Project #1

Due Date: 03/2/2022

In this project, you will experience a close to the real world evaluation individual level utility maximization and construction of algorithms aimed at helping consumers make decisions in complex marketplaces.

You have a real dataset containing the scanner data listing all transactions in (an unspecified) supermarket. Each row of the attached dataset corresponds to an SKU (stock keeping unit) of an item purchased by a given consumer. If there are multiple items purchased within the same transaction, they will share the same transaction id. Notably, the "sales amount" in the data is the total dollar value of the line item and thus to get the unit price you will need to divide the value by the "quantity" of the same products (SKUs) within the same transaction.

Suppose that the utility model is as follows. If consumer i purchases *one unit* (which means you need to decompose a multi-item transaction to multiple one-item transactions) of product j, she obtains utility

$$u_{ij} = \beta_{0j} + \beta_{1j}x_1^i + \ldots + \beta_{kj}x_k^i - \alpha_j p_j + \epsilon_{ij},$$

where: (a) $x_1^i, \ldots, x_k^i$ are consumer-specific *feature variables*; (b) p_j is the unit price of the product; (c) $\beta_{0j}, \beta_{1j}, \ldots, \beta_{kj}$ and α_j are the to-be-learned model parameters for product j. Random variable ϵ_{ij} is independent across products and consumers and follows Type I extreme value distribution.[1]

The main *open-ended* component in the above model is the consumer-specific feature variables $x_1^i, \ldots, x_k^i$, which *you will need to extract from the provided real data by yourself.* The data contains the purchase behavior of each customer i, and it is up to you to distill useful "features" (including its dimension k) for each customer that are predictive about this customer's utility and preferences. One simple example feature is the average expense of this customer on a transaction; another example feature is how many times this customer purchased some particular product A. However, it will be up to you to think about more.

If consumer does not purchase anything, she obtains utility

$$u_{i0} = \epsilon_{i0},$$

where ϵ_{i0} is an independent Type I extreme value distributed variable.

A nice property of the Type I extreme value distribution is that the probability that consumer i purchases product j (with respect to the distribution of ϵ_{ij}) can be computed as

$$P(\text{purchase product } j) = \frac{\exp(\beta_{0j} + \beta_{1j}x_1^i + \ldots + \beta_{kj}x_k^i - \alpha_j p_j)}{1 + \sum_{j=1}^{J} \exp(\beta_{0j} + \beta_{1j}x_1^i + \ldots + \beta_{kj}x_k^i - \alpha_j p_j)}, \tag{1}$$

where J is the total number of products sold in the supermarket. This probability model is called the multinomial logit model and it can be estimated using any standard statistical software package.

Task Part I: Estimating Model Parameters for Each Product j

[1] You may refer to wikipedia for what a Type I extreme value distribution is, but for this project it is not super important for you to know its concrete format since we already provide you the resultant probability of choices in Equation (1) due to this randomness.

1. Using a Jupyter notebook to import the csv file as pandas dataframe.

2. The fact that consumer does not purchase anything can be interpreted as that she chose an outside option. Given that the fact that she chose an outside option is not recorded in this dataset, argue how you would construct a proxi variable for the choice of an outside option. Add such a proxi variable to your dataframe.

   Hint: you can use information that some consumers do not appear in the data every week.

3. Given that we do not have *explicit* consumer feature vectors $\mathbf{x}^i = (x_1^i, \cdots, x_k^i)$ in the data, discuss how you would construct such feature vectors for each consumer i from the given data. Add your constructed characteristics to your dataframe.

   Hint: you can use transaction history and argue that past shopping patterns may give a good characterization for a given consumer.

4. Produce the utility parameters $\beta_{0j}, \beta_{1j}, \cdots \beta_{kj}$ and α_j for every product j by estimating a multinomial logit model from your constructed dataset.

Task Part II: Online Learning and Regret

In this part, we stand at a customer's point of view and consider the consumer's online learning problem (to purchase her favorite product). Here, suppose that when customer i purchases item j, the true utility of each is fixed and is equal to the deterministic part of the utility that you obtained from your dataframe, i.e.

$$\widehat{u}_{ij} = \beta_{0j} + \beta_{1j} x_1^i + \ldots + \beta_{kj} x_k^i - \alpha_j p_j$$

where $\beta_{0j}, \beta_{1j}, \cdots \beta_{kj}$ and α_j are the parameter you learned in Par I. Moreover, assume

$$\widehat{u}_{i0} = 0.$$

1. Construct a multi-armed bandit algorithm such that

   - It is randomly initialized at first and selects *one* product out of j available products.
   - It updates $\beta_{0j}, \beta_{1j}, \cdots \beta_{kj}$ and α_j over time by observing the utility $\widehat{u}_{ij}$ of each product j it selected in the past and selects new products

2. Draw 1000 random consumers from your data. For each consumer, run your online learning algorithm for 100 steps. Note that this is a simulation process — i.e., your algorithm itself does not know $\beta_{0j}, \beta_{1j}, \cdots \beta_{kj}$ and α_j, but can only observe the $\widehat{u}_{ij}$ for any product j that the algorithm pulled (i.e., purchased).

   For each randomly picked consumer i, compute the difference Δ_i between the maximum utility $\max_j \widehat{u}_{ij}$ (i.e., consumer i's utility for her favorite product) and the average utility that your algorithm achieved at the 100th step. Compute the average of Δ_i over those 1000 consumers, and explain why there is such a difference.

Note: For each of the question, please add some print or graph drawing commands to show your results in a clear way and also necessary analyses and demonstrations to help people who are not in your group understand your logics and results.