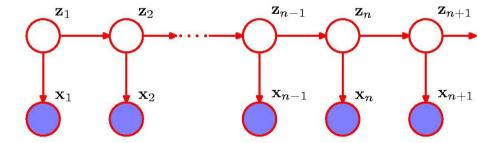
#### **Hidden Markov Models**

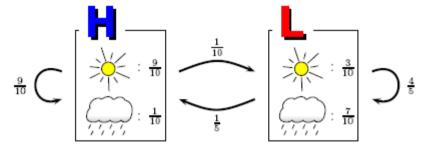
Selecting the initial model parameters
Using HMMs for gene finding



# HMMs as a generative model

A HMM generates a sequence of observables by moving from latent state to latent state according to the transition probabilities and emitting an observable (from a discrete set of observables, i.e. a finite alphabet) from each latent state visited according to the emission probabilities of the state ...





A run follows a sequence of states:



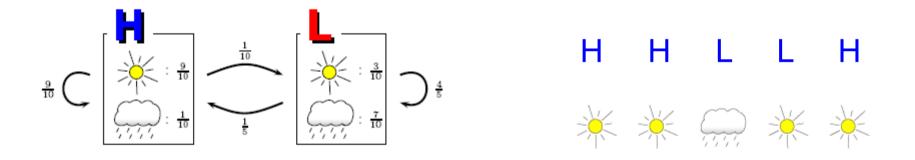
And emits a sequence of symbols:



For a HMM that generates finite strings (e.g. a HMM with an end-state), the language  $L = \{X \mid p(X) > 0\}$  is regular ...

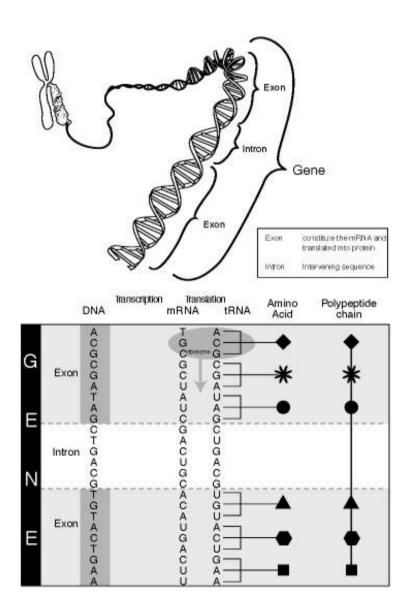
# Selecting initial model parameters

The initial selection of transition and emission probabilities, i.e. A,  $\pi$ ,  $\Phi$ , should model (how we see) the underlying structure of the observations, i.e. the syntax of possible sequences of observations, recall that the language L = {x | P(x |  $\theta$ ) > 0} is regular.



The initial selection of parameters is essential just to decide which parameters are 0 (or 1), i.e. to decide which transitions of emission should never (or always) be possible ...

# Example – Gene finding



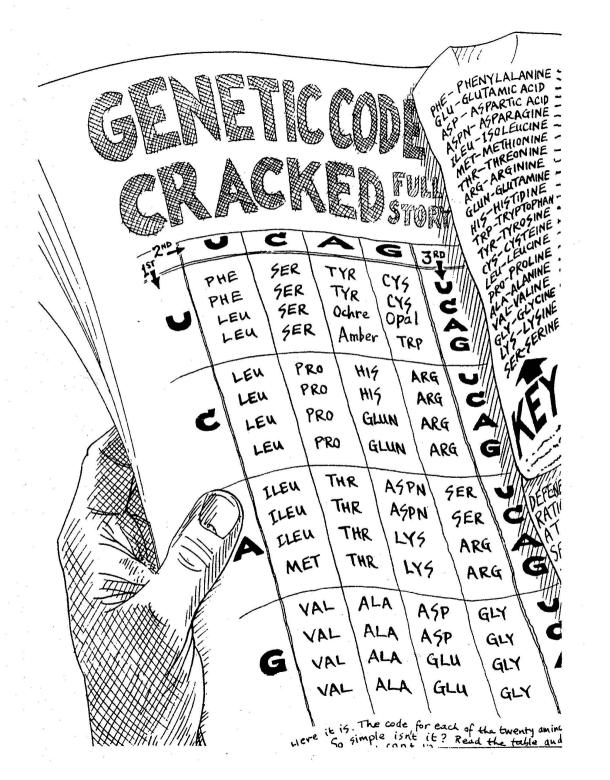
Each protein is encoded in a stretch of DNA. A gene ...

Which is expressed when the protein is needed ...

#### Important problem

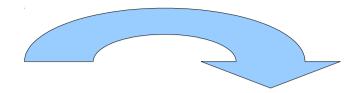
Locating genes on the genome and determining how they get expressed ...

Recognizing the patterns that indicates a gene ...



>NC 002737.1 Streptococcus pyogenes M1 GAS TTGTTGATATTCTGTTTTTTTTTTTTTTTTTCCACATGAAAAATAGTTGAAAACAATA GCGGTGTCCCCTTAAAATGGCTTTTCCACAGGTTGTGGAGAACCCAAATTAACAGTGTTA ATTTATTTCCACAGGTTGTGGAAAAACTAACTATTATCCATCGTTCTGTGGAAAACTAG AATAGTTTATGGTAGAATAGTTCTAGAATTATCCACAAGAAGGAACCTAGTATGACTGAA AATGAACAAATTTTTTGGAACAGGGTCTTGGAATTAGCTCAGAGTCAATTAAAACAGGCA ACTTATGAATTTTTTGTTCATGATGCCCGTCTATTAAAGGTCGATAAGCATATTGCAACT ATTTACTTAGATCAAATGAAAGAGCTCTTTTGGGAAAAAAATCTTAAAGATGTTATTCTT ACTGCTGGTTTTGAAGTTTATAACGCTCAAATTTCTGTTGACTATGTTTTCGAAGAAGAC CTAATGATTGAGCAAAATCAGACCAAAATCAACCAAAAACCTAAGCAGCAAGCCTTAAAT TCTTTGCCTACTGTTACTTCAGATTTAAACTCGAAATATAGTTTTGAAAACTTTATTCAA GGAGATGAAAATCGTTGGGCTGTTGCTGCTTCAATAGCAGTAGCTAATACTCCTGGAACT ACCTATAATCCTTTGTTTATTTGGGGTGGCCCTGGGCTTGGAAAAACCCATTTATTAAAT GCTATTGGTAATTCTGTACTATTAGAAAATCCAAATGCTCGAATTAAATATATCACAGCT GAAAACTTTATTAATGAGTTTGTTATCCATATTCGCCTTGATACCATGGATGAATTGAAA GAAAAATTTCGTAATTTAGATTTACTCCTTATTGATGATATCCAATCTTTAGCTAAAAAA ACGCTCTCTGGAACACAAGAAGAGTTCTTTAATACTTTTAATGCACTTCATAATAATAAC AAACAAATTGTCCTAACAAGCGACCGTACACCAGATCATCTCAATGATTTAGAAGATCGA TTAGTTACTCGTTTTAAATGGGGATTAACAGTCAATATCACACCTCCTGATTTTGAAACA CGAGTGGCTATTTTGACAAATAAAATTCAAGAATATAACTTTATTTTTCCTCAAGATACC ATTGAGTATTTGGCTGGTCAATTTGATTCTAATGTCAGAGATTTAGAAGGTGCCTTAAAA GATATTAGTCTGGTTGCTAATTTCAAACAAATTGACACGATTACTGTTGACATTGCTGCC GAAGCTATTCGCGCCAGAAAGCAAGATGGACCTAAAATGACAGTTATTCCCATCGAAGAA ATTCAAGCGCAAGTTGGAAAATTTTACGGTGTTACCGTCAAAGAAATTAAAGCTACTAAA CGAACACAAAATATTGTTTTAGCAAGACAAGTAGCTATGTTTTTAGCACGTGAAATGACA GATAACAGTCTTCCTAAAATTGGAAAAGAATTTGGTGGCAGAGACCATTCAACAGTACTC CATGCCTATAATAAAATCAAAAACATGATCAGCCAGGACGAAAGCCTTAGGATCGAAATT GAAACCATAAAAACAAAATTAAATAACATGTGGAAAAGAATATCTTTTATGAAATAGTT 

#### Viterbi decoding



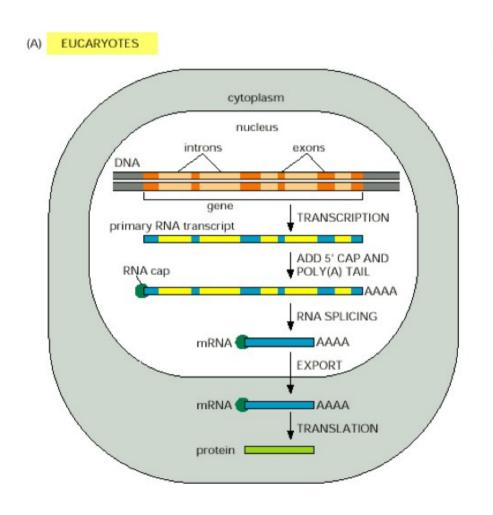
>NC 002737.1 Streptococcus pyogenes M1 GAS TTGTTGATATTCTGTTTTTTCTTTTTTAGTTTTCCACATGAAAAATAGTTGAAAACAATA GCGGTGTCCCCTTAAAATGGCTTTTCCACAGGTTGTGGAGAACCCAAATTAACAGTGTTA ATTTATTTTCCACAGGTTGTGGAAAAACTAACTATTATCCATCGTTCTGTGGAAAACTAG AATAGTTTATGGTAGAATAGTTCTAGAATTATCCACAAGAAGGAACCTAGTATGACTGAA AATGAACAAATTTTTTGGAACAGGGTCTTGGAATTAGCTCAGAGTCAATTAAAACAGGCA ACTTATGAATTTTTTGTTCATGATGCCCGTCTATTAAAGGTCGATAAGCATATTGCAACT ATTTACTTAGATCAAATGAAAGAGCTCTTTTGGGAAAAAAATCTTAAAGATGTTATTCTT ACTGCTGGTTTTGAAGTTTATAACGCTCAAATTTCTGTTGACTATGTTTTCGAAGAAGAC CTAATGATTGAGCAAAATCAGACCAAAATCAACCAAAAACCTAAGCAGCAAGCCTTAAAT TCTTTGCCTACTGTTACTTCAGATTTAAACTCGAAATATAGTTTTGAAAACTTTATTCAA GGAGATGAAAATCGTTGGGCTGTTGCTGCTTCAATAGCAGTAGCTAATACTCCTGGAACT ACCTATAATCCTTTGTTTATTTGGGGTGGCCCTGGGCTTGGAAAAACCCATTTATTAAAT GCTATTGGTAATTCTGTACTATTAGAAAATCCAAATGCTCGAATTAAATATATCACAGCT GAAAACTTTATTAATGAGTTTGTTATCCATATTCGCCTTGATACCATGGATGAATTGAAA GAAAAATTTCGTAATTTAGATTTACTCCTTATTGATGATATCCAATCTTTAGCTAAAAAA ACGCTCTCTGGAACACAAGAAGAGTTCTTTAATACTTTTAATGCACTTCATAATAATAAC AAACAAATTGTCCTAACAAGCGACCGTACACCAGATCATCTCAATGATTTAGAAGATCGA TTAGTTACTCGTTTTAAATGGGGATTAACAGTCAATATCACACCTCCTGATTTTGAAACA CGAGTGGCTATTTTGACAAATAAAATTCAAGAATATAACTTTATTTTTCCTCAAGATACC ATTGAGTATTTGGCTGGTCAATTTGATTCTAATGTCAGAGATTTAGAAGGTGCCTTAAAA GATATTAGTCTGGTTGCTAATTTCAAACAAATTGACACGATTACTGTTGACATTGCTGCC GAAGCTATTCGCGCCAGAAAGCAAGATGGACCTAAAATGACAGTTATTCCCATCGAAGAA ATTCAAGCGCAAGTTGGAAAATTTTACGGTGTTACCGTCAAAGAAATTAAAGCTACTAAA CGAACACAAATATTGTTTTAGCAAGACAAGTAGCTATGTTTTTAGCACGTGAAATGACA GATAACAGTCTTCCTAAAATTGGAAAAGAATTTGGTGGCAGAGACCATTCAACAGTACTC CATGCCTATAATAAAATCAAAAACATGATCAGCCAGGACGAAAGCCTTAGGATCGAAATT GAAACCATAAAAAACAAAATTAAATAACATGTGGAAAAGAATATCTTTTATGAAATAGTT 

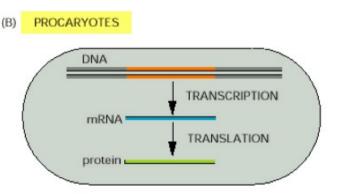
>NC 002737.1 gene annotation Streptococcus pyogenes M1 GAS 

#### Design a HMM that models the syntax of genes

### **Gene structure**

Depends on the organism (eucaryote or procaryote)

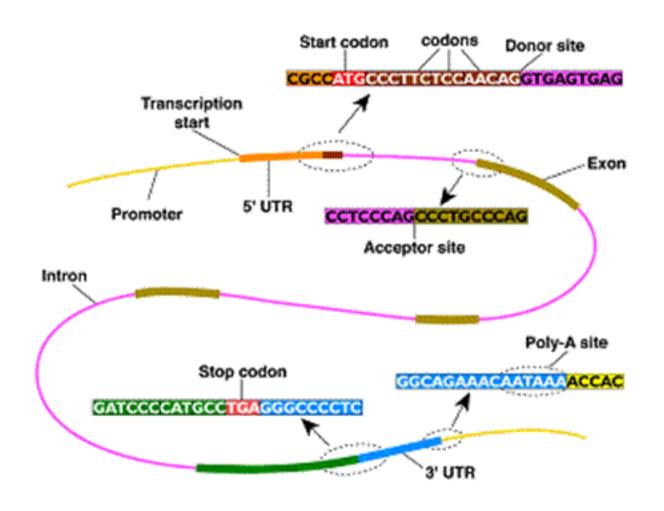




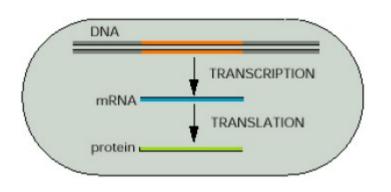
Smaller genomes and high coding density.

Large genomes. Intron/exon structure and low coding density

## Gene structure in eukaryotes



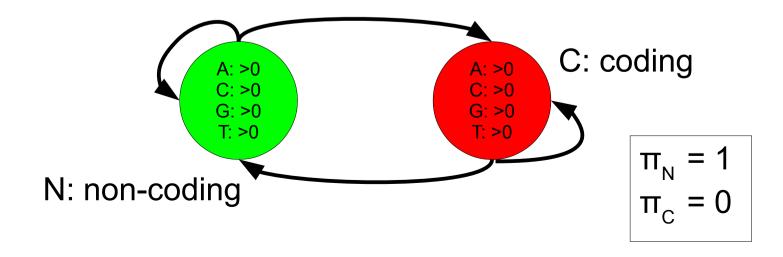
Eukaryotic gene structure in more details

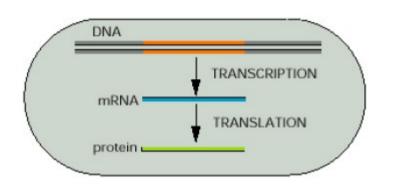


#### **Biological facts**

The gene is a substring of the DNA sequence of A,C,G,T's

X: acgatgcgctaatatgtccgatgacgtgagcataagcgacatgcag

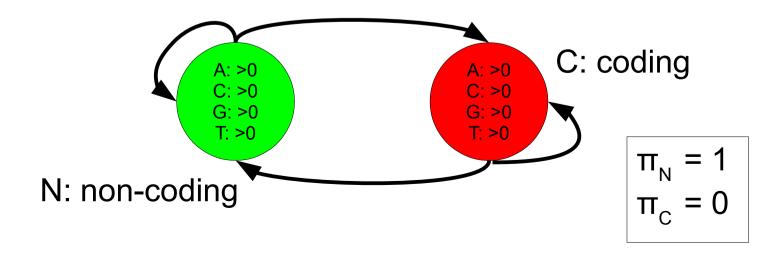


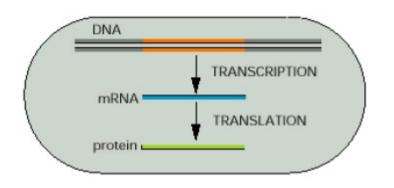


#### **Biological facts**

- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon atg

X: acgatgcgctaatatgtccgatgacgtgagcataagcgacatgcag





#### **Biological facts**

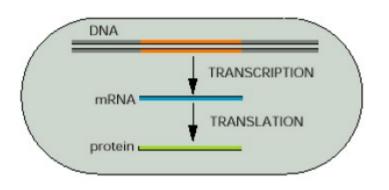
The gene is a substring of the DNA sequence of A,C,G,T's

C: coding

The gene starts with a start-codon atg

X: acgatgcgctaatatgtccgatgacgtgagcataagcgacat  $\begin{array}{c} \pi_N = 1 \\ \pi_C = 0 \\ \hline \\ A: > 0 \\ C: > 0 \\ G: > 0 \\ C: > 0 \\ G: > 0 \\ T: > 0 \\ \hline \\ T: > 0 \\ \hline \end{array}$ 

N: non-coding



#### **Biological facts**

The gene is a substring of the DNA sequence of A,C,G,T's

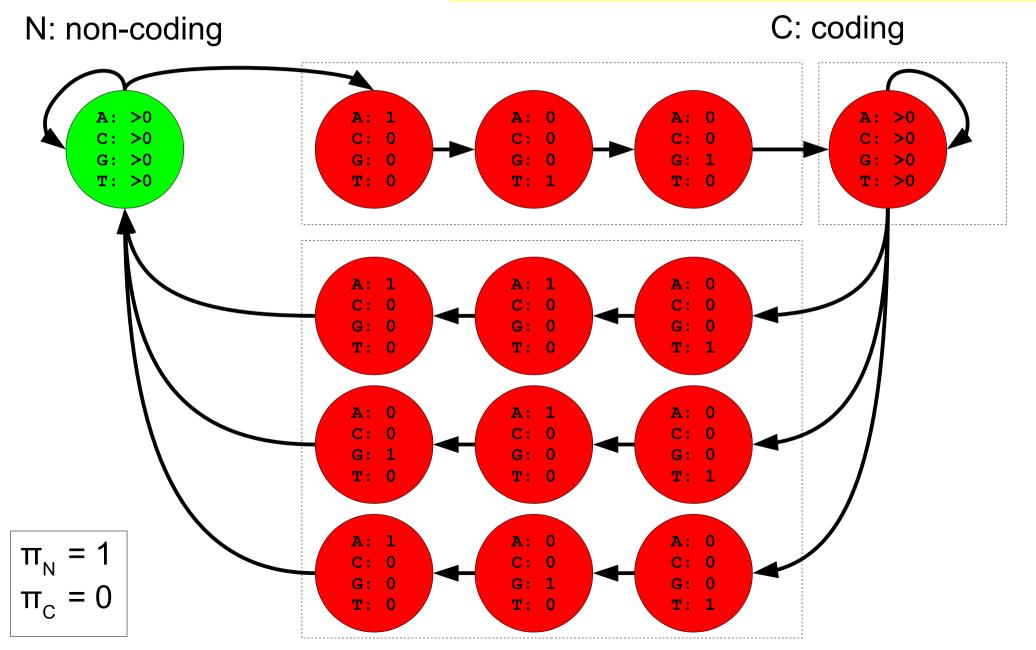
C: coding

- The gene starts with a start-codon atg
- The gene ends with a stop-codon taa, tag or tga

N: non-coding

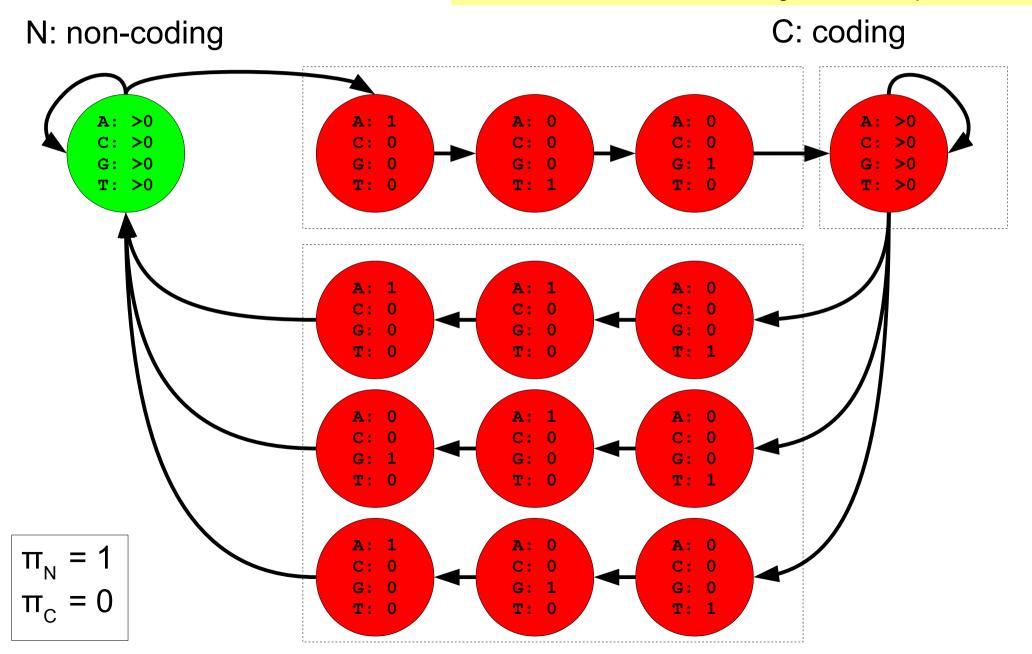
- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon atg

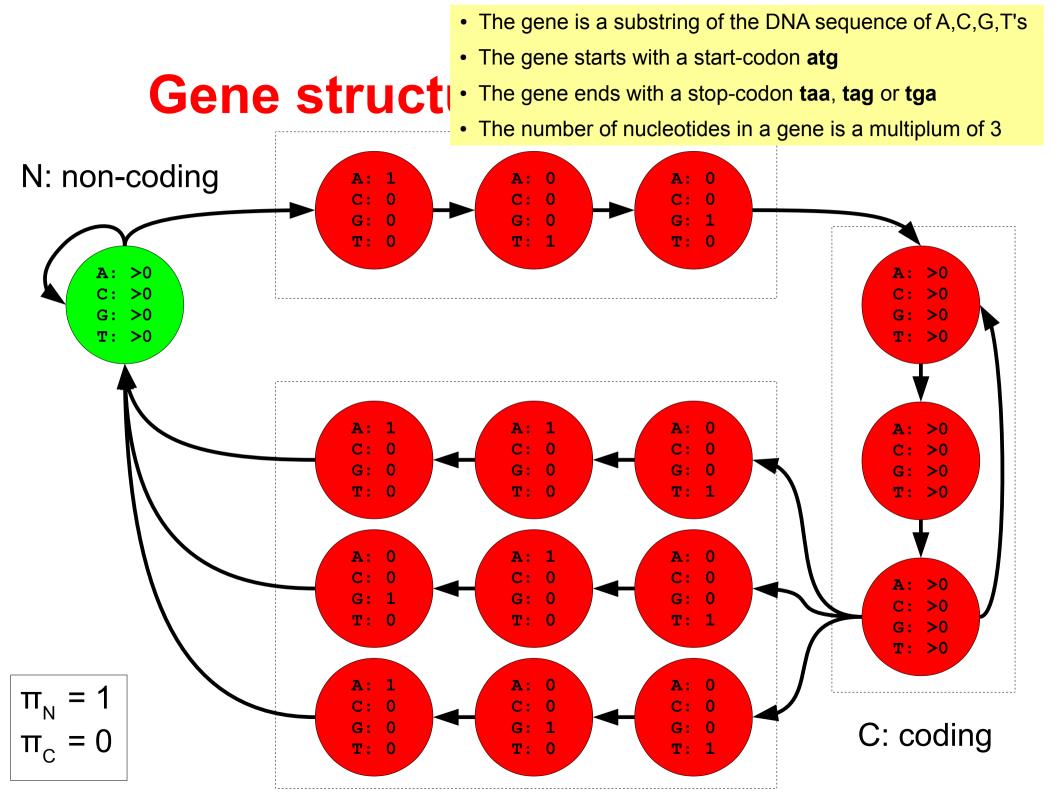
Gene struct • The gene ends with a stop-codon taa, tag or tga

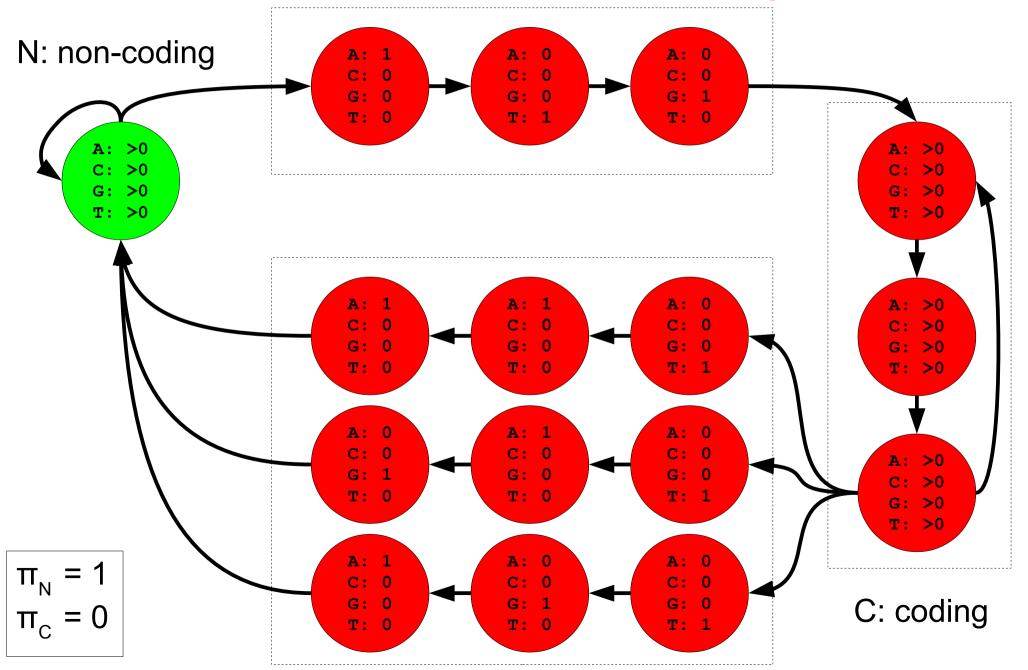


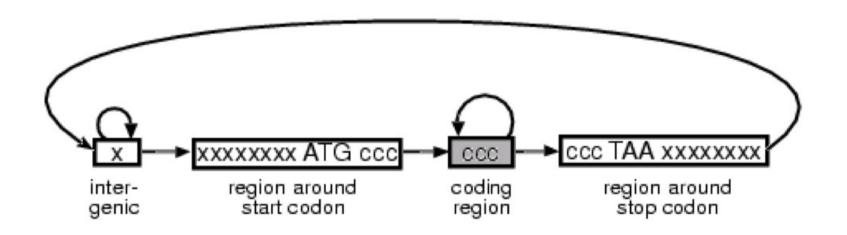
## Gene struct • The gene ends with a stop-codon taa, tag or tga

- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon atg
- The number of nucleotides in a gene is a multiplum of 3



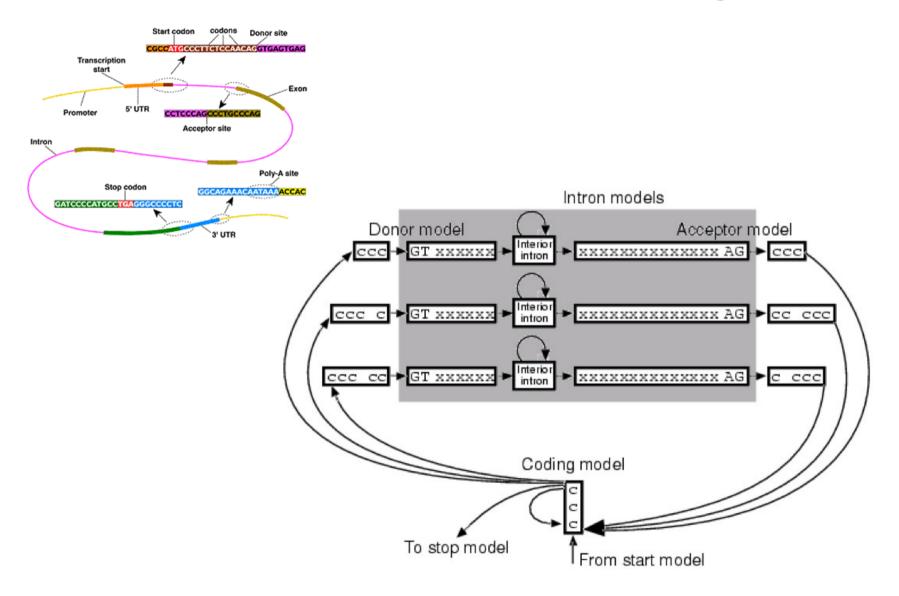




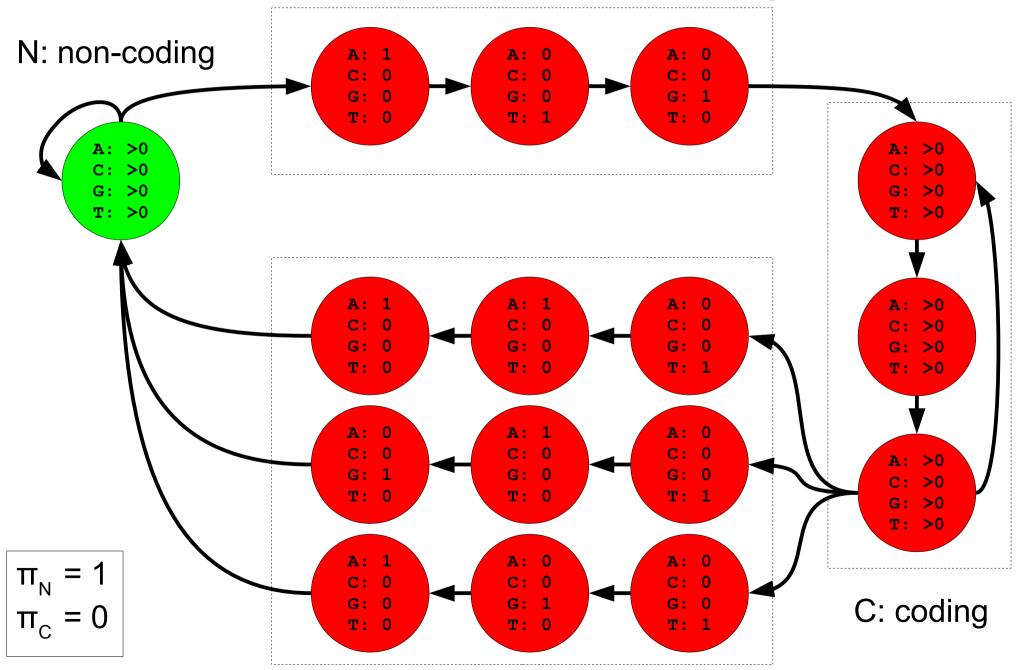


From "An Introduction to HMMs for Biological Sequences", A. Krogh, 1998

## Gene structure in eukaryotes



From "An Introduction to HMMs for Biological Sequences", A. Krogh, 1998

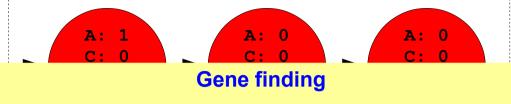


N: non-coding

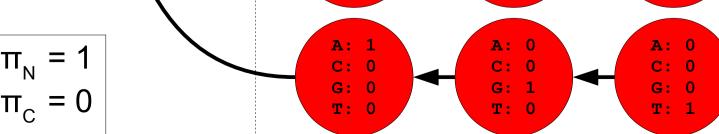
>0

G: >0

T: >0



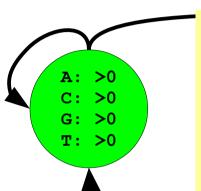
- Select initial model structure (e.g. as done here)
- Select model parameters by training. Either "by counting" from examples of (**X**,**Z**)'s, i.e. genes with known structure, or by EM- or Viterbi-training from examples of **X**, i.e. sequences which are known to contain a gene (as we will see later)
- Given a new sequence **X**, predict its gene structure using the Viterbi algorithm for finding the most likely sequence of underlying latent states, i.e. its gene structure

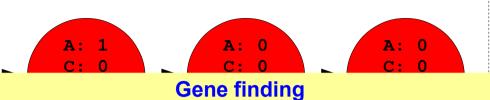


C: >0G: >0T: >0 A: >0 C: >0G: >0T: > 0A: >0 C: >0 G: >0 T: > 0

# **Example – Gene finding**

#### N: non-coding

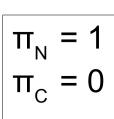




- Select initial model structure (e.g. as done here)
- Select model parameters by training. Either "by counting" from examples of (X,Z)'s, i.e. genes with known structure, or by EM- or Viterbi-training from examples of X, i.e. sequences which are known to contain a gene (as we will

#### **Even more biology**

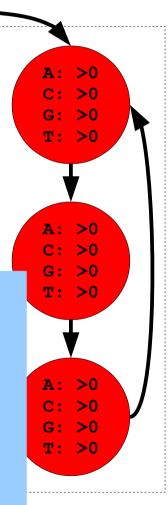
- There can be genes in both directions (and over lapping)
- There are more possible start-codons atg, gtg, and ttg
- Internal codons cannot be start- or stop-codons
- And a lot more ...



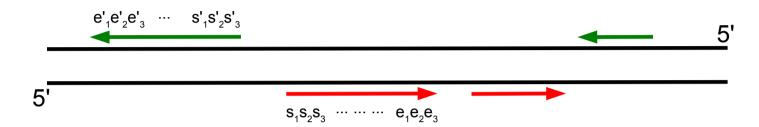
T: 0

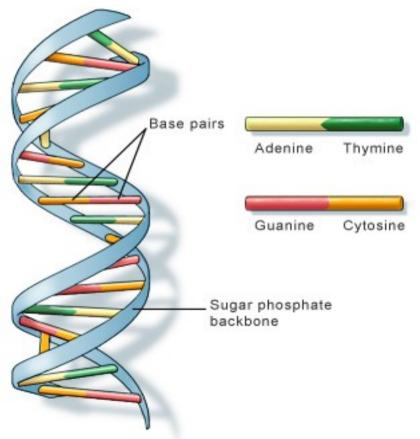
T: 0

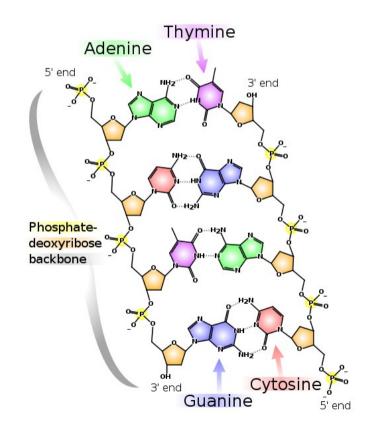
T: 1



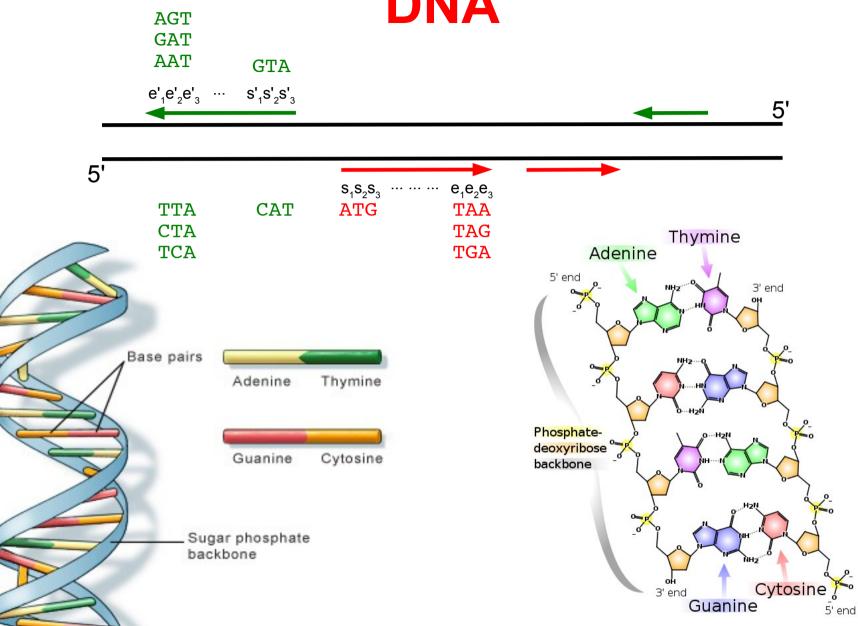
## **DNA**

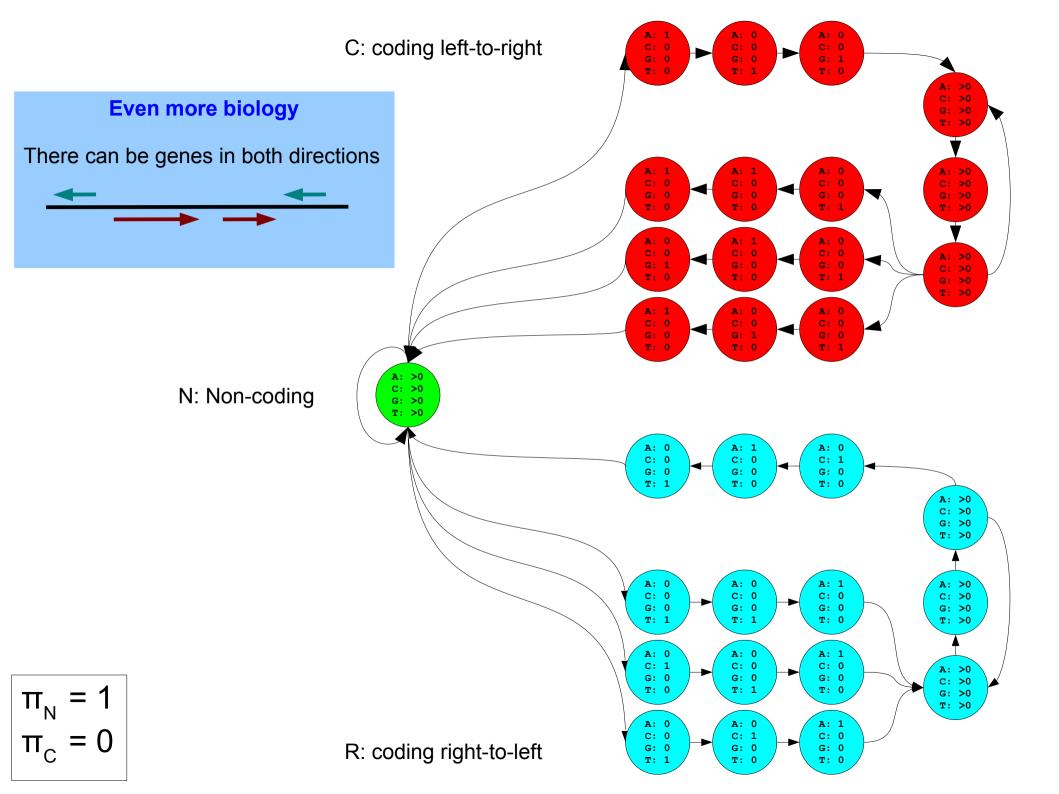




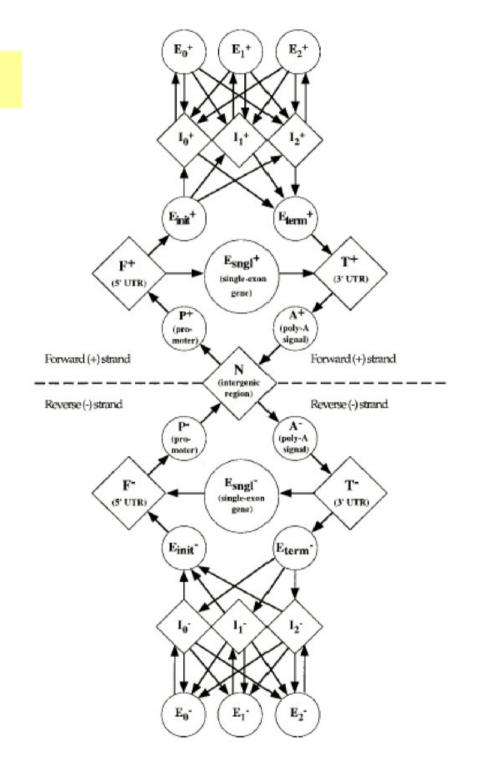


## **DNA**

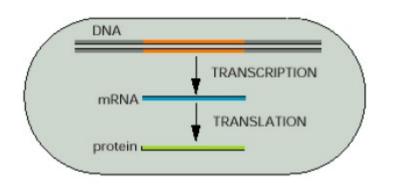




Eukaryotic gene structure in both directions (GenScan)



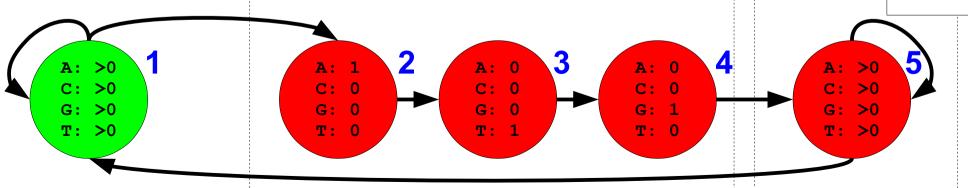
## Problem: From annotation to Z



#### **Biological facts**

- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon atg

**X:** acgatgcgctaatatgtccgatgacgtgagcataagcgacat $\pi_c = 0$ 



N: non-coding

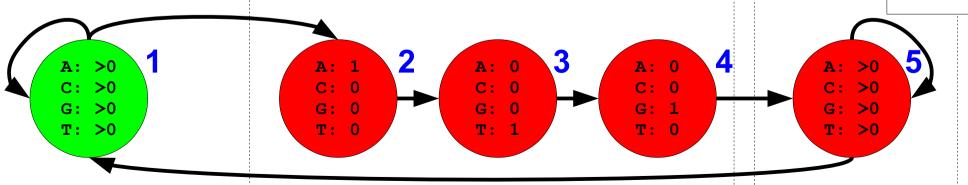
## Problem: From annotation to Z

**Problem:** The string **Z=NNNCCC....** is not a prober sequence of states in the illustrated HMM, but is can easily be converted into one.

#### **Biological facts**

a substring of the DNA sequence of A,C,G,T's tarts with a start-codon **atg** 

X: acgatgcgctaatatgtccgatgacgtgagcataagcgacat $\pi_N = 1$  $\pi_C = 0$ 



N: non-coding

protein

## Problem: From annotation to Z

**Problem:** The string **Z=NNNCCC....** is not a prober sequence of states in the illustrated HMM, but is can easily be converted into one.

#### **Biological facts**

a substring of the DNA sequence of A,C,G,T's tarts with a start-codon atg

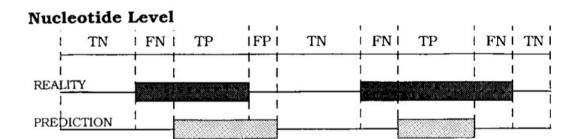
protein

#### 

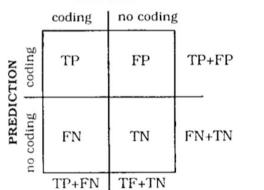
X: acgatgcgctaatatgtccgatgacgtgagcataagcgacat

N: non-coding

## **Evaluating performance**







$$Sn = \frac{TP}{TP + FN}$$

Sensitivity

$$Sp = \frac{TP}{TP + FP}$$

Specificity

$$CC = \frac{(TP \times TN) - (FN \times FP)}{\sqrt{(TP + FN) \times (TN + FP) \times (TP + FP) \times (TN + FN)}}$$
 Correlation Coefficient

$$ACP = \frac{1}{4} \left[ \frac{TP}{TP + FN} + \frac{TP}{TP + FP} + \frac{TN}{TN + FP} + \frac{TN}{TN + FN} \right]$$

$$AC = (ACP - 0.5) \times 2$$

 $AC = (ACP - 0.5) \times 2$  Approximate Correlation