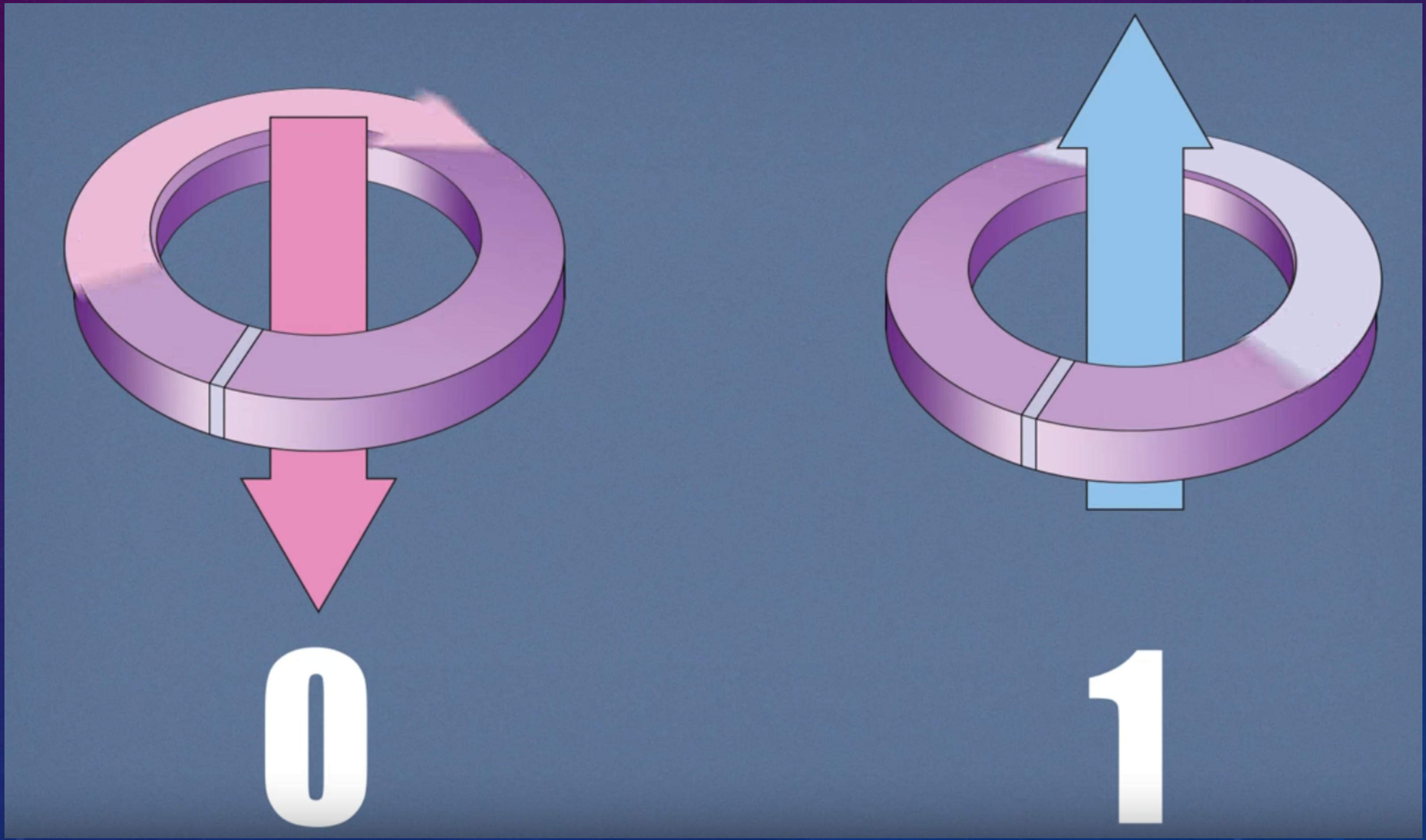
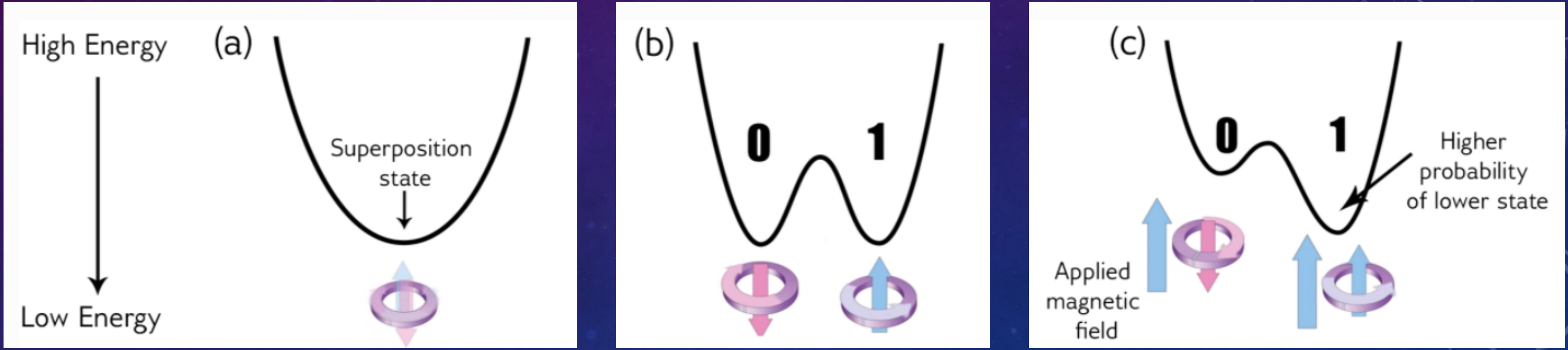
QUANTUM ANNEALING USING DWAVE OCEAN SDK

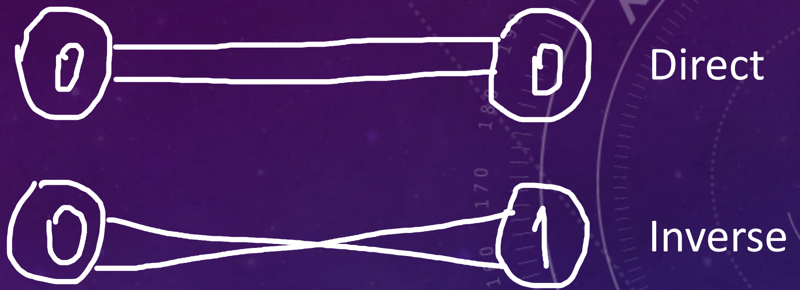
THE HARDWARE

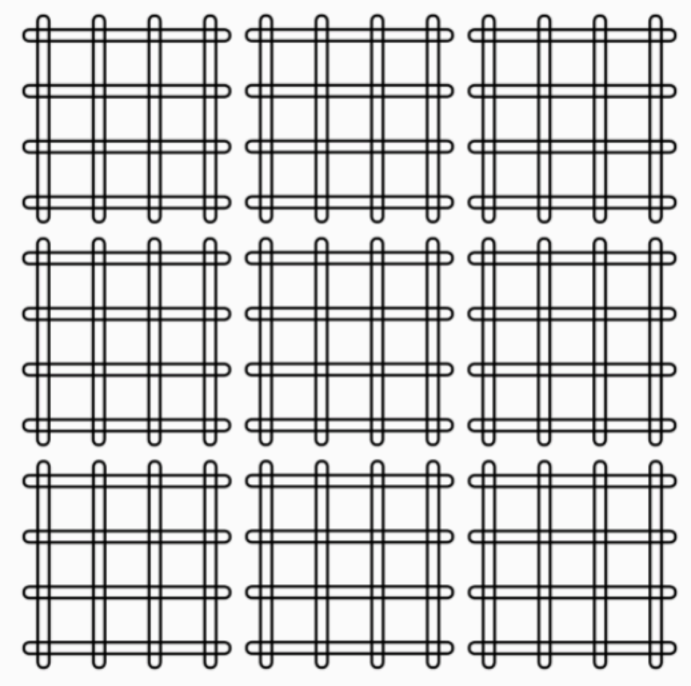


THE PROCESS

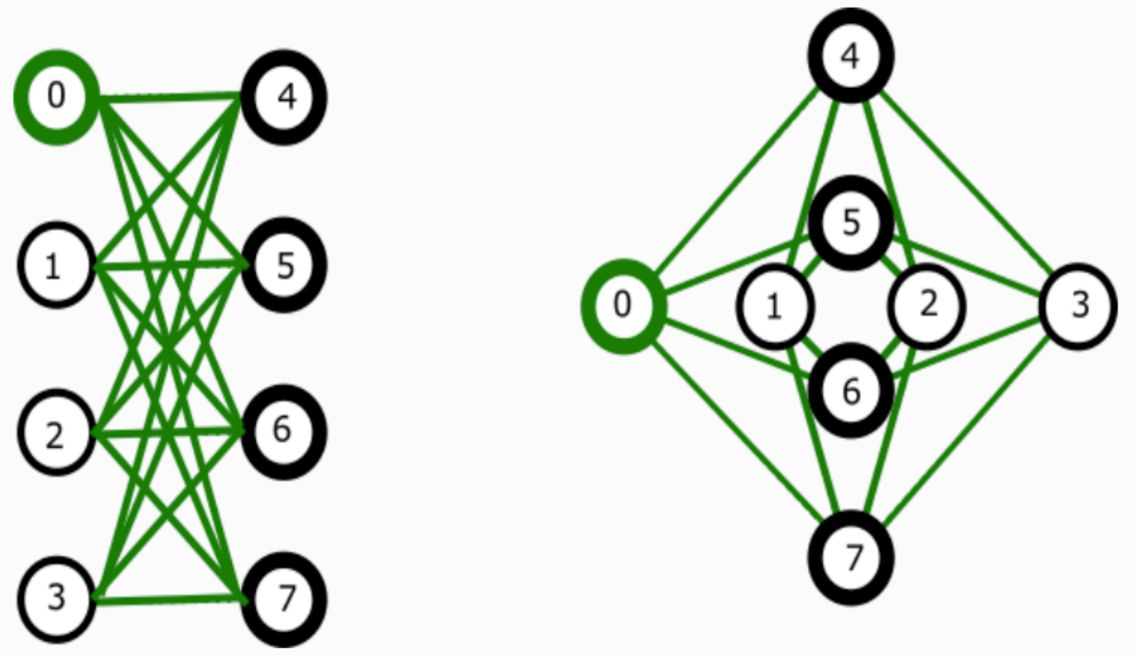

High Energy
Low Energy
(a)
Superposition state
(b)
0
1
(c)
0
1
Higher probability of lower state
Applied magnetic field

COUPLERS





Chimera Graph

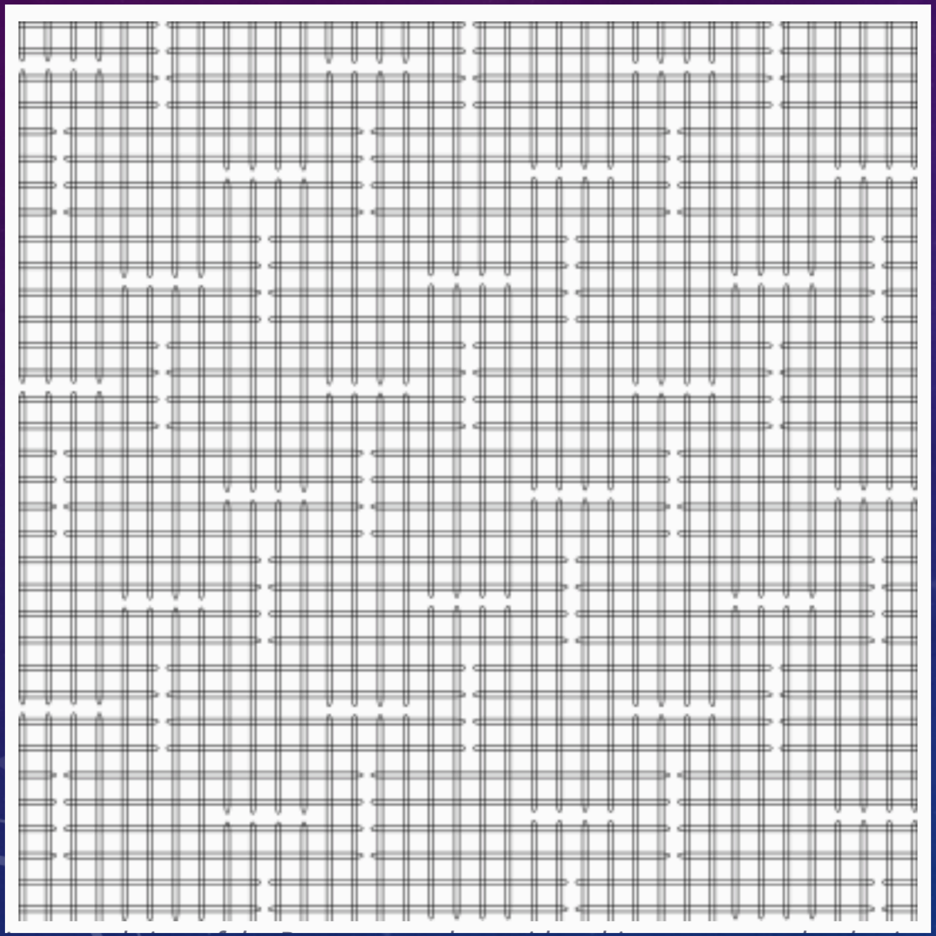


$K_{4,4}$ bipartite graph

COUPLERS

Direct

Inverse



Pegasus Graph

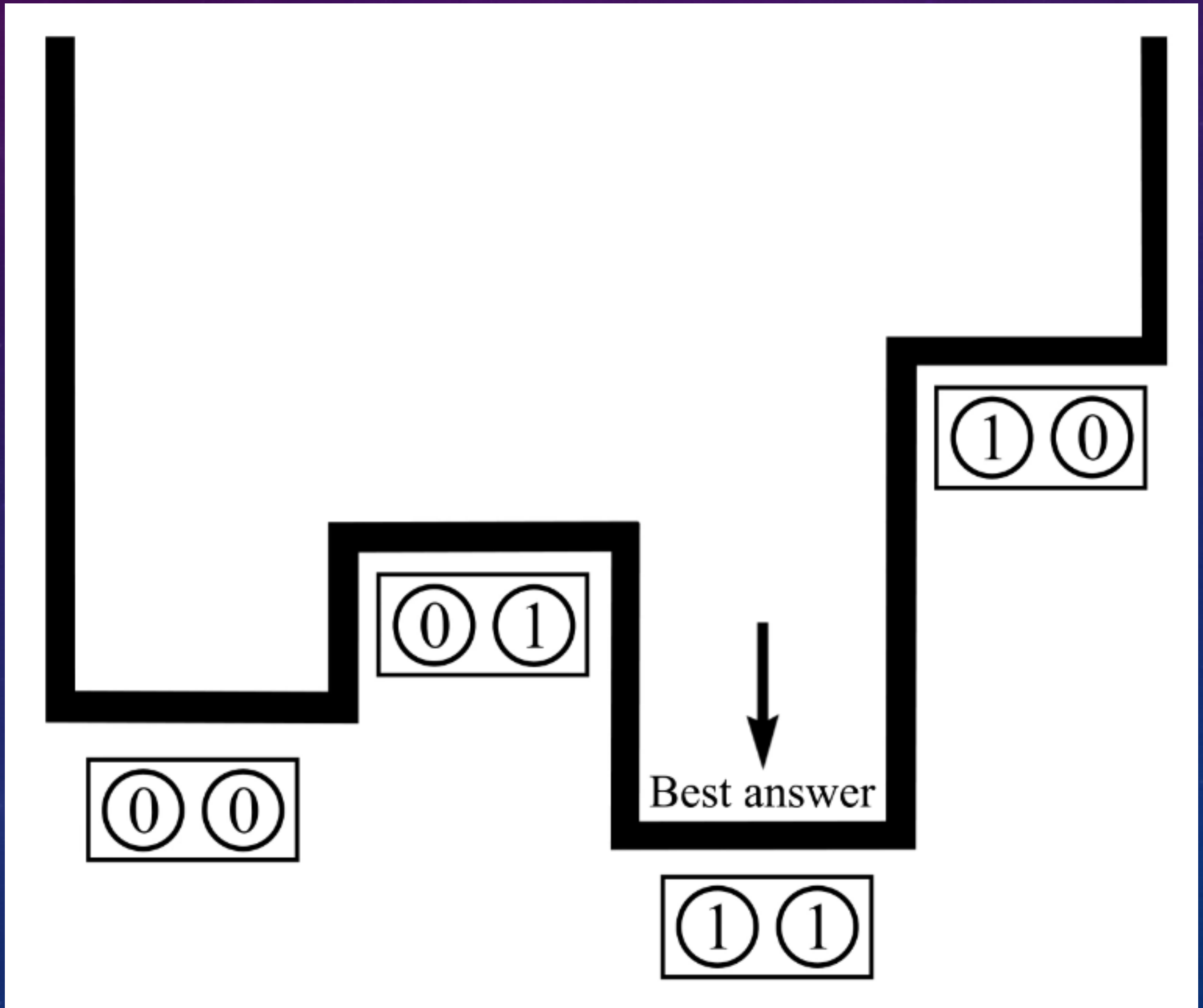
Pegasus features qubits of degree 15

and native

K_4 and $K_{6,6}$ subgraphs.

Future Work!!!

ENERGY LANDSCAPE



UNVEILING QUANTUM

HAMILTONIANS

- Hamiltonians are functions which convert the eigenstates of the system to the corresponding energy values.

- $\hat{H}|\psi\rangle = \hat{E}|\psi\rangle$

$$\mathcal{H}_{ising} = \underbrace{-\frac{A(s)}{2}\left(\sum_i \hat{\sigma}_x^{(i)}\right)}_{\text{Initial Hamiltonian}} + \underbrace{\frac{B(s)}{2}\left(\sum_i h_i \hat{\sigma}_z^{(i)} + \sum_{i>j} J_{i,j}\hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(j)}\right)}_{\text{Final Hamiltonian}}$$

BINARY QUADRATIC MODEL

- Quadratic Unconstrained Binary Optimization (QUBO) Model
  - Information represented in binary form
  - 0 / 1

$$\mathrm{E}_{qubo}(a_i, b_{i,j}; q_i) = \sum_i a_i q_i + \sum_{i<j} b_{i,j} q_i q_j.$$

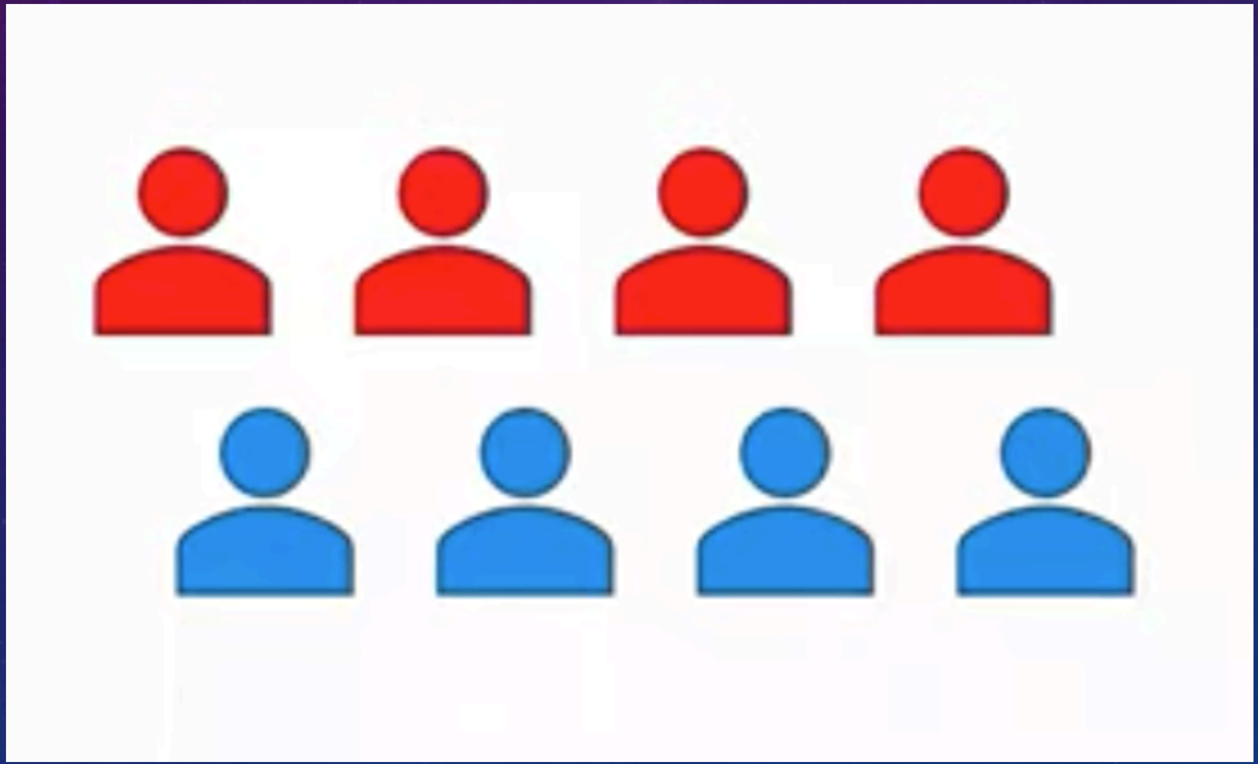
- Ising Model
  - Information represented in terms of spin information
  - +1 / -1

$$\mathrm{E}_{ising}(\boldsymbol{s}) = \sum_{i=1}^{N} h_i s_i + \sum_{i=1}^{N} \sum_{j=i+1}^{N} J_{i,j} s_i s_j$$

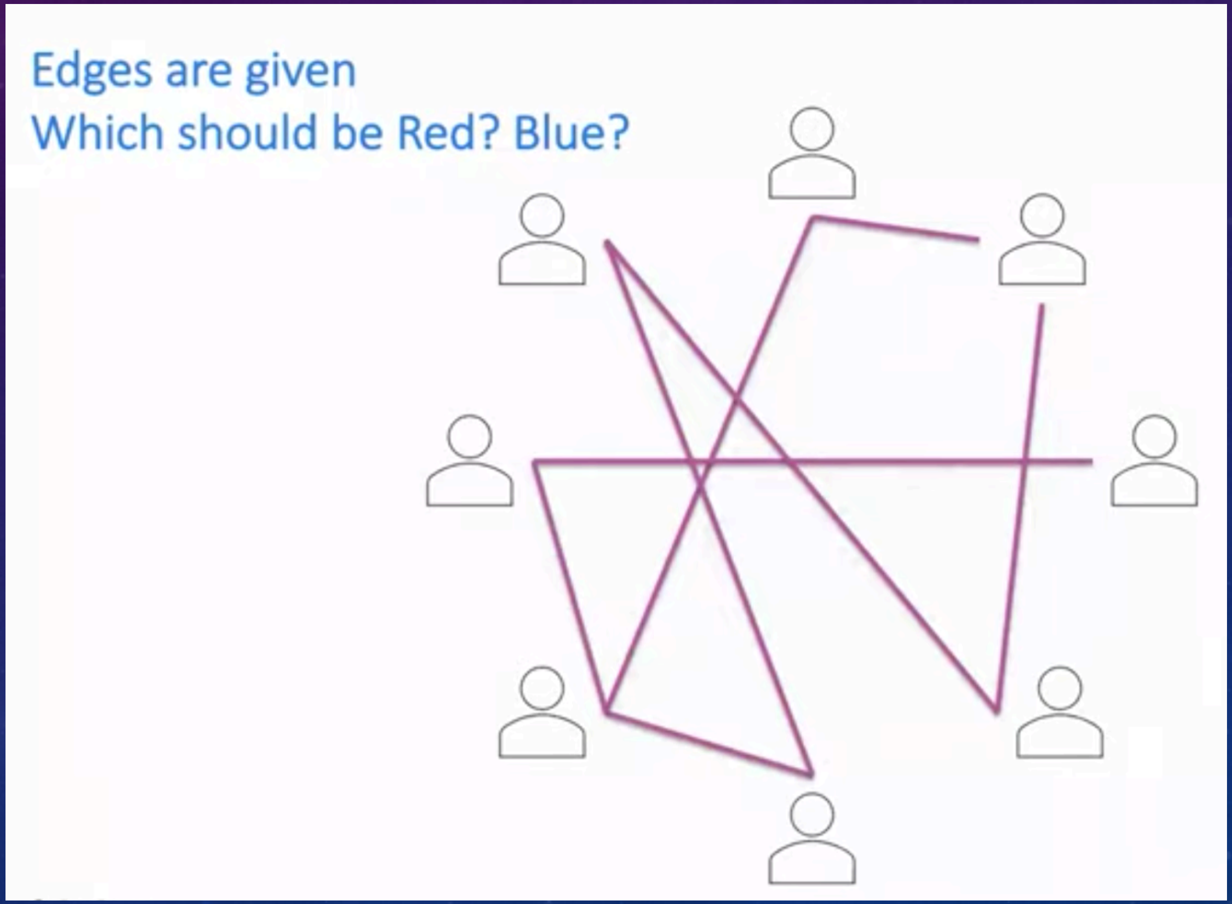
BQM DEMO

EXAMPLE PROBLEM

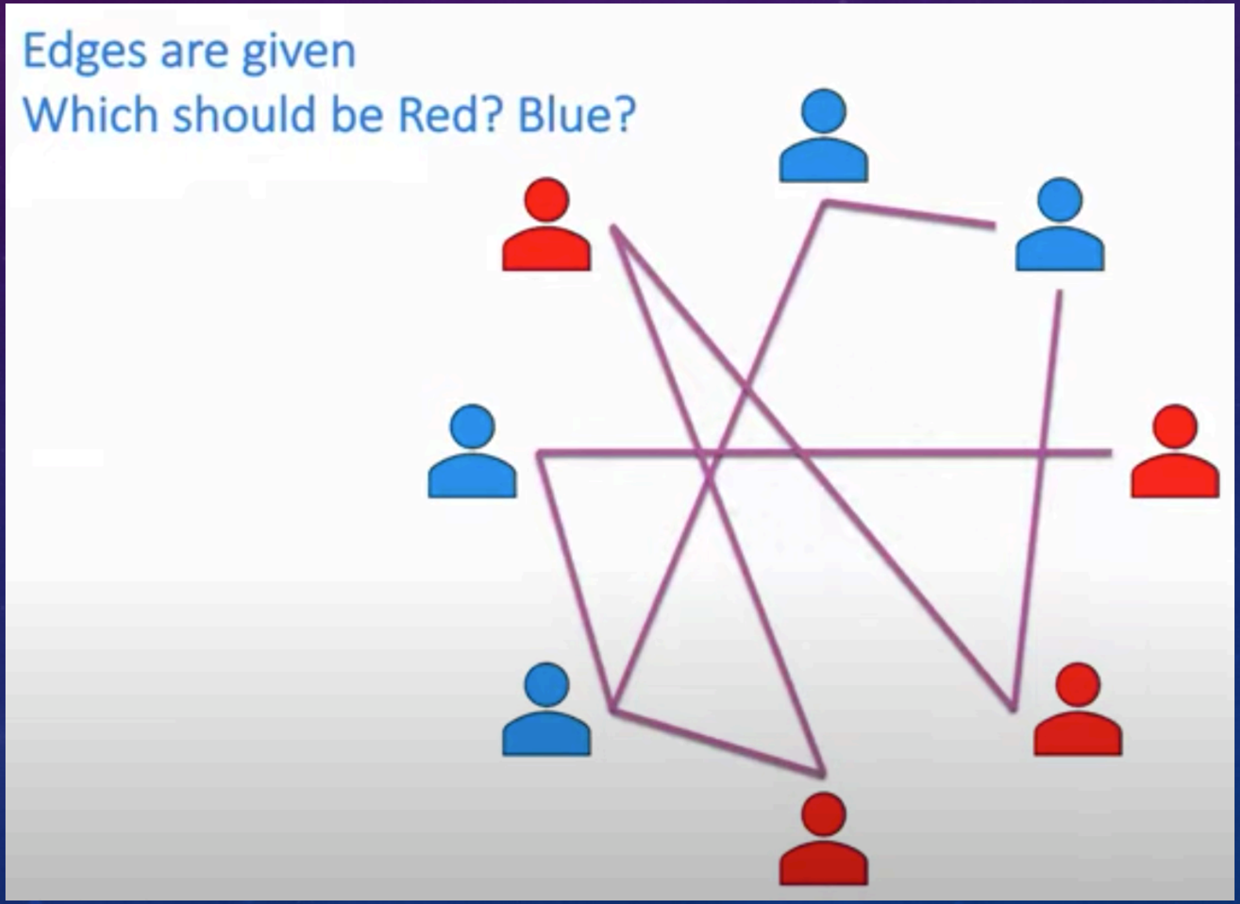
EXAMPLE PROBLEM



EXAMPLE PROBLEM



EXAMPLE PROBLEM



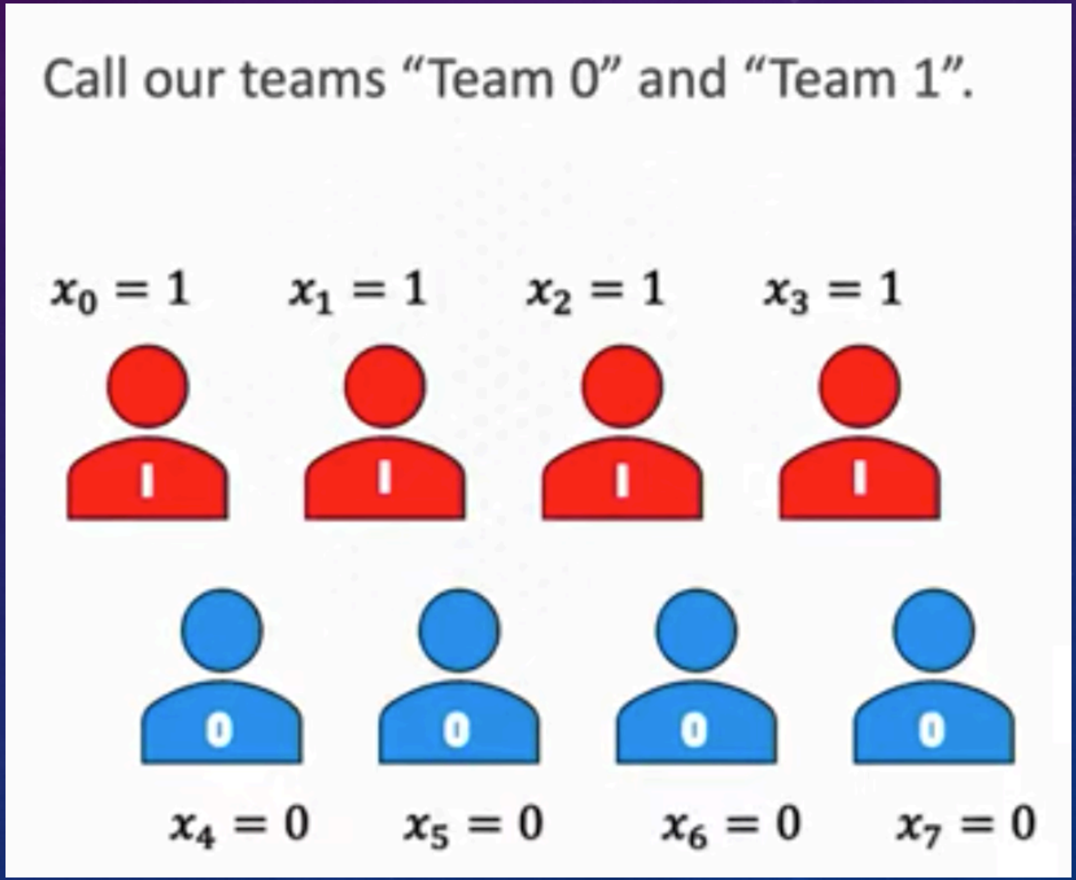
BQM DEV PROCESS

- Write out the objectives and constraints in your problem domain
- Convert your objective and constraints into math statements with binary variables
- Make your objective and constraints “QUBO or Ising appropriate”
  - Objective is a minimization function
  - Constraints are satisfied at their minimum values
- Combine your objective and constraints

FORMULATING THE PROBLEM

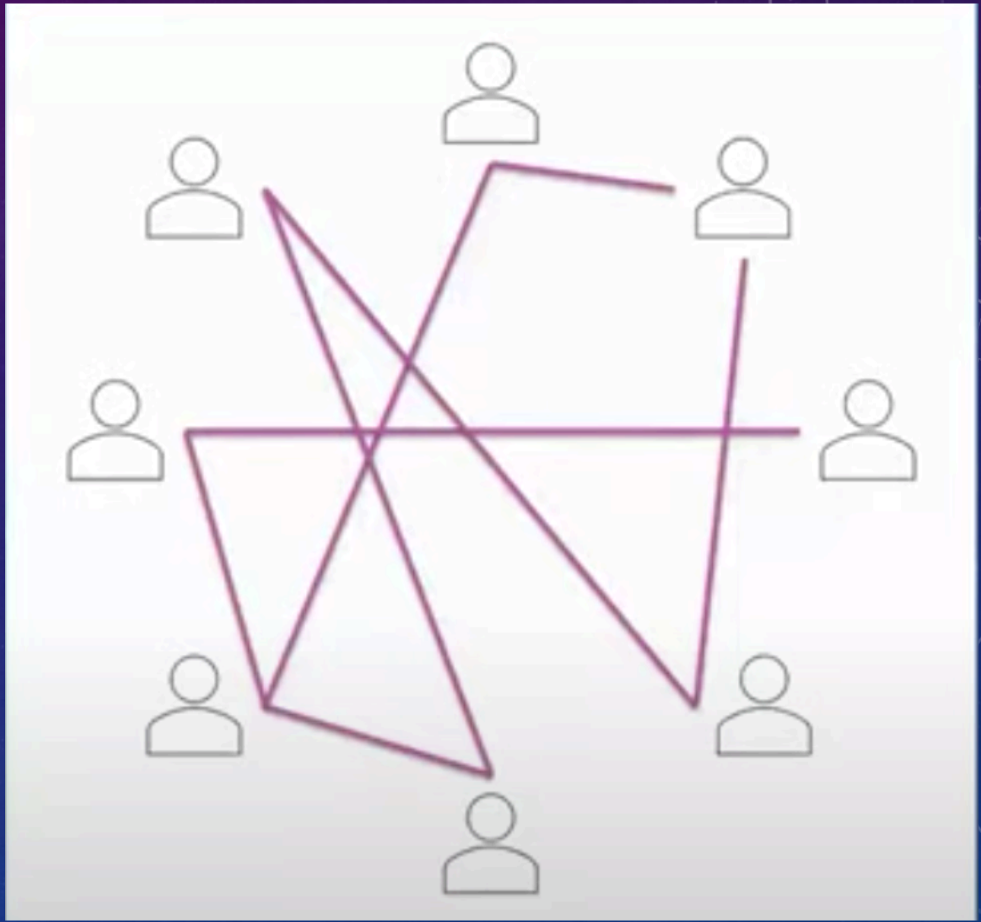
- BQM has two parts:
  - **Objective:** Minimize the number of friends that are split up. That is, minimize the number of edges between red and blue people.
  - **Constraint:** Two teams of same size.

NAMING CONVENTION



OBJECTIVE

- **Words:** Minimize friends that are split up
- **Model:** Minimize edges with different color endpoints

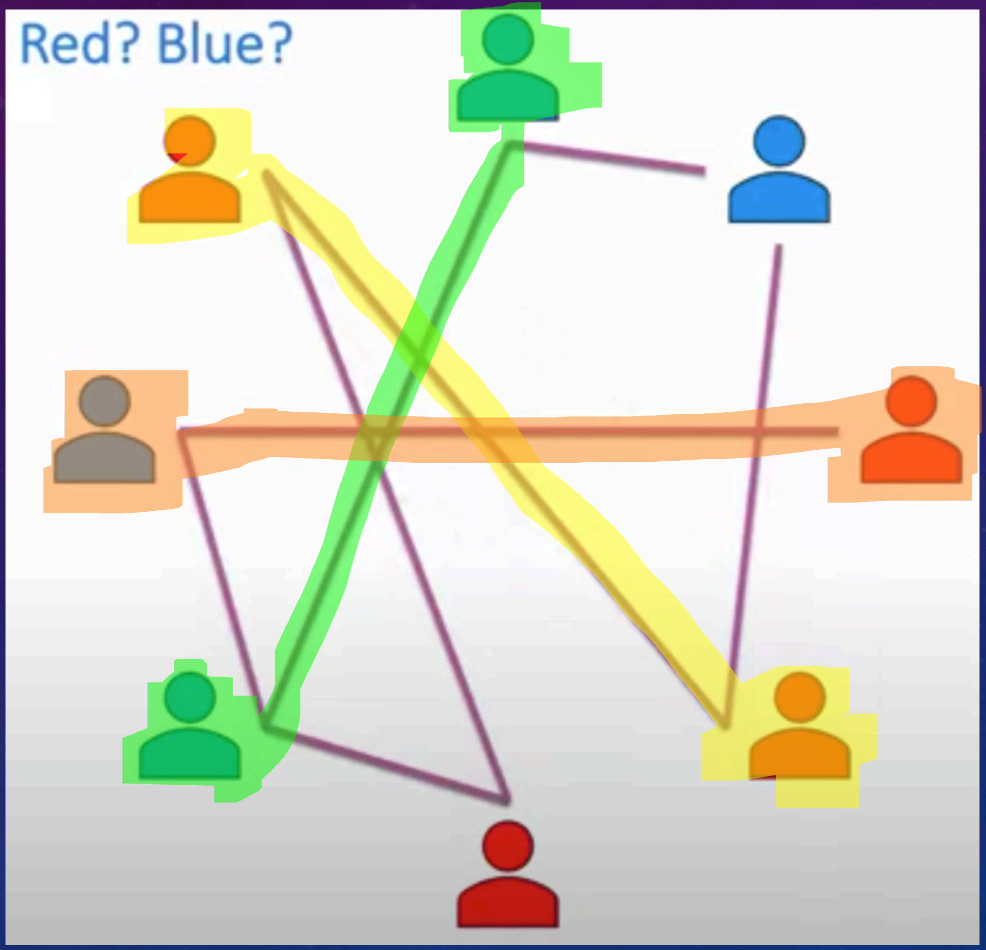


GOOD OR BAD?

Good

Good

Bad



Player i	Player j	Remarks
Blue	Blue	Good
Blue	Red	Bad
Red	Blue	Bad
Red	Red	Good

X_i	X_j	Value
0	0	0
0	1	1
1	0	1
1	1	0

CONVERTING TO QUBO

General form of QUBO

$$a_i x_i + a_j x_j + b_{i,j} x_i x_j + C$$

For one pair of friends:

$$\min(x_i + x_j - 2x_i x_j)$$

For all pairs of friends:

$$\min \sum_{(i,j) \in E} (x_i + x_j - 2x_i x_j)$$

x_i	x_j	Value	Equation
0	0	0	$a_i \cdot 0 + a_j \cdot 0 + b_{i,j} \cdot 0 \cdot 0 + C$
0	1	1	$a_i \cdot 0 + a_j \cdot 1 + b_{i,j} \cdot 0 \cdot 1 + C$
1	0	1	$a_i \cdot 1 + a_j \cdot 0 + b_{i,j} \cdot 1 \cdot 0 + C$
1	1	0	$a_i \cdot 1 + a_j \cdot 1 + b_{i,j} \cdot 1 \cdot 1 + C$

$$0 = C$$

$$\cancel{1 = a_j + C} \rightarrow 1 = a_j$$

$$\cancel{1 = a_i + C} \rightarrow 1 = a_i$$

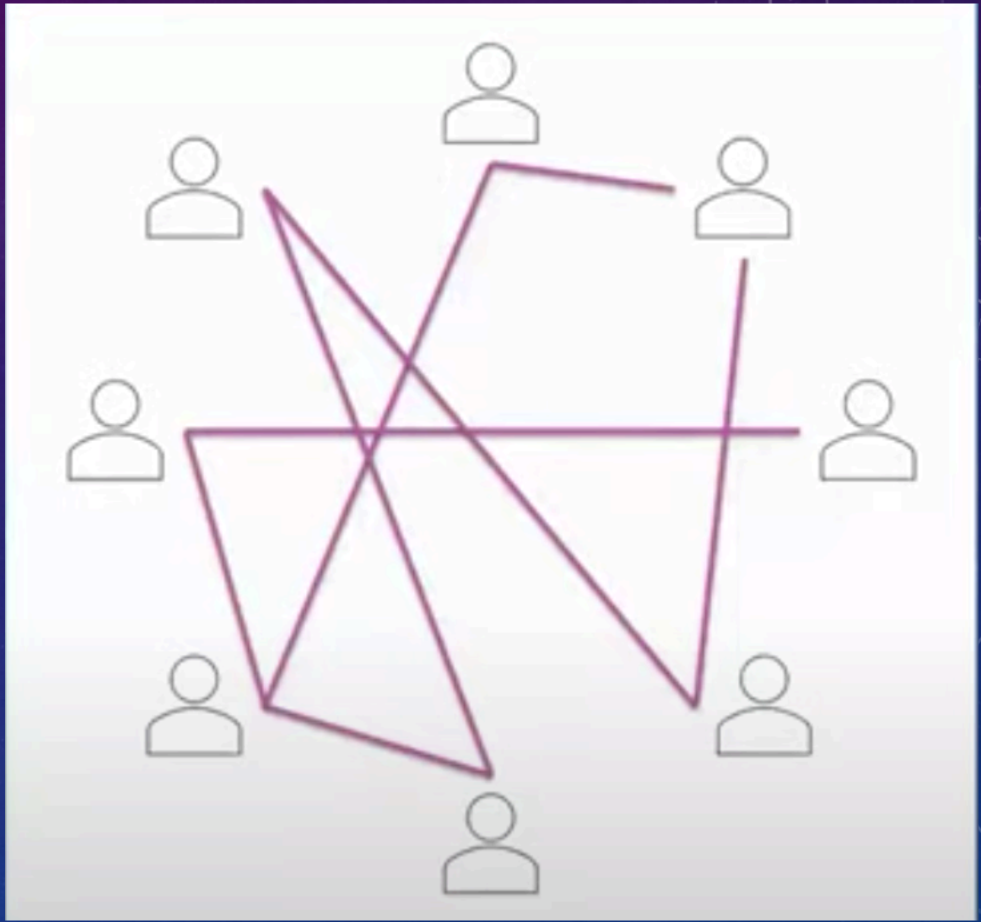
$$\cancel{0 = a_i + a_j + b_{i,j} + C} \rightarrow -2 = b_{i,j}$$

CONSTRAINT

- **Words:** Both teams are the same size
- **Model:** Half of the variables have value = 1 and half have value = 0.
- **Equation**

$$\sum_{i=0}^{7} x_i = 4$$

$$\left(\left(\sum_{i=0}^{7} x_i\right) - 4\right)^2$$



FINAL QUBO

Lagrange parameter

QUBO:

$$\min\left(\sum_{(i,j)\in E} x_i + x_j - 2x_ix_j\right) + \gamma\left(\left(\sum_{i=0}^{7} x_i\right) - 4\right)^2$$

a_0	$b_{0,1}$	$b_{0,2}$	$b_{0,3}$	$b_{0,4}$	$b_{0,5}$	$b_{0,6}$	$b_{0,7}$
	a_1	$b_{1,2}$	$b_{1,3}$	$b_{1,4}$	$b_{1,5}$	$b_{1,6}$	$b_{1,7}$
		a_2	$b_{2,3}$	$b_{2,4}$	$b_{2,5}$	$b_{2,6}$	$b_{2,7}$
			a_3	$b_{3,4}$	$b_{3,5}$	$b_{3,6}$	$b_{3,7}$
				a_4	$b_{4,5}$	$b_{4,6}$	$b_{4,7}$
					a_5	$b_{5,6}$	$b_{5,7}$
						a_6	$b_{6,7}$
							a_7

$$\left(\left(\sum_{i=0}^{7} x_i\right) - 4\right)^2 = \sum_{i=0}^{7} x_i^2 + 2\sum_{i=0}^{7}\sum_{j=i+1}^{7} x_ix_j - 8\sum_{i=0}^{7} x_i + 16$$

ADDING THE CONSTRAINT

$$\sum_{i=0}^{7} x_i^2 \boxed{+ 2\sum_{i=0}^{7} \sum_{j=i+1}^{7} x_i x_j} - 8\sum_{i=0}^{7} x_i + 16$$

$$\begin{matrix} 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\ & & 0 & 2 & 2 & 2 & 2 & 2 \\ & & & 0 & 2 & 2 & 2 & 2 \\ & & & & 0 & 2 & 2 & 2 \\ & & & & & 0 & 2 & 2 \\ & & & & & & 0 & 2 \\ & & & & & & & 0 \end{matrix}$$

ADDING THE CONSTRAINT

$$\sum_{i=0}^{7} x_i^2 + 2\sum_{i=0}^{7}\sum_{j=i+1}^{7} x_i x_j \boxed{-8\sum_{i=0}^{7} x_i} + 16$$

$$\begin{array}{cccccccc}
-8 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
 & -8 & 2 & 2 & 2 & 2 & 2 & 2 \\
 & & -8 & 2 & 2 & 2 & 2 & 2 \\
 & & & -8 & 2 & 2 & 2 & 2 \\
 & & & & -8 & 2 & 2 & 2 \\
 & & & & & -8 & 2 & 2 \\
 & & & & & & -8 & 2 \\
 & & & & & & & -8
\end{array}$$

ADDING THE CONSTRAINT

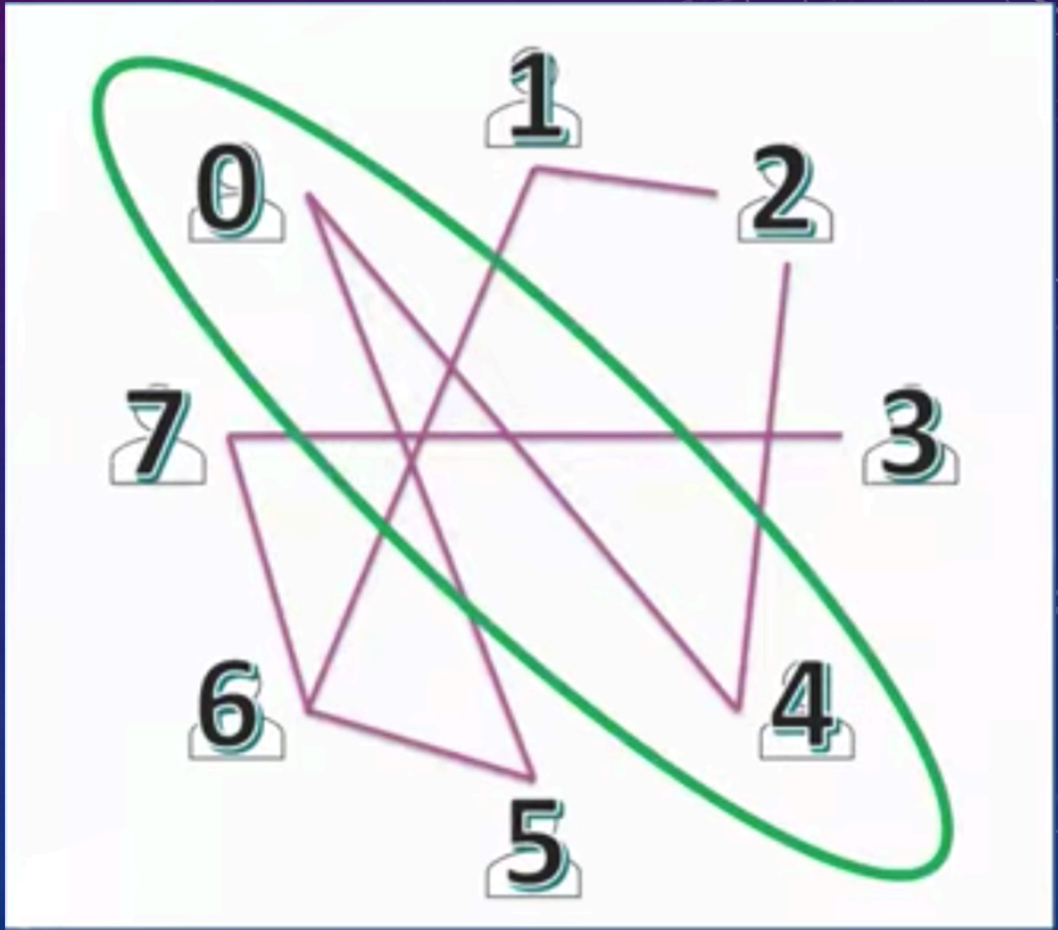
$$\boxed{\sum_{i=0}^{7} x_i^2} + 2\sum_{i=0}^{7}\sum_{j=i+1}^{7} x_i x_j - 8\sum_{i=0}^{7} x_i + 16$$

$$\begin{matrix}
-7 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
 & -7 & 2 & 2 & 2 & 2 & 2 & 2 \\
 & & -7 & 2 & 2 & 2 & 2 & 2 \\
 & & & -7 & 2 & 2 & 2 & 2 \\
 & & & & -7 & 2 & 2 & 2 \\
 & & & & & -7 & 2 & 2 \\
 & & & & & & -7 & 2 \\
 & & & & & & & -7
\end{matrix}$$

OBJECTIVE

$$\sum_{(i,j)\in E} x_i + x_j - 2x_i x_j$$

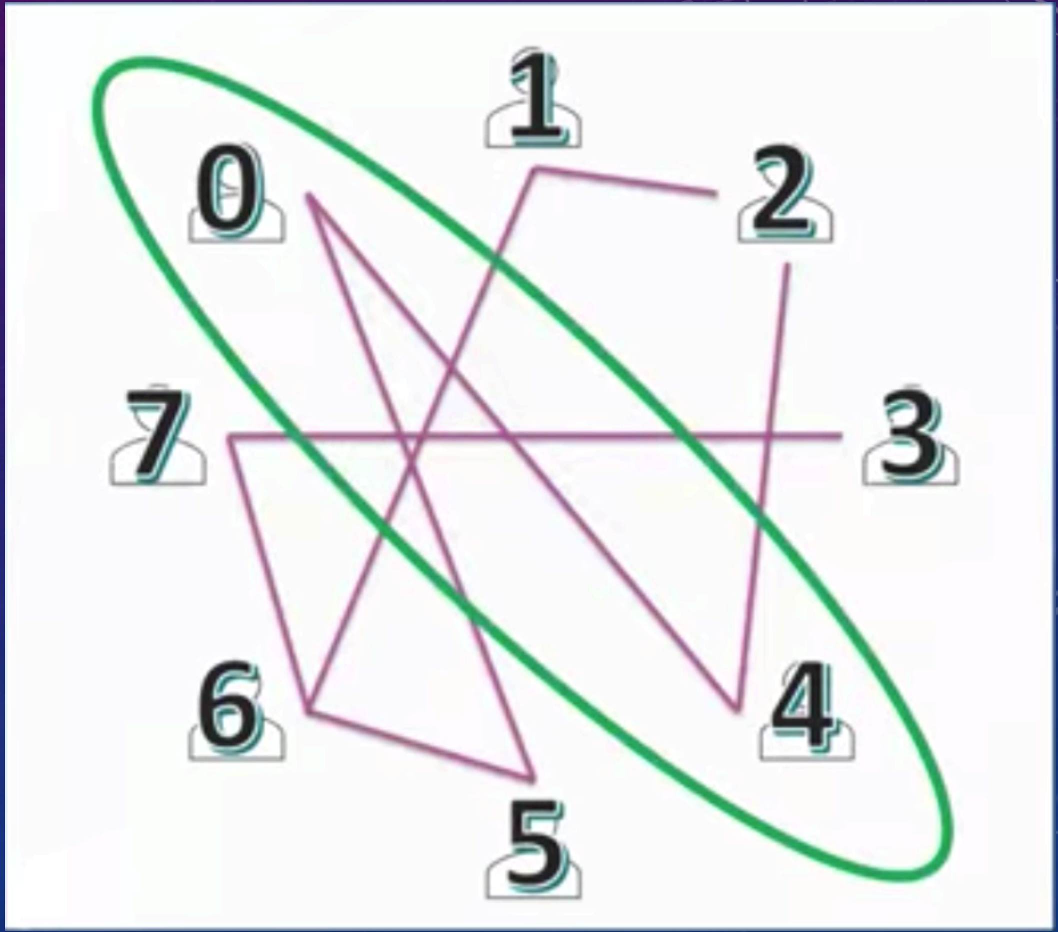
$$\begin{matrix}
-7 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
 & -7 & 2 & 2 & 2 & 2 & 2 & 2 \\
 & & -7 & 2 & 2 & 2 & 2 & 2 \\
 & & & -7 & 2 & 2 & 2 & 2 \\
 & & & & -7 & 2 & 2 & 2 \\
 & & & & & -7 & 2 & 2 \\
 & & & & & & -7 & 2 \\
 & & & & & & & -7
\end{matrix}$$



OBJECTIVE

$$\sum_{(i,j)\in E} x_i + x_j - 2x_i x_j$$

$$\begin{matrix}
-6 & 2 & 2 & 2 & 0 & 2 & 2 & 2 \\
 & -7 & 2 & 2 & 2 & 2 & 2 & 2 \\
 & & -7 & 2 & 2 & 2 & 2 & 2 \\
 & & & -7 & 2 & 2 & 2 & 2 \\
 & & & & -6 & 2 & 2 & 2 \\
 & & & & & -7 & 2 & 2 \\
 & & & & & & -7 & 2 \\
 & & & & & & & -7
\end{matrix}$$



LET’S RUN THE PROBLEM

THE LAGRANGE PARAMETER

We can weight our constraint more heavily using a **Lagrange parameter.**

QUBO:

$$\min\left(\sum_{(i,j)\in E} x_i + x_j - 2x_i x_j\right) + \gamma\left(\left(\sum_{i=0}^{7} x_i\right) - 4\right)^2$$

Let’s set the lagrange parameter = 4

LET’S RUN THE PROBLEM… AGAIN

CHAIN STRENGTH

- This is a property which defines the strength of entanglement between the qubits.

- It uses the hardware property of couplers to manipulate chain strength.

Let’s set the chain strength = 10

LET'S RUN THE PROBLEM…
FOR THE FINAL TIME

How do we know this is an optimal solution?

Remember the constant: 16 * lagrange

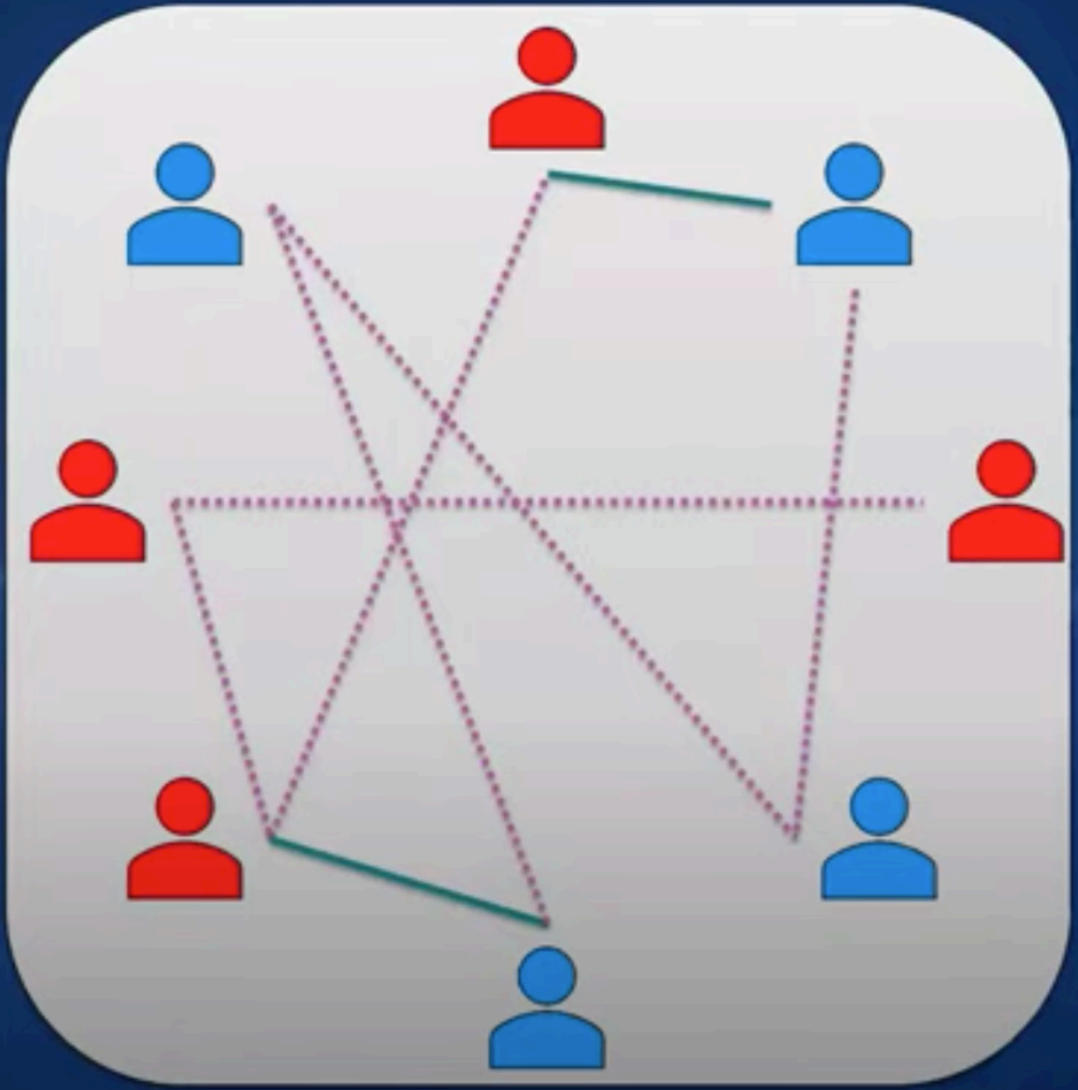
If we add the constant, the minimum energy is +2

Does this sound correct to you?

There are 2 links between Red and Blue

Remember that each Red-Blue link has energy 1

This is an optimal solution!



THANK YOU! :D