

Declarative programming

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(Programming languages and tools)

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[1.2 ,2.2-2.3]

Function definitions

- So far: predefined functions
- Functions can also be defined (use the definition area of DrRacket)
- Why define functions?

```
> (/ (+ 12 17) 2)
```

```
14.5
```

```
> (/ (+ 100 200) 2)
```

```
150
```

Function definitions

- So far: predefined functions
- Functions can also be defined (use the definition area of DrRacket)
- Why define functions?

```
> (/ (+ 12 17) 2)
```

```
14.5
```

```
> (/ (+ 100 200) 2)
```

```
150
```

Avoidance of
redundancy

Better
comprehensibility
through abstract
names

Function definitions

- Keyword "define" to differentiate between expressions

```
(define (average x y) (/ (+ x y) 2))
```

- Self-defined functions can be used in the same way as predefined functions

```
> (average 12 17)
```

```
14.5
```

```
> (average 100 200)
```

```
150
```

Function definitions

(define (FunctionName InputName1 InputName2 ...) BodyExpression)

- define
 - Keyword introduces function definition
- FunctionName
 - Name via which the function can be called
- InputName1, InputName2, ...
 - Names for the passed arguments
- BodyExpression
 - Expression that determines what the application of the function to the arguments means
 - Usually uses InputName1, InputName2, ...
 - Also "implementation" or "body" of the function

Function definitions

- Function definitions are not expressions
 - Cannot be evaluated
 - Cannot be passed as an argument

Calling user-defined functions

(FunctionName ArgumentExpression1 ArgumentExpression2 ...)

- FunctionName
 - Name of a defined function
- ArgumentExpression1, ArgumentExpression2, ...
 - Expressions to which the function is to be applied
- Calls to user-defined and predefined functions do not differ

Reduction with function definitions

- Reduction of an expression e

1. If e has the form $(f\ v_1\ \dots\ v_n)$, f is a "**primitive**" function and the
(PRIM) application of f to $v_1\ \dots\ v_n$ has the value v ,
then $(f\ e_1\ \dots\ e_n) \rightarrow v$ applies.
2. If e has the form $(f\ v_1\ \dots\ v_n)$, f is **not a primitive** function and the
(FUN) "context" contains the function definition of f :
(define ($f\ x_1\ \dots\ x_n$) e_{Body}),
then $(f\ v_1\ \dots\ v_n) \rightarrow_{eNewBody}$ applies, whereby $e_{NewBody}$ is created from
 e_{Body} by replacing all x_i with v_i ($i = 1\ \dots\ n$).
3. If e has a sub-expression e_1 in an evaluation item
(KONG) with $e_1 \rightarrow e_1'$,
then $e \rightarrow e'$ applies, whereby e' is generated from e by replacing
 e_1 with e_1' .

Reduction with function definitions

- Reduction of an expression e

- (PRIM) If e has the form $(f\ v_1\ \dots\ v_n)$, f is a "**primitive**" function and the application of f to $v_1\ \dots\ v_n$ has the value v then $(f\ e_1\ \dots\ e_n) \rightarrow v$ applies.
- (FUN) If e has the form $(f\ v_1\ \dots\ v_n)$, "context" contains the function definition $(\text{define } (f\ x_1\ \dots\ x_n) e_{Body})$, then $(f\ v_1\ \dots\ v_n)$ applies, whereby $e_{NewBody}$ is created from e_{Body} by replacing x_i with v_i ($i = 1 \dots n$).
- (KONG) If e has the form $(f\ e_1\ \dots\ e_n)$ and e_1 is in an evaluation item then $e \rightarrow e'$ applies, whereby e' is generated from e by replacing e_1 with e_1' .

This means
"predefined" or
"built-in".

Search space for all
known definitions.

Context of function definitions

- Function definitions are part of the program
- Function definitions must be found when **evaluating expressions**
- Search in context
- Context = entire program up to the current printout

Context of function definitions

- Function definitions are part of the program
- Function definitions must be found when **evaluating expressions**

Definition of functions is not an evaluation!
Therefore, function definitions may refer to functions that are defined later.







- Search in context
- Context = entire program up to the current printout

Context of function definition

- Convention: all definitions are at the beginning
- Then: Context = whole program
- This means that there is only one "global context"
- This simplifies the formal definition of the reduction rules

Example

```
(define (g y z) (+ (f y) y (f z)))
(define (f x) (* x 2))
```

$(g (+ 2 3) 4)$  Rule KONG
 $\rightarrow (g 5 4)$  Rule FUN
 $\rightarrow (+ (f 5) 5 (f 4))$  Rule KONG
 $\rightarrow (+ (* 5 2) 5 (f 4))$  Rule KONG
 $\rightarrow (+ 10 5 (f 4))$  Rule KONG
 $\rightarrow (+ 10 5 8)$  PRIM rule
 $\rightarrow 23$

Example

```
(define (g y z) (+ (f y) y (f z)))
(define (f x) (* x 2))
```

(g (+ 2 3) 4)



Rule KONG

(+ 2 3) → 5 Rule PRIM

→ (g 5 4)



Rule FUN

→ (+ (f 5) 5 (f 4))



Rule KONG

→ (+ (* 5 2) 5 (f 4))



Rule KONG

→ (+ 10 5 (f 4))



Rule KONG

→ (+ 10 5 8)



PRIM rule

→ 23

Example

```
(define (g y z) (+ (f y) y (f z)))
(define (f x) (* x 2))
```

Replace y with 5 and z with 4.

(g (+ 2 3) 4)

↪ Rule KONG

→ (g 5 4)

↪ (g 5 4) □ Rule FUN

→ (+ (f 5) 5 (f 4))

↪ Rule KONG

→ (+ (* 5 2) 5 (f 4))

↪ Rule KONG

→ (+ 10 5 (f 4))

↪ Rule KONG

→ (+ 10 5 8)

↪ PRIM rule

→ 23

Example

```
(define (g y z) (+ (f y) y (f z)))
(define (f x) (* x 2))
```

Replace x with 5.

(g (+ 2 3) 4)

↪ Rule KONG

→ (g 5 4)

↪ Rule FUN

→ (+ (f 5) 5 (f 4))

↪ Rule KONG

(f 5) □ Rule FUN

→ (+ (* 5 2) 5 (f 4))

↪ Rule KONG

→ (+ 10 5 (f 4))

↪ Rule KONG







→ (+ 10 5 8)


↪ PRIM rule

→ 23

Example

```
(define (g y z) (+ (f y) y (f z)))
(define (f x) (* x 2))
```

$(g (+ 2 3) 4)$  Rule KONG
 $\rightarrow (g 5 4)$  Rule FUN
 $\rightarrow (+ (f 5) 5 (f 4))$  Rule KONG
 $\rightarrow (+ (* 5 2) 5 (f 4))$  Rule KONG
 $\rightarrow (+ 10 5 (f 4))$  Rule KONG
 $\rightarrow (+ 10 5 8)$  PRIM rule
 $\rightarrow 23$



Example

```
(define (g y z) (+ (f y) y (f z)))
(define (f x) (* x 2))
```

Replace x with 4.

$(g (+ 2 3) 4)$ \hookrightarrow Rule KONG
 $\rightarrow (g 5 4)$ \hookrightarrow Rule FUN
 $\rightarrow (+ (f 5) 5 (f 4))$ \hookrightarrow Rule KONG
 $\rightarrow (+ (* 5 2) 5 (f 4))$ \hookrightarrow Rule KONG
 $\rightarrow (+ 10 5 (f 4))$ \hookrightarrow Rule KONG
 $\rightarrow (+ 10 5 8)$ \hookrightarrow PRIM rule
 $\rightarrow 23$

(f 4) \square Rule FUN

Example

```
(define (g y z) (+ (f y) y (f z)))
(define (f x) (* x 2))
```

(g (+ 2 3) 4) ↪ Rule KONG

→ (g 5 4) ↪ Rule FUN

→ (+ (f 5) 5 (f 4)) ↪ Rule KONG

→ (+ (* 5 2) 5 (f 4)) ↪ Rule KONG

→ (+ 10 5 (f 4)) ↪ Rule KONG

→ (+ 10 5 8) ↪ (+ 10 5 8) → 23 Rule PRIM

→ 23

Definition sequence

```
(define (g y z) (+ (f y) y (f z)))  
(define (f x) (* x 2))  
(g (+ 2 3) 4)
```

Function definition may refer to function defined later.

Definition sequence

```
(define (g y z) (+ (f y) y (f z)))
```

```
(f 4)
```

```
(define (f x) (g x x x))
```

Expression must **not access**
a function defined later.

Definition sequence

```
(define (g y z) (+ (f y) y (f z)))
```

```
(g (+ 2 3) 4)
```

```
(define (f x) (
```

What happens?

A function that refers to another function may not be used in an expression if the other function is not yet defined.

Context contains g but not f.

Tasks

What is the result of the following programs?

1. `(define (g x y) (* (f x) (f y)))`
`(define (f x) (+ x 1))`
`(g 2 3)`

- a) 12
- b) 5
- c) A mistake

2. `(define (g x y) (* (f x) (f y)))`
`(define (f x) (g x 1))`
`(define (h x) (/ x 2))`
`(h 6)`

- a) 12
- b) 3
- c) A mistake

Live Vote



<https://ilias.uni-marburg.de/vote/2053>

Data type: Images

- In DrRacket images are values
- Insert via copy/paste or insert → Insert image
- Images are literals and evaluate themselves
- Further functions for images

DEMO

Data type: Images

- In DrRacket images are values
- Insert via copy/paste or insert → Insert image
- Images are literals and evaluate themselves
- Further functions for images

> (* (image-width  image-height))

600

> (circle 10 "solid" "red")



- Etc.

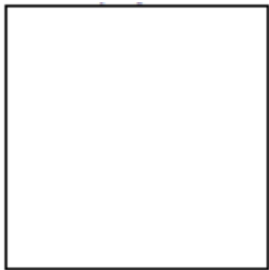
Useful functions for images

- A "Scene" allows images to be combined

Width

Height

```
>(empty-scene 100 100)
```



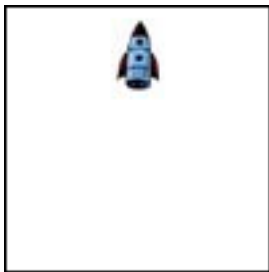
Useful functions for images

- Insert an image into a scene

From the
left

From above

>(place-image  20 (empty-scene 100 100))



Useful functions for images

- Determine the size of an image

>(image-width 
20


>(image-height 
30

Useful functions for images

- Insert an image into a scene

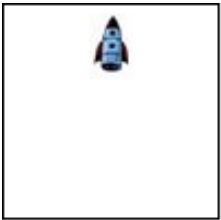
>(place-image  20 (empty-scene 100 100))

Reads like (almost) an English sentence:

place-the-image  at x-position 50 and y-position 20 into an empty-scene with width 100 and height 100.

Program with pictures

>(place-image  20 (empty-scene 100 100))



>(place-image  40 (empty-scene 100 100))



>(place-image  60 (empty-scene 100 100))



Program with pictures

```
(place-image 🚀 20 (empty-scene 100 100))  
(place-image 🚀 40 (empty-scene 100 100))  
(place-image 🚀 60 (empty-scene 100 100))
```

The only difference

- Redundancy
 - Complex
 - Changes difficult

Function definitions avoid redundancy

- Function definitions make commonality reusable

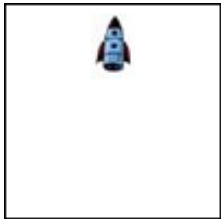
```
(define (create-rocket-scene height)  
  (place-image 50 height (empty-scene 100 100)))
```



- Function definitions make differences to parameters

Program with pictures

> (create-rocket-scene 20)



> (create-rocket-scene 40)



> (create-rocket-scene 60)



Easier to read:

create-rocket-in-scene at height 20.
create-rocket-in-scene at height 40.
create-rocket-in-scene at height 60.

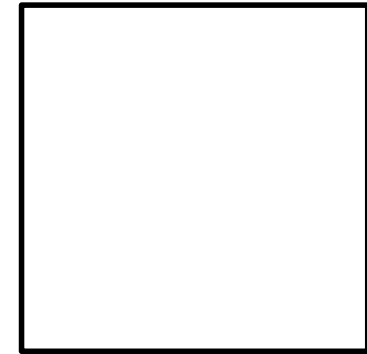
Useful functions for images

- Animate a scene

```
>(animate create-rocket-scene)  
138
```

Number of frames
until abort.

Called 28 times per
second with counter
passed as argument.



Useful functions for images

- Animate a scene

```
>(animate create-rocket-scence)  
138
```

Passing a function name as an argument. Formal meaning comes much later!