Contents

Chapter 1

Binary Phase-Shift Keying (BPSK)

FOR NON-TECHNICAL READERS

BPSK is like Morse code with a twist—instead of turning a signal on and off, you flip the wave upside-down to send 1s and 0s.
Simple idea:

- Bit 0 = wave pointing “up” ↑
- Bit 1 = wave pointing “down” ↓ (flipped $180°$)

Real use: GPS satellites use BPSK. Your phone detects whether the signal is normal or flipped.
Why flip instead of on/off? More reliable in noise, works with constant power, less interference. Trade-off: Simple but slow (1 bit per symbol).

1.1 Overview

Binary Phase-Shift Keying (BPSK) is the simplest form of phase modulation, where binary data is encoded by shifting the carrier phase between two states: $0°$ and $180°$.

KEY CONCEPT

BPSK provides **3 dB better performance** than On-Off Keying (OOK) at the same signal-to-noise ratio, making it the optimal choice for power-limited channels such as satellite and deep-space communications.

BPSK forms the foundation for higher-order phase shift keying schemes including QPSK (4 phases), 8PSK (8 phases), and beyond.

1.2 Mathematical Description

1.2.1 Time-Domain Signal

The BPSK waveform is expressed as:

$$s(t) = A\cos(2\pi f_c t + \phi_n) \tag{1.1}$$

where:

- A = carrier amplitude
- f_c = carrier frequency (Hz)
- $\phi_n \in \{0°, 180°\}$ = phase for bit n

Phase encoding:

$$\phi_n = \begin{cases} 0^\circ & \text{if bit = 0} \\ 180^\circ & \text{if bit = 1} \end{cases} \tag{1.2}$$

Alternative representation using the cosine identity $\cos(\theta + 180^\circ) = -\cos(\theta)$:

$$s(t) = A \cdot d_n \cdot \cos(2\pi f_c t) \tag{1.3}$$

where $d_n \in \{+1, -1\}$ is the bipolar data symbol:

- Bit 0 $\rightarrow d_n = +1 \rightarrow 0^\circ$ phase
- Bit 1 $\rightarrow d_n = -1 \rightarrow 180^\circ$ phase (inverted carrier)

Physical Interpretation

BPSK is effectively **amplitude modulation with bipolar data**. The carrier polarity flips between positive and negative, which is equivalent to a 180° phase shift. This representation simplifies both mathematical analysis and hardware implementation.

1.3 IQ Representation

The baseband complex representation of BPSK is:

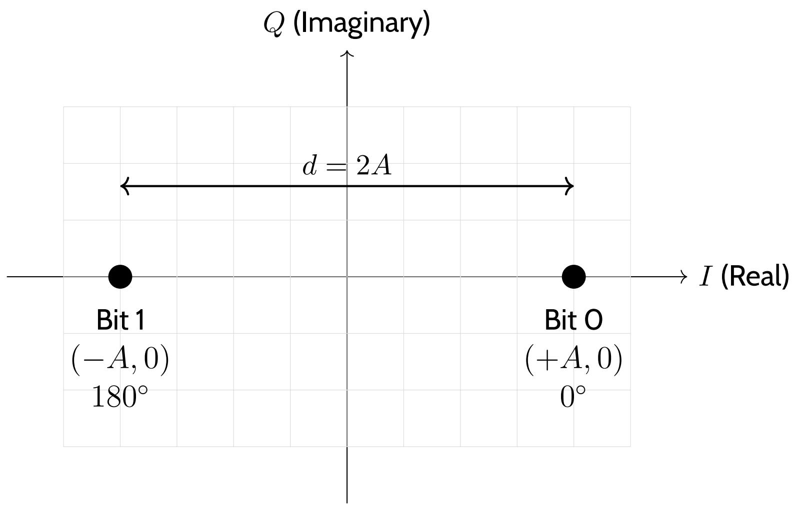
$$s(t) = \mathrm{Re}\{A \cdot d_n \cdot e^{j2\pi f_c t}\} \tag{1.4}$$

IQ components:

- **I (In-phase):** $I_n = A \cdot d_n$ (either $+A$ or $-A$)
- **Q (Quadrature):** $Q_n = 0$ (BPSK uses only the I axis)

1.3.1 Constellation Diagram

The BPSK constellation consists of two points on the real axis separated by maximum distance $d = 2A$:



This maximum Euclidean separation between symbols provides optimal noise immunity for binary modulation schemes.

1.4 Modulation and Demodulation

1.4.1 Transmitter (Modulator)

The BPSK modulator consists of three stages:

Process:

1. **NRZ encoding:** Map bits to bipolar symbols
   - Bit $0 \rightarrow d_n = +1$
   - Bit $1 \rightarrow d_n = -1$
2. **Multiply by carrier:** $s(t) = A \cdot d_n \cdot \cos(2\pi f_c t)$
3. **Pulse shaping:** Apply raised-cosine filter to:
   - Limit occupied bandwidth
   - Prevent intersymbol interference (ISI)
   - Meet spectral mask requirements

1.4.2 Receiver (Coherent Detector)

Local Oscillator;

[->,thick] (input) – (mult); [->,thick] (lo) – (mult); [->,thick] (mult) – (lpf); [->,thick] (lpf) – (sample); [->,thick] (sample) – (thresh); [->,thick] (thresh) – (output);

CRITICAL REQUIREMENT

Phase synchronization is critical. The local oscillator must be exactly in phase with the transmitter carrier. A phase offset ϕ_e reduces detected signal by $\cos(\phi_e)$. At $\phi_e = 90°$, complete signal loss occurs.

Detection process:

1. **Multiply by local carrier** (frequency f_c, phase $\phi = 0$):
$$r(t) = s(t) \cdot 2\cos(2\pi f_c t) = 2Ad_n \cos^2(2\pi f_c t) \tag{1.5}$$
2. **Apply trigonometric identity** $\cos^2(x) = \frac{1}{2}[1 + \cos(2x)]$:
$$r(t) = Ad_n[1 + \cos(4\pi f_c t)] \tag{1.6}$$
3. **Lowpass filter** removes $2f_c$ component, leaving baseband:
$$y(t) = Ad_n \tag{1.7}$$
4. **Sample at bit period** T_b: $y_n = Ad_n + n(t)$ where $n(t)$ is AWGN

5. **Threshold decision:**

$$\hat{d}_n = \begin{cases} +1 & \text{if } y_n > 0 \quad \text{(decode as bit 0)} \\ -1 & \text{if } y_n < 0 \quad \text{(decode as bit 1)} \end{cases} \tag{1.8}$$

1.5 Carrier Recovery

The receiver must generate a local oscillator **exactly in phase** with the transmitter carrier. This is the primary challenge in coherent BPSK detection.

1.5.1 Problem: Phase Ambiguity

A phase offset ϕ_e between transmitter and receiver carriers causes:

$$y(t) = Ad_n \cos(\phi_e) + n(t) \tag{1.9}$$

Effects:

- $\phi_e = 0°$: Full signal strength (optimal)
- $\phi_e = 45°$: Signal reduced by 3 dB
- $\phi_e = 90°$: Complete signal loss
- $\phi_e = 180°$: Inverted data (all bits flipped)

1.5.2 Carrier Recovery Techniques

1. Pilot Tone

✓ Simple implementation
✓ Accurate phase reference
× Wastes power (typically 10–20% of total)
× Reduces data throughput

2. Costas Loop

PLL-based carrier recovery using I/Q demodulation:

✓ No pilot tone required
✓ Optimal for BPSK and QPSK
× Complex analog circuitry
× Acquisition time required

3. Squaring Loop

Exploits $d_n^2 = 1$ to remove data modulation:

$$[d_n \cos(2\pi f_c t)]^2 = \frac{1}{2}[1 + \cos(4\pi f_c t)] \tag{1.10}$$

PLL locks to $2f_c$, then divides by 2 to recover f_c.

✓ Completely removes data modulation
✓ Robust in low SNR
× $180°$ phase ambiguity (requires differential encoding)

1.5.3 Differential BPSK (DBPSK)

Principle: Encode data in **phase transitions**, not absolute phase.

Encoding rule:

$$\phi_n = \phi_{n-1} + \Delta\phi_n \quad \text{where} \quad \Delta\phi_n = \begin{cases} 0^\circ & \text{if bit = 0} \\ 180^\circ & \text{if bit = 1} \end{cases} \tag{1.11}$$

Decoding: Compare consecutive symbols:

$$\hat{b}_n = \begin{cases} 0 & \text{if } \operatorname{sgn}(y_n) = \operatorname{sgn}(y_{n-1}) \\ 1 & \text{if } \operatorname{sgn}(y_n) \neq \operatorname{sgn}(y_{n-1}) \end{cases} \tag{1.12}$$

Trade-off:

- ✓ No carrier recovery needed
- ✓ Simpler receiver
- × Approximately 3 dB performance penalty
- × Error propagation (single error affects two bits)

1.6 Bit Error Rate (BER) Performance

1.6.1 Coherent BPSK in AWGN Channel

For ideal coherent detection with perfect synchronization:

$$\text{BER} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{1.13}$$

where:

- $E_b = \frac{A^2 T_b}{2}$ = energy per bit (joules)
- N_0 = noise power spectral density (W/Hz)
- $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt$ (Gaussian Q-function)

Performance benchmarks:

E_b/N_0 (dB)	BER	Practical Meaning
0 dB	7.9×10^{-2}	1 error in 13 bits
5 dB	9.7×10^{-4}	1 error in 1,000 bits
10 dB	3.9×10^{-6}	1 error in 250,000 bits
15 dB	6.9×10^{-10}	1 error in 1.4 billion bits

1.6.2 Comparison: BPSK vs OOK

At the same $E_b/N_0 = 10$ dB:

Modulation Scheme	BER	Performance Ratio
OOK (non-coherent)	4.0×10^{-3}	Baseline
BPSK (coherent)	$\mathbf{3.9 \times 10^{-6}}$	**1000× better**

KEY CONCEPT

Why is BPSK 3 dB better than OOK?

1. **Full signal space utilization:** BPSK uses $\pm A$ (both polarities), while OOK uses $\{0, A\}$ (one polarity). This doubles the Euclidean distance between symbols.
2. **Coherent detection:** Correlating with a known carrier phase is the optimal detection strategy (maximum likelihood).
3. **Constant envelope:** Energy is transmitted continuously, not just during "on" bits.

1.6.3 Differential BPSK Performance

DBPSK trades synchronization complexity for performance:

$$\mathrm{BER}_{\mathrm{DBPSK}} \approx \frac{1}{2} e^{-E_b/N_0} \tag{1.14}$$

At $E_b/N_0 = 10$ dB: BER $\approx 5 \times 10^{-6}$ (approximately 1.3 dB penalty versus coherent BPSK).

1.7 Bandwidth Efficiency

The occupied bandwidth (99% power) for rectangular pulses is:

$$B \approx \frac{1}{T_b} = R_b \tag{1.15}$$

where R_b is the bit rate (bps) and T_b is the bit period (seconds).

With **raised-cosine pulse shaping** (roll-off factor α):

$$B = R_b(1 + \alpha) \tag{1.16}$$

Typical value: $\alpha = 0.35$ gives $B = 1.35 R_b$

Spectral efficiency:

$$\eta = \frac{R_b}{B} = \frac{1}{1 + \alpha} \approx 0.74 \text{ bps/Hz} \tag{1.17}$$

Example: 1 Mbps BPSK System

- Data rate: $R_b = 1$ Mbps
- Roll-off: $\alpha = 0.35$
- Required bandwidth: $B = 1 \times (1 + 0.35) = 1.35$ MHz
- Spectral efficiency: $\eta = 1/1.35 = 0.74$ bps/Hz

1.8 Practical Implementations

1.8.1 IEEE 802.15.4 (Zigbee)

Low-rate wireless personal area networks (868/915 MHz bands):

- **Modulation:** BPSK with Direct-Sequence Spread Spectrum (DSSS)

- **Chip rate:** 300 kcps (868 MHz), 600 kcps (915 MHz)
- **Data rate:** 20 kbps (868 MHz), 40 kbps (915 MHz)
- **Spreading gain:** 15:1 to 20:1 (improves interference rejection)

1.8.2 Satellite Telemetry

Deep-space missions (Voyager, Mars rovers) use BPSK for maximum power efficiency:

- **Modulation:** BPSK or QPSK
- **Coding:** Concatenated (Convolutional + Reed-Solomon)
- **Data rate:** 10 bps to 10 kbps (extreme path loss)
- **Rationale:** Every dB matters at interplanetary distances

Example: Voyager 1 at 24 Billion km

TX power	23 W
TX antenna gain	48 dBi (3.7 m dish)
RX antenna	70 m Deep Space Network dish (74 dBi)
Free-space path loss	310 dB
Received power	-196 dBm
Link budget	Barely positive with FEC
Achieved BER	$\sim 10^{-5}$

1.8.3 RFID Backscatter

Passive RFID tags use backscatter modulation (effectively BPSK):

- **Mechanism:** Tag switches antenna impedance (reflection vs absorption)
- **Binary encoding:** Reflection = bit 0, absorption = bit 1
- **Data rate:** 40–640 kbps (EPC Gen2 standard)
- **Power source:** Harvested from reader's carrier

1.9 Advantages and Disadvantages

Advantages

1. **Optimal binary modulation:** Best BER performance for 1 bit/symbol (3 dB better than OOK)
2. **Constant envelope:** Compatible with nonlinear amplifiers (no AM-PM distortion)
3. **Simple constellation:** Two points simplify visualization and analysis
4. **Foundation for higher PSK:** Concepts extend naturally to QPSK, 8PSK, etc.

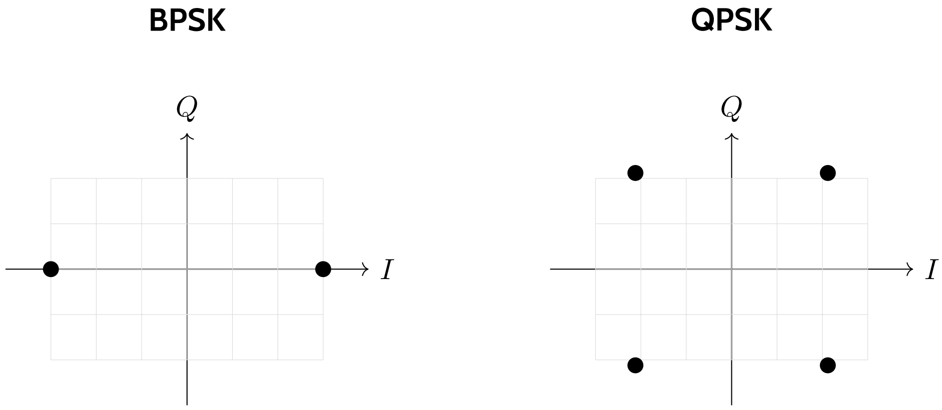
Disadvantages

1. **Carrier synchronization required:** Costas loop or squaring loop adds complexity
2. **DBPSK penalty:** Avoiding synchronization costs 3 dB in performance
3. **Low spectral efficiency:** 1 bit/symbol = maximum 1 bps/Hz
4. **Outperformed at high SNR:** QPSK, 16-QAM more efficient when SNR permits

1.10 Transition to Higher-Order Modulation

BPSK uses only the I-axis (real axis) with two constellation points.

Natural extension: Utilize **both I and Q axes** to create QPSK:



2 points, 1 bit/symbol   4 points, 2 bits/symbol

QPSK = Two independent BPSK channels (I and Q) operating in parallel, doubling spectral efficiency to $\sim$2 bps/Hz.

1.11 Worked Example: Satellite Link Budget

Scenario: Geostationary satellite downlink to 1 m ground station

Given Parameters

TX power	$P_t = 10$ W = 40 dBm
TX antenna gain	$G_t = 30$ dBi
Distance	$d = 36{,}000$ km (GEO orbit)
Frequency	$f = 12$ GHz (Ku-band)
RX antenna gain	$G_r = 40$ dBi (1 m dish)
System noise temp	$T_s = 150$ K
Bandwidth	$B = 1$ MHz
Required BER	10^{-6}

Step 1: Free-Space Path Loss

$$\text{FSPL [dB]} = 20\log_{10}(d_{\text{km}}) + 20\log_{10}(f_{\text{MHz}}) + 32.45 \tag{1.18}$$

$$\text{FSPL} = 20\log_{10}(36{,}000) + 20\log_{10}(12{,}000) + 32.45 = 205.5 \text{ dB} \tag{1.19}$$

Step 2: Received Signal Power

$$P_r = P_t + G_t + G_r - \text{FSPL} \tag{1.20}$$

$$P_r = 40 + 30 + 40 - 205.5 = -95.5 \text{ dBm} \tag{1.21}$$

Step 3: Noise Power

$$N = kT_sB = (1.38 \times 10^{-23})(150)(10^6) = 2.07 \times 10^{-15}\ \mathrm{W} \tag{1.22}$$

$$N = 10\log_{10}(2.07 \times 10^{-15}/10^{-3}) = -117\ \mathrm{dBm} \tag{1.23}$$

Step 4: Signal-to-Noise Ratio

$$\mathrm{SNR} = P_r - N = -95.5 - (-117) = 21.5\ \mathrm{dB} \tag{1.24}$$

Step 5: Energy-per-Bit to Noise Ratio

Assuming data rate $R_b = 500$ kbps:

$$\frac{E_b}{N_0} = \mathrm{SNR} + 10\log_{10}\left(\frac{B}{R_b}\right) \tag{1.25}$$

$$\frac{E_b}{N_0} = 21.5 + 10\log_{10}\left(\frac{1{,}000{,}000}{500{,}000}\right) = 21.5 + 3.0 = 24.5\ \mathrm{dB} \tag{1.26}$$

Step 6: Link Margin Calculation

- **Required E_b/N_0 for BER $= 10^{-6}$:** 10.5 dB
- **Available E_b/N_0:** 24.5 dB
- **Link margin:** $24.5 - 10.5 = 14.0$ dB

Link Budget Summary

Result: Link closes with 14 dB margin
This comfortable margin accommodates:

- Rain fade ($\sim$5–8 dB at Ku-band)
- Implementation losses ($\sim$2–3 dB)
- Pointing errors ($\sim$1–2 dB)
- Aging and component degradation

Conclusion: Link is viable for reliable 500 kbps BPSK transmission.

1.12 Summary

Parameter	Value
Bits per symbol	1
Constellation points	2 (0°, 180°)
Spectral efficiency	$\sim$0.7–1.0 bps/Hz
BER @ 10 dB E_b/N_0	3.9×10^{-6}
Carrier recovery	Required (Costas/squaring loop)
Implementation	Moderate complexity
Best application	Power-limited channels
Typical uses	Satellite, deep-space, RFID

1.13 Further Reading

- **Chapter 5:** On-Off Keying (OOK)—simpler but inferior performance
- **Chapter 6:** Frequency-Shift Keying (FSK)—alternative binary scheme
- **Chapter 7:** Quadrature Phase-Shift Keying (QPSK)—2 bits/symbol extension
- **Chapter 12:** Constellation Diagrams—visualization techniques
- **Chapter 13:** IQ Representation—complex baseband mathematics
- **Chapter 18:** Bit Error Rate Analysis—performance measurement
- **Chapter 22:** Forward Error Correction—coding for BER improvement
- **Chapter 25:** Carrier Recovery Techniques—synchronization methods