

1. Likelihood Function:

$$\text{Let } \theta_1 = \mu \quad \theta_2 = \sigma^2$$

$$L(\theta_1, \theta_2 | n_1, \dots, n_n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n_1 - \mu)^2}{2\sigma^2}} \times \dots \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n_n - \mu)^2}{2\sigma^2}}$$

Take log on both sides

$$\ln[L(\theta_1, \theta_2 | n_1, \dots, n_n)] = \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n_1 - \mu)^2}{2\sigma^2}} \times \dots \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n_n - \mu)^2}{2\sigma^2}}\right)$$

$$= \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n_1 - \mu)^2}{2\sigma^2}}\right) + \dots + \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n_n - \mu)^2}{2\sigma^2}}\right)$$

$$= -\frac{1}{2} \ln(2\pi) - \ln(\sigma) - \frac{(n_1 - \mu)^2}{2\sigma^2} - \dots - \frac{1}{2} \ln(2\pi) - \ln(\sigma) - \frac{(n_n - \mu)^2}{2\sigma^2}$$

$$\ln[L(\theta_1, \theta_2 | n_1, \dots, n_n)] = -\frac{n}{2} \ln(2\pi) - n \ln(\sigma) - \frac{(n_1 - \mu)^2}{2\sigma^2} - \dots - \frac{(n_n - \mu)^2}{2\sigma^2}$$

$$\frac{\partial}{\partial \mu} \ln [L(\theta_1, \theta_2 | n_1, \dots, n_n)] = 0 = 0 + \frac{(n_1 - \mu)}{\sigma^2} + \dots + \frac{(n_n - \mu)}{\sigma^2}$$

$$= \frac{1}{\sigma^2} [(n_1 + \dots + n_n) - n\mu] \dots \text{--- (1)}$$

$$\frac{\partial}{\partial \sigma} \ln [L(\theta_1, \theta_2 | n_1, \dots, n_n)] = 0 = -\frac{n}{\sigma} + \frac{1}{\sigma^3} [(n_1 - \mu)^2 + \dots + (n_n - \mu)^2] \dots \text{--- (2)}$$

$$\frac{\partial}{\partial \mu} \ln [L(\theta_1, \theta_2 | n_1, \dots, n_n)] = 0$$

$$n_1 + n_2 + \dots + n_n = n\mu$$

$$\theta_1 / \mu = \frac{n_1 + n_2 + \dots + n_n}{n}$$

$$\frac{\partial}{\partial \sigma} \ln [L(\theta_1, \theta_2 | n_1, \dots, n_n)] = 0$$

$$\sigma^2 n = (n_1 - \mu)^2 + \dots + (n_n - \mu)^2$$

$$\theta_2 / \sigma^2 = \frac{(n_1 - \mu)^2 + \dots + (n_n - \mu)^2}{n}$$

$$\sigma = \sqrt{\frac{(n_1 - \mu)^2 + \dots + (n_n - \mu)^2}{n}}$$

2. Likelihood Function:

$$L(\theta | n_1, \dots, n_n) = \prod_{i=1}^n \binom{m}{n_i} \theta^{n_i} (1-\theta)^{m-n_i}$$

$$\ln [L(\theta | n_1, \dots, n_n)] = \sum_{i=1}^n \left( \ln \binom{m}{n_i} + n_i \log \theta + (m-n_i) \log (1-\theta) \right)$$

$$\frac{\partial}{\partial \theta} [\ln [L(\theta | n_1, \dots, n_n)]] = 0$$

$$\sum_{i=1}^n \frac{n_i}{\theta} - \sum_{i=1}^n \frac{(m-n_i)}{1-\theta} = 0$$

$$\frac{1-\theta}{\theta} = \frac{nm - \sum n_i}{\sum n_i}$$

$$\frac{1}{\theta} = \frac{nm}{\sum n_i}$$

$$\boxed{\theta = \frac{\sum n_i}{nm}}$$