PadhAI: Backpropagation - the light math version

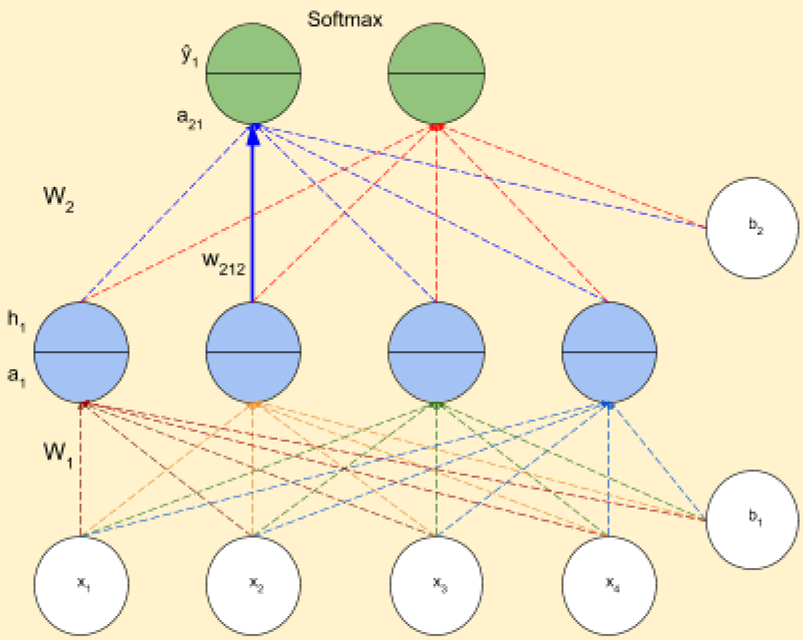
One Fourth Labs

Partial Derivatives with respect to a

Part 1

How do we compute partial derivatives

1. The following neural network will be used to demonstrate the calculations



2. Here are the parameters of the network
   a. $b = [\,0.5 \;\; 0.3\,]$
   b.

$$W_1 = \begin{bmatrix} 0.1 & 0.3 & 0.8 & -0.4 \\ -0.3 & -0.2 & 0.5 & 0.5 \\ -0.3 & 0 & 0.5 & 0.4 \\ 0.2 & 0.5 & -0.9 & 0.7 \end{bmatrix}$$

   c.

$$W_2 = \begin{bmatrix} 0.5 & 0.8 & 0.2 & 0.4 \\ 0.5 & 0.2 & 0.3 & -0.5 \end{bmatrix}$$

   d. $x = [\,2 \;\; 5 \;\; 3 \;\; 3\,]$ true distribution $y = [\,1 \;\; 0\,]$

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3. Now, we want to find the partial derivative w.r.t w_{212} as highlighted in the figure $\frac{\partial L}{\partial w_{212}}$
4. $\frac{\partial L}{\partial w_{212}} = (\frac{\partial L}{\partial a_{21}}).(\frac{\partial a_{21}}{\partial w_{212}}) = (\frac{\partial L}{\partial \hat{y}_1}).(\frac{\partial \hat{y}_1}{\partial a_{21}}).(\frac{\partial a_{21}}{\partial w_{212}})$
5. We will solve the above equation sequentially
   a. Consider square error loss function L
   b. $\frac{\partial L}{\partial \hat{y}_1} = \Sigma^2{}_{i=1}(y_i - \hat{y}_i)^2$
      i. $\frac{\partial L}{\partial \hat{y}_1} = \frac{\partial}{\partial y_1}[(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2]$
      ii. Here, the y_2 terms get cancelled, leaving $\frac{\partial}{\partial y_1}[(y_1 - \hat{y}_1)^2] = -2(y_1 - \hat{y}_1)$