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- ▶ Linear polarized light is a transverse electromagnetic wave (homogeneous Maxwell-Equations)

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- In the complex plane (light is propagating perpendicular to it):

$$\mathbf{E} = E_0 \cdot [e^{i(k'_l x - \omega t)} + e^{-i(k'_r x - \omega t)}] = \mathbf{E}_l + \mathbf{E}_r \quad (4)$$

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$$\Rightarrow k'_\pm := k'(\omega \pm \omega_L) \approx k'(\omega) \pm \frac{dk'}{d\omega}(\omega) \cdot \omega_L := k' \pm \frac{dk'}{d\omega} \omega_L$$

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► combined with (4) one obtains:

$$\mathbf{E} = E_0 \cdot [e^{i \frac{dk'}{d\omega} \omega_L \cdot x} (e^{i(k'x - \omega t)} + e^{-i(k'x - \omega t)})]$$

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$$\begin{aligned} \mathbf{E} &= E_0 \cdot \left[e^{i \frac{dk'}{d\omega} \omega_L \cdot x} (e^{i(k'x - \omega t)} + e^{-i(k'x - \omega t)}) \right] \\ \Rightarrow \beta &= \frac{dk'}{d\omega} \omega_L \cdot \Delta x = \frac{dk'}{d\omega} \frac{e}{2m_e} B \cdot \Delta x \end{aligned}$$

($\Delta x \equiv$ length of the interval the light has travelled in the medium
 $g = 1$ und $q = e$ in (2) for simplicity)

- Using the chainrule and the dispersion relation:

$$\frac{dk'}{d\omega} = \frac{2\pi}{\lambda} \cdot \frac{dn}{d\lambda} \frac{d\lambda}{d\omega} = \frac{2\pi}{\lambda} \cdot \frac{dn}{d\lambda} \cdot \frac{-\lambda^2}{2\pi c} = -\frac{\lambda}{c} \cdot \frac{dn}{d\lambda} \quad (5)$$

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$$\Rightarrow \beta = -\frac{\lambda}{c} \frac{dn}{d\lambda} \frac{e}{2m_e} \cdot \Delta x \cdot B = \Delta x \cdot V \cdot B \quad (6)$$

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- For non homogeneous **B**-field we let $\Delta x \rightarrow 0$ to get the more general formula for β

$$\beta_{l_1 \rightarrow l_2} = V \cdot \int_{l_1}^{l_2} B dx \quad (7)$$

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Left/right-handed circular polarized light changes to right/left-handed circular polarized light by changing the direction of propagation and you see, that eq. (4) is invariant under the transformation

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As a consequence, we obtain $\beta = \beta'$
(the specific difference from the Faraday-Effect to other double refraction phenomena)