

### Partial Derivatives with respect to a

#### Part 3

How do we compute partial derivatives?

1. Solving the equation sequentially
  - a. Let's look at the third partial derivative  $\frac{\partial a_{21}}{\partial w_{212}}$ 
    - i. Here  $a_{21} = w_{211}h_{11} + w_{212}h_{12} + w_{213}h_{13} + w_{214}h_{14}$
    - ii.  $\frac{\partial a_{21}}{\partial w_{212}} = h_{12}$ , as all other terms cancel out.
2. Consider the following output values
  - a.  $a_1 = W_1 * x + b_1 = [2.9 \ 1.4 \ 2.1 \ 2.3]$
  - b.  $h_1 = \text{sigmoid}(a_1) = [0.95 \ 0.80 \ 0.89 \ 0.91]$
  - c.  $a_2 = W_2 * h_1 + b_2 = [1.66 \ 0.45]$
  - d.  $\hat{y} = \text{softmax}(a_2) = [0.77 \ 0.23]$
  - e. Squared error loss  $L(\Theta) = (1 - 0.77)^2 + (1 - 0.23)^2 = 0.1058$
3. Substituting these values in our formulae
  - a.  $\frac{\partial L}{\partial \hat{y}_1} = -2(y_1 - \hat{y}_1) = -0.46$
  - b.  $\frac{\partial \hat{y}_1}{\partial a_{21}} = \hat{y}_1(1 - \hat{y}_1) = 0.1771$
  - c.  $\frac{\partial a_{21}}{\partial w_{212}} = h_{12} = 0.8$
  - d.  $\frac{\partial L}{\partial w_{212}} = (-2(y_1 - \hat{y}_1)) * (\hat{y}_1(1 - \hat{y}_1)) * (h_{12}) = (-0.46) * (0.1771) * (0.8) = -0.065$
4. Now we can calculate the updated value of  $w_{212}$
5.  $w_{212} = w_{212} - \eta(\frac{\partial L}{\partial w_{212}})$ 
  - a.  $w_{212} = 0.8 - (1) * (-0.065)$
  - b.  $w_{212} = 0.865$
6. We can repeat this process for each weight.