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Observer-based fuzzy adaptive consensus tracking control for nonlinear multi-agent systems with actuator and sensor faults

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Abstract This paper studies the distributed consensus tracking control problem for a class of nonlinear multi-agent systems with actuator faults, sensor faults, unknown disturbances and model uncertainties. An observer-based fuzzy adaptive fault-tolerant control scheme based on the hierarchical design thought is proposed. First, a differentiator-based fully distributed leader's predictor is constructed to estimate the trajectory of the leader for each follower to avert the phenomenon of faults propagation. Second, a novel fuzzy observer based on actual measurement values is designed to estimate the unmeasured states and the actuator fault, respectively. Thirdly, based on the information from the predictor and the observer, an adaptive output feedback controller is designed by employing the sensor fault compensation mechanism. It is ensured that the tracking error can be reduced to a small neighborhood of the origin, and all the signals of agents are uniformly bounded. Finally, simulation examples indicate the effectiveness of the proposed fault-tolerant approach.

Keywords nonlinear multi-agent systems, backstepping control, fuzzy logic systems, actuator and sensor faults, state observer

Citation

1 Introduction

In recent years, the distributed cooperative control of multi-agent systems (MASs) [1] has attracted significant attention because of its extensive applications in various engineering fields, such as unmanned aerial vehicles, attitude alignment of spacecrafts, smart grids, and wireless sensor networks [2–5]. As an important and fundamental problem of distributed cooperative control, the consensus of MASs has been widely investigated in the control field. According to the available literature, the consensus problem of MASs may be classified as leaderless consensus and leader-following consensus. The leaderless consensus denotes that all agents achieve a common interest value by designing a suitable control protocol based on neighbor information. The leader-following consensus means that a team of agents follow the desired trajectory generated by the leader. Many valuable results have been reported on the consensus problem of MASs by developing different control protocols [6–9].

Previous research on consensus control mostly concentrated on linear MASs [10–12]. In fact, many real-world plants exhibit highly nonlinear dynamics, such as single-link robots [13], and unmanned aerial vehicles [14]. Therefore, the consensus problem of nonlinear MASs has more theoretical and practical significance. Recently, several novel consensus control methods of nonlinear MASs have been proposed in [15–18]. In [15], the H_∞ consensus problem was studied for nonlinear second-order MASs based on the inequality techniques. A finite-time distributed controller was designed for nonlinear MASs with Lipschitz condition in [16]. In [17], by employing an event-triggered scheme, a distributed sliding mode controller was developed for second-order nonlinear MASs. In [18], a fuzzy H_∞ controller based on Takagi-Sukeno fuzzy model was proposed to settle the leader-following consensus problem of nonlinear MASs

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with external disturbances. In addition, when the dynamics of many systems are not only nonlinear, but also uncertain, it is of more challenging to study the consensus problem of uncertain nonlinear MASs. Thankfully, fuzzy logic systems (FLS) and neural network (NN) have been proven as universal approximation tools to handle the uncertain nonlinear systems. In [19], the unknown nonlinear functions were estimated by FLS, and the consensus control problem was considered for MASs with matched nonlinear functions. The finite-time adaptive fuzzy consensus control method was addressed for the heterogeneous nonlinear MASs by FLS and Lyapunov theory [20]. In [21], the consensus tracking control protocol was presented for nonlinear MASs by adaptive NN approach and backstepping technique.

On the other hand, with the wide application of MASs in engineering fields, the reliability and security requirements of MASs are further increasing. In fact, the effect of any fault on an agent may diffuse to neighbor agents via the communication network, and performance degradation or instability for the entire systems would be arised [22]. Therefore, the fault-tolerant control (FTC) of MASs has attracted the attention of many scholars [23–29]. For the multiplicative fault of actuator, a distributed adaptive event-triggered fault-tolerant consensus was studied for general linear MASs in [30]. [31] considered fault-tolerant consensus tracking control of MASs with incipient and abrupt actuator faults under fixed and switching topologies. In [32], the distributed adaptive FTC was developed for nonlinear MASs with actuator biased faults. For partial loss of effectiveness fault and bias fault occurring on actuator, a cooperative FTC was explored for MASs with switching directed topology in [33]. An adaptive NN event-triggered control scheme was developed for nonlinear MASs against sensor faults, where the fault types cover bias fault, drift fault, and loss of accuracy [34]. It is worth noting that the FTC methods mentioned above just solve single-component fault. For the simultaneous faults of sensor and actuator, several novel resilient state-feedback-based consensus tracking protocols were designed in [35]. In [36], an adaptive fuzzy consensus FTC method is proposed for a class of second-order MASs with uncertain dynamics and unknown failure on actuator and sensor. However, as far as we know, the FTC problem for nonlinear MASs with actuator and sensor faults has not been fully studied. Based on the above arguments, this paper will investigate the distributed fault-tolerant consensus tracking control problem for unknown nonlinear MASs with unknown actuator and sensor faults. Compared with existing FTC strategies for MASs, the main contributions of this paper are summarized as follows:

1) By using the hierarchical design thought, the distributed fault-tolerant consensus tracking control problem for nonlinear MASs is uncoupled into the distributed consensus tracking of the predictor, and the FTC of single system. Different from the most existing FTC approaches, this approach not only averts the effect of faults propagation, but also ensures the expected performance of the whole MASs.

2) Based on actual measurement values and the estimated value of the sensor fault, a novel fuzzy observer is designed to effectively estimate actuator fault and unmeasured states, respectively. At the same time, the FLS is used to approximate the uncertain nonlinear functions.

3) An observer-based adaptive output feedback controller based on the information from the predictor is designed to eliminate the effects of sensor fault and ensure the stability of the closed-loop system.

4) A fully distributed leader's predictor based on the tracking differentiator is constructed in a directed communication network to estimate the reference trajectory of each follower.

The remainder of this article is organized as follows. The preliminaries and problem statement is provided in Section 2. Section 3 derives the main results. Section 4 provides the simulation results. Finally, section 5 presents the conclusion.

Notations: R, R^n and $R^{n\times m}$ represent the set of real numbers, set of n-dimensional column vector and real $n \times m$ matrices, respectively. $|\cdot|$ is the absolute value of a number. $\|\cdot\|$ is the norm of a vector or a matrix. $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ are the minimum and maximum eigenvalues of matrix $X \in R^{n\times n}$. $\sigma_{\min}(\cdot)$ is the minimum singular of a matrix. X^T is the transposed matrix of X.

2 Preliminary and problem statement

2.1 Algebraic graph theory

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ denote a directed topological graph. $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$, $\mathcal{E} = \{(v_i, v_j)|\, v_i, v_j \in \mathcal{V}\}$ and $\mathcal{A} = [a_{ij}]_{N\times N}$ represent the node set, the edge set, and an adjacency matrix, respectively. When node v_i receives information from node v_j, an edge $(v_j, v_i) \in \mathcal{E}$ is found, $a_{ij} = 1$, and vice versa. $\mathcal{N}_i = \{v_j|\,(v_j, v_i) \in \mathcal{E}\}$ is the neighbor set of node v_i. $\mathcal{L} = [l_{ij}]_{N\times N}$ indicates the Laplacian matrix of the digraph $\mathcal{G}$ satisfying $l_{ij} = -a_{ij}$ for $i \neq j$ and $l_{ii} = \sum_{j \in \mathcal{N}_i} a_{ij}$. If a directed path exists from a root node to every other node, then the digraph $\mathcal{G}$ is said to have a directed spanning tree. A graph $\bar{\mathcal{G}}$ describes the leader-following network graph, including a leader node v_0 and a node set $\mathcal{V}$. The diagonal matrix is defined as $\mathcal{B} = diag\{b_1, b_2, \cdots, b_n\}$, where $b_i = 1$ denotes that the follower node v_i obtains information from the leader node v_0, and otherwise $b_i = 0$.

2.2 Problem statement

Consider a group of nonlinear MASs consisting of N followers and a leader. The nonlinear dynamic model of the ith follower is given as

$$\begin{aligned} \dot{x}_{i,m} &= x_{i,m+1} + f_{i,m}(\bar{x}_{i,m}) + d_{i,m}, m = 1, \ldots, n_i - 1, \\ \dot{x}_{i,n_i} &= u_i + f_{i,n_i}(\bar{x}_{i,n_i}) + d_{i,n_i}, i = 1, \ldots, N, \\ y_i &= x_{i,1}, \end{aligned} \tag{1}$$

where $\bar{x}_{i,m} = [x_{i,1}, x_{i,2}, \cdots, x_{i,m}]^T \in R^m$, $u_i \in R$ and $y_i \in R$ are the system state, control input and output of the ith follower, respectively. $f_{i,m}$ denotes an uncertain smooth nonlinear function. $d_{i,m}$ is the system disturbance and satisfies $|d_{i,m}| \leqslant d^*_{i,m}$ with $d^*_{i,m}$ being unknown constants. $y_d \in R$ denote the leader's trajectories.

In this paper, the ith follower agent is subjected to unknown time-varying actuator and sensor faults, simultaneously. The faulty models are described as

$$y_i^f = \beta_{si}(t) y_i + f_{si}(t), \tag{2}$$
$$u_i^f = \beta_{ai}(t) u_i + f_{ai}(t), \tag{3}$$

where y_i^f is the measured value of the system output y_i under sensor fault, and $0 < \beta_{si}(t) \leqslant 1$ represents the sensor sensitivity. u_i^f is the actuator output, and $0 < \beta_{ai}(t) \leqslant 1$ denotes the loss of effectiveness fault. $f_{si}(t)$ and $f_{ai}(t)$ are bounded unknown functions, which depict uncertain bias fault of sensor and actuator, respectively.

Motivated by [38], eqautions (2) and (3) can be rewritten as

$$y_i^f = y_i + \delta_{si}, \tag{4}$$
$$u_i^f = u_i + \delta_{ai}, \tag{5}$$

where $\delta_{si} = (\beta_{si}(t) - 1) y_i + f_{si}(t)$, $\delta_{ai} = (\beta_{ai}(t) - 1) u_i + f_{ai}(t)$.

In accordance with (4) and (5), the system (1) is rewritten as

$$\begin{aligned} \dot{x}_{i,m} &= x_{i,m+1} + f_{i,m}(\bar{x}_{i,m}) + d_{i,m}, \\ \dot{x}_{i,n_i} &= u_i + \delta_{ai} + f_{i,n_i}(\bar{x}_{i,n_i}) + d_{i,n_i}, \\ y_i^f &= y_i + \delta_{si}. \end{aligned} \tag{6}$$

Lemma 1. (see [42]) Suppose $f(x)$ is a continuous function defined on a compact set Ω. Given any constant $\varepsilon > 0$, a FLS exists such that

$$\sup_{x \in \Omega} \left| f(x) - \theta^T \phi(x) \right| \leqslant \varepsilon, \tag{7}$$

where $\theta^T = [\theta_1, \theta_2, \cdots, \theta_N]$ is the parameter vector. $\phi^T(x) = [\phi_1(x), \phi_2(x), \cdots, \phi_N(x)] / \sum_{i=1}^N \phi_i(x)$ is the fuzzy basis function vector, and $\phi_i(x)$ are defined as

$$\phi_i(x) = \exp\left[\frac{-(x - b_i)^T (x - b_i)}{a_i^2} \right],$$

where a_i is the width of the Gaussian function, and $b_i = [b_{i,1}, b_{i,2}, \cdots, b_{i,l}]^T$ is the center vector.

Suppose $\hat{\bar{x}}_{i,m} = [\hat{x}_{i,1}, \hat{x}_{i,2}, \cdots, \hat{x}_{i,m}]^T$ is the estimated value of $\bar{x}_{i,m} = [x_{i,1}, x_{i,2}, \cdots, x_{i,m}]^T$, the system (6) can be rewritten as follows:

$$\begin{aligned} \dot{x}_{i,m} &= x_{i,m+1} + f_{i,m}(\hat{\bar{x}}_{i,m}) + \Delta f_{i,m} + d_{i,m}, \\ \dot{x}_{i,n_i} &= u_i + \delta_{ai} + f_{i,n_i}(\hat{\bar{x}}_{i,n_i}) + \Delta f_{i,n_i} + d_{i,n_i}, \\ y_i^f &= y_i + \delta_{si}, \end{aligned} \tag{8}$$

where $\Delta f_{i,m} = f_{i,m}(\bar{x}_{i,m}) - f_{i,m}(\hat{\bar{x}}_{i,m})$.

In accordance with Lemma 2, the unknown function $f_{i,m}(\hat{\bar{x}}_{i,m})$ in system (8) is approximated by the following FLS.

$$\hat{f}_{i,m}(\hat{\bar{x}}_{i,m}) = \theta_{i,m}^T \phi_{i,m}(\hat{\bar{x}}_{i,m}). \tag{9}$$

The optimal parameter vector $\theta_{i,m}^*$ is determined as

$$\theta_{i,m}^* = \arg \min_{\theta_{i,m} \in \Omega_{i,m}} [\sup_{\hat{\bar{x}}_{i,m} \in U_{i,m}} \left| \hat{f}_{i,m}(\hat{\bar{x}}_{i,m} | \theta_{i,m}) - f_{i,m}(\hat{\bar{x}}_{i,m}) \right|], \tag{10}$$

where $\Omega_{i,m}$ and $U_{i,m}$ are the compact regions for $\theta_{i,m}$ and $\hat{\bar{x}}_{i,m}$, respectively. Then, we have

$$\varepsilon_{i,m} = f_{i,m}(\hat{\bar{x}}_{i,m}) - \hat{f}_{i,m}(\hat{\bar{x}}_{i,m} | \theta_{i,m}^*), \tag{11}$$

$$\varpi_{i,m} = f_{i,m}(\hat{\bar{x}}_{i,m}) - \hat{f}_{i,m}(\hat{\bar{x}}_{i,m} | \theta_{i,m}), \tag{12}$$

where $\varepsilon_{i,m}$ is the fuzzy minimum approximation error , $\varpi_{i,m}$ is the fuzzy approximation error.

Assumption 1. (see [40]) There exist positive constants $\varepsilon_{i,m}^* > 0$ and $\varpi_{i,m}^* > 0$ such that $|\varepsilon_{i,m}| \leqslant \varepsilon_{i,m}^*$ and $|\varpi_{i,m}| \leqslant \varpi_{i,m}^*$, $1 \leqslant i \leqslant N$, $1 \leqslant m \leqslant n_i$.

According to (11), the system (8) can be rewritten as

$$\begin{aligned} \dot{x}_{i,m} &= x_{i,m+1} + \hat{f}_{i,m}(\hat{\bar{x}}_{i,m} | \theta_{i,m}^*) + \varepsilon_{i,m}(\hat{\bar{x}}_{i,m}) + \Delta f_{i,m} + d_{i,m}, \\ \dot{x}_{i,n_i} &= u_i + \delta_{ai} + \hat{f}_{i,n_i}(\hat{\bar{x}}_{i,n_i} | \theta_{i,n_i}^*) + \varepsilon_{i,n_i}(\hat{\bar{x}}_{i,n_i}) + \Delta f_{i,n_i} + d_{i,n_i}, \\ y_i^f &= y_i + \delta_{si}. \end{aligned} \tag{13}$$

Objective: According to the hierarchical design thought, an observer-based fuzzy adaptive FTC scheme is presented for uncertain nonlinear MASs (6) subjected to actuator and sensor faults, and unknown disturbance, such that the tracking error $y_i - y_d$ can be reduced to a small neighborhood of the origin, and all the signals of agents are uniformly bounded. To achieve this objective, the following assumptions and preliminaries are given for subsequent development.

Assumption 2. The augmented graph $\bar{\mathcal{G}}$ includes a directed spanning tree with the leader agent as the root node.

Assumption 3. The functions $f_{i,m}(\cdot)$ for $i = 1, 2, \ldots, N$ and $m = 1, 2, \ldots, n_i$ satisfies the Lipschitz condition, There exists a set of positive constants $\gamma_{i,m} > 0$ such that the inequality $|f_{i,m}(X) - f_{i,m}(Y)| \leqslant \gamma_{i,m} \|X - Y\|$ holds.

Assumption 4. Time-varying fault functions δ_{si} and δ_{ai} and their derivatives are bounded, that $\bar{\delta}_{si}$, $\bar{\bar{\delta}}_{si}$, $\bar{\delta}_{ai}$, and $\bar{\bar{\delta}}_{ai}$ are found, such that $|\delta_{si}| \leqslant \bar{\delta}_{si}$, $\left|\dot{\delta}_{si}\right| \leqslant \bar{\bar{\delta}}_{si}$, $|\delta_{ai}| \leqslant \bar{\delta}_{ai}$, and $\left|\dot{\delta}_{ai}\right| \leqslant \bar{\bar{\delta}}_{ai}$.

Assumption 5. y_d is piecewise continuous, and y_d, $\dot{y}_d$, and $\ddot{y}_d$ are bounded.

Remark 1. Assumption 2 is a normal condition to achieve the consensus tracking for MASs in many studies [19,21]. Assumption 4 is the basic assumption for the output feedback control of nonlinear systems in many studies [37,40]. Assumption 3 is reasonable to guarantee the FTC, which has been investigated in [32,41]. Assumption 5 is relatively common in literature [37] .

Lemma 2. (see [39]) If the graph $\bar{\mathcal{G}}$ contains a directed spanning tree, the matrix $\mathcal{L} + \mathcal{B}$ is invertible, and all its eigenvalues have positive real part.

3 Main results

3.1 Differentiator-based distributed leader's predictor design

In this section, to isolate the faults. The differentiator-based distributed leader's predictor is designed as

$$\begin{aligned} &\dot{\psi}_{i,1} = \psi_{i,2}, \\ &\dot{\psi}_{i,2} = -\iota^2 \mathrm{sgn}(\psi_{i,1} - y_{\mathcal{N}_i})|\psi_{i,1} - y_{N_i}|^{\beta} - \iota\psi_{i,2}, \\ &\dot{\bar{y}}_i = \frac{1}{\rho_i}\left(\psi_{i,2} - k_0 \sum_{j=1}^{N} a_{ij}(\bar{y}_i - \bar{y}_j) - k_0 b_i(\bar{y}_i - y_d)\right), \end{aligned} \tag{14}$$

where $\psi_{i,1}$ and $\psi_{i,2}$ are states of the tracking differentiator (TD), $\bar{y}_i$ is the predictor's state, $y_{\mathcal{N}_i} = \sum_{j=1}^{N} a_{ij}\bar{y}_j + b_i y_d$ is a known signal, $k_0 > 0$ denotes the coupling gain, and the scalar $\rho_i = \sum_{j=1}^{N} a_{ij} + b_i$. When the parameters ι and β satisfy the inequalities $\iota > 0$ and $0 < \beta < 1$, $\psi_{i,1}$ and $\psi_{i,2}$ can track $y_{\mathcal{N}_i}$ and $\dot{y}_{\mathcal{N}_i}$, respectively. The parameters selection of TD is given in [44].

Lemma 3. Consider the distributed leader's predictor given in (14). Then, $\bar{y}_i$ can track y_d with a small tracking error.

Proof. The consensus error is defined as

$$e_i = \sum_{j=1}^{N} a_{ij}(\bar{y}_i - \bar{y}_j) + b_i(\bar{y}_i - y_d), i = 1, 2, ..., N. \tag{15}$$

From (15), the global error vector is expressed as

$$e = (\mathcal{L} + \mathcal{B})(\bar{y} - 1_N y_d), \tag{16}$$

where $e^T = [e_1, e_2, \cdots, e_N]$, $\bar{y}^T = [\bar{y}_1, \bar{y}_2, \cdots, \bar{y}_N]$ and 1_N is the N-vector of ones.

Differentiating (15) results in

$$\dot{e}_i = \psi_{i,2} - k_0 e_i - \dot{y}_{N_i}. \tag{17}$$

From (17), we obtain

$$e_i = e^{-k_0 t} e_i(0) + \int_0^t e^{-k_0(t-\tau)}(\psi_{i,2} - \dot{y}_{N_i})d\tau. \tag{18}$$

where $e_i(0)$ is the initial conditions of the neighborhood synchronization error.

According to TD, there exists a constant $\varsigma_i > 0$ such that the inequality $\lim_{t\to\infty} |\psi_{i,2} - \dot{y}_{N_i}| \leqslant \varsigma_i$ holds. Equation (18) implies $\lim_{t\to\infty} |e_i| \leqslant \varsigma_i/k_0$. Using Lemma 1 and (16) yields $\|\bar{y} - 1_N y_d\| \leqslant \|e\| / \sigma_{\min}(\mathcal{L} + \mathcal{B})$, where $\sigma_{\min}(\mathcal{L} + \mathcal{B})$ is the minimum singular of $(\mathcal{L} + \mathcal{B})$. Then, $\bar{y}_i$ can track y_d with a small tracking error by selecting suitable parameters. The proof is completed.

Remark 2. It is worth noting that the distributed leader predictor does not require any global information of the communication network graph, so it is fully distributed. Meanwhile, the faults of the follower and its neighbors does not affect the whole system through the communication network.

3.2 fuzzy observer design

In this section, to deal with the unmeasured states and the actuator fault, a novel fuzzy observer is designed as:

$$\begin{aligned} &\dot{\hat{x}}_{i,m} = \hat{x}_{i,m+1} + \hat{f}_{i,m}(\hat{\bar{x}}_{i,m}\,|\theta_{i,m}) + k_m \hat{l}^m (y_i^f - \hat{\delta}_{si} - \hat{y}_i), \\ &\dot{\hat{x}}_{i,n_i} = u_i + \hat{\delta}_{ai} + \hat{f}_{i,n_i}(\hat{\bar{x}}_{i,n_i}\,|\theta_{i,n_i}) + k_{n_i}\hat{l}^{n_i}(y_i^f - \hat{\delta}_{si} - \hat{y}_i), \\ &\dot{\hat{\delta}}_{ai} = -\sigma_{ai}\upsilon_{ai}\hat{\delta}_{ai} + k_{ai}\hat{l}^{-1}(y_i^f - \hat{\delta}_{si} - \hat{y}_i), \\ &\hat{y}_i = \hat{x}_{i,1}, \end{aligned} \tag{19}$$

where $k_m > 0$, $\hat{l} > 0$, $\sigma_{ai} > 0$, $\upsilon_{ai} > 0$, $k_{ai} > 0$, $\hat{\delta}_{ai}$ is the estimation of δ_{ai}, and $\hat{\delta}_{si}$ denotes the estimation of δ_{si}.

3.3 Adaptive fault-tolerant controller design and stability analysis

In this section, an adaptive fault-tolerant controller is designed based on the above the state observer, the actuator fault observer, and the distributed leader's predictor by using the backstepping method. Due to the effects of sensor fault, the actual measured value is unavailable for controller design. To solve this problem, a new coordinate transformation is introduced as:

$$\begin{aligned} z_{i,1} &= y_i - \bar{y}_i = y_i^f - \bar{y}_i - \hat{\delta}_{si} - \tilde{\delta}_{si} = \tilde{z}_{i,1} - \tilde{\delta}_{si}, \\ z_{i,m} &= \hat{x}_{i,m} - \zeta_{i,m}, \\ \chi_{i,m} &= \zeta_{i,m} - \alpha_{i,m-1}, i = 1, 2, \cdots, N, m = 2, \cdots, n_i, \end{aligned} \tag{20}$$

where $\tilde{z}_{i,1} = y_i^f - \bar{y}_i - \hat{\delta}_{si}$. $\alpha_{i,m-1}$ are the virtual controllers. $\zeta_{i,m}$ and $\chi_{i,m}$ are the output and output errors of the following first-order filter, respectively. A first-order filter is given to avoid the problem of "explosion of complexity". Here, $\alpha_{i,m-1}$ is regarded as the filter input.

$$\tau_{i,m}\dot{\zeta}_{i,m} + \zeta_{i,m} = \alpha_{i,m-1}, \zeta_{i,m}(0) = \alpha_{i,m-1}(0), i = 1, 2, \cdots, N, m = 2, \cdots, n_i, \tag{21}$$

where $\tau_{i,m}$ is a given constant.

Theorem 1. For the nonlinear MASs (6), by designing the distributed output predictor (14), the state observer (19), the virtual controllers (**??**), (**??**), (**??**), the sensor fault estimation adaptive law (**??**), the actual controller (**??**), and the parameters adaptive laws (**??**), (**??**), (**??**), (**??**), all the signals of the MASs are bounded, and the consensus tracking error is proven to converge to a neighborhood of the origin.

Proof. Consider a Lyapunov function as:

$$V = \sum_{i=1}^{N} V_{i,n_i}. \tag{22}$$

From (**??**), we can obtain

$$\dot{V} \leqslant -CV + D, \tag{23}$$

where $C = \min\{C_i,\ i = 1, 2, \cdots, N\}$ and $D = \sum_{i=1}^{N} D_i$. Integrating the two sides of (23) yields

$$0 \leqslant \|z_{i,1}\|^2 \leqslant V(t) \leqslant e^{-Ct}V(0) + \tfrac{D}{C}(1 - e^{-Ct}). \tag{24}$$

Apparently, inequality (24) shows that the output $y_{i,1}$ can be controlled to track the virtual signal $\bar{y}_i$ of the leader's predictor with a small tracking error by selecting suitable parameters. The output of all followers in graph $\mathcal{G}$ tracks to the leader trajectory with bounded residual errors, and all the variables of the closed-loop MAS are bounded. The proof is completed.

Remark 3. Inequality (24) indicates that some parameters affect the performance of the MASs, and it is easy to see that increasing C and decreasing D can improve the convergence of the MASs and steady-state performance. However, adjusting parameters μ_i, υ_{si}, $\kappa_{i,m}$, and $\eta_{i,m}$ can affect C_i and D_i at the same time. Increasing μ_i, $\kappa_{i,m}$ and decreasing υ_{si}, $\eta_{i,m}$ may increase C_i and D_i. Then, optimizing the response speed and steady-state performance is impossible. In practical applications, appropriate parameters are designed in accordance with the performance requirements of MASs.

4 Simulation results

In this section, a MASs with one leader labeled as 0 and four followers is considered, as shown Fig.1. Two simulation examples are provided to verify the feasibility and superiority of the aforementioned control strategy. On the basis of algebraic graph theory, the adjacency matrix $\mathcal{A}$, the Laplacian matrix $\mathcal{L}$, and the matrix $\mathcal{B}$ of the subgraph $\mathcal{G}$ can be given as

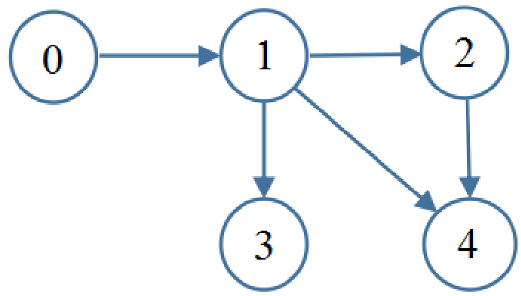

Figure 1 Directed topological topology

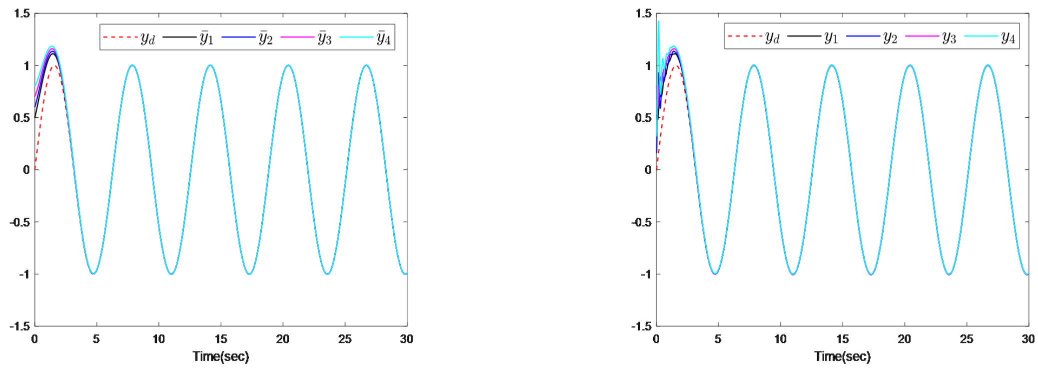

Figure 2 The output consensus tracking of all followers. (a) The trajectories of leader and predictors; (b) The Output trajectories of leader and followers.

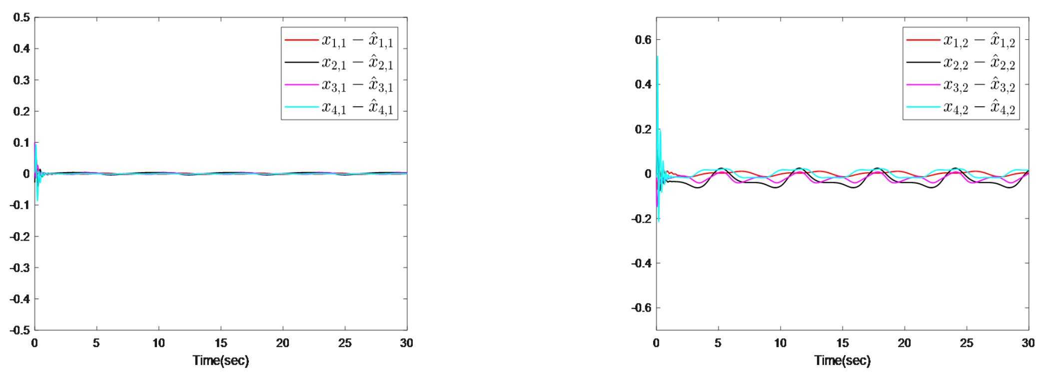

Figure 3 The observer error evolution of all followers. (a) The observer error $x_{i,1}-\hat{x}_{i,1}$ of all followers; (b)The observer error $x_{i,2}-\hat{x}_{i,2}$ of all followers.

$$\mathcal{A}=\begin{bmatrix}0&0&0&0\\1&0&0&0\\1&0&0&0\\1&1&0&0\end{bmatrix},\ \mathcal{L}=\begin{bmatrix}0&0&0&0\\-1&1&0&0\\-1&0&1&0\\-1&-1&0&2\end{bmatrix},\ \mathcal{B}=\begin{bmatrix}1&0&0&0\end{bmatrix}^T.$$

Example 1: Consider the dynamics of the heterogeneous followers as

$$\begin{aligned}\dot{x}_{i,1}&=x_{i,2}+f_{i,1}(x_{i,1})+d_{i,1}\\ \dot{x}_{i,2}&=\beta_{ai}(t)u_i+f_{ai}(t)+f_{i,2}(\bar{x}_{i,2})+d_{i,2}\\ y_i^f&=\beta_{si}(t)x_{i,1}+f_{si}(t), i=1,2,\cdots,4,\end{aligned}\tag{25}$$

where the nonlinear functions are given as $f_{1,1}(x_{1,1})=0$, $f_{1,2}(\bar{x}_{1,2})=2\sin(x_{1,1}x_{1,2})$, $f_{2,1}(x_{2,1})=$

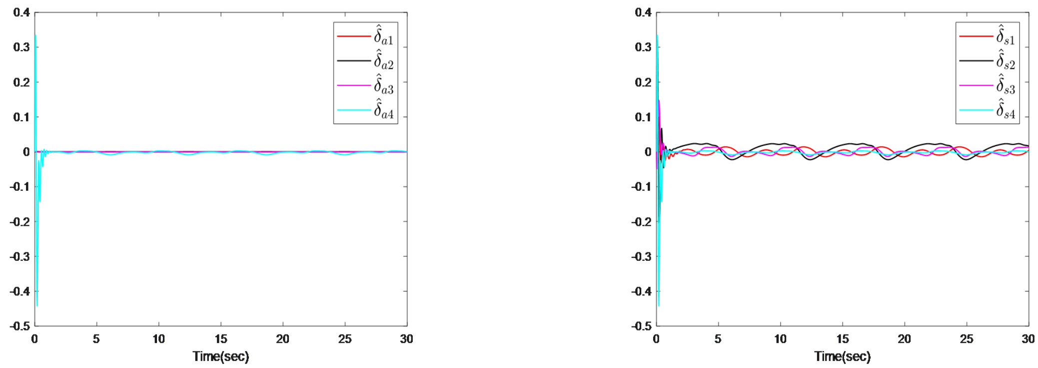

Figure 4 The estimation of actuator and sensor faults. (a) Trajectories $\hat{\delta}_{ai}$ of actuator fault; (b) Trajectories $\hat{\delta}_{si}$ of sensor fault.

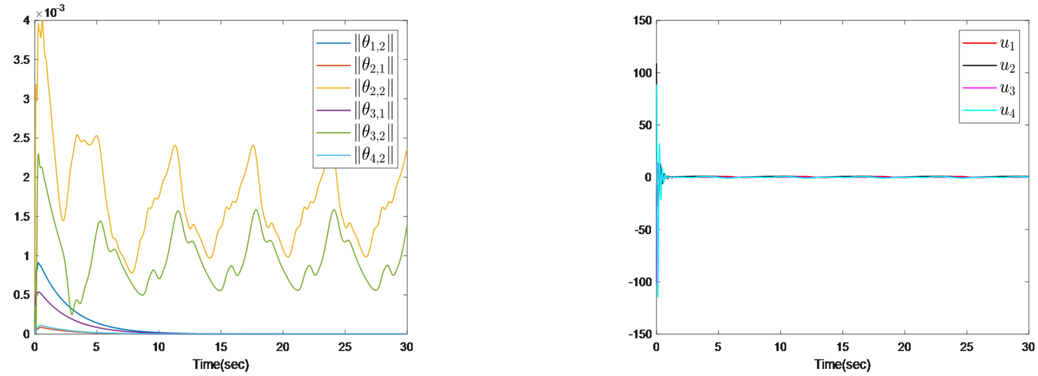

Figure 5 The trajectories of parameters adaptive law and controllers. (a) The trajectories of parameters adaptive law $\|\theta_{i,j}\|$; (b)The trajectories of control input u_i.

$x_{2,1}\cos(x_{2,1})$, $f_{2,2}(\bar{x}_{2,2}) = 2\sin(x_{2,1}x_{2,2}) + x_{2,2}$, $f_{3,1}(x_{3,1}) = \cos(x_{3,1})e^{-x_{3,1}^2}$, $f_{3,2}(\bar{x}_{3,2}) = \sin(x_{3,1}x_{3,2})e^{-x_{3,1}^2}$, and $f_{4,1}(x_{4,1}) = \sin(x_{4,1})e^{-x_{4,1}^2}$, $f_{4,2}(\bar{x}_{4,2}) = x_{4,1}\sin(x_{4,1}x_{4,2}) + x_{4,2}^2$. The external disturbances are chosen as $d_{i,1} = 0.1\sin(t)$ and $d_{i,2} = 0.2\cos(t)$. The trajectory of leader is expressed as $y_d = \sin(t)$. The faulty parameters are chosen as $\beta_{a1} = \beta_{a2} = \beta_{a3} = \beta_{s1} = \beta_{s4} = 0.8$, $\beta_{a4} = \beta_{s2} = \beta_{s3} = 0.2\sin(t) + 0.8$, $f_{a1} = f_{a2} = f_{s1} = 0.2$, $f_{a4} = f_{s2} = 0$, and $f_{a3} = f_{s3} = f_{s4} = 0.2\sin(t)$. Choose the fuzzy basis functions as $\mu_{F_i^l}(\hat{x}_{i,m}) = \exp(-((\hat{x}_{i,m} + \frac{\pi}{3} - (l-1)\frac{\pi}{6}/\frac{\pi}{12})^2)$, $l = 1, 2, \cdots, 5$.

The design parameters are given as $l = 5$, $k_1 = 8$, $k_2 = 20$, $k_{ai} = 0.5$, $\upsilon_{ai} = 2$, $\sigma_{ai} = 5$, $g_{i,1} = 10$, $g_{i,2} = 9$, $\eta_{i,1} = \eta_{i,2} = 0.1$, $\mu_{si} = 25$, $\kappa_{i,1} = \kappa_{i,2} = 0.4$, and $\tau_{i,2} = 0.01 (i = 1, \cdots, 4)$. The initial conditions of the followers are taken by $x_1(0) = [0.4,\ 0.5]^T$, $x_2(0) = [0.2,\ 0.3]^T$, $x_3(0) = [1,\ 0.5]^T$, and $x_4(0) = [0.3,\ 0.4]^T$). The initial conditions of the predictors and the fuzzy observers are given as $\bar{y}(0) = [0.5,\ 0.6,\ 0.7,\ 0.8]^T$ and $\hat{x}_i(0) = [0.3,\ 0.5]^T (i = 1, \cdots, 4)$, respectively. The initial conditions of the parameter vectors and the faults are chosen as $\theta_{i,m}(0) = 0$ and $\hat{\delta}_{ai}(0) = \hat{\delta}_{si}(0) = 0 (i = 1, \cdots, 4)$.

The simulation results are displayed in Figs.2-5. Figure 2(a) shows that the distributed normal predictor output $\bar{y}_i$ can track the trajectory y_d of the leader. Figure 2(b) shows that the output of followers can track the leader trajectory. The two diagrams in Fig.3 exhibit that the observer error converges into a small region of zero. The estimations of the actuator and sensor faults are shown to compensate the unknown faults in Fig.4. Figure 5 shows that the control input $u_i (i = 1, \cdots, 4)$ of the follower and the adaptive parameters $\|\theta_{i,j}\| \,(i = 1, \cdots, 4,\ j = 1, 2)$ are bounded.

Example 2: In practical engineering applications, Lagrangian dynamic models have been widely used in industrial system modeling, such as vehicle motion systems, inverted pendulum systems, and robot manipulators [43]. Then, the second-order Lagrangian dynamics in [39] is used as an example. We use Figure 1 to describe the communication network of example 2. The trajectory of leader is given as $y_d = \sin(0.5t) + \sin(1.5t)$. Four single-link robotic arms act as followers. Each single-link robotic arm consists of rigid connecting rods coupled to the DC motor through a gear train, as shown in Figure 6. Different from literature [39] , the follower agents suffer from external disturbances, the time-varying

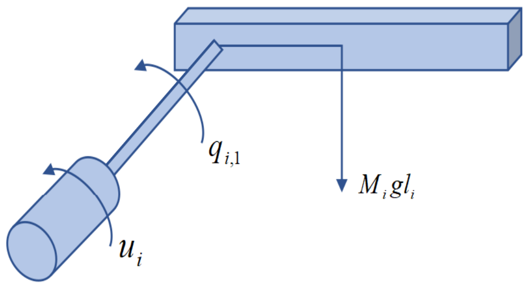

Figure 6 Single-link robot arm.

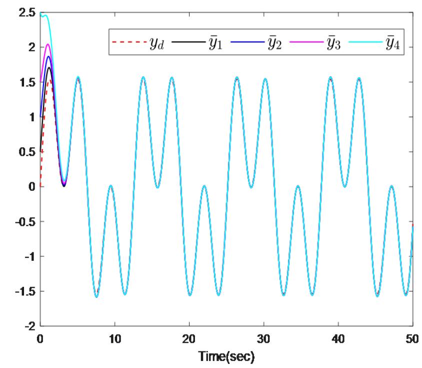

Figure 7 The output trajectories of leader and predictors.

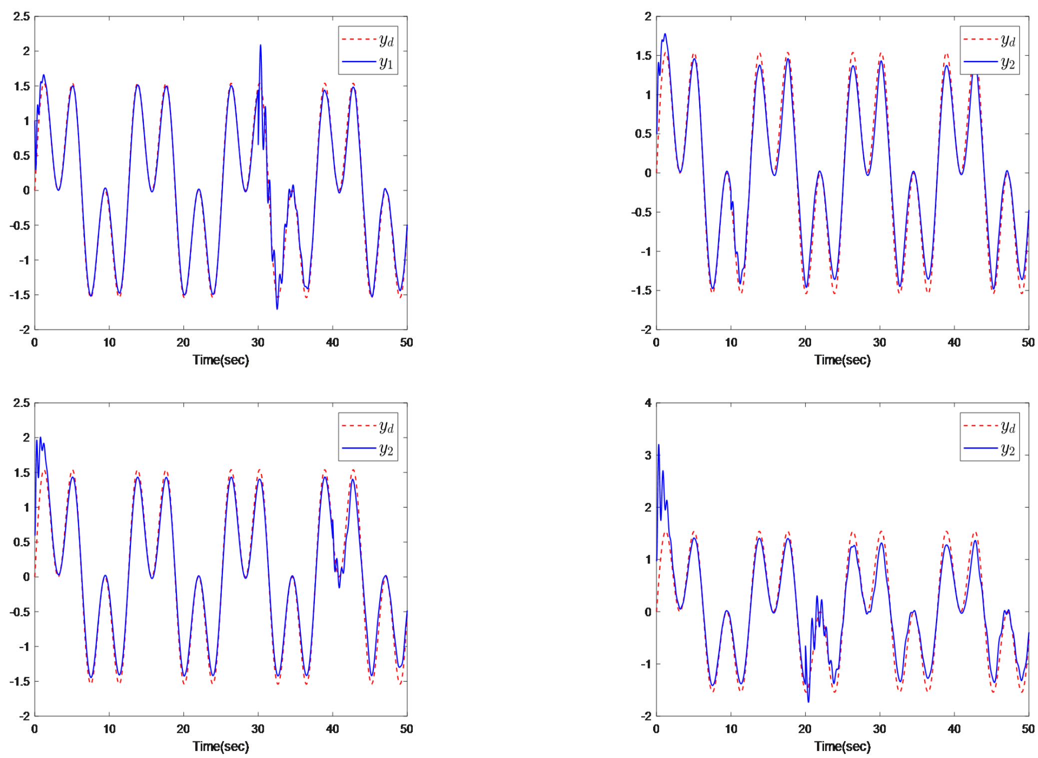

Figure 8 The output consensus tracking of all followers in case of faults at different times. (a) Trajectories of y_d and y_1 in occur of faults at time $t = 30$; (b)Trajectories of y_d and y_2 in occur of faults at time $t = 10$; (c) Trajectories of y_d and y_3 in occur of faults at time $t = 40$; (d) Trajectories of y_d and y_4 in occur of faults at time $t = 20$.

actuator and sensor faults. The dynamics of the followers are expressed as

$$\begin{aligned}
\dot{q}_{i,1} &= q_{i,2} + d_{i,1} \\
\dot{q}_{i,2} &= J_i^{-1}(\beta_{ai}(t)u_i + f_{ai}(t) + B_i q_{i,2} - M_i g l_i \sin(q_{i,1}) + d_{i,2}) \\
y_i^f &= \beta_{si}(t) q_{i,1} + f_{si}(t), i = 1, 2, \cdots, 4,
\end{aligned} \tag{26}$$

where $q_{i,1}$ and $q_{i,2}$ denote the angle and angular velocity of the link, respectively. J_i is the inertia of the link and motor, B_i is the overall damping coefficient, M_i is the link total mass, g is the gravity coefficient, and l_i is the location of the link center of mass. The disturbances are expressed as $d_{i,1} = 0.1\sin(t)$ and $d_{i,2} = 0.2\cos(t)$. The time-varying actuator and sensor faults are expressed as $\beta_{ai}(t) = 0.5$, $f_{ai}(t) = 0.5\cos(t)$, $\beta_{si}(t) = 0.8$ and $f_{si}(t) = 0.5\sin(t), i = 1, 2, \cdots, 4$.

For the simulation, the parameters of the followers are given as $[J_1, J_2, J_3, J_4]^T = [6.9667, 7.7, 8.46, 10.2]^T$, $B_i = 30.5, i = 1, 2, \cdots, 4$, $g = 9.8$ and $[l_1, l_2, l_3, l_4]^T = [0.6, 0.8, 1, 1.2]^T$. In accordance with Theorem 1, the design parameters are taken as $l = 30$, $k_1 = 5$, $k_2 = 10$, $k_{ai} = 2$, $\upsilon_{ai} = 2$, $\sigma_{ai} = 10$, $g_{i,1} = 16$, $g_{i,2} = 10$, $\eta_{i,1} = \eta_{i,2} = 0.1$, $\mu_{si} = 25$, $\kappa_{i,1} = \kappa_{i,2} = 0.4$, and $\tau_{i,2} = 0.01(i = 1, \cdots, 4)$. The initial conditions of the followers are expressed as $q_1(0) = [1,\ 0.5]^T$, $q_2(0) = [0.5,\ 0.2]^T$, $q_3(0) = [0.6,\ 0]^T$, and $q_4(0) = [1,\ 0]^T$. The initial conditions of the predictor are chosen as $\bar{y}(0) = [0.5,\ 0.6,\ 0.7,\ 0.8]^T$. The initial conditions of the fuzzy observer are expressed as $\hat{q}_i(0) = [0.3,\ 0.5]^T(i = 1, \cdots, 4)$. Choose The initial conditions of parameter vectors as $\theta_{i,m}(0) = 0$. The initial conditions of the actuator and sensor faults adaptive laws are expressed as $\hat{\delta}_{ai}(0) = \hat{\delta}_{si}(0) = 0(i = 1, \cdots, 4)$. Figure 7 shows that all the output predictors are synchronized to the leader's trajectory. Figure 8 shows that the output of all followers tracks the leader's trajectory in case of faults at different times.

5 Conclusion

In this study, a fuzzy adaptive fault-tolerant consensus tracking control problem has been investigated for nonlinear MASs with disturbances, the time-varying actuator and sensor faults. A differentiator-based distributed leader's predictor has been proposed to isolate faults. The unmeasured states of the system, the unknown actuator fault and the unknown nonlinear functions have been estimated by state observer and fault observer, respectively. An adaptive controller and the compensation algorithm of sensor fault have been presented by the backstepping technique and Lyapunov stability theory. In the future work, we will investigate the cooperative control problem of nonlinear MASs with communication link fault and physical fault.

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