Earth-Mars Cycler Trajectories

ASEN5050 Research Paper
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Abstract

In the final few pages of his 1989 novel *Men from Earth*, Buzz Aldrin briefly discussed the idea of using Earth-Mars trajectories requiring small or no propulsive maneuvers, which he called cycler trajectories, for transporting astronauts from the Earth to Mars. Although NASA's current plan for future manned missions to Mars favors a Earth-Moon trajectory followed by a Moon-Mars trajectory, the lower cost option of cyclic trajectories may become more favorable if budget constraints become more restrictive in the future. For this reason, interest still remains today in investigating optimal solutions to this trajectory problem. Since Aldrin first introduced the concept of an Earth-Mars cycler orbit repeating every synodic period, a number of studies have expanded upon his original work to construct trajectories in addition to the original Aldrin cycler. This paper reviews one of the more recently discovered Earth-Mars cycler trajectories and as an extension investigates a possible modification to the original Aldrin cycler to make it a more favorable candidate for Earth-Mars missions.

Introduction

Cycler trajectories are not an entirely new concept, as others (McConaghy et al. (2002)) have cited references to them in the literature from as early as the 1960s. Though, these trajectories don't seem to have garnered much popular attention until the 1980s when Buzz Aldrin began suggesting the utility of such a trajectory for future manned missions to Mars. Since Buzz lent his high profile name to their cause, numerous other researchers have contributed to the effort of searching for and analyzing potential cycler trajectories for future manned missions to Mars. One group especially, led by T. Troy McConaghy at Purdue University, seems to be driving most of the most recent developments in the area of Earth-Mars cycler trajectories, and my work on this paper has depended heavily on their publications.

Fundamentally, a cycler trajectory is an orbit that intersects the orbits of both Earth and Mars in a periodic way. For example, the cycler Aldrin originally proposed, now commonly referred to as the Aldrin cycler (no surprise), has an orbit approximately equal to the alignment period of Earth and Mars. This alignment period, known as the Earth-Mars synodic period, is approximately $2\frac{1}{7}$ years in duration, and is purely a product of the differences in orbit velocity between Earth and Mars. The purpose, then, of a cycler trajectory is to provide an orbit that revisits the Earth at least once every one or more synodic periods. At these times, the position of Mars relative to the Earth will be equivalent, so if the trajectory initially goes from Earth to Mars on the first revolution it will continue to do so for every revolution after that. Ideally vehicles in these trajectories require no additional fuel expenditures once they've achieve their final cycler orbit, though in reality perturbations due to other gravitating bodies, solar radiation pressure, etc. will at the very least drive the need for periodic maneuver corrections. Overall, though, these trajectories offer a low cost, low maintenance option for persistent transportation between Earth and Mars, and are worthy of consideration for future planned manned missions to Mars.

Literature Search

The following three sources served as the primary references for this paper, though additional references were also consulted and are listed in the references section at the conclusion of this paper.

McConaghy et al. (2002)

This was undoubtedly the primary source for this paper. The method for constructing Earth-Mars cycler trajectories is explained in great detail, and characteristics for many of the cycler trajectories in the space of n = 1:4, r = 1:8. These results served as a good check of my Lambert solver algorithm, and provided key suggestions for expanding the analysis to include gravity assists.

McConaghy et al. (2006)

Builds upon the previous paper and narrows the focus to a single cycler, S1L1-B. The cycler is modeled in a simplified solar-system model (replicated in the following section using my Lambert solver), and then in a higher fidelity simulation using JPL ephemerides. Detailed itineraries are constructed for varying launch dates, and corrective delta V maneuvers were modeled to optimize the overall solution.

Shen et al. (2003)

Provides multi-revolution analogues to the equations originally proposed by Battin (1999) for solving the Lambert problem. Also introduces the method of reverse substitution, which reverses the order of the Battin algorithm and allows for convergence to a solution when the initial guess is within the divergent region for the other solution. These methods were implemented in my algorithm for solving the Lambert problem, and proved to be critical in finding the long period solutions for the multi-revolution cases.

Problem of Interest

Not since the 1970s has humankind ventured beyond the confines of low earth orbit, and in order for this feat to be accomplished again new trajectories will need to be designed for future missions to the Moon, Mars and beyond. Missions to other bodies in the solar system are by no means a novel venture for our scientists and engineers, though the inclusion of humans on the vehicle traversing these trajectories places strict constraints on the mission that might make previously favored trajectories unfavorable. Time of flight certainly becomes an important consideration, as do free return options and time of stay on destination planet. As always cost will also weigh heavily on mission design, and this time probably much more than it did for the original Apollo missions. Cycler trajectories provide an elegant solution to the cost/safety problems for these missions. Ideally, once launched and placed into their final orbit, these cycler vehicles will continue on their trajectories with minimal corrective maneuvers required. They can be reused so long as they maintain their trajectories and cabin environment, and constellations as small as two vehicles could guarantee an outbound and inbound opportunity once every synodic period. Compared to other trajectories, these cycler trajectories also perform well against safety requirements. By their definition as cycler trajectories they provide a free return option in the case of an aborted rendezvous at the target planet, and for the cases of ballistic trajectories few if any burns of the engine are required to maintain the orbit. And finally, since vehicles on these trajectories will likely persist for long periods of time, they would make it much easier to continue space exploration during periods when funding for missions is constrained.

With specific regard to the field of astrodynamics, the problem of solving for these trajectories is both a challenging and critical task. Accuracy in modeling and optimizing these trajectories is perhaps more important for these trajectories than those for other missions, as the design lifetime for these vehicles will likely be extremely long. Furthermore, the cycler vehicles will likely have to be quite large (as compared to the command modules used in the Apollo missions) and on-board fuel will be at a premium, so any maneuvers required to correct inaccuracies in the trajectory design could be quite costly. Given these considerations, this astrodynamics problem of designing Earth-Mars cycler trajectories seems worthy of our consideration.

(Note: this method described below is drawn heavily from McConaghy et. Al (2002, 2006).)

Our problem of interest is constructing trajectories between Earth and Mars which have a repeat period equal to an integer multiple of the Earth-Mars synodic period. When constructing these Earth-Mars "cycler" trajectories, the following assumptions are often made:

[1] The Earth-Mars synodic period is, for simplicity, $2\frac{1}{7}$ years. This implies a Mars orbit period of $1\frac{7}{8}$ years.

[2] Earths' orbit, Mars' orbit, and the cycler trajectory lie in the ecliptic plane.

[3] Earth and Mars have circular orbits.

[4] The cycler trajectory is conic and prograde (direct).

[5] Only the Earth has sufficient mass to provide gravity-assist maneuvers.

[6] Gravity-assist maneuvers occur instantaneously.

Under these assumptions, the conditions for the trajectory problem are as follows:

$$\begin{aligned}
&r(t=0) = \left[a_E,\ 0\right] \\
&r(t=nS) = \left[a_E \cos(2\pi nS),\ a_E \sin(2\pi nS)\right] \\
&\Delta t = nS
\end{aligned}$$

where $a_E = 1\ \ AU$ is the semi-major axis of Earth's orbit, $S = 2\frac{1}{7}\ \ yrs$ is the Earth-Mars synodic period, and $n > 0$ is the number of periods elapsed before the next Earth rendezvous. With these conditions, the trajectories can be found as solutions to a Lambert problem: given the initial and final positions and the time of flight, find an orbit that connects these two points. If the transfer is direct, then only one solution exists, but if multiple revolutions, r, are permitted then there are two solutions for each count of revolutions. The two solutions are distinguished by the length of the semi-major axis of their orbits, which can also be thought of as characterizing by period. That is, for a given number of permitted revolutions greater than zero, two orbits exist which connect the two points in the desired time of flight and differ in orbit period. This point is illustrated in the figure below, which plots all of the trajectory solutions for the n = 1 case and highlights those having a time of flight equal to the Earth-Mars synodic period. The Aldrin cycler is also highlighted, which is this context is the long period solution for the single revolution solution pair.

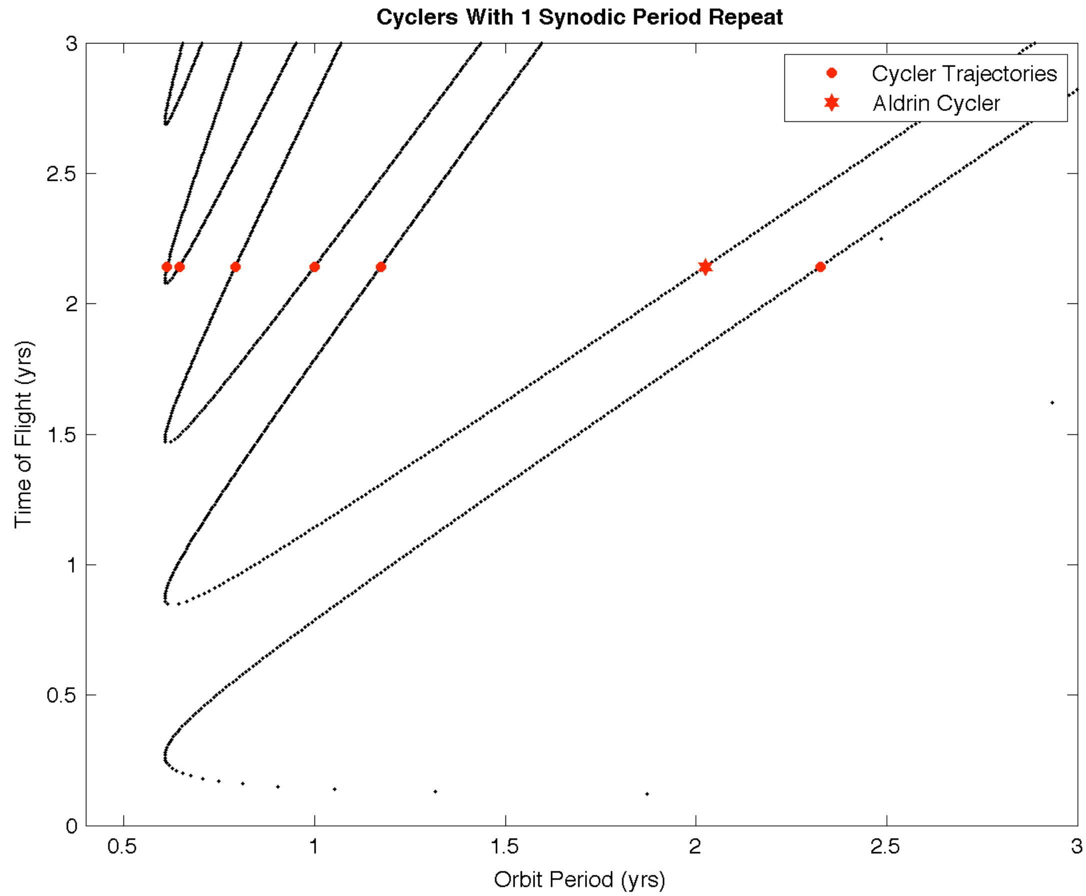

Figure 1: Trajectories for n = 1 and cycler trajectories for 1 synodic period repeat highlighted. The profile on the far right is for r = 0 revolutions, and profile to the immediate left of that is for r = 1, and so on…

While the r = 0 solution is valid for the conditions as they were structured for the Lambert problem, it is not an interesting solution in our context because it does not repeat (i.e. it fails the implicit "cycler" condition). The plot shows that there are six possible cycler trajectories for the n = 1 case, and indicates that more will become available as the repeat period is increased since higher rev solutions would qualify.

Absent perturbations to the orbit (other gravitational bodies, solar radiation pressure, drag, etc…), the cycler vehicles will continue on these trajectories without needing any additional maneuvers. However, in order to maintain the shape of the orbit relative to our objective planets (Mars & Earth), the orbit will have to be rotated to match the drifting alignment of the Earth-Mars system. This magnitude of this angle is given by the following equation:

$$\Delta\Psi = \mathrm{mod}\left(\frac{n}{7}\cdot 2\pi,\ 2\pi\right)\ rad$$

This rotation may be implemented via gravity assist without the use of fuel for a delta V maneuver, but only if the perigee radius of the required fly-by is both realistic and practical. Given a hyperbolic excess velocity, v_∞, specified by the trajectory solution and a required turn angle given by the above equation, the fly-by altitude is given by the following equations

$$e = \left(\sin\left(\frac{\delta}{2}\right)\right)^{-1}$$

$$r_p = \frac{\mu(e-1)}{v_\infty^2}$$

$$h_p = r_p - R$$

where δ is the turn angle between the hyperbolic excess velocity vectors in the gravitating body's frame, e is the eccentricity of the hyperbolic orbit, r_p is the perigee radius of the hyperbolic trajectory, R is the radius of the gravitating body, and h_p is the perigee height above the surface of the gravitating body. The acceptable lower bound for h_p can vary depending on the gravitational body, but at a minimum for any body the perigee height must be greater than zero (i.e. trajectory must be above the surface of the body). Note that we can relate the hyperbolic turn angle to the inertial turn angle using the following expression

$$\vec{v}(t_f) = \begin{bmatrix} -\cos(\Delta\Psi) & \sin(\Delta\Psi) \\ \sin(\Delta\Psi) & \cos(\Delta\Psi) \end{bmatrix} \vec{v}(t_0)$$

$$\delta = \frac{\vec{v}_\infty(t_f)\cdot\vec{v}_\infty(t_0)}{\left|\vec{v}_\infty(t_f)\right|\left|\vec{v}_\infty(t_0)\right|}$$

Trajectories for which the required turn angle is less than the maximum turn angle (defined as turn angle at minimum perigee radius) are referred to as ballistic cyclers, because they require no additional delta V to maintain the orientation of the orbit relative to the objective planets. If a delta V is required to assist the gravity fly-by in completing the rotation of the line of apsides, then the cycler is referred to as powered.

The preceding solutions assume the vehicle stays on the same trajectory for the entire repeat period, but solutions for which the vehicle traverses multiple trajectories over the period are also possible and valid. The interchange from one trajectory to another can be made via instantaneous gravity assist, impulsive delta V, or a combination of the two. The conditions for a trajectory using Earth gravity assists to move from one leg of the trajectory to another are listed below.

$$r(t=0)=[a_E,\ 0]$$
$$r(t=\tau)=[a_E\cos(2\pi\tau),\ a_E\sin(2\pi\tau)]$$
$$r(t=nS)=[a_E\cos(2\pi nS),\ a_E\sin(2\pi nS)]$$
$$\Delta t_1=\tau$$
$$\Delta t_2=nS-\tau$$

Note that τ is the time of the transition from leg 1 to leg 2 of the trajectory. Solutions are found separately for legs 1 and 2 of the trajectory, and then the transitions are checked to verify that a gravity assist can provide the required turn angle. Note that if the incoming and outgoing v_∞ are not equal for the transition, then a delta V is required; a gravity assist cannot change the hyperbolic excess velocity in the frame of the gravitating body. If the turn angles can be accomplished via gravity assist and the transition hyperbolic velocities are equal, then the trajectory is of the ballistic type mentioned earlier. Otherwise, the cycler trajectory is powered, and the cycler vehicle will have to periodically expend fuel to maintain the orbit.

To illustrate the practice of this method, we shall analyze one of the notable cycler trajectories found recently, S1L1-B. The name indicates that the first leg is a Short-period 1-rev solution, the second leg is a Long-period 1-rev solution, and the entire trajectory is in the Ballistic class. In McConaghy et al. (2006), τ for the S1L1-B cycler is given to be ~2.8276 years, so the conditions for the trajectory are:

Leg 1:

$$r_1=[a_E,\ 0]$$
$$r_2=[0.4690a_E,\ -0.8832a_E]$$
$$\Delta t_1=2.8276\ \ yr$$

Leg 2:

$$r_2=[0.4690a_E,\ -0.8832a_E]$$
$$r_2=[-0.2225a_E,\ 0.9749a_E]$$
$$\Delta t_1=1.4581\ \ yr$$

Using the original Battin method (Vallado 2007, Battin 1999) and the multiple revolution modifications and convergence techniques suggested by Shen et al (2003), I wrote a Lambert solver to compute solutions for the Lambert problem for multiple revolutions and both directions: $t_m=+1$ (short way), $t_m=+1$ (long way). The solutions found for each of these legs using the Lambert solver are plotted in the figures below.

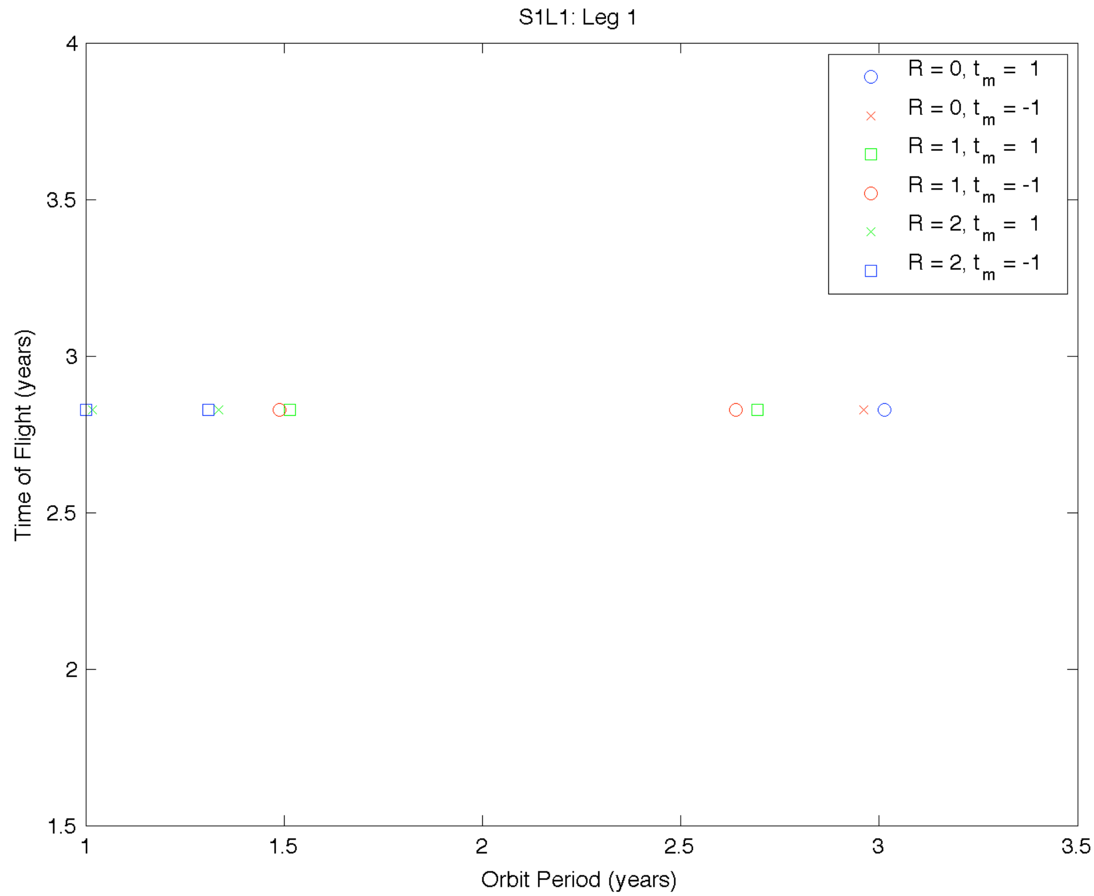

Figure 2: S1L1-B Cycler – Possible solutions for Leg 1

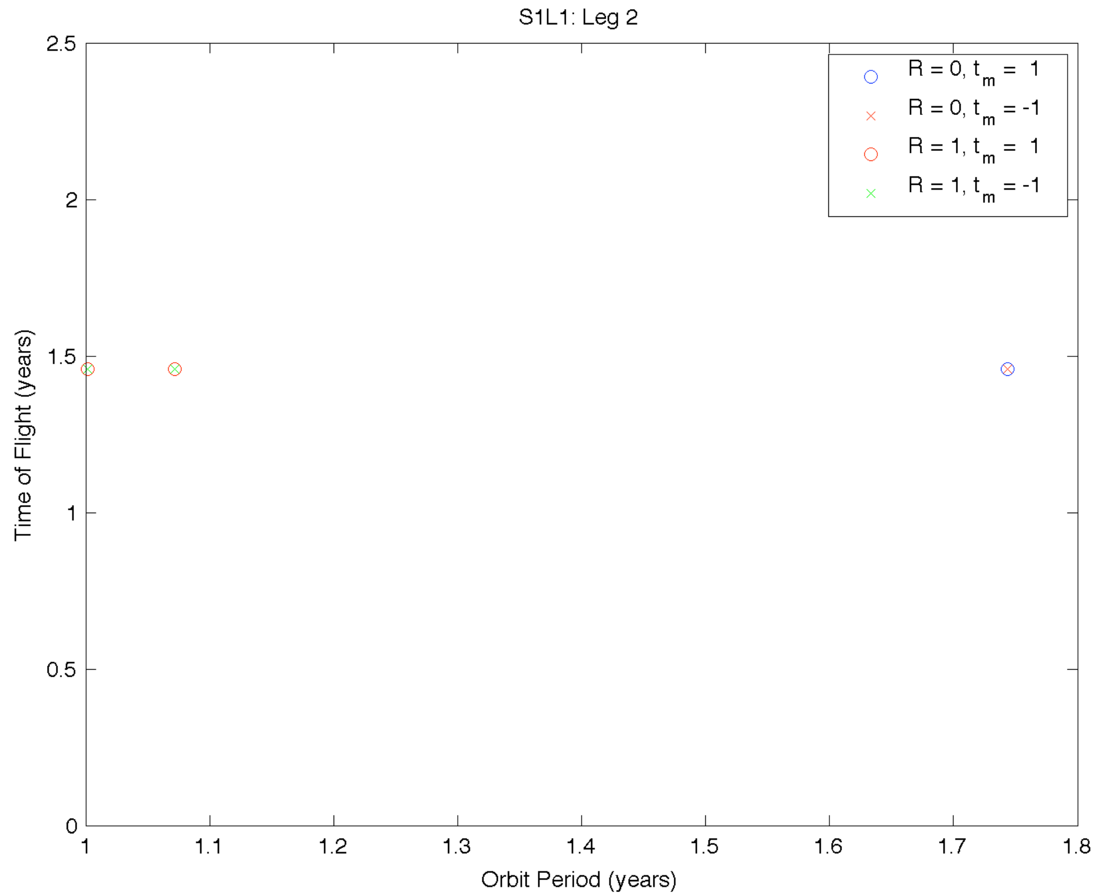

Figure 3: S1L1-B Cycler - Possible solutions for Leg 2

It is interesting to note that Earth's orbit appears to be a solution for both legs of the trajectory, though these solutions are obviously not useful for this application. McConaghy et al. (2006) gives the period of the orbit for the first leg as 1.4889 yrs for the first leg, and 1.0733 yrs for the second leg. Examination of the data points in the plots provided the parameters needed to reconstruct the orbit and characterize its properties. The trajectory property values determined by my Lambert solver agree well with the results published in McConaghy et al. (2006).

Leg 1: $[t_m,\ R] = [-1,\ 1]$

Leg 2: $[t_m,\ R] = [+1,\ 1]$

Table 1: S1L1-B Cycler Characteristics

Characteristic	Value
Earth V infinity	4.7071 km/sec

Mars V infinity	5.0042 km/sec
Earth-Mars transfer time	153.3494 days
Repeat time	30/7 yr

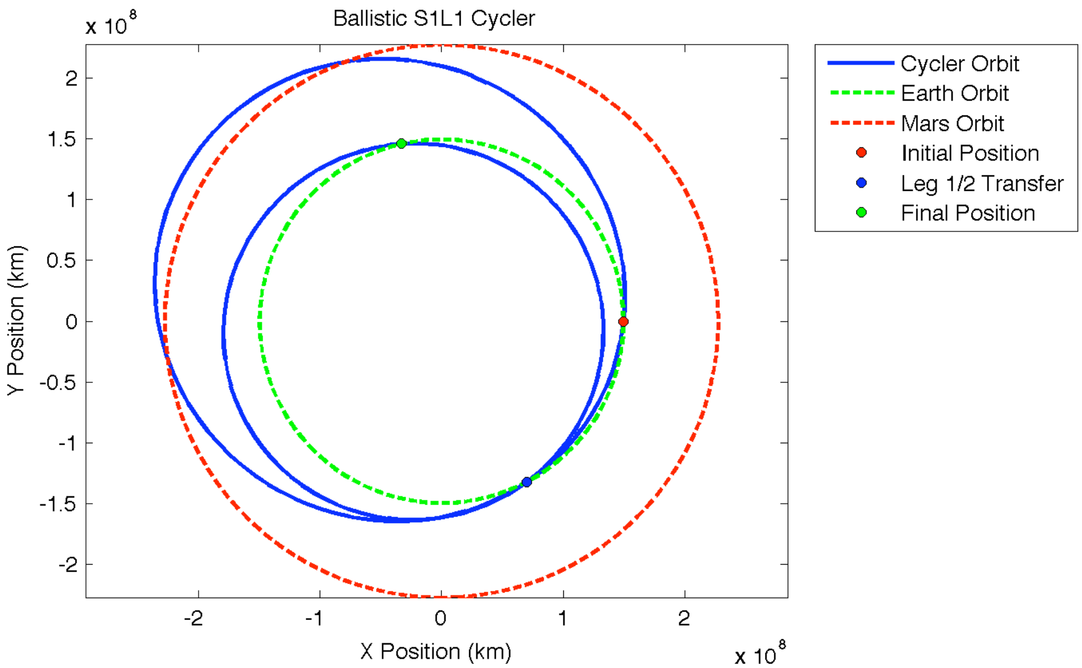

Figure 4: S1L1-B Cycler Trajectory

Now that the method has been described and the accuracy of the model has been demonstrated, an alternative cycler trajectory will be proposed and analyzed as an extension to the current research in the literature.

Extension

Cycler trajectories having a small number of revs (for instance, 2 or fewer) per synodic repeat period are often considered unfavorable because their Earth v_∞ and Mars v_∞ are typically quite large, making the rendezvous problem for outgoing and incoming trajectories more difficult. Further, the max turn angle for gravity fly-bys

(δ_{max}) is reduced because the velocity magnitudes are large. As the equation below shows, for a constant fly-by altitude (nominally assigned to be 1000 km) the turn angle decreases with increasing v_∞.

$$\sin\left(\frac{\delta}{2}\right) = \left(1 + \frac{r_p v_\infty^2}{\mu}\right)^{-1}$$

For some trajectories the required turn angle is greater than the max turn angle, which implies that a delta V must be applied to complete the rotation of the line of apsides. The Aldrin cycler is such a case of a cycler that requires a delta V during Earth fly-by to maintain the shape of its orbit relative to Earth and Mars.

Lower rev cyclers, however, do have the advantage of shorter transit times from Earth to Mars (and vice versa), and those cyclers that repeat every synodic period only require a two vehicle constellation in order to provide an inbound and outbound vehicle every synodic period. (Note that in general $2n$ vehicles are required to meet this requirement, where n is the period of the repeat interval, expressed as an integer multiple of synodic periods.) Also, if a Mars rendezvous opportunity is aborted, the duration of the free return trajectory back to Earth is typically longer for higher rev cyclers than lower rev cyclers. For example, the free return duration for the S1L1 cycler is 2.8276 years, whereas for the Aldrin cycler the duration is 1.7429 years.

With this in mind, the objective of this extension is investigate whether a low rev cycler, the Aldrin cycler in particular, can be modified to reduce the required delta V and v_∞ for planetary fly-bys. To begin, a characterization of the original Aldrin cycler is presented.

At the conclusion of his biography *Men from Earth*, Buzz Aldrin briefly discussed cycler vehicles on repeating trajectories as an alternative to the historical approach of using one-use-only vehicles on direct trajectories to ferry astronauts from the Earth to Mars and back again. Since his original suggestion referenced a trajectory whose period would be equal to the Earth-Mars synodic period, the long period (~2.02 years) solution to the Lambert problem for a single synodic period (~2.14 years) time of flight has been referred to as the Aldrin cycler. A plot of the Aldrin cycler trajectory as viewed from above the ecliptic is shown in the figure below, and characteristics of the orbit (as calculated by my simulation) are listed in the table below. Note that the assumptions for the modeling of this trajectory are the same as those used in the previous section.

Table 2: Original Aldrin Cycler Characteristics

Characteristic	Value
Earth V infinity	6.5318 km/sec
Earth fly-by altitude	2.6795 km/sec
Earth delta V required	1000 km
Mars V infinity	9.7371 km/sec
Mars flyby altitude	Infinity (no GA required)
Mars delta V required	0 km/sec
Earth-Mars transfer time	145.7158 days
Repeat time	15/7 yr

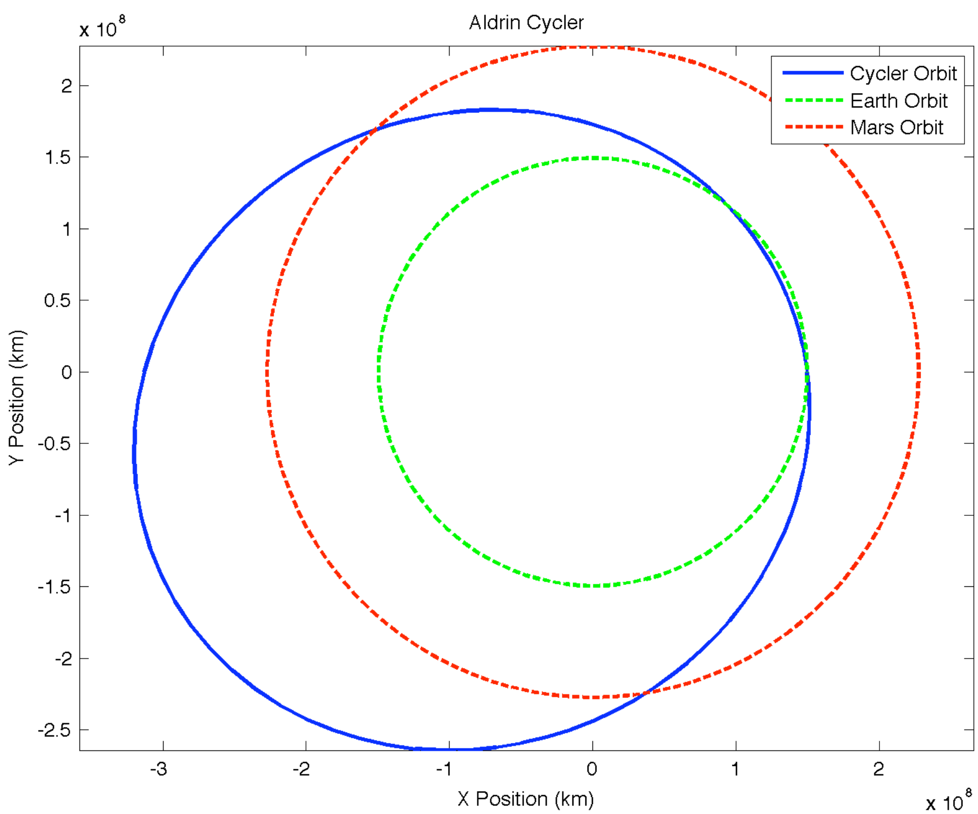

Figure 5: Original Aldrin Cycler

In order to model the delta V required at each Earth fly-by to maintain the shape of the orbit relative to Earth and Mars, I assumed a delta V was applied as the cycler vehicle entered the Earth SOI, and then an equal and opposite delta V was applied as the cycler vehicle left the Earth SOI. The magnitude of this delta V was determined as the difference in the magnitudes of the initial v_∞ and the v_∞ required (assuming a fly-by altitude of 1000 km) to produce the turn angle needed to maintain the shape of the orbit (~86.3 deg).

The table indicates that a Mars gravity assist is not required to maintain this trajectory. Because of this constraint, the entire rotation of the orbit line of apsides must be applied during the Earth fly-by. I hypothesized that the inclusion of a gravity assist maneuver at Mars could reduce the delta V required to complete the rotation of the line of apsides at Earth fly-by, as well reduce the v_∞ at each of the planetary rendezvous points.

To model this modified Aldrin cycler trajectory, I broke up the original solution into two legs. The first leg constituted a trajectory from Earth to Mars at the original rendezvous point (i.e. the position vector for the first Mars orbit crossing was held constant between the original and modified solutions) and extended the transfer time beyond 145.7158 days in increments of 1 hour. Only times of flight greater than the original were considered because trajectory solutions for those times would immediately reduce the required turn angle for the Earth fly-by and the v_∞ for the Mars fly-by. Values for planetary v_∞, fly-by altitude, required turn angle, and required delta V are plotted in the figure below against the Earth-Mars time of flight for the first leg.

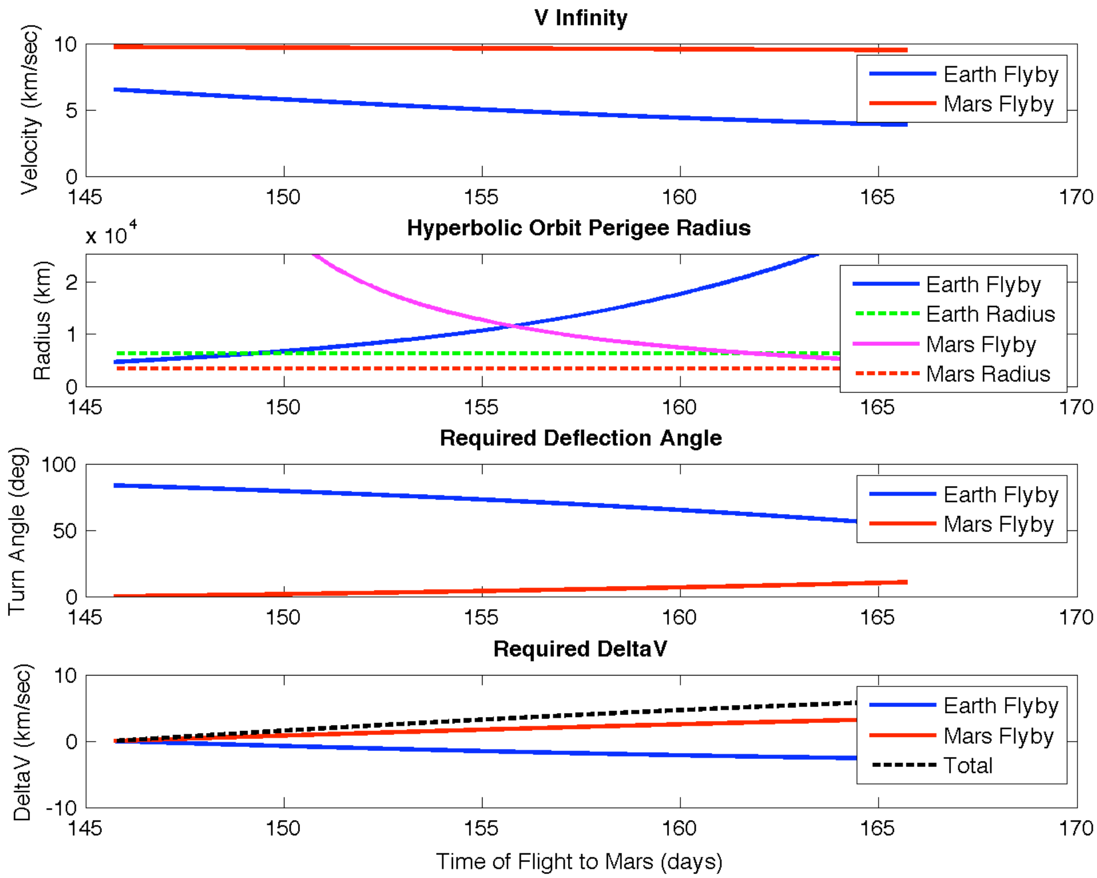

Figure 6: Modified Aldrin Trajectories

Note that the values for delta V plotted in the figure are simply the difference in magnitude between the v_∞^+ and v_∞^- for each fly-by. For example, if magnitude of the final velocity for the solution to the first leg trajectory and the magnitude of the initial velocity for the solution to the second leg trajectory are equal, then the delta V shown is zero. This is the case for both the Mars and Earth fly-by in the original Aldrin cycler (left-most data point in all of the plots), but as was mentioned earlier a delta-V would have to be used to complete the apsidal rotation since the max turn angle is less than the required turn angle.

The plots demonstrate that as the time of flight for the first leg trajectory is increased, the required turn angle and v_∞ at both Earth and Mars is decreased. Fortunately, there also appears to be a 10-day period in the space of Earth-Mars transfer times for which the perigee radius of the fly-by is greater than the planet radius for both Earth and Mars. This indicates that for those trajectories no additional delta V would be required to complete the rotation of the orbit. Since the required delta V increases with offset from the original Mars time of flight, I selected for further investigation a trajectory with a time of flight at the close to the beginning of this period in order to minimize the overall delta V requirements. A plot of the selected trajectory is shown below, as is a characterization table comparing the original and modified Aldrin cycler, as well as the S1L1 cycler.

Table 2: Cycler Characteristics

Characteristic	Aldrin	Modified Aldrin	S1L1*
Earth V infinity	6.5318 km/sec	5.3116 km/sec	4.7 km/sec
Earth fly-by altitude	1000 km	2497.0522 km	31,809 km
Earth delta V required	2.6795 km/sec	-1.221 km/sec	0 km/sec
Mars V infinity	9.7371 km/sec	9.6528 km/sec	5.0 km/sec
Mars flyby altitude	Infinity (no GA required)	13,468.8598 km	Infinity (no GA required)
Mars delta V required	0 km/sec	1.3642 km/sec	0 km/sec
Earth-Mars transfer time	145.7158 days	153 days	153.15 days
Repeat time	15/7 yr	15/7 yr	30/7 yr
Free return period	1.7436 yr	1.7237 yr	2.8276 yr

* Values taken from McConaghy et al. (2006).

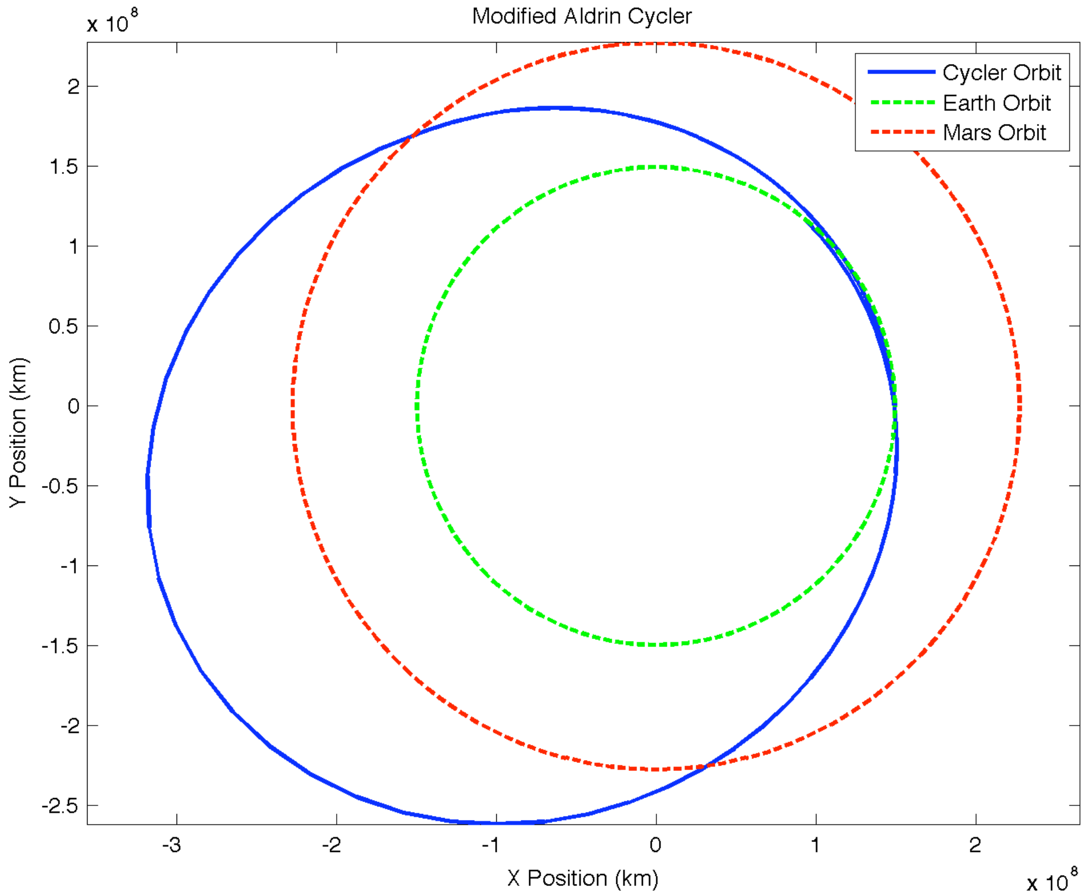

Figure 7: Modified Aldrin Cycler

The plot shows that the increase in the time of flight for the Earth-Mars transfer and inclusion of a gravity assist maneuver for the Mars fly-by yields a slight rotation of the orbit before the cycler vehicle approaches the Earth rendezvous point. The savings in total delta V for this modified Aldrin cycler compared to the original trajectory is almost 0.1 km/sec less (the magnitude of the difference will increase if the perigee radius for the Earth fly-by in the original trajectory is increased) and the v_∞ at both Earth and Mars is reduced, with the reduction at Earth amounting to more than 1 km/sec.

Based on this analysis, it would appear that the modified Aldrin cycler could potentially be a good candidate for a cycler trajectory mission to Mars. Further work should investigate other solutions within the interval of realistic perigee radii, as well as to repeat the analysis for varying Mars rendezvous points. These investigations should help to produce increasingly more refined modifications to the Aldrin cycler and continue to make it compare more favorably to other cyclers like the S1L1-B.

Summary & Conclusions

This paper has defined and investigated the astrodynamics research problem of determining optimal Earth-Mars cycler trajectories. The reasons why cycler trajectories are a favorable option for manned missions to Mars were discussed, and the importance of this problem to astrodynamics was explained. A tool was built to solve the general Lambert problem, and previously discovered trajectories were confirmed. Finally, a modification to the original Aldrin cycler was proposed, and the results due to that modification were analyzed and compared to the original Aldrin cycler and the S1L1-B cycler.

Based on the results shown in this paper, it seems clear that cycler trajectories should constitute a viable option for manned missions to Mars. Ultimately, the degree to which we can accurately model these trajectories will determine how competitive they are among the other options for manned mission trajectories. The S1L1-B cycler appears to be the latest preferred option for an Earth-Mars-cycler trajectory, mostly because it requires little or no delta V to maintain. I have suggested a modification to the original Aldrin cycler that simultaneously reduces its hyperbolic velocity at Earth and Mars and the overall delta V required to maintain the orbit. Further research into this and other modifications to the Aldrin cycler should continue to make it compare more favorably to the S1L1-B cycler and possibly make it a more competitive candidate for the final Mars manned mission trajectory design.

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Appendix

Matlab Code