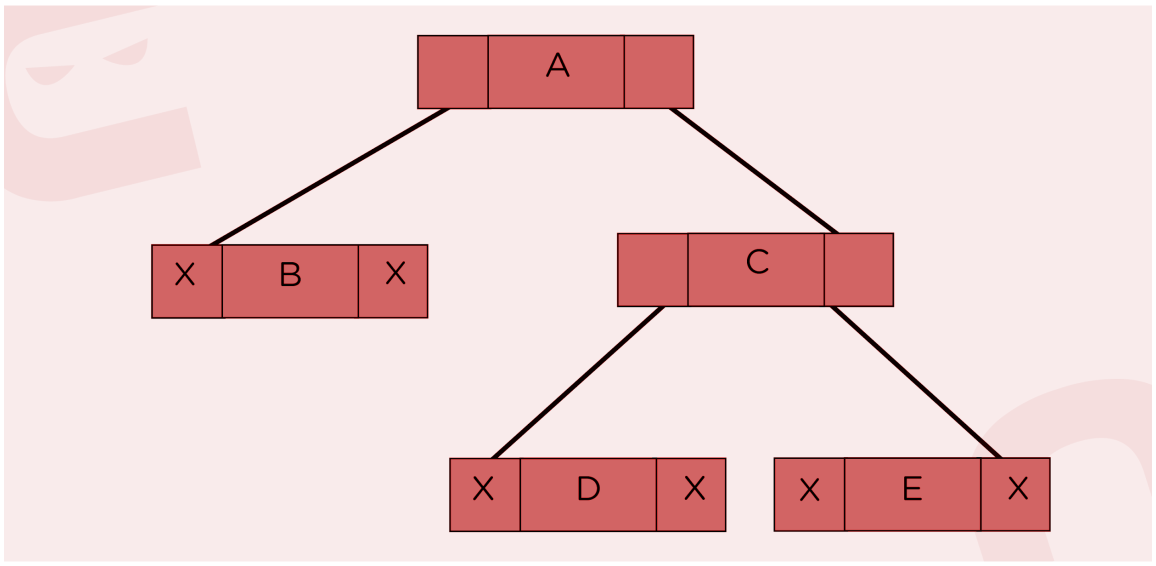
As discussed above, we see that the left and right child of each node is stored at locations **2*(nodePosition)** and **2*(nodePosition)+1**, respectively.

For Example, The location of node 3 in the array is 3. So its left child will be placed at **2*3 = 6**. Its right child will be at the location **2*3 +1 = 7**. As we can see in the array, children of 3, which are 6 and 7, are placed at locations 6 and 7 in the array.

Note: The sequential representation of the tree is not preferred due to the massive amount of memory consumption by the array.

Linked list representation:

In this type of model, a linked list is used to store the tree nodes. The nodes are connected using the parent-child relationship like a tree. The following diagram shows a linked list representation for a tree.

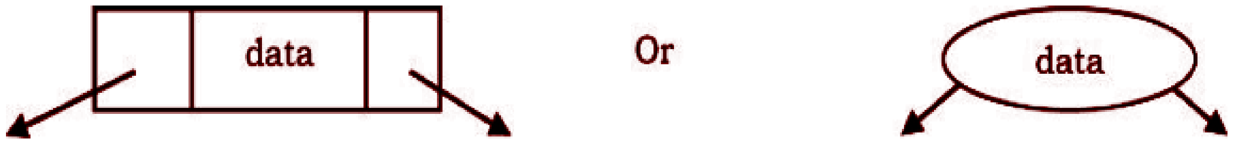


As shown in the above representation, each linked list node has three components:

- Pointer to the left child
- Data
- Pointer to the right child

Note: If there are no children for a given node (leaf node), then the left and right pointers for that node are set to **None**.

Let's now check the implementation of the **Binary tree class**.



```
class BinaryTreeNode:
    def __init__(self, data):
        self.left = None #To store data
        self.right = None #For storing the reference to left pointer
        self.data = data #For storing the reference to right pointer
```

Operations on Binary Trees

Basic Operations

- Inserting an element into a tree
- Deleting an element from a tree
- Searching for an element
- Traversing the tree

Auxiliary Operations

- Finding the size of the tree
- Finding the height of the tree
- Finding the level which has the maximum sum and many more...

Print Tree Recursively

Let's first write a program to print a binary tree recursively. Follow the comments in the code below:

```
def printTree(root):
      root == None: #Empty tree
            return
      print(root.data, end ":") #Print root data
      if root.left != None:
            print("L", root.left.data, end=",") #Print left child
      if root.right != None:
            print("R", root.right.data, end="") #Print right child
      Print #New line
```

```
    printTree(root.left) #Recursive call to print left subtree
    printTree(root.right) #Recursive call to print right subtree
```

Input Binary Tree

We will be following the level-wise order for taking input and -1 denotes the **None** pointer.

```
def treeInput():
    rootData = int(input())
    if rootData == -1: #Leaf Node is denoted by -1
        return None

    root = BinaryTreeNode(rootData) #Create a tree node
    leftTree =treeInput() #Take input for left subtree
    rightTree = treeInput() #Take input for right subtree
    root.left = leftTree #Assign the left subtree to the left child
    root.right = rightTree #Right subtree to the right child
    return root
```

Count nodes

- Unlike the Generic trees, where we need to traverse the children vector of each node, in binary trees, we just have at most left and right children for each node.
- Here, we just need to recursively call on the right and left subtrees independently with the condition that the node pointer is not None.
- Follow the comments in the upcoming code for better understanding:

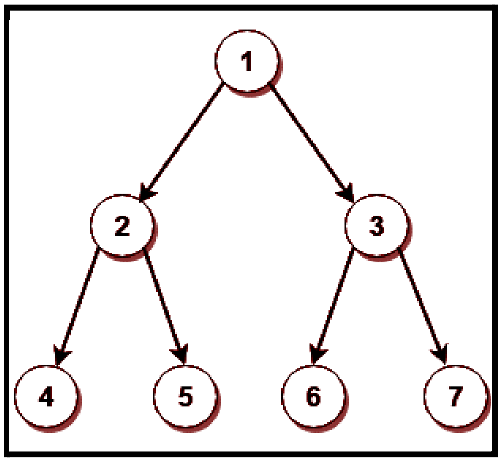
```
def count_nodes(node):
    if node is None: #Check if root node is None
        return 0
    return 1 + count_nodes(node.left) + count_nodes(node.right)
#Recursively count number of nodes in left and right subtree and add
```

Binary tree traversal

Following are the ways to traverse a binary tree and their orders of traversal:

- **Preorder traversal** : ROOT -> LEFT -> RIGHT
- **Postorder traversal** : LEFT -> RIGHT-> ROOT
- **Inorder traversal** : LEFT -> ROOT -> RIGHT

Some examples of the above-stated traversal methods:



❖ **Preorder traversal:** 1, 2, 4, 5, 3, 6, 7

❖ **Postorder traversal:** 4, 5, 2, 6, 7, 3, 1

❖ **Inorder traversal:** 4, 2, 5, 1, 6, 3, 7

Let’s look at the code for inorder traversal, below:

```
# A function to do inorder tree traversal
def printInorder(root):
    if root:#If tree is not empty
        printInorder(root.left) # First recur on left child
        print(root.val)# Then print the data of the node
        printInorder(root.right) # Now recur on right child
```

Now, from this inorder traversal code, try to code preorder and postorder traversal yourselves. If you get stuck, refer to the solution tab for the same.

Node with the Largest Data

In a Binary Tree, we must visit every node to figure out the maximum. So the idea is to traverse the given tree and for every node return the maximum of 3 values:

- Node's data.
- Maximum in node's left subtree.
- Maximum in node's right subtree.

Below is the implementation of the above approach.

```
def findMaximum(root):
    # Base case
    if (root == None):
        return float('-inf') #**

    # Return maximum of 3 values:
    # 1) Root's data 2) Max in Left Subtree
    # 3) Max in right subtree
    max = root.data
    lmax = findMaximum(root.left) #Maximum of left subtree
    rmax = findMaximum(root.right) #Maximum of right subtree
    if (lmax > max):
        max = lmax
    if (rmax > max):
        max = rmax
    return max
```

Note**: In python, float values can be used to represent an infinite integer. One can use **float('-inf')** as an integer to represent it as “Negative” infinity or the smallest possible integer.

Binary Trees- 2

Construct a Binary Tree from Preorder and Inorder Traversal

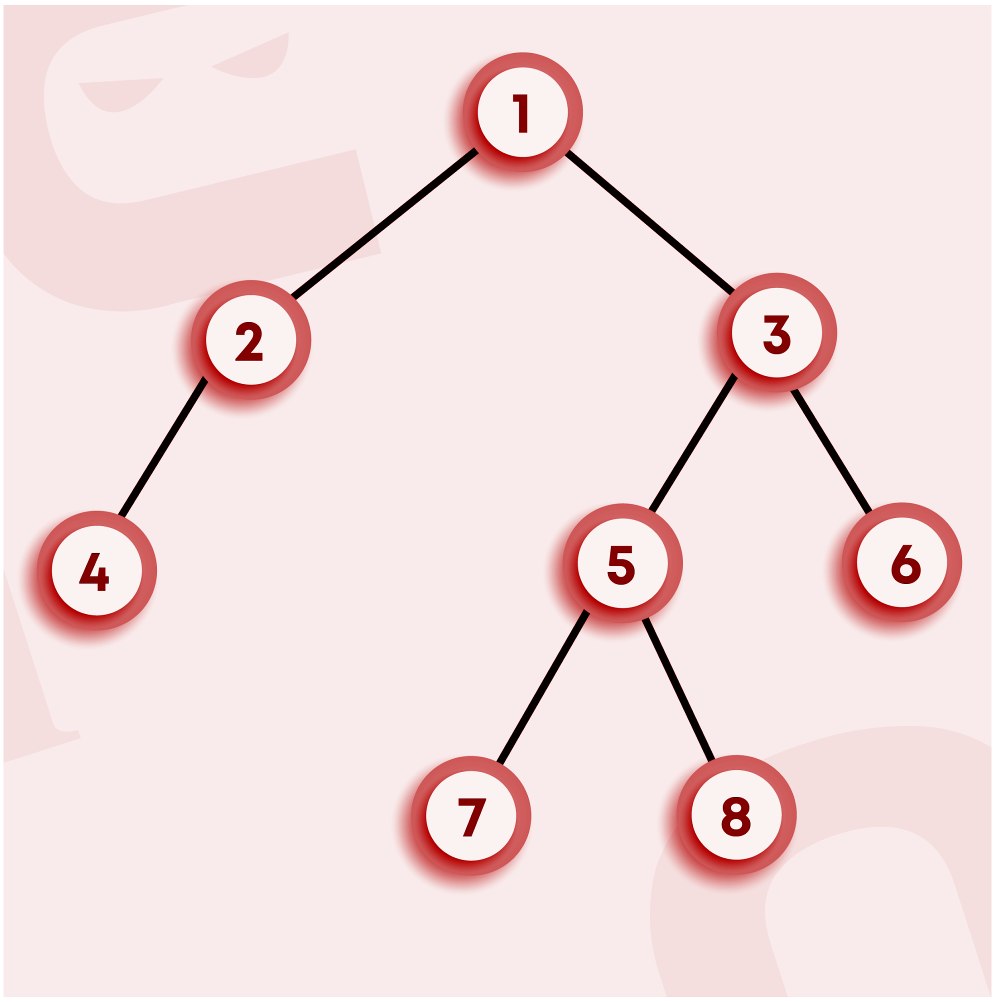
Consider the following example to understand this better.

Input:

Inorder traversal : {4, 2, 1, 7, 5, 8, 3, 6}

Preorder traversal : {1, 2, 4, 3, 5, 7, 8, 6}

Output: Below binary tree...



- The idea is to start with the root node, which would be the first item in the preorder sequence, and find the boundary of its left and right subtree in the inorder array.
- Now, all keys before the root node in the inorder array become part of the left subtree, and all the indices after the root node become part of the right subtree.
- We repeat this recursively for all nodes in the tree and construct the tree in the process.

To illustrate, consider below inorder and preorder sequence-
Inorder: {4, 2, 1, 7, 5, 8, 3, 6}
Preorder: {1, 2, 4, 3, 5, 7, 8, 6}

- The root will be the first element in the preorder sequence, i.e. 1.
- Next, we locate the index of the root node in the inorder sequence.
- Since 1 is the root node, all nodes before 1 must be included in the left subtree, i.e., {4, 2}, and all the nodes after one must be included in the right subtree, i.e. {7, 5, 8, 3, 6}.
- Now the problem is reduced to building the left and right subtrees and linking them to the root node.

Thus we get:

Left subtree:	**Right subtree:**
Inorder : {4, 2}	**Inorder :** {7, 5, 8, 3, 6}
Preorder : {2, 4}	**Preorder :** {3, 5, 7, 8, 6}

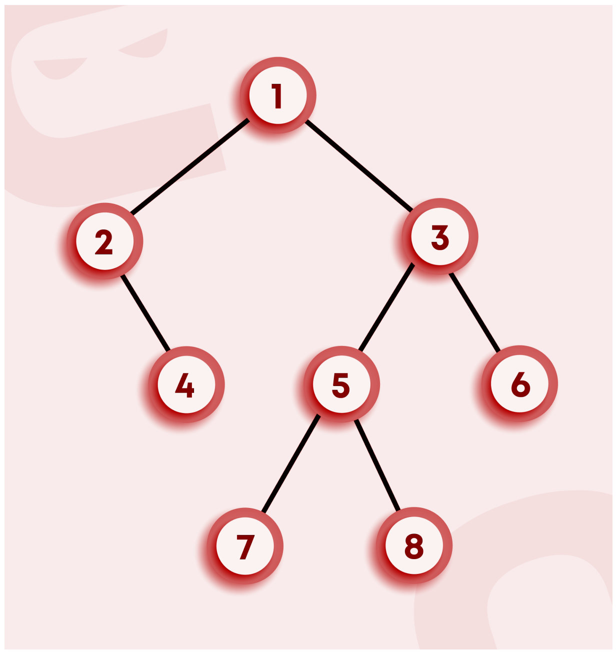
Follow the above approach until the complete tree is constructed. Now let us look at the code for this problem.

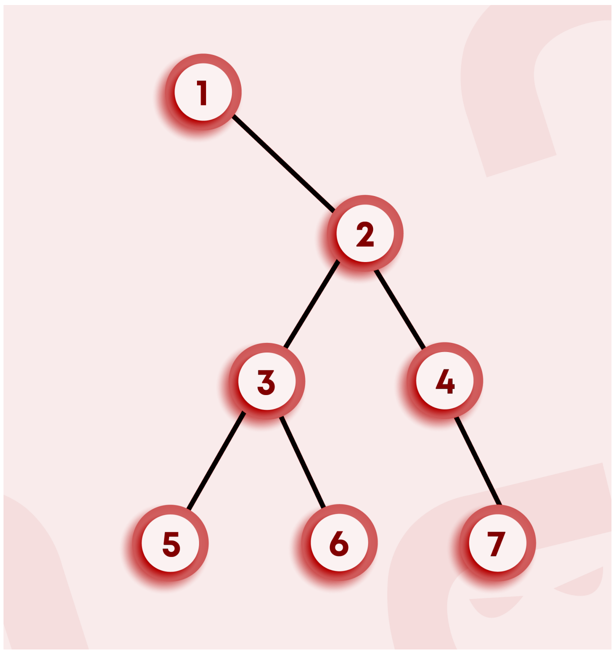
Note: The code written from the above insights can be accessed in the solution tab in the question itself.

The Diameter of a Binary tree

- The **diameter** of a tree (sometimes called the width) is the number of nodes on the longest path between two child nodes.
- The diameter of the binary tree may pass through the root (not necessary).

For example, the figure below shows two binary trees having diameters 6 and 5, respectively. The diameter of the binary tree shown on the left side passes through the root node while on the right side, it doesn't.





There are three possible paths of the diameter:

1. The diameter could be the sum of the left height and the right height.
2. It could be the left subtree's diameter.
3. It could be the right subtree's diameter.

We will pick the one with the maximum value.

Now let's check the code for this...

```python
def height(node):

    # Base Case : Tree is empty
    if node is None:
        return 0

    # If tree is not empty then height = 1 + max of left
    # height and right heights
    return 1 + max(height(node.left), height(node.right))

def diameter(root):

    # Base Case when tree is empty
    if root is None:
        return 0

    # Get the height of left and right subtrees
    leftH= height(root.left)
    rightH = height(root.right)

    # Get the diameter of left and right subtrees
    leftD = diameter(root.left)
    rightD = diameter(root.right)

    # Return max of the three
    return max(leftH + rightH + 1, max(leftD, rightD))
```

The time complexity for the above approach:

- Height function traverses each node once; hence time complexity will be **O(n).**
- Option2 and Option3 also traverse on each node, but for each node, we are calculating the height of the tree considering that node as the root node, which makes time complexity equal to **O(n*h)**. (worst case with skewed trees, i.e., a type of binary tree in which all the nodes have only either one child or no child.) Here, **h** is the height of the tree, which could be **O(n^2)**.

This could be reduced if the height and diameter are obtained simultaneously, which could prevent extra **n** traversals for each node.

The Diameter of a Binary tree: Better Approach

In the previous approach, for each node, we were finding the height and diameter independently, which was increasing the time complexity. In this approach, we will find the height and diameter for each node at the same time, i.e., we will store the height and diameter using a pair class where the **first** pointer will be storing the height of that node and the **second** pointer will be storing the diameter. Here, also we will be using recursion.

Let's focus on the **base case**: For a NULL tree, height and diameter both are equal to 0. Hence, the pair class will store both of its values as zero.

Now, moving to **Hypothesis**: We will get the height and diameter for both left and right subtrees, which could be directly used.

Finally, the **induction step**: Using the result of the Hypothesis, we will find the height and diameter of the current node:

Height = `max(leftHeight, rightHeight)`

Diameter = `max(leftHeight + rightHeight, leftDiameter, rightDiameter)`

To create a pair class, follow the syntax below:

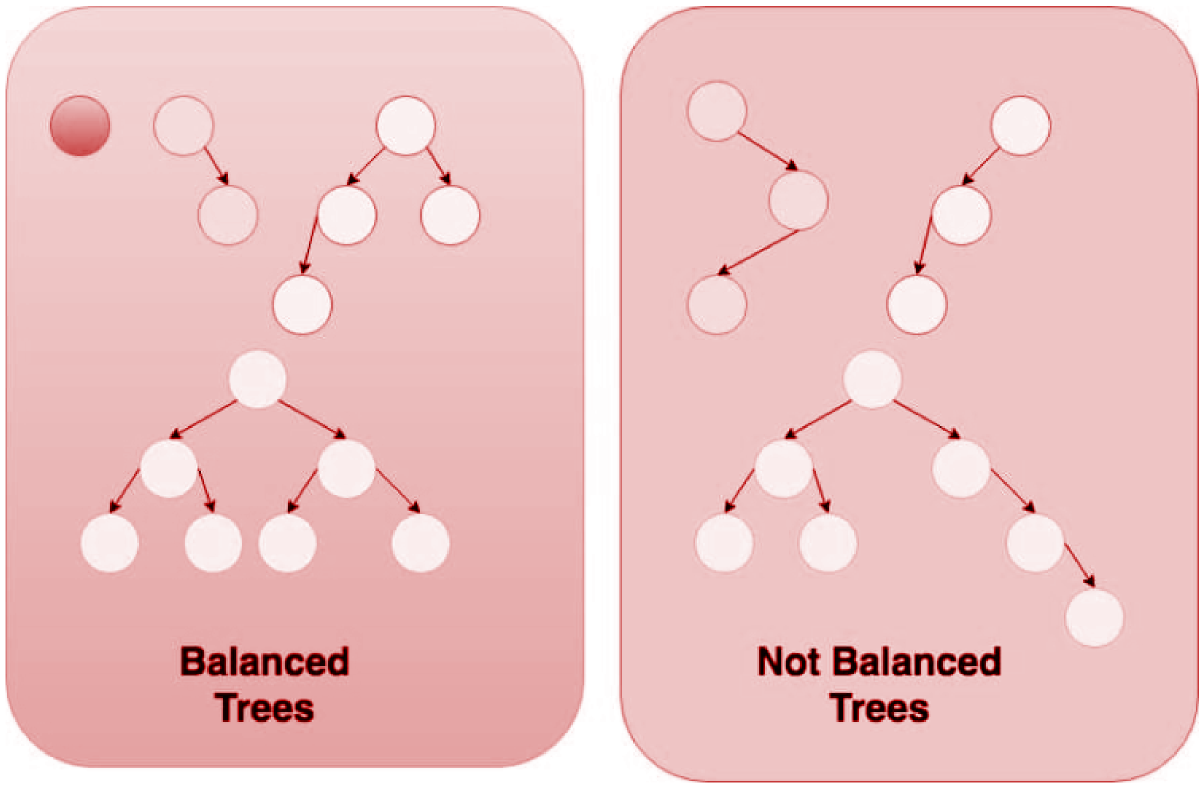
```python
class Pair:
    def __init__(self, data): # Constructor to create a new Pair
        self.data = data
        self.left = self.right = None
```

It can be observed that we are just traversing each node once while making recursive calls and the rest of all other operations are performed in constant time, hence the time complexity of this program is **O(n)**, where **n** is the number of nodes.

Note: The code written from the above insights can be accessed in the solution tab in the question itself.

Balanced Binary Tree

A binary tree is balanced if, for each node in the tree, the difference between the height of the right subtree and the left subtree is at most one.



Check if a Binary Tree is Balanced

From the definition of a balanced tree, we can conclude that a binary tree is balanced if at every node in the tree:

- The right subtree is balanced.
- The left subtree is balanced.
- The difference between the height of the left subtree and the right subtree is at most 1

Let's define a recursive function **is_balanced()** that takes a root node as an argument and returns a boolean value that represents whether the tree is balanced or not.

Let's also define a helper function **get_height()** that returns the height of a tree. Notice that **get_height()** is also implemented recursively

```
def get_height(root):#Returns height of the tree
    if root is None:
        return 0
    return 1 + max(get_height(root.left , get_height(root.right))

def is_balanced(root):
    # a None tree is balanced
    if root is None:
        return True
    return is_balanced(root.right) and is_balanced(root.left) and
abs(get_height(root.left) - get_height(root.right)) <= 1
```

The **is_balanced()** function returns true if the right subtree and the left subtree are balanced, and if the difference between their height does not exceed 1.

Check if a Binary Tree is Balanced- Improved Solution

Here, we are using two recursive functions: one that checks if a tree is balanced, and another one that returns the height of a tree. Can we achieve the same goal by using only one recursive function?

Let us define our recursive function **is_balanced_helper()** to be a function that takes one argument, the tree root and returns an integer such that:

- If the tree is balanced, return the height of the tree
- If the tree is not balanced, return -1

Notice that this new **is_balanced_helper()** can be easily implemented recursively as well by following these rules:

- Apply is_balanced_helper on both the right and left subtrees
- If either the right or left subtrees returns -1, then we should return -1 (because our tree is not balanced if either subtree is not balanced)
- If both subtrees return an integer value (indicating the heights of the subtrees), then we check the difference between these heights.
- If the difference doesn't exceed 1, then we return the height of this tree. Otherwise, we return -1

Let us now look at its implementation:

```
def is_balanced_helper(root):
    if root is None: # a None tree is balanced
        return 0

 # if the left subtree is not balanced, then tree is not balanced
    left_height = is_balanced_helper(root.left)
    if left_height == -1:
        return -1

    # if the right subtree is not balanced,then tree is not balanced
    right_height = is_balanced_helper(root.right)
    if right_height == -1:
        return -1

    # If the difference in heights is greater than 1
    if abs(left_height - right_height) > 1:
        return -1

    # this tree is balanced, return its height
    return max(left_height, right_height) + 1
```

Finally we know that, if **is_balanced_helper()** returns a number that is greater than -1, the tree is balanced, otherwise, it is not.

```
def is_balanced(root):
    return is_balanced_helper(root) > -1
```

Practice problems:

- https://www.hackerrank.com/challenges/tree-top-view/problem
- https://www.codechef.com/problems/BTREEKK
- https://www.spoj.com/problems/TREEVERSE/
- https://www.hackerearth.com/practice/data-structures/trees/binary-and-nary-trees/practice-problems/approximate/largest-cycle-in-a-tree-9113b3ab/