Faraday Effekt

weekly report group B

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What is the Faraday-Effect

- The Faraday-Effect describes an interaction between light, as an electromagnetic wave, and a given magnetic field **B** in a nonconductive transparent medium
- It causes a rotation β of the plane of polarization, which is proportional to the component of the magnetic field in the direction of propagation

Assumptions

- There are microscopic magnetical moments in every nontransparent and nonconductive medium in an atomic scale

  Classical: Electrons orbiting the nucleus of the atom
  QM: The total angular momentum $\vec{\mathbf{J}}$ of Electrons

- We have a linear transverse electromagnetic wave (homogene Maxwell-Equations)

Derivation of the rotation angle β

- Starting with the gyroscopic equation:

$$\dot{\vec{\mu}} = \gamma \cdot \vec{\mu} \times \vec{\mathbf{B}} \tag{1}$$

$\Rightarrow$ Precession of the magnetical moments

QM : $\gamma = g\frac{q}{2m}$, g the Lande-Faktor (Classically: $g = 1$)

- w.l.o.g. $\vec{\mu} \perp \vec{\mathbf{B}}$, so we obtain the Larmor frequency

$$\omega_L = \Delta E_{m_J}/\hbar = g\frac{q}{2m}B \qquad (B = |\vec{\mathbf{B}}|) \tag{2}$$

as the precession frequency of $\vec{\mu}$ from eq. (1)

- To describe the linear transverse electromagnetic wave we start with the dispersion relation:

$$\omega = k'(\omega) \cdot c = k \cdot n(\omega) \cdot c \tag{3}$$

and obtain (after moving to the complex plane for simplicity)

$$\mathbf{E} = E_0 \cdot [e^{i(k_l' x - \omega t)} + e^{-i(k_r' x - \omega t)}] = \mathbf{E}_\mathrm{l} + \mathbf{E}_\mathrm{r} \tag{4}$$

where the light is propagating perpendicular to the complex plane (we will just look at the **E**-field in the following)

- So we optain a superposition of a right- and left-handed circular wave with total amplitude $2E_0$

- We have to move in the system of the electrons, to get a picture of the wave properties
- As we see in eq. (1) the electons precess with the magnetical moments counterclockwise around $\vec{\mathbf{B}}$

  $\Rightarrow$ In the **perspective of the electron**:
  $k_l' = k'(\omega + \omega_L)$ for a left-handed circular wave and
  $k_r' = k'(\omega - \omega_L)$ for a right-handed circular wave
- Inserted in (4) we obtain:

$$\mathbf{E} = E_0 \cdot [e^{i(k'(\omega+\omega_L)x-\omega t)} + e^{-i(k'(\omega-\omega_L)x-\omega t)}] \tag{5}$$

- Assuming $\omega_L << \omega \Rightarrow$

$$k'_{\pm} := k'(\omega \pm \omega_L) \approx k'(\omega) \pm \frac{\mathrm{d}k'}{\mathrm{d}\omega}(\omega) \cdot \omega_L := k' \pm \frac{\mathrm{d}k'}{\mathrm{d}\omega}\omega_L$$

- combined with (5) we get:

$$\begin{aligned} \mathbf{E} &= E_0 \cdot [e^{i((k'+\frac{\mathrm{d}k'}{\mathrm{d}\omega}\omega_L)x-\omega t)} + e^{-i((k'-\frac{\mathrm{d}k'}{\mathrm{d}\omega}\omega_L)x-\omega t)}] \\ &= E_0 \cdot [e^{i\frac{\mathrm{d}k'}{\mathrm{d}\omega}\omega_L \cdot x}(e^{i(k'x-\omega t)} + e^{-i(k'x-\omega t)})] \end{aligned} \tag{6}$$

- As a consequence of eq. (6), with Δx being the length of the interval the light has travelled in the medium, we obtain:

$$\beta = \frac{\mathrm{d}k'}{\mathrm{d}\omega}\omega_L \cdot \Delta x = \frac{\mathrm{d}k'}{\mathrm{d}\omega}\frac{e}{2m_e}B \cdot \Delta x \tag{7}$$

(by using $g = 1$ und $q = e$ in (2) for simplicity)

- With (3) we see

$$\frac{\mathrm{d}k'}{\mathrm{d}\omega} = \frac{2\pi}{\lambda} \cdot \frac{\mathrm{d}n}{\mathrm{d}\lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}\omega} = \frac{2\pi}{\lambda} \cdot \frac{\mathrm{d}n}{\mathrm{d}\lambda} \cdot \frac{-\lambda^2}{2\pi c} = -\frac{\lambda}{c} \cdot \frac{\mathrm{d}n}{\mathrm{d}\lambda} \tag{8}$$

and get with eq. (7)

$$\beta = -\frac{\lambda}{c}\frac{\mathrm{d}n}{\mathrm{d}\lambda}\frac{e}{2m_e} \cdot \Delta x \cdot B = \Delta x \cdot V \cdot B \tag{9}$$

where V is the Verdet constant of the medium

- For non homogeneous **B**-field we let $\Delta x \to 0$ to get the more general formula for β

$$\beta_{l_1 \to l_2} = V \cdot \int_{l_1}^{l_2} B dx \tag{10}$$

Comments to characteristic features of the Faraday-Effect

- With a strong **B**-Field we know $g > 1$, so the precession frequency will be bigger than the classical Larmor frequency
- The Verdet constant dependens on the temperature of the medium ($\frac{\mathrm{d}n}{\mathrm{d}\lambda}(T)$ in the area of small variations negligible)
- β ist independent on the direction of propagation of the light. Because left/right-handed circular polarized light changes to right/left-handed circular polarized light by changing the direction of propagation and we see, that eq. (4) is invariant under the transformation

$$\begin{pmatrix} k_r \\ k_l \\ \omega \end{pmatrix} \rightarrow \begin{pmatrix} -k_l \\ -k_r \\ -\omega \end{pmatrix} \tag{11}$$

As a consequence, we get $\beta = \beta'$
(**the specific difference from the Faraday-Effect to other double refraction phenomena**)