Work : If a force of $F(x)$ moves an object in $a \le x \le b$, the work done is $W = \int_a^b F(x)\,dx$

Average Function Value : The average value of $f(x)$ on $a \le x \le b$ is $f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

Arc Length Surface Area : Note that this is often a Calc II topic. The three basic formulas are,

$L = \int_a^b ds$ $\quad SA = \int_a^b 2\pi y\,ds$ (rotate about *x*-axis) $\quad SA = \int_a^b 2\pi x\,ds$ (rotate about *y*-axis)

where *ds* is dependent upon the form of the function being worked with as follows.

$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ if $y = f(x),\ a \le x \le b$ $\qquad ds = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$ if $x = f(t), y = g(t),\ a \le t \le b$

$ds = \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$ if $x = f(y),\ a \le y \le b$ $\qquad ds = \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$ if $r = f(\theta),\ a \le \theta \le b$

With surface area you *may* have to substitute in for the *x* or *y* depending on your choice of *ds* to match the differential in the *ds*. With parametric and polar you will always need to substitute.

Improper Integral

An improper integral is an integral with one or more infinite limits and/or discontinuous integrands. Integral is called convergent if the limit exists and has a finite value and divergent if the limit doesn't exist or has infinite value. This is typically a Calc II topic.

Infinite Limit

1. $\int_a^\infty f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx$
2. $\int_{-\infty}^b f(x)\,dx = \lim_{t\to-\infty}\int_t^b f(x)\,dx$
3. $\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^c f(x)\,dx + \int_c^\infty f(x)\,dx$ provided BOTH integrals are convergent.

Discontinuous Integrand

1. Discont. at *a*: $\int_a^b f(x)\,dx = \lim_{t\to a^+}\int_t^b f(x)\,dx$
2. Discont. at b : $\int_a^b f(x)\,dx = \lim_{t\to b^-}\int_a^t f(x)\,dx$
3. Discontinuity at $a < c < b$: $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ provided both are convergent.

Comparison Test for Improper Integrals : If $f(x) \ge g(x) \ge 0$ on $[a,\infty)$ then,

1. If $\int_a^\infty f(x)\,dx$ conv. then $\int_a^\infty g(x)\,dx$ conv.
2. If $\int_a^\infty g(x)\,dx$ divg. then $\int_a^\infty f(x)\,dx$ divg.

Useful fact : If $a > 0$ then $\int_a^\infty \frac{1}{x^p}\,dx$ converges if $p > 1$ and diverges for $p \le 1$.

Approximating Definite Integrals

For given integral $\int_a^b f(x)\,dx$ and a *n* (must be even for Simpson's Rule) define $\Delta x = \frac{b-a}{n}$ and divide $[a,b]$ into *n* subintervals $[x_0,x_1]$, $[x_1,x_2]$, … , $[x_{n-1},x_n]$ with $x_0 = a$ and $x_n = b$ then,

Midpoint Rule : $\int_a^b f(x)\,dx \approx \Delta x\left[f(x_1^*)+f(x_2^*)+\cdots+f(x_n^*)\right]$, x_i^* is midpoint $[x_{i-1},x_i]$

Trapezoid Rule : $\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_0)+2f(x_1)++2f(x_2)+\cdots+2f(x_{n-1})+f(x_n)\right]$

Simpson's Rule : $\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)\right]$