

Faraday Effekt

weekly report group B

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Theory for the Faraday-Effect

What is the Faraday-Effect

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What is the Faraday-Effect

- ▶ The Faraday-Effect describes an interaction between light, as an electromagnetic wave, and a given magnetic field \mathbf{B} in a nonconductive transparent medium
- ▶ It causes a rotation β of the plane of polarization, which is proportional to the component of the magnetic field in the direction of propagation

Assumptions

- ▶ There are microscopic magnetical moments in every nontransparent and nonconductive medium in an atomic scale

Classical: Electrons orbiting the nucleus of the atom

QM: The total angular momentum \vec{J} of Electrons

- ▶ We have a linear transverse electromagnetic wave (homogene Maxwell-Equations)

Derivation of the rotation angle β

- ▶ Starting with the gyroscopic equation:

$$\dot{\vec{\mu}} = \gamma \cdot \vec{\mu} \times \vec{\mathbf{B}} \quad (1)$$

\Rightarrow Precession of the magnetical moments

QM : $\gamma = g \frac{e}{2m}$, g the Lande-Faktor (Classically: $g = 1$)

- ▶ w.l.o.g. $\vec{\mu} \perp \vec{\mathbf{B}}$, so we obtain the Larmor frequency

$$\omega_L = \Delta E_{m_J} / \hbar = g \frac{q}{2m} B \quad (B = |\vec{\mathbf{B}}|) \quad (2)$$

as the precession frequency of $\vec{\mu}$ from eq. (1)

- ▶ To describe the linear transverse electromagnetic wave we start with the dispersion relation:

$$\omega = k'(\omega) \cdot c = k \cdot n(\omega) \cdot c \quad (3)$$

and obtain (after moving to the complex plane for simplicity)

$$\mathbf{E} = E_0 \cdot [e^{i(k_l'x - \omega t)} + e^{-i(k_r'x - \omega t)}] = \mathbf{E}_l + \mathbf{E}_r \quad (4)$$

where the light is propagating perpendicular to the complex plane (we will just look at the \mathbf{E} -field in the following)

- ▶ So we obtain a superposition of a right- and left-handed circular wave with total amplitude $2E_0$

- ▶ We have to move in the system of the electrons, to get a picture of the wave properties
- ▶ As we see in eq. (1) the electrons precess with the magnetical moments counterclockwise around $\vec{\mathbf{B}}$

⇒ In the **perspective of the electron**:

$k'_l = k'(\omega + \omega_L)$ for a left-handed circular wave and

$k'_r = k'(\omega - \omega_L)$ for a right-handed circular wave

- ▶ Inserted in (4) we obtain:

$$\mathbf{E} = E_0 \cdot [e^{i(k'(\omega + \omega_L)x - \omega t)} + e^{-i(k'(\omega - \omega_L)x - \omega t)}] \quad (5)$$

- Assuming $\omega_L \ll \omega \Rightarrow$

$$k'_{\pm} := k'(\omega \pm \omega_L) \approx k'(\omega) \pm \frac{dk'}{d\omega}(\omega) \cdot \omega_L := k' \pm \frac{dk'}{d\omega} \omega_L$$

- combined with (5) we get:

$$\begin{aligned} \mathbf{E} &= E_0 \cdot [e^{i((k' + \frac{dk'}{d\omega} \omega_L)x - \omega t)} + e^{-i((k' - \frac{dk'}{d\omega} \omega_L)x - \omega t)}] \\ &= E_0 \cdot [e^{i \frac{dk'}{d\omega} \omega_L \cdot x} (e^{i(k'x - \omega t)} + e^{-i(k'x - \omega t)})] \end{aligned} \quad (6)$$

- As a consequence of eq. (6), with Δx being the length of the interval the light has travelled in the medium, we obtain:

$$\beta = \frac{dk'}{d\omega} \omega_L \cdot \Delta x = \frac{dk'}{d\omega} \frac{e}{2m_e} B \cdot \Delta x \quad (7)$$

(by using $g_J = 1$ und $q = e$ in (2) for simplicity)

- With (3) we see

$$\frac{dk'}{d\omega} = \frac{2\pi}{\lambda} \cdot \frac{dn}{d\lambda} \frac{d\lambda}{d\omega} = \frac{2\pi}{\lambda} \cdot \frac{dn}{d\lambda} \cdot \frac{-\lambda^2}{2\pi c} = -\frac{\lambda}{c} \cdot \frac{dn}{d\lambda} \quad (8)$$

and get with eq. (7)

$$\beta = -\frac{\lambda}{c} \frac{dn}{d\lambda} \frac{e}{2m_e} \cdot \Delta x \cdot B = \Delta x \cdot V \cdot B \quad (9)$$

where V is the Verdet constant of the medium

- For non homogeneous **B**-field we let $\Delta x \rightarrow 0$ to get the more general formula for β

$$\beta_{l_1 \rightarrow l_2} = V \cdot \int_{l_1}^{l_2} B dx \quad (10)$$

Comments to characteristic features of the Faraday-Effect

- ▶ With a strong **B**-Field we know $g > 1$, so the precession frequency will be bigger than the classical Larmor frequency
- ▶ The Verdet constant depends on the temperature of the medium ($\frac{dn}{d\lambda}(T)$ in the area of small variations negligible)
- ▶ β is independent on the direction of propagation of the light. Because left/right-handed circular polarized light changes to right/left-handed circular polarized light by changing the direction of propagation and we see, that eq. (4) is invariant under the transformation

$$\begin{pmatrix} k_r \\ k_l \\ \omega \end{pmatrix} \rightarrow \begin{pmatrix} -k_l \\ -k_r \\ -\omega \end{pmatrix} \quad (11)$$

As a consequence, we get $\beta = \beta'$
(the special feature of the Faraday-Effect)