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- Linear polarized light is a transverse electromagnetic wave (homogene Maxwell-Equations)

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- In the complex plane (light is propagating perpendicular to it):

$$\mathbf{E} = E_0 \cdot [e^{i(k_l' x - \omega t)} + e^{-i(k_r' x - \omega t)}] = \mathbf{E}_\mathrm{l} + \mathbf{E}_\mathrm{r} \tag{4}$$

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- Assuming $\omega_L << \omega$

$$\Rightarrow k'_\pm := k'(\omega \pm \omega_L) \approx k'(\omega) \pm \frac{\mathrm{d}k'}{\mathrm{d}\omega}(\omega) \cdot \omega_L := k' \pm \frac{\mathrm{d}k'}{\mathrm{d}\omega}\omega_L$$

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$$\Rightarrow \beta = \frac{\mathrm{d}k'}{\mathrm{d}\omega}\omega_L \cdot \Delta x = \frac{\mathrm{d}k'}{\mathrm{d}\omega}\frac{e}{2m_e}B \cdot \Delta x$$

($\Delta x \equiv$ length of the interval the light has travelled in the medium
$g = 1$ und $q = e$ in (2) for simplicity)

- Using the chainrule and the dispersion relation:

$$\frac{\mathrm{d}k'}{\mathrm{d}\omega} = \frac{2\pi}{\lambda} \cdot \frac{\mathrm{d}n}{\mathrm{d}\lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}\omega} = \frac{2\pi}{\lambda} \cdot \frac{\mathrm{d}n}{\mathrm{d}\lambda} \cdot \frac{-\lambda^2}{2\pi c} = -\frac{\lambda}{c} \cdot \frac{\mathrm{d}n}{\mathrm{d}\lambda} \tag{5}$$

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- For non homogeneous **B**-field we let $\Delta x \to 0$ to get the more general formula for β

$$\beta_{l_1 \to l_2} = V \cdot \int_{l_1}^{l_2} B dx \tag{7}$$

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  Left/right-handed circular polarized light changes to right/left-handed circular polarized light by changing the direction of propagation and you see, that eq. (4) is invariant under the transformation

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  As a consequence, we obtain $\beta = \beta'$
  (**the specific difference from the Faraday-Effect to other double refraction phenomena**)