Discovery: Period of $\tan\left(\frac{\pi}{n}[x]\right)$

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Definition

Let $f(x) = \tan\left(\frac{\pi}{n}[x]\right)$, where $[x]$ denotes the greatest integer function (GIF), and n is a positive integer ($n \geq 2$).

Observation

The function $\tan(\theta)$ is periodic with period π, but when we apply it on $\frac{\pi}{n}[x]$, the input becomes piecewise constant, changing only at integer points. This leads to a new periodicity pattern.

Periodicity

For integer values of x, $[x]$ increases by 1. So for $x \in [k, k+1)$, $f(x)$ is constant as:

$$f(x) = \tan\left(\frac{\pi}{n}k\right)$$

Now observe how this pattern repeats.

Let's compute the smallest T such that:

$$f(x+T) = f(x) \quad \forall x$$

This requires:

$$\tan\left(\frac{\pi}{n}[x+T]\right) = \tan\left(\frac{\pi}{n}[x]\right)$$

For this to hold, $[x+T] = [x] + n$ must make $\frac{\pi}{n}[x+T] - \frac{\pi}{n}[x] = \pi$, i.e., the tangent function completes a full period.

So, $T = n$ works.

Examples

- For $n = 2$: $f(x) = \tan\left(\frac{\pi}{2}[x]\right)$ period $= 2$
- For $n = 3$: $f(x) = \tan\left(\frac{\pi}{3}[x]\right)$ period $= 3$
- For $n = 4$: $f(x) = \tan\left(\frac{\pi}{4}[x]\right)$ period $= 4$

This has been experimentally verified using graphs and values.

Visual Illustration

(Optional: You may insert a TikZ or Desmos-generated image here to show the repetition for different values of n.)

Conclusion

This result is a special case of combining a periodic function with a step function. The periodic nature of $\tan(\theta)$ and the step behavior of $[x]$ lead to a clean, predictable pattern.

Therefore, for all $n \geq 2$:

$$\boxed{f(x) = \tan\left(\frac{\pi}{n}[x]\right) \text{ has period } n}$$

This rule is not commonly highlighted in standard school or JEE-level materials and represents a novel generalization discovered by Arpit through logic and pattern recognition.

In a world driven by formulas, it's the patterns we recognize that spark real mathematics. This discovery is a reminder that mathematics isn't just about solving problems — it's about seeing deeper truths where others see only numbers.

Keep discovering. Keep questioning.
And remember — even the greatest mathematicians once began with a small idea that no one had noticed before.

Passionate learner. Keen observer. Future innovator.