6.逻辑回归

在训练的初始阶段，我们将要构建一个逻辑回归模型来预测，某个学生是否被大学录取。 设想你是大学相关部分的管理者，想通过申请学生两次测试的评分，来决定他们是否被录取。 现在你拥有之前申请学生的可以用于训练逻辑回归的训练样本集。对于每一个训练样本，你有他们两次测试的评分和最后是被录取的结果。

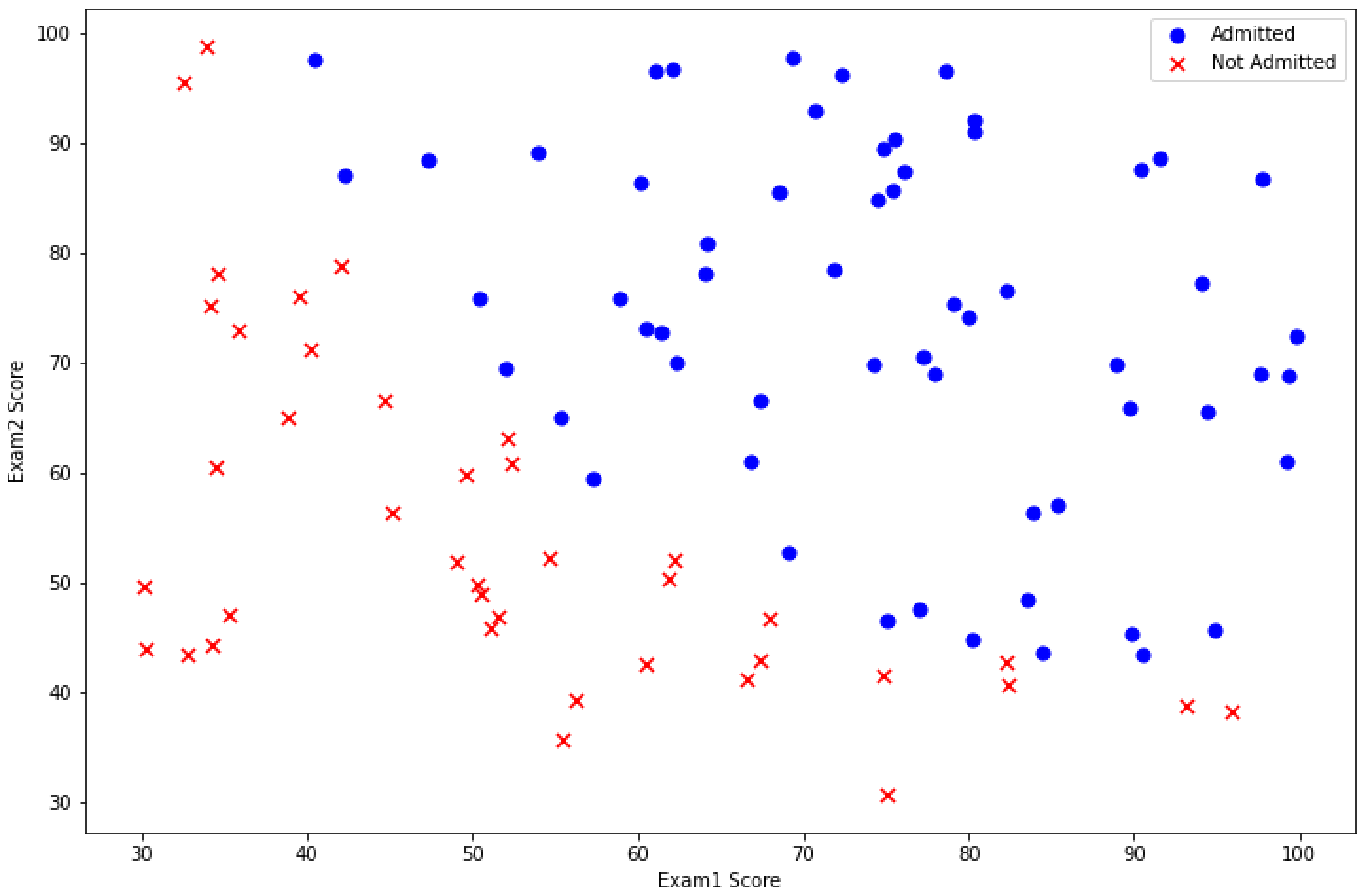
In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

In [2]:
```
data=pd.read_csv('6.1.txt',header=None,names=['Exam1','Exam2','Admitted'])
data.head()
```

Out[2]:

	Exam1	Exam2	Admitted
0	34.623660	78.024693	0
1	30.286711	43.894998	0
2	35.847409	72.902198	0
3	60.182599	86.308552	1
4	79.032736	75.344376	1

In [3]:
```
positive = data[data['Admitted'].isin([1])]
negative = data[data['Admitted'].isin([0])]

fig, ax = plt.subplots(figsize=(12,8))
ax.scatter(positive['Exam1'], positive['Exam2'], s=50, c='b', marker='o', label='Admitted')
ax.scatter(negative['Exam1'], negative['Exam2'], s=50, c='r', marker='x', label='Not Admitted')
ax.legend()
ax.set_xlabel('Exam1 Score')
ax.set_ylabel('Exam2 Score')
plt.show()
```



In [4]:
```
#sigmoid 函数
def sigmoid(z):
    return 1/(1 + np.exp(-z))
```

In [5]:
```
#实现代价函数
def cost(theta,X,y):
    theta=np.matrix(theta)
    X=np.matrix(X)
    y=np.matrix(y)
    first=np.multiply(-y,np.log(sigmoid(X*theta.T)))
    second=np.multiply((1-y),np.log(1-sigmoid(X*theta.T)))
    return np.sum(first-second)/(len(X))
```

In [6]:
```
#初始化X，y，θ

# 加一列常数列
data.insert(0, 'Ones', 1)

# 初始化X，y，θ
cols = data.shape[1]
X = data.iloc[:,0:cols-1]
y = data.iloc[:,cols-1:cols]
theta = np.zeros(3)

# 转换X，y的类型
X = np.array(X.values) #array:产生数组
y = np.array(y.values) #values:返回值
```

In [7]:
```
# 检查矩阵的维度
X.shape, theta.shape, y.shape
```

Out[7]: ((100, 3), (3,), (100, 1))

In [8]:
```
# 用初始θ计算代价
cost(theta, X, y)
```

Out[8]: 0.6931471805599453

In [9]:
```
# 实现梯度计算的函数（并没有更新θ）
def gradient(theta, X, y):
    theta = np.matrix(theta) #数组化
    X = np.matrix(X)
    y = np.matrix(y)

    parameters = int(theta.ravel().shape[1]) #ravel：将多维数组转换为一维数组，参数theta的数量
    grad = np.zeros(parameters)

    error = sigmoid(X * theta.T) - y

    for i in range(parameters):
        term = np.multiply(error, X[:,i])
        grad[i] = np.sum(term) / len(X)
```

```
    return grad
```

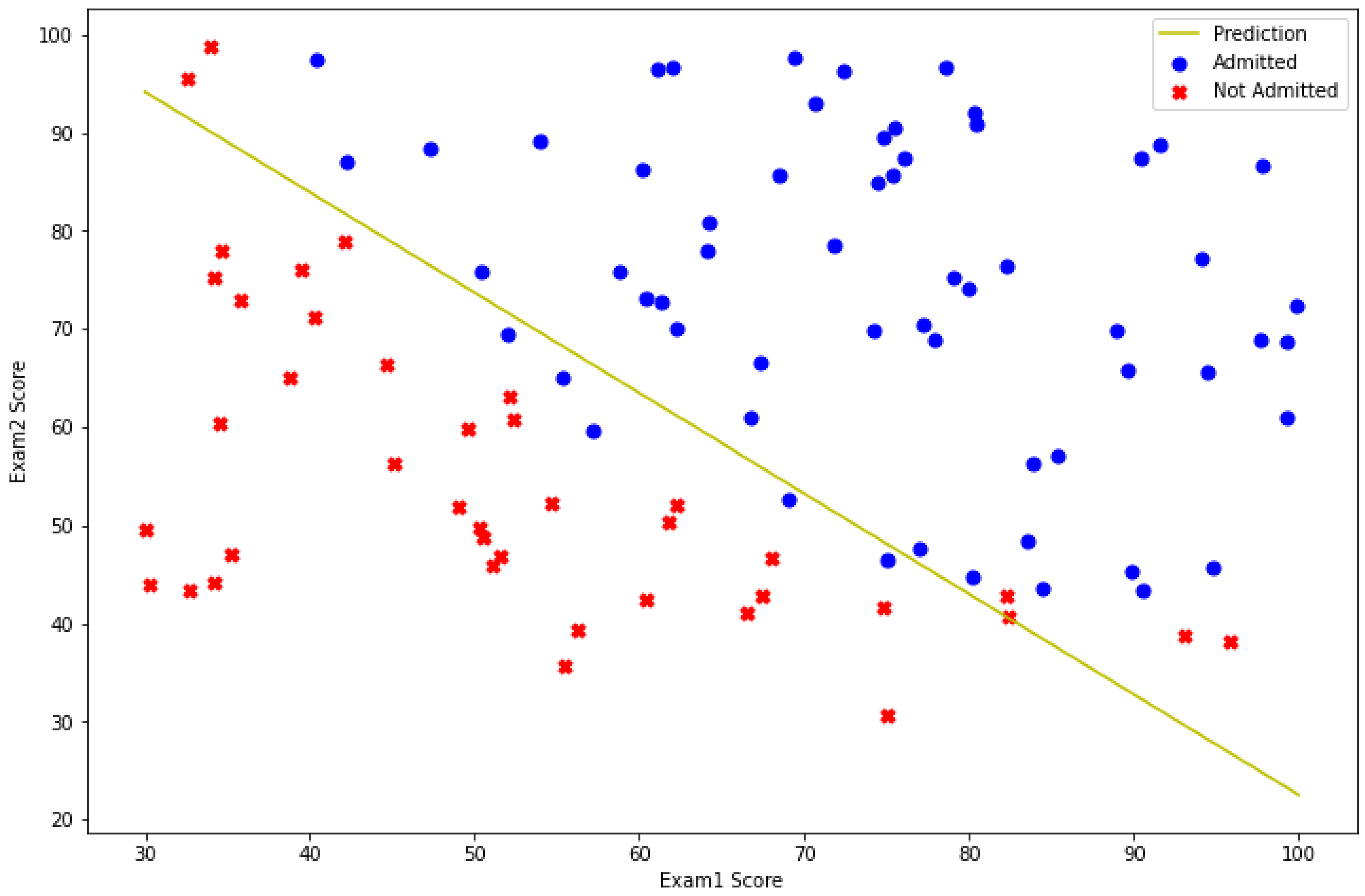
In [10]:
```
import scipy.optimize as opt
result=opt.fmin_tnc(func=cost,x0=theta,fprime=gradient,args=(X,y))
#调用一个已有的库，不用自己定义迭代次数和步长，功能会直接告诉我们最优解
#参数：func:优化的目标函数，x0:初值，fprime:提供优化函数func的梯度函数，否则优化函数func必须返回函数值和梯度，或者设置approx_grad=True
#approx_grad :如果设置为True，会给出近似梯度，args：元组，是传递给优化函数的参数
#返回：x：θ,nfeval：整数，function evaluations的数目，在一次迭代过程中会有多次function evaluation
result
```

Out[10]: (array([-25.16131863,   0.20623159,   0.20147149]), 36, 0)

In [11]:
```
# 用θ的计算结果代回代价函数计算
cost(result[0], X, y)
```

Out[11]: 0.20349770158947458

In [12]:
```
#画出决策曲线
plotting_x1=np.linspace(30,100,100) #等差数列，表示在区间[30, 100]之间取100个点作为横坐标   X
plotting_h1=(-result[0][0]-result[0][1]*plotting_x1)/result[0][2] #θt*X=0为决策边界(x0=1)  y
#result[0]是theta3个值的一个数组，result[0][0]表示从theta的三个值中取第一个值
fig,ax=plt.subplots(figsize=(12,8)) #创建绘图空间
ax.plot(plotting_x1,plotting_h1,'y',label='Prediction') #画线
ax.scatter(positive['Exam1'], positive['Exam2'], s=50, c='b', marker='o', label='Admitted')
ax.scatter(negative['Exam1'], negative['Exam2'], s=50, c='r', marker='X', label='Not Admitted')
ax.legend()
ax.set_xlabel('Exam1 Score')
ax.set_ylabel('Exam2 Score')
plt.show()
```



In [13]:
```
# 实现hθ
def hfunc1(theta, X):
    return sigmoid(np.dot(theta.T, X))
hfunc1(result[0],[1,45,85])
```

Out[13]: 0.7762906240463825

In [14]:
```
# 定义预测函数
def predict(theta, X):
    probability = sigmoid(X * theta.T) #dot是乘法，*是点乘
    return [1 if x >= 0.5 else 0 for x in probability]
```

In [15]:
```
# 统计预测正确率
theta_min = np.matrix(result[0]) #取θ数组
predictions = predict(theta_min, X)
correct = [1 if ((a == 1 and b == 1) or (a == 0 and b == 0)) else 0 for (a, b) in zip(predictions, y)] #zip：压缩，保留一一对应的关系
accuracy = (sum(map(int, correct)) % len(correct)) #map()会根据提供的函数对指定的顺序做映射。
print ('accuracy = {0}%'.format(accuracy))
```

accuracy = 89%

7.正则化逻辑回归

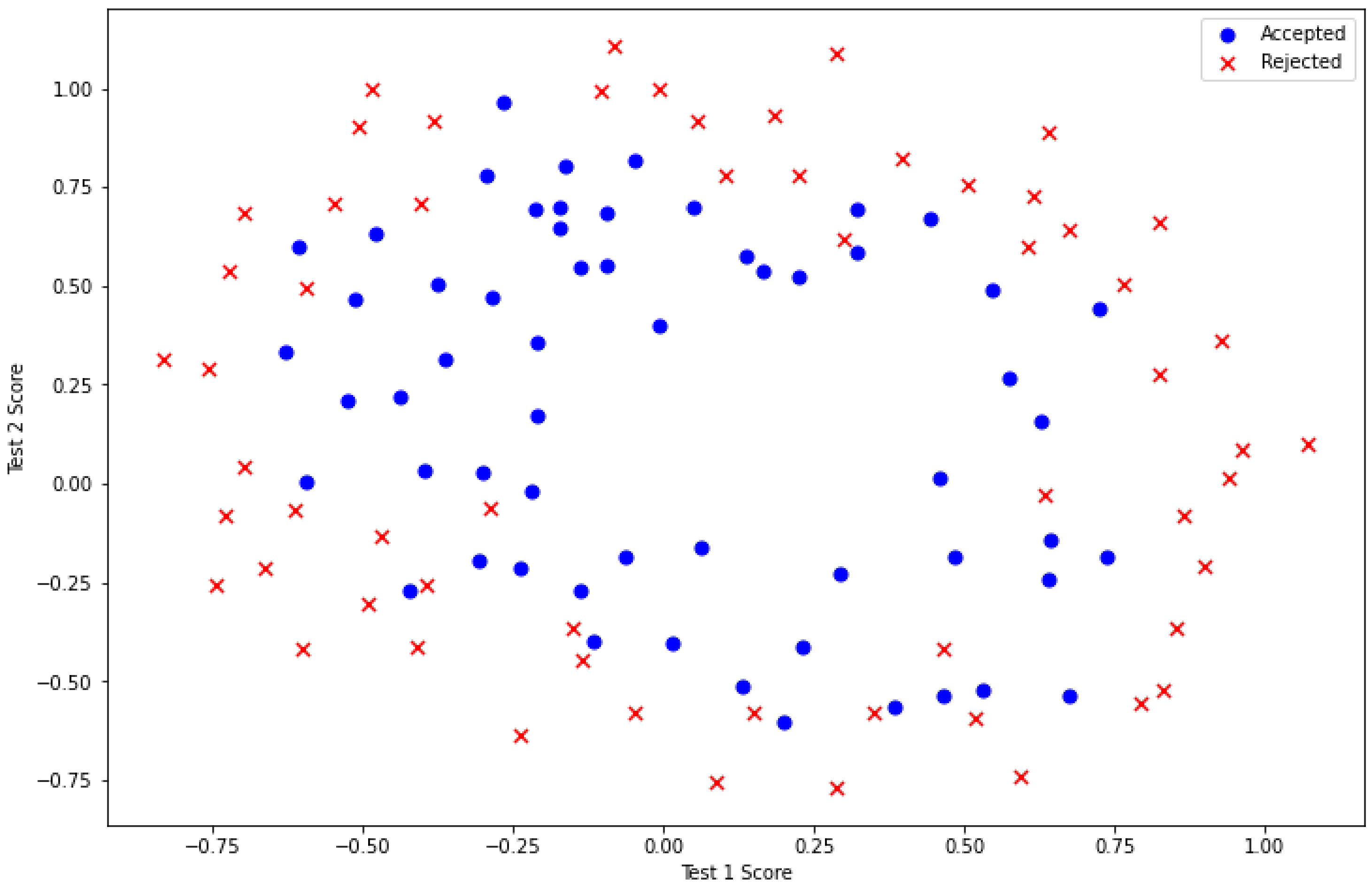
在训练的第二部分，我们将实现加入正则项提升逻辑回归算法。 设想你是工厂的生产主管，你有一些芯片在两次测试中的测试结果，测试结果决定是否芯片要被接受或抛弃。 你有一些历史数据，帮助你构建一个逻辑回归模型。

In [16]:
```
data_init = pd.read_csv('6.2.txt', header=None, names=['Test 1', 'Test 2', 'Accepted'])
data_init.head()
```

Out[16]:

	Test 1	Test 2	Accepted
0	0.051267	0.69956	1
1	-0.092742	0.68494	1
2	-0.213710	0.69225	1
3	-0.375000	0.50219	1
4	-0.513250	0.46564	1

In [17]:
```
positive2 = data_init[data_init['Accepted'].isin([1])]
negative2 = data_init[data_init['Accepted'].isin([0])]

fig, ax = plt.subplots(figsize=(12,8))
ax.scatter(positive2['Test 1'], positive2['Test 2'], s=50, c='b', marker='o', label='Accepted')
ax.scatter(negative2['Test 1'], negative2['Test 2'], s=50, c='r', marker='x', label='Rejected')
ax.legend()
ax.set_xlabel('Test 1 Score')
ax.set_ylabel('Test 2 Score')
plt.show()
```



特征映射

一种更好的使用数据集的方式是为每组数据创造更多的特征。所以我们为每组 x_1,x_2 添加了最高到6次幂的特征

In [18]:

```
degree = 6
data2 = data_init
x1 = data2['Test 1']
x2 = data2['Test 2']

data2.insert(3, 'Ones', 1)

for i in range(1, degree+1):
    for j in range(0, i+1):
        data2['F' + str(i-j) + str(j)] = np.power(x1, i-j) * np.power(x2, j) #power: 求幂

data2.drop('Test 1', axis=1, inplace=True) #drop:删除指定行列
data2.drop('Test 2', axis=1, inplace=True)

data2.head()
```

Out[18]:

	Accepted	Ones	F10	F01	F20	F11	F02	F30	F21	F12	...	F23	F14	F05	F60	F51	F42	F33	F24	F15	F06
0	1	1	0.051267	0.69956	0.002628	0.035864	0.489384	0.000135	0.001839	0.025089	...	0.000900	0.012278	0.167542	1.815630e-08	2.477505e-07	0.000003	0.000046	0.000629	0.008589	0.117206
1	1	1	-0.092742	0.68494	0.008601	-0.063523	0.469143	-0.000798	0.005891	-0.043509	...	0.002764	-0.020412	0.150752	6.362953e-07	-4.699318e-06	0.000035	-0.000256	0.001893	-0.013981	0.103256
2	1	1	-0.213710	0.69225	0.045672	-0.147941	0.479210	-0.009761	0.031616	-0.102412	...	0.015151	-0.049077	0.158970	9.526844e-05	-3.085938e-04	0.001000	-0.003238	0.010488	-0.033973	0.110047
3	1	1	-0.375000	0.50219	0.140625	-0.188321	0.252195	-0.052734	0.070620	-0.094573	...	0.017810	-0.023851	0.031940	2.780914e-03	-3.724126e-03	0.004987	-0.006679	0.008944	-0.011978	0.016040
4	1	1	-0.513250	0.46564	0.263426	-0.238990	0.216821	-0.135203	0.122661	-0.111283	...	0.026596	-0.024128	0.021890	1.827990e-02	-1.658422e-02	0.015046	-0.013650	0.012384	-0.011235	0.010193

5 rows × 29 columns

代价函数和梯度

In [19]:

```
# 实现正则化的代价函数
def costReg(theta, X, y, learningRate): #learningRate: 正则化参数
    theta = np.matrix(theta)
    X = np.matrix(X)
    y = np.matrix(y)
    first = np.multiply(-y, np.log(sigmoid(X * theta.T)))
    second = np.multiply((1 - y), np.log(1 - sigmoid(X * theta.T)))
    reg = (learningRate / (2 * len(X))) * np.sum(np.power(theta[:,1:theta.shape[1]], 2)) #正则化项，不取θ0
    #数组切片: 逗号“,”分隔各个维度，“:”表示各个维度内的切片，只有:表示取这个维度的全部值
    #X[:2,1:] 第一维取下标2之前的，即第2行之前（0，1两行），列上从第一列开始取，不要第0列
    return np.sum(first - second) / len(X) + reg
```

In [20]:

```
# 实现正则化的梯度函数
def gradientReg(theta, X, y, learningRate):
    theta = np.matrix(theta)
    X = np.matrix(X)
    y = np.matrix(y)

    parameters = int(theta.ravel().shape[1]) #shape[1]:theta的列数，即参数θ的数量
    grad = np.zeros(parameters)

    error = sigmoid(X * theta.T) - y

    for i in range(parameters):
        term = np.multiply(error, X[:,i])

        if (i == 0):
            grad[i] = np.sum(term) / len(X) #θ0不正则化
        else:
            grad[i] = (np.sum(term) / len(X)) + ((learningRate / len(X)) * theta[:,i]) #正则化梯度下降

    return grad
```

In [21]:

```
# 初始化X, y, θ
cols = data2.shape[1]
X2 = data2.iloc[:,1:cols]
y2 = data2.iloc[:,0:1]
theta2 = np.zeros(cols-1)

# 进行类型转换
X2 = np.array(X2.values)
y2 = np.array(y2.values)

# λ设为1
learningRate = 1
```

In [22]:

```
# 计算初始代价
costReg(theta2, X2, y2, learningRate)
```

Out[22]:

```
0.6931471805599454
```

In [23]:

```
result2 = opt.fmin_tnc(func=costReg, x0=theta2, fprime=gradientReg, args=(X2, y2, learningRate))
result2
```

Out[23]:

```
(array([ 1.27271027,  0.62529965,  1.18111686, -2.01987399, -0.9174319 ,
        -1.43166929,  0.12393227, -0.36553118, -0.35725403, -0.17516292,
        -1.45817009, -0.05098418, -0.61558552, -0.27469165, -1.19271298,
        -0.24217841, -0.20603297, -0.04466178, -0.27778952, -0.29539513,
        -0.45645981, -1.04319155,  0.02779373, -0.29244872,  0.01555761,
```

```
        -0.32742406, -0.1438915 , -0.92467487]),
 32,
 1)
```

In [24]:

```python
#预测函数查看在训练数据上的准确度
theta_min = np.matrix(result2[0])
predictions = predict(theta_min, X2)
correct = [1 if ((a == 1 and b == 1) or (a == 0 and b == 0)) else 0 for (a, b) in zip(predictions, y2)]
accuracy = (sum(map(int, correct)) % len(correct))
print ('accuracy = {0}%'.format(accuracy))
```

```
accuracy = 98%
```

In [25]:

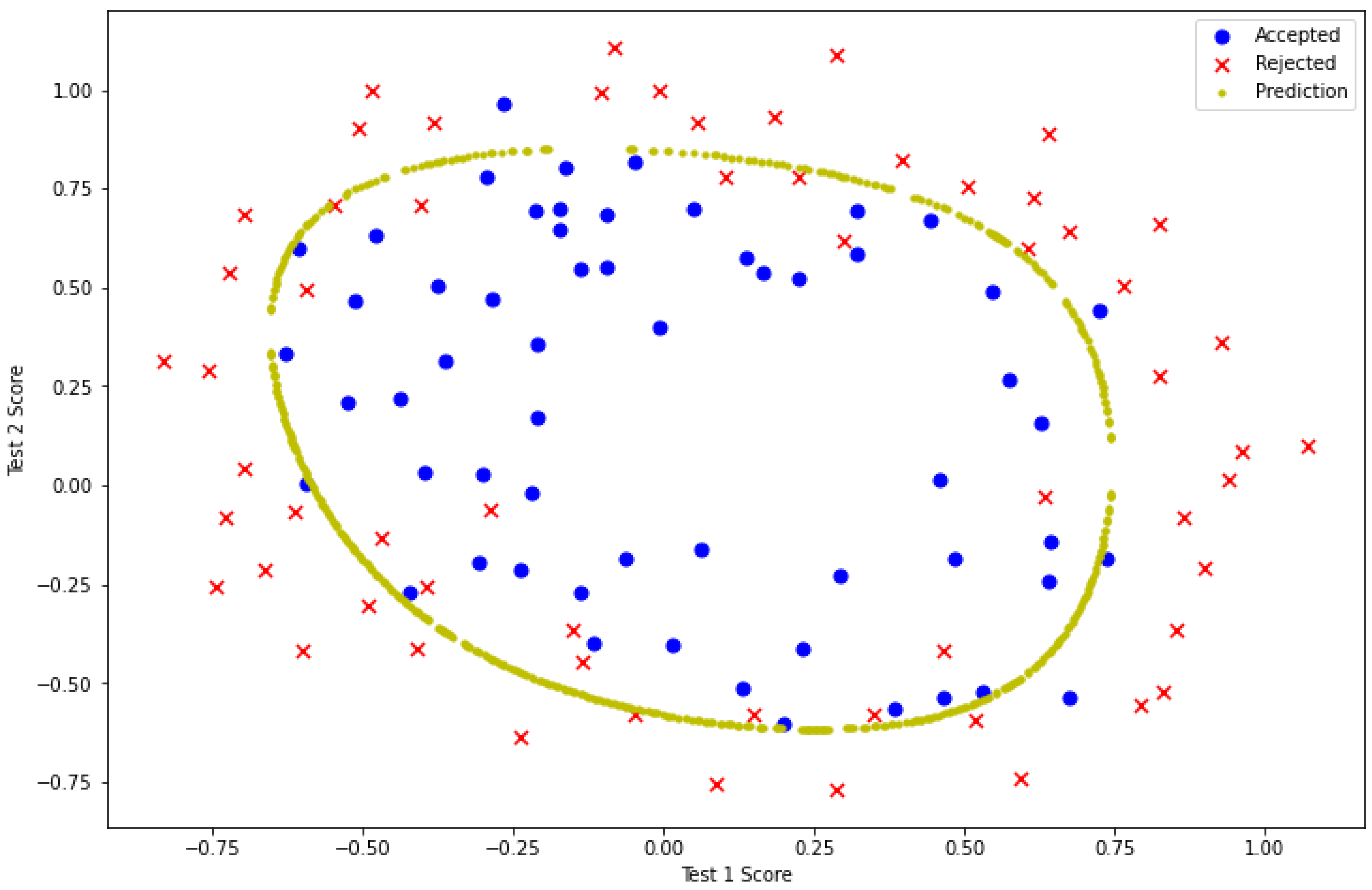
```python
#画出决策的曲线
def hfunc2(theta, x1, x2):
    temp = theta[0][0]
    place = 0
    for i in range(1, degree+1):
        for j in range(0, i+1):
            temp+= np.power(x1, i-j) * np.power(x2, j)* theta[0][place+1]  #假设函数
            place+=1
    return temp
```

In [26]:

```python
#找决策边界
def find_decision_boundary(theta):
    t1 = np.linspace(-1, 1.5, 1000) #在指定的间隔内返回均匀间隔的数字 1000为样本数
    t2 = np.linspace(-1, 1.5, 1000)

    cordinates = [(x, y) for x in t1 for y in t2]
    x_cord, y_cord = zip(*cordinates) #zip(*):将原来的行的列表转换为列的列表; zip():将两个不同序列的元素以元组形式一一配对
    h_val = pd.DataFrame({'x1':x_cord, 'x2':y_cord}) #DataFrame是由多种类型的列构成的二维标签数据结构
    h_val['hval'] = hfunc2(theta, h_val['x1'], h_val['x2']) #代入假设函数

    decision = h_val[np.abs(h_val['hval'])< 2 * 10**-3 ] #abs(): 函数返回数字的绝对值; ** : 乘方 ; 2 * 10**-1: 边界宽度（描点）
    return decision.x1, decision.x2
```

In [27]:

```python
fig, ax = plt.subplots(figsize=(12,8))
ax.scatter(positive2['Test 1'], positive2['Test 2'], s=50, c='b', marker='o', label='Accepted')
ax.scatter(negative2['Test 1'], negative2['Test 2'], s=50, c='r', marker='x', label='Rejected')
ax.set_xlabel('Test 1 Score')
ax.set_ylabel('Test 2 Score')

x, y = find_decision_boundary(result2) #把θ代入
plt.scatter(x, y, c='y', s=10, label='Prediction')
ax.legend()
plt.show()
```



In [28]:

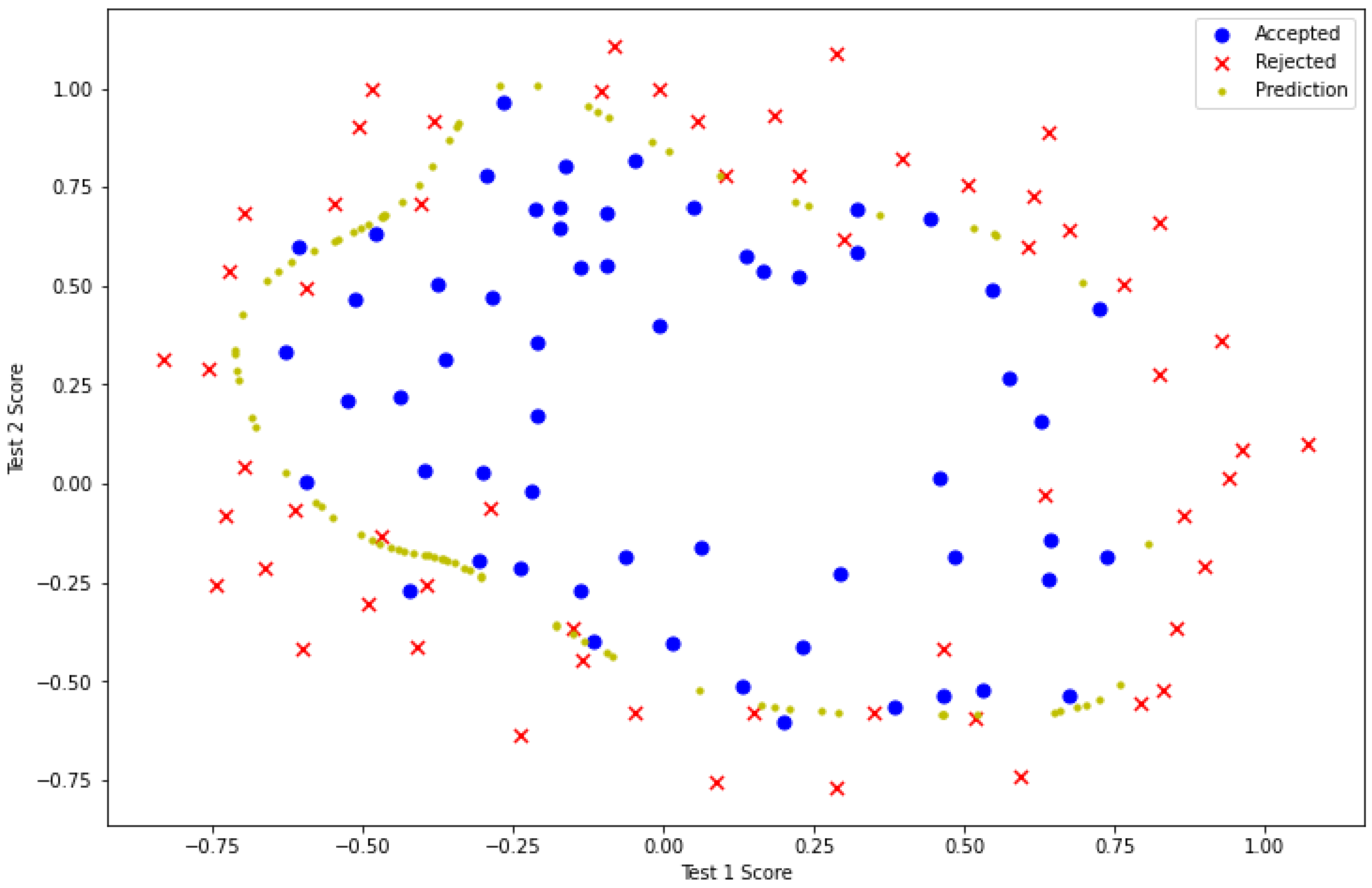
```python
#改变λ, 观察决策曲线
#过拟合
learningRate2 = 0
result3 = opt.fmin_tnc(func=costReg, x0=theta2, fprime=gradientReg, args=(X2, y2, learningRate2))
result3
```

Out[28]:

```
(array([ 9.11192364e+00,  1.18840465e+01,  6.30828094e+00, -8.39706468e+01,
        -4.48639810e+01, -3.81221435e+01, -9.42525756e+01, -8.14257602e+01,
        -4.22413355e+01, -3.52968361e+00,  2.95734207e+02,  2.51308760e+02,
         3.64155830e+02,  1.61036970e+02,  5.70100234e+01,  1.71716716e+02,
         2.72109672e+02,  3.12447535e+02,  1.41764016e+02,  3.22495698e+01,
        -1.75836912e-01, -3.58663811e+02, -4.82161916e+02, -7.49974915e+02,
        -5.03764307e+02, -4.80978435e+02, -1.85566236e+02, -3.83936243e+01]),
 280,
 3)
```

In [29]:

```python
fig, ax = plt.subplots(figsize=(12,8))
ax.scatter(positive2['Test 1'], positive2['Test 2'], s=50, c='b', marker='o', label='Accepted')
ax.scatter(negative2['Test 1'], negative2['Test 2'], s=50, c='r', marker='x', label='Rejected')
ax.set_xlabel('Test 1 Score')
ax.set_ylabel('Test 2 Score')

x, y = find_decision_boundary(result3)
plt.scatter(x, y, c='y', s=10, label='Prediction')
ax.legend()
plt.show()
```



In [30]:

```
#欠拟合
learningRate3 = 100
result4 = opt.fmin_tnc(func=costReg, x0=theta2, fprime=gradientReg, args=(X2, y2, learningRate3))
```

In [31]:

```
fig, ax = plt.subplots(figsize=(12,8))
ax.scatter(positive2['Test 1'], positive2['Test 2'], s=50, c='b', marker='o', label='Accepted')
ax.scatter(negative2['Test 1'], negative2['Test 2'], s=50, c='r', marker='x', label='Rejected')
ax.set_xlabel('Test 1 Score')
ax.set_ylabel('Test 2 Score')

x, y = find_decision_boundary(result4)
plt.scatter(x, y, c='y', s=10, label='Prediction')
ax.legend()
plt.show()
```