Preliminary Scaling Report

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Introduction

Background

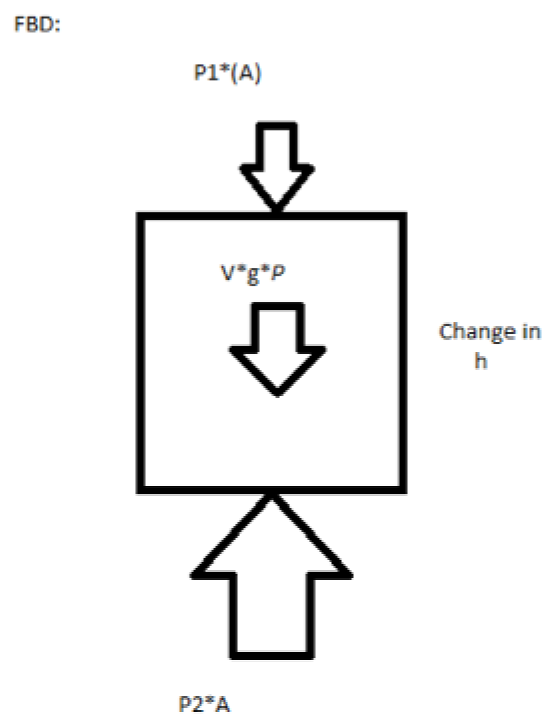
Our goal for this project is to effectively design a prototype pipe that could be used to contain the 2010 Deepwater Horizon oil spill. On April 20, 2010 in the Gulf of Mexico, the spill became the largest marine oil spill to date when an explosion in the main drilling oil rig lead to the failure of the rig and a corresponding 87 day oil leak (Avery). Early attempts to combat the spill included dispersants (chemicals to improve separation of particles) and top kills (plugging the well with thick mud). The well was finally capped on July 15, 2010, but not before authorities had discharged approximately 4.9 million barrels of oil into the Gulf of Mexico (On Scene Coordinator Report 1).

Approach

To fully understand this problem we need to obtain experimental data on the pipe and flow. However, because of the extremely large magnitude of the pipe, we will not be able to build a full scale model, and instead, will need to build a scaled down model for testing purposes. Pi terms (dimensionless terms used for scaling) will be used for our dimensional analysis. This uses the concept of similitude, which relates measurements made in one system to describe the behavior of other, similar systems (Fundamentals of Fluid Mechanics 346). We will use a theoretical testing facility to design the specifications for an experiment which would yield data explaining the complicated flow mechanics. The concept of similitude can then apply these observations to the real world scale. We looked at this problem as a system made up of 2 parts: external flow and internal considerations. The external force is almost entirely due to the ocean current velocity, as the hydrostatic pressure cancels out on all sides of the pipe. Internal effects on the pipe include differences in water and oil density, oil-water mixture flow rate, and the ratio of water to oil to prevent clogging of the pipe. After defining a working fluid and determining an optimal diameter using control volume analysis, we manipulated dimensionless pi terms to find corresponding model diameters for both external and internal testing environments.

Discussion of Physical Phenomena

The Deepwater Horizon incident was caused by a high-pressure methane gas bubble trapped inside the oil well. Once inside, the methane bubble rose up through the piping to the drilling rig, where it combusted and engulfed the platform in flames. The aftermath saw a destroyed platform and an uncapped pipe leaking approximately 50,000 barrels of oil a day. The phenomena that caused the methane bubble to rise to the surface is the same phenomena that caused the oil to rise ocean surface, a difference in density. The force that caused the ascension of these two compounds is due to a difference in density, which we denote as the buoyant force. Buoyancy becomes important when submerging one material with a specific density into another material with a different density. When descending into any medium there is a pressure gradient that causes an increase in pressure for an increase in depth. In this case the medium is water, an incompressible substance, which means that under compressive loading water will not experience a change in density. Incompressible fluids allow for a simplified analysis of pressure with respect to depth because for every increment of depth descended there remains a constant amount of fluid per unit length. We define this as a linear increase in pressure with respect to depth. Furthermore, we can compute this pressure with respect to depth using the following equation: pressure=(density of medium)*(gravity)*(change in height), or p=ρgh. When objects are submerged there is a pressure discrepancy between the top and bottom surface; due to its depth, the bottom surface is experiencing a greater pressure than the top surface. A free body diagram, such as the one to the right, helps illustrate why density plays a role in this buoyant force. Mathematically, we analyze a cube with arbitrary volume 'V,' area on the top and bottom surface defined as 'A,' and a material density 'ρ_c.' The pressure at the top and bottom surface are $p_1 = \rho_w g h_1$ and $p_2 = \rho_w g h_2$, respectively, where $h_2 > h_1$. Computing a force balance: F=ma , $m*a = p_2A - p_1A - V\rho g$. If $\rho_c = \rho_w$, then all of the terms on the right side will cancel out and the cube will remain stagnant in the water because the



acceleration the cube is experiencing is equal to zero. If $\rho_c > \rho_w$, the cube will experience a downwards acceleration, meaning the volume will sink in water. If $\rho_c < \rho_w$, as with methane and oil in water, the cube will rise in the water. The buoyant force acting on the water in the upward direction is due to this difference in densities. Furthermore, the static pressure properties of water allow us to make some simplifications, such as disregarding the general position in water when computing buoyancy. Whether our cube of oil is submerged 1 meter or 1500 meters, the buoyant force is a factor of the difference in h_1 & h_2, which is equal to the height of the cube.

Now that the parameter buoyancy is established, it is apparent that the need of a suction mechanism is not required to gather the spilled oil because it will naturally rise to the ocean surface. If the spill conveniently took place in a body of water with a stagnant current (and no viscous effects), the oil could be immediately contained before breaking the surface using a pipe equal in diameter to the leak diameter located directly 1500m of above the leak. But this is not the case. The location of the deepwater horizon has fairly mild currents but because the oil has to cover 1500m before reaching the surface current, it can dramatically affect oil dispersion rates. In the context of this project, in order to prevent this dispersion, a pipe long enough to span the entire 1500m gap is required to contain the leak. The use of such a long mechanical device in the ocean introduces additional important external parameters such as supports/anchors, material of the pipe, and the corresponding drag force. The supports of the pipe and material of the pipe are problems to be left to other professions, and for the sake of the analysis we will assume the pipe does not deflect and remains vertical. The only external parameter directly related to fluid mechanics is drag, which is dependent on the velocity of the current and the cross sectional area perpendicular to the current. The average velocity passing the pipe can be computed using geographical data gather near the Deepwater Horizon oil rig. The cross-sectional area is A=(length of the pipe)*(diameter of the pipe). The length of the pipe is 1500 m and the diameter will be optimized based on balanced consideration of external and internal effects.

To analyze the internal physical parameters we must model the physical parameters of the fluid approaching the pipe opening and later moving through the pipe. The first of these governing parameters is the requirement that the oil-water mixture must consist of at least 96% water, 4% oil. This stipulation must be accommodated in order to prevent the pipe from clogging. Thus, the immediate issue that arises is, how does the water move up the pipe when its density is the same as its environment? The physical phenomenon that accounts for this is viscosity, which is a consequence of friction between neighboring particles moving at different velocities. Provided the pipe diameter is large enough, the large volume of oil rising through the water causes some of the surrounding water to be pulled along into the pipe. Once inside, the oil continues to pull the water along with a force due to viscosity and the mixture pushed along as more of the oil-water solution continues to enter the pipe. In reality, this phenomenon will not provide a uniformly distributed mixture, as the fluid inside the pipe would be highly inconsistent. The oil would coagulate, forming bubbles of various sizes and moving with different speeds relative to the surrounding water. The analysis of this non-uniform mixture is beyond the scope of this course and, for the sake of computation, it will be assumed that the oil droplets will be evenly dispersed throughout the water and traveling with the same upward velocity, which nullifies any surface tension considerations. We will only be analyzing viscosity and corresponding density to select the working fluid, which will be used to simulate the oil-in-water conditions. The last two internal parameters that need to be addressed are diameter of the pipe and the velocity of the fluid in the pipe. The relation of the two must always satisfy the mass flow rate of the oil-water mixture of at least 2.3003 m^3/s (that is with the minimum 96% water compared to 50,000 barrels of oil). Assuming that the oil will carry along the required amount of water regardless of its velocity or the pipe diameter, there is some leeway when choosing the model diameter, as the other will be determined from the selection. The parameter that we will choose to determine value and explicit value for will be depend on the restrictions of the lab we will be conducting our tests in.

Other parameters that can be ignored are temperature and several other variables relating to material properties. Temperature can be ignored because the temperature difference between the seafloor and the surface is insignificantly small. Also, variables relating to material properties such as the Young's modulus, pipe mass, corrosion, moment of inertia, stress, strain, cable tension, etc. are out of the scope of this assignment.

Dimensional Parameters and Analysis

External Parameters		Variable	Unit
Independent	Current velocity	V_c	LT^{-1}
	*Density of water	ρ_w	ML^{-3}
	*Dynamic viscosity of water	μ_w	$ML^{-1}T^{-1}$
	*Height	h	L
	Diameter	D	L
Dependent	Net Force	F	MLS^{-2}

$$\Pi_1 = \frac{V_c D \rho_w}{\mu_w}$$

$$\Pi_2 = \frac{h}{D}$$

$$\Pi_3 = \frac{F \rho_w}{\mu_w^2}$$

Internal Parameters		Variable	Unit
Independent	*Density of water	ρ_w	ML^{-3}
	Density of Oil	ρ_o	ML^{-3}
	Diameter	D	L
	*Height	h	L
	*Combined Dynamic Viscosity	μ	$ML^{-1}T^{-1}$
	Gravity	g	LT^{-2}
	Seawater Volume Fraction	ϕ_w	-
Dependent	Volumetric flow rate	Q	L^3T^{-1}

$$\Pi_1 = \frac{Q}{g^{1/2} D^{5/2}}$$

$$\Pi_2 = \frac{\rho_o}{\rho_w}$$

$$\Pi_3 = \frac{h}{D}$$

$$\Pi_4 = \frac{\mu}{g^{1/2} \rho_w D^{3/2}}$$

$$\Pi_5 = \phi_w$$

Buckingham Pi Analysis

As described above, we created a list of possible variables that might be significant to this problem and eliminated unnecessary variables and the variables outside of the project scope. We also separated the variables into external and internal parameters. There are five independent variables and one dependent variable for the external parameters. There are eight independent variables and one dependent variable for the internal parameters. Using the Buckingham pi theorem, we determined that three pi terms were needed to describe the external variables while five pi terms were needed to describe the internal variables. We picked three repeating variables for the internal portion; gravity, height and density of the water. These were chosen based on our ability to vary them in testing and their dimensional incompatibilities. We then formed pi terms by multiplying the repeating variables with each non-repeating variable and determining their respective exponents. A sample calculation is shown below for our first internal pi term.

$\prod_1 = Q$

$[L^3T^{-1}][LT^{-2}]^a[L]^b[ML^{-3}]^c = M^0L^0T^0$

$3+a+b-3c=0$

$3-.5+b=0$

$-1-2a=0$

$c=0$

$a=-.5$

$b=-5/2$

Significance and Interpretation

The physical interpretations of the pi terms listed above give us an idea of how our flow will behave when certain variables are changed. For the external problem, $\prod_1$ is the more important term because $\prod_2$ can be reasonably neglected and accounted for by using smaller height segments. $\prod_1$ is a form of the Reynolds number, which relates inertial and viscous forces in the flow. In other words, it indicates whether the flow is laminar or turbulent. Similarly, for the internal problem, some of the pi terms can be neglected. $\prod_3$ was ignored for the same reasons as described for external problem and $\prod_5$ was ignored because it is ultimately accounted for by $\prod_1$, which uses the combined flow rate for the oil and the water. The remaining pi terms are either practically or theoretically significant. The presence of $\prod_2$ explains the primary driving force behind the flow, a difference in density. $\prod_1$ and $\prod_4$ explain how the combined flow rate and combined viscosity will depend on the diameter of the pipe. If we increase the pipe diameter, both values will decrease.

In order to model a scaled-down version of the system for experimental purposes, evaluated the optimal diameter and physical flow parameters. We designed both external and internal experiments to simulate a representative test. The external test relied on $\prod_1$. The internal test, however, relied on $\prod_1$,$\prod_2$, and $\prod_4$ because they are the most dynamic and significant of the internal pi terms. All other pi terms will not play a significant role when optimizing the simulation.

Analytical Model Development for Pipe Flow and Drag

Control Volume Analysis for Pipe Flow

$$Bernoulli's\,Equation: p + \frac{1}{2}\rho U^2 + \rho g H = Constant$$

$$Conservation\,of\,Mass: Q = A_1 U_1 = Constant$$

$$Conservation\,of\,Momentum:$$

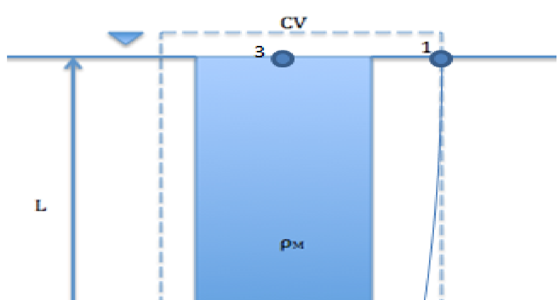
$$\frac{d}{dt}\iiint_{CV} \rho \vec{U} dV + \iint_{CS} \rho \vec{U}(\vec{U}\cdot\hat{n}) dA = \iint_{CS} p\hat{n} dA + \iiint_{CV} \rho \vec{g} dV + \iint_{CS} \bar{\bar{\tau}} dA + \sum F_{reaction}$$

$$Friction\,from\,pipe\,wall: F_{friction} = \pi D L \left(f \frac{1}{8}\rho U^2\right)$$

$$Darcy\,friction\,factor: f = \begin{cases} \frac{64}{Re}\ if\ Re < 2500 \\ 0.02\ if\ Re > 2500 \end{cases}$$

Assumptions:

- Developing length is much less than total length
- Steady flow
- Fully developed for most of the pipe
- Incompressible substance (water)



The first step in this problem is finding the pressure at the entrance of the pipe. We do this by using Bernoulli's equation from points 1 and 2 as shown in the diagram to the left.

$$p_1 + \frac{1}{2}\rho_A U_1^2 + \rho_A g H_1 = p_2 + \frac{1}{2}\rho_A U_2^2 + \rho_A g H_2$$

Where V_1 is much smaller than V_2, p_1 is equal to 0 gage pressure, and H_1-H_2=L. Therefore, our equation is simplified to:

$$\rho_A g L - \frac{1}{2}\rho_A U_2^2 = p_2$$

Next we use the conservation of mass equation to relate the inlet (point 2) and outlet (point 3) velocities of the pipe. However, since $A_1 = A_2$, we simply state that

$$V_2 = V_3$$

This greatly simplifies our conservation of momentum equation. Taking the momentum in the y direction, our conservation of momentum equation is:

$$\frac{d}{dt}\iiint_{CV} \rho_m U_y dV + \iint_{CS} \rho_m U_y (U_y \cdot \hat{n}) dA = -\iint_{CS} p_y \hat{n} dA + \iiint_{CV} \rho_m g_y dV + \iint_{CS} \bar{\bar{\tau}} dA$$

However due to our assumptions we can cancel out both terms on the left hand side of the equation. The time rate of change of our flow is equal to 0 due to our steady flow condition, and the momentum flux in is equal to the momentum flux out because the velocities at points 2 and 3 are equal, which we found from conservation of mass. Therefore the equation is simply:

$$0 = +\iint_{CS} p_y \hat{n} dA + \iiint_{CV} \rho_m g_y dV + \iint_{CS} \bar{\bar{\tau}} dA$$

The first term is the force due to pressure differences. The pressure difference between points 2 and 3 is simply the pressure at point 2 when we take point 3 to equal 0 gage pressure, and we solved for the pressure at point two earlier with Bernoulli's equation. Therefore, the first term is simply:

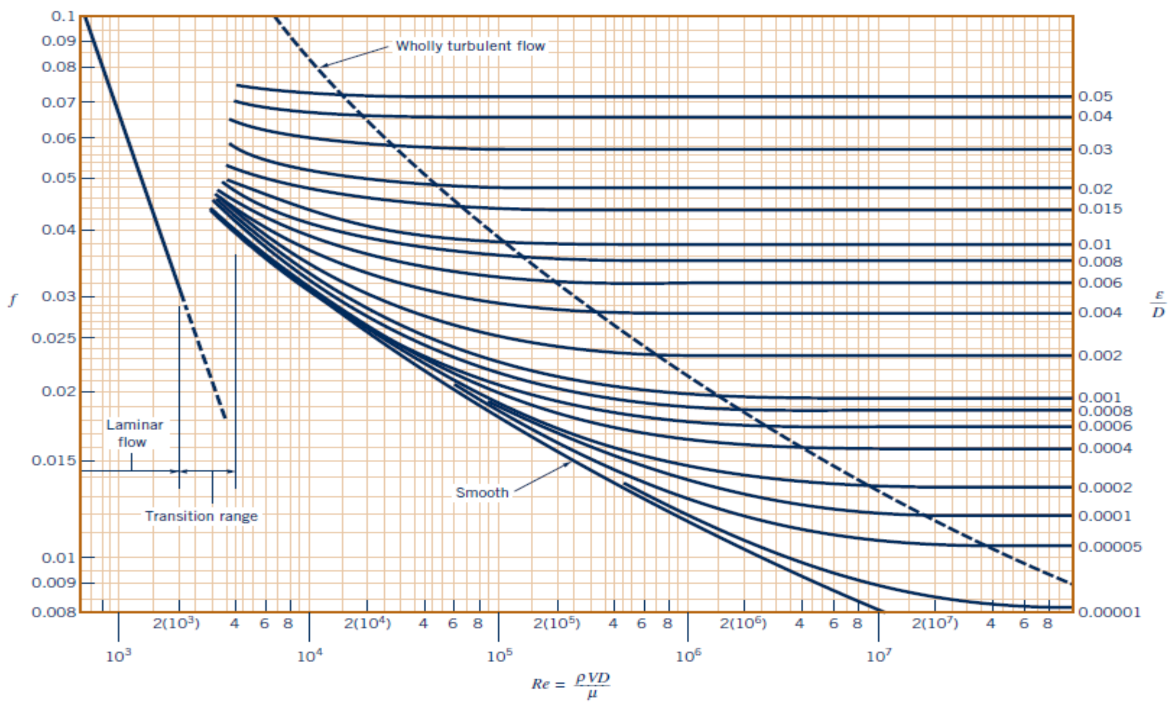
$$\iint_{CS} p_y \hat{n} dA = \frac{\pi D^2}{4}(\rho g L - \frac{1}{2}\rho U_2^2)$$

The second term is the force of gravity on the fluid within the pipe. The density of this term is the density of the mixture between our fluids A and B. Since the density ρ is constant and gravity only minimally changes with height, the second term is simply the product between the density, gravity, and the volume of the pipe, which is given by:

$$\iiint_{CV} \rho_m g_y dV = -\rho_m g L \frac{\pi D^2}{4}$$

The final term is given to us in terms of equation 4. We assume the shear stress is equal at all points in the pipe, thus leaving us with

$$\iint_{CS} \bar{\bar{\tau}} dA = -\pi D L \left(f \frac{1}{8} \rho_m U^2 \right)$$



From the Moody chart we estimated our f factor to be approximately 0.02, using an estimated surface roughness per diameter of 0.001, corresponding to an almost completely smooth pipe.

Putting all of our terms together we get the equation:

$$0 = \frac{\pi D^2}{4}\left(\rho_A g L - \frac{1}{2}\rho_A U_2^2\right) - \rho_m g L \frac{\pi D^2}{4} - \pi D L \left(f \frac{1}{8} \rho_m U^2 \right)$$

Next we assume the dynamic pressure term in this equation to be negligible, simplifying the equation more, and thusly allowing us to solve for the velocity U to be:

$$U = \sqrt{\frac{2D}{f\rho_m}(\rho_A - \rho_m)g}$$

From here we can plug this back into our conservation of mass equation to find an equation for volumetric flow rate, Q, in terms of our diameter of the pipe, D.

$$Q = A_2 U_2$$

$$Q(D) = \frac{\pi}{4}D^2\sqrt{\frac{2D}{f\rho_m}(\rho_A - \rho_m)g}$$

Drag Force

The external flow problem is relatively simple. The only force on the pipe in the direction of the streamlines, and the only force that we are looking for, is the drag force. The drag force is given by

$$\mathfrak{D} = C_D \frac{1}{2}\rho U^2 A$$

Where C_D for a smooth infinitely long cylinder at our Reynolds number is approximately 1.2, the density is equal to 1025 kg/m^3, and the velocity U is equal to the maximum ocean current that we can predict, which is equal to 0.347 m/s. Thus, our drag force is calculated to be:

$$\mathfrak{D} = 263.45\ kN$$

The results in a moment from a restraint on the surface to the center of mass of the pipe of

$$Moment\ about\ the\ end\ of\ the\ pipe = M = \mathfrak{D} * y_c = 197587.5\ kN - m$$

Model Scaling Specification

Our actual and model scaling parameters are as follows:

Actual Value	**Model**	**Model Values-Internal**	**Model Values-External**
D=1.03m	μ (mixed) =0.001058791 kg/m-s	D_m=0.916 m	Dm=0.36 m
h=1500 m	μ (ambient fluid) =0.001 kg/m-s	h_m=9.9m	hm=1m
Q=2.3003 m^3/s	ρ (working fluid) =889 kg/m3	Injection Rate=0.0686 m^3/s	Vcm=0.885 m/s
μ=0.001262 kg/ms	ρ (ambient fluid) =1000kg/m^3		
$V_{c\,max}$=0.347 m/s			
ρ_o=825 kg/m3			
ρ_w=1025 kg/m^3			

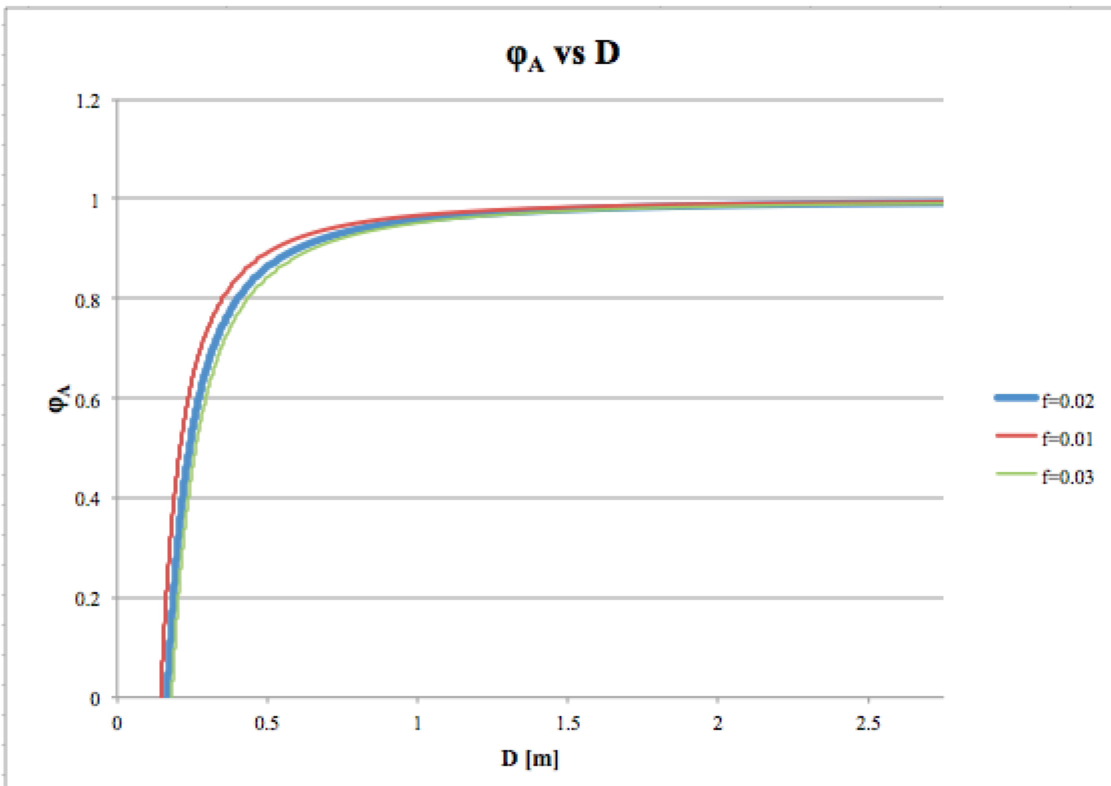
Actual Diameter Determination

The total flow rate of our fluid in the pipe is equal to 2.3003 m^3/s and we calculated the combined density to be 1017 kg/m^3 using the volumetric flow rate fraction. By applying the Refutas Equation, we found the combined viscosity to be 0.001262. We then plugged these values into the equation below, which was derived from the previously defined control volume flow rate equation, to find the optimal diameter:

$$D=\left\{\frac{f\left[\rho_w\phi_w+(1-\phi_w)\rho_o\right]\left[Q_o\left(1+\frac{\phi_w}{1-\phi_w}\right)\right]^2\left(\frac{4}{\pi}\right)^2}{2g\left(\rho_w-\rho_o\phi_w-(1-\phi_w)\rho_o\right)}\right\}^{\left(\frac{1}{5}\right)}$$

$$D = 1.03\ meters$$

The below graph is a visual interpretation of our equation as evaluated for three separate friction coefficients. Ultimately, we used f=0.02 because the diameter values did not vary significantly at the 96% seawater volume fraction.



Internal Model Scaling

To find the internal model diameter, we manipulated our internal dimensionless Pi terms. After choosing our working fluid (Tetrahydrofuran), we were able to iteratively determine the optimal model diameter by matching the combined viscosity of our working fluid with the expected model combined viscosity from $\prod_{4.}$ This combined viscosity of the ambient fluid and tetrahydrofuran is μ=0.000100873 kg/(m-s). Once we determined the model diameter, we were able to manipulate $\prod_1$ to find the injection rate for the working fluid. The model diameter and working fluid injection rate are as follows:

$$D = 0.916 \quad m$$

$$Q = 0.0686 \quad m^3/s$$

This is assuming that the 1m by 1m cross section is only for the external drag test facility, as shown in the the project problem statement.

The process for the model scale flow control volume is identical to the process for the full scale problem. The combined viscosity μ=0.000100873 kg/(m-s), the model flow rate is Q=1.7514 m^3/s, and the combined density ρ=992.19 kg/m^3. This gives us a necessary diameter for the model scale pipe to be:

$$D = 0.909 \; meters$$

This is similar to the model diameter that we chose from the pi terms.

External Model Scaling

Similarly, we obtained the external model diameter by iteratively evaluating the model current velocity. We accomplished this by using an actual current velocity of V=0.347 m/s, which we determined using the data from the National Data Buoy Center. The only remaining specification was that the model current velocity was restricted by a maximum value of 1 m/s. The results of this analysis are as follows:

$$D = 0.36 \quad m$$

$$V = 0.885 \quad m^3/s$$

Critical Review of Results

In order to verify the validity of our internal flow analysis, we plugged the model diameter of D=0.916m into the control volume equation for model flow rate. We then compared the resultant value to the model flow rate predicted by the pi terms ($\prod_1$). The comparison is as follows:

$$Q_{control\ \ volume} = 1.751 \ \ {m^3}/{s}$$

$$Q_{Pi\ \ terms} = 1.716 \ \ {m^3}/{s}$$

The results of this test show us that our scaling matches with the theoretical control volume interpretation of the pipe flow. Because these values differ only minimally, we can assume that our actual pipe diameter of D=1.01m would fulfil the requirement of the problem specifications for internal flow.

For the external flow problem, the calculated drag force and resulting moment about either end of the pipe indicate that we will require strong materials both for the pipe and the mooring cables. The drag force (F=263.45 kN) seemed to be overly cautious, as the value would require that the maximum ocean current we assumed would be uniform along the entire length of the pipe. However, this should be assumed as a worst case scenario, and the pipe could be fabricated with this in mind in order to minimize cost.

Summary and Conclusions

The Deepwater Horizon Oil Spill was just one of many problems that engineers are required to solve in the "real world." We applied a control volume analysis to our pipe system to evaluate internal pipe flow in our project. From this analysis we obtained a function for diameter in terms of oil flow rate and seawater volume fraction. From this we calculated the optimal diameter using a seawater volume fraction of 96%. We then used dimensional analysis to effectively translate the large scale problem into a manageable experimental size, at which level we could investigate and interpret complexities of this pipe flow. From this analysis, we were able to iteratively determine effective model scaling parameters for both the internal and external model testing environments. The results of this analysis yielded working diameters of 0.916 meters for the internal flow testing and 0.36 meters for external testing. Also, for our internal flow testing, we chose Tetrahydrofuran to be our working fluid because it exhibits relatively comparable density and viscosity of the real world oil and saltwater mixture when mixed with the ambient testing fluid. If we were able to conduct the internal and external experiments, we would be able translate the results back into a real world scale in order to verify our physical understanding of the fluid mechanics behind this particular problem and allow us to solve the problem of the Deepwater Horizon oil spill.

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