3388 Problem Set 1

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Ex.1 Show that the basis vector ˆk is orthogonal to both ˆi and ˆj.

Solution
If a vector v is orthogonal to another vector n, then v • n = 0
Thus if ^k • ^i = 0 and ^k • ^j = 0, then ^k is orthogonal to ^i and ^j
.
^k • ^i = [0, 0, 1] • [1, 0, 0] = (0*1) + (0*0) + (1*0) = 0
^k • ^j = [0, 0, 1] • [0, 1, 0] = (0*0) + (0*1) + (1*0) = 0

Therefore ˆk is orthogonal to both ˆi and ˆj.

Ex.2 Show that the basis vector ˆk is orthogonal to both ˆi and ˆj.

Solution

$$\begin{bmatrix} 1 & -4 & 8 \\ 11 & 2 & 24 \\ 12 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} -9 & 8 & 6 \\ 0 & 15 & 2 \\ 3 & 14 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} -9+0+24 & 8-60+112 & 6-8+0 \\ -99+0+72 & 88+30+336 & 66+4+0 \\ -108+0+3 & 96+60+14 & 72+8+0 \end{bmatrix}$$
$$= \begin{bmatrix} 15 & 60 & -2 \\ -27 & 454 & 70 \\ -105 & 170 & 80 \end{bmatrix}$$

Ex.3 Find a vector that is in the same direction as the reflection of v across n

We can use the formula for the reflection of a vector across another vector:

$$\vec{r} = \vec{v} - 2\frac{\vec{v}\cdot\vec{n}}{||n||^2}\cdot\vec{n}$$

$$\vec{r} = (2,\, 3) - 2\frac{(2,\, 3)\cdot(-1,\, 2)}{||(-1,\, 2)||^2}\cdot(-1,\, 2)$$

$$\vec{r} = (2,\, 3) - 2\frac{4}{5}\cdot(-1,\, 2)$$

$$\vec{r} = (2,\, 3) - \left(-\frac{8}{5},\, \frac{16}{5}\right)$$

$$\vec{r} = \left(\frac{18}{5},\, -\frac{1}{5}\right)$$

Here, since we don't need the reflection vector itself, but just a vector collinear to the reflection, we can use (18, -1) for r instead, to make things simpler.
Now in order to find the angles in between the vectors, we can use the dot product formula.

$$|\vec{r}| = \sqrt[2]{18^2+(-1)^2} = \sqrt[2]{325}$$

$$|\vec{v}| = \sqrt[2]{2^2+3^2} = \sqrt[2]{13}$$

$$|\vec{n}| = \sqrt[2]{2^2+(-1)^2} = \sqrt[2]{5}$$

$$\cos(\theta) = \frac{\vec{v}\cdot\vec{n}}{|v||n|} = \frac{4}{\sqrt[2]{65}}$$

$$\theta = 60.255118\ldots^\circ$$

$$\cos(\theta) = \frac{r\cdot\vec{n}}{|r||n|} = \frac{-20}{5\sqrt[2]{65}} = \frac{-4}{\sqrt[2]{65}}$$

$$\theta = 119.744881\ldots^\circ$$

Since the 2 angles are on different sides of the axis, we need to subtract from 180 to see if they are equal
180 - 60.255188… = 119.744881…

Therefore the angles are the same, so r is a reflection of v across n.

Ex.4

In order to find the normal vector, first we get 2 vectors using the 3 points, in this case we used pq and pr. Call these vectors a and b respectively

Then we take the cross product of a and b to get their normal vector.

Since we are asked for the normalized normal vector, we find the magnitude of n, and then divide n by its magnitude to get the normalized vector n.

$$\vec{p} = (3,\ 2,\ 4)$$
$$\vec{q} = (1, -3,\ 4)$$
$$\vec{r} = (1,\ 3, -1)$$
$$\vec{a} = \vec{pq} = (1-3,\ -3-2,\ 4-4) = (-2, -5,\ 0)$$
$$\vec{b} = \vec{pr} = (1-3,\ 3-2,\ -1-4) = (-2,\ 1, -5)$$
$$\vec{n} = \vec{a} \times \vec{b} = [(-5 \cdot -5) - 0,\ 0 - (-2 \cdot -5),\ (-2 - 10)]$$
$$\vec{n} = (25,\ -10,\ -12)$$
$$|\vec{n}| = \sqrt[2]{25^2 + 10^2 + 12^2} = \sqrt[2]{869}$$
$$\vec{n} = \frac{(25,\ -10,\ -12)}{\sqrt[2]{869}}$$