

Declarative programming

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(Programming languages and tools)

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[Script 2.4 - 2.5]

Conditional expressions

- Case-dependent functions
 - Various possible expressions
 - Selection based on condition (condition)
 - Condition: Value of an expression is **true**

Conditional expressions

```
(define (note points)
  (cond
    [(>= points 90) 1]
    [(>= points 80) 2]
    [(>= points 70) 3]
    [(>= points 60) 4]
    [(>= points 50) 5]
    [(< points 50) 6]))
```

```
> (note 95)
```

```
1
```


```
> (note 73)
```

```
3
```

```
> (note 24)
```

```
6
```

Meaning of conditional expressions

(cond  Keyword
[*eCondition*₁ *eResult*₁]
...
[*eCondition*_{*n*} *eResult*_{*n*}])

- Informal meaning
 - Evaluation of *eCondition*_{*i*} in sequence
 - For the first *i* with *eCondition*_{*i*} **true**, *eResult*_{*i*} is evaluated
 - The result is the value of the total expression

Errors with conditional expressions

- Expression for condition must return **true** or **false** (data type truth value)

```
>(cond [(+ 2 3) 4])
```

cond: question result is not true or false: 5

- One of the conditions must be **true**

```
>(cond [(< 5 3) 77]  
      [(> 2 9) 88])
```

cond: all question results were false

Errors with conditional expressions

- Note the order of the conditions
- Put stronger conditions first

```
(define (note points)
  (cond
    [(< points 50) 6]
    [(>= points 50) 5]
    [(>= points 60) 4]
    [(>= points 70) 3]
    [(>= points 80) 2]
    [(>= points 90) 1]))

> (note 95)
```

Errors with conditional expressions

- Note the order of the conditions
- Put stronger conditions first

```
(define (note points)
  (cond
    [(< points 50) 6]
    [(>= points 50) 5]
    [(>= points 60) 4]
    [(>= points 70) 3]
    [(>= points 80) 2]
    [(>= points 90) 1]))
```

> (note 95)
5

(>= points 50) → true
However, this result is
undesirable

Reduction with conditional expressions

- Reduction of an expression e

1. (PRIM) If e has the form $(f\ v_1\ \dots\ v_n)$, f is a **"primitive"** function and the application of f to $v_1\ \dots\ v_n$ has the value v , then $(f\ e_1\ \dots\ e_n) \rightarrow v$ applies.
2. (FUN) If e has the form $(f\ v_1\ \dots\ v_n)$, f is **not a primitive** function and the "context" contains the function definition of f :
 $(\text{define } (f\ x_1\ \dots\ x_n)\ e_{\text{Body}})$,
 then $(f\ v_1\ \dots\ v_n) \rightarrow_{e_{\text{NewBody}}}$ applies, whereby e_{NewBody} is created from e_{Body} by replacing all x_i with v_i ($i = 1\ \dots\ n$).
3. (COND-false) If e has the form $(\text{cond } [\text{false } e_1] [e_2\ e_3] \dots [e_{n-1}\ e_n])$, then $(\text{cond } [\text{false } e_1] [e_2\ e_3] \dots [e_{n-1}\ e_n]) \rightarrow (\text{cond } [e_2\ e_3] \dots [e_{n-1}\ e_n])$ applies.
4. (COND-true) If e has the form $(\text{cond } [\text{true } e_1] [e_2\ e_3] \dots [e_{n-1}\ e_n])$, then $(\text{cond } [\text{true } e_1] [e_2\ e_3] \dots [e_{n-1}\ e_n]) \rightarrow e_1$
5. (KONG) If e has a sub-expression e_1 in an evaluation item^{*)} with $e_1 \rightarrow e_1'$, then $e \rightarrow e'$ applies, whereby e' is generated from e by replacing e_1 with e_1' .

Reduction with conditional expressions

- Reduction of an expression e

1. (PRIM) If e has the form $(f\ v_1\ \dots\ v_n)$, f is a **"primitive"** function and the application of f to $v_1\ \dots\ v_n$ has the value v ,
then $(f\ e_1\ \dots\ e_n) \rightarrow v$.
2. (FUN) If e has the form $(\text{define } (f\ x_1\ \dots\ x_n)\ e_{\text{Body}})$, then $(f\ v_1\ \dots\ v_n) \rightarrow v$ if v is the value of e_{Body} from e_{Body} by replacing all x_i with v_i ($i = 1\ \dots\ n$).

*) In an expression of the form $(\text{cond } [e_0\ e_1]\ \dots\ [e_{n-1}\ e_n])$ the "context" contains only e_0 an evaluation item.
3. (COND-false) If e has the form $(\text{cond } [\text{false}\ e_1]\ [e_2\ e_3]\ \dots\ [e_{n-1}\ e_n])$, then $(\text{cond } [\text{false}\ e_1]\ [e_2\ e_3]\ \dots\ [e_{n-1}\ e_n]) \rightarrow (\text{cond } [e_2\ e_3]\ \dots\ [e_{n-1}\ e_n])$ applies.
4. (COND-true) If e has the form $(\text{cond } [\text{true}\ e_1]\ [e_2\ e_3]\ \dots\ [e_{n-1}\ e_n])$, then $(\text{cond } [\text{true}\ e_1]\ [e_2\ e_3]\ \dots\ [e_{n-1}\ e_n]) \rightarrow e_1$.
5. (KONG) If e has a sub-expression e_1 in an evaluation item*) with $e_1 \rightarrow e_1'$, then $e \rightarrow e'$ applies, where e' is generated from e by replacing e_1 with e_1' .

Example: Reduction

(note 83)

→ (cond [(>= 83 90) 1] [(>= 83 80) 2] ...)

Rule FUN

```
(define (note points)
```

```
  (cond
```

```
    [(>= points 90) 1]
```

```
    [(>= points 80) 2]
```

```
    [(>= points 70) 3]
```

```
    [(>= points 60) 4]
```

```
    [(>= points 50) 5]
```

```
    [(< points 50) 6]))
```

Example: Reduction

(note 83)

→ (cond [(>= 83 90) 1] [(>= 83 80) 2] ...)

→ (cond [false 1] [(>= 83 80) 2] ...)

Rule KONG with rule
PRIM:

(>= 83 90) → false

```
(define (note points)
  (cond
```

```
    [(>= points 90) 1]
```

```
    [(>= points 80) 2]
```

```
    [(>= points 70) 3]
```

```
    [(>= points 60) 4]
```

```
    [(>= points 50) 5]
```

```
    [(< points 50) 6]))
```

Example: Reduction

(note 83)

→ (cond [(>= 83 90) 1] [(>= 83 80) 2] ...)

→ (cond [false 1] [(>= 83 80) 2] ...)

Rule COND-false

→ (cond [(>= 83 80) 2] ...)

(define (note points)

(cond

[(>= points 90) 1]

[(>= points 80) 2]

[(>= points 70) 3]

[(>= points 60) 4]

[(>= points 50) 5]

[(< points 50) 6]))

Example: Reduction

(note 83)

→ (cond [(>= 83 90) 1] [(>= 83 80) 2] ...)

→ (cond [false 1] [(>= 83 80) 2] ...)

→ (cond [(>= 83 80) 2] ...)

→ (cond [true 2] ...)

Rule KONG with rule
PRIM

(>= 83 80) → true

(define (note points)

(cond

[(>= points 90) 1]

[(>= points 80) 2]

[(>= points 70) 3]

[(>= points 60) 4]

[(>= points 50) 5]

[(< points 50) 6]))

Example: Reduction

(note 83)

→ (cond [(>= 83 90) 1] [(>= 83 80) 2] ...)

→ (cond [false 1] [(>= 83 80) 2] ...)

→ (cond [(>= 83 80) 2] ...)

→ (cond [true 2] ...)

→ 2

Rule COND-true

```
(define (note points)
  (cond
```

```
    [(>= points 90) 1]
    [(>= points 80) 2]
    [(>= points 70) 3]
    [(>= points 60) 4]
    [(>= points 50) 5]
    [(< points 50) 6]))
```

Example: Evaluation items

```
(define (rate amount total)
  (cond [(= total 0) 100]
        [true (* (/ amount total) 100)]))
```

```
>(rate 3 9)
```

```
33.3
```

```
>(rate 3 0)
```

```
100
```

If this were an evaluation item, the second printout would fail.

Task

1. Given a program:

```
(define (my-fun x)
  (cond [(and (> x 10) (< x 20)) (- 22 x)]
        [true (+ 22 x)]))
```

What is a valid reduction of the following expression:
(my-fun 5)

- a) 27
- b) (cond [(and (> 5 10) (< 5 20)) (- 22 5)] [true (+ 22 5)])
- c) (cond [(and (> 5 10) (< 5 20)) 17] [true (+ 22 5)])
- d) (cond [(and (> 5 10) (< 5 20)) (- 22 5)] [true 27])
- e) (cond [(and (> 5 10) true) (- 22 5)] [true (+ 22 5)])
- f) (cond [true (+ 22 5)])

Live Vote



<https://ilias.uni-marburg.de/vote/IQXW>

Syntactic sugar

- Some program constructs appear frequently
- Independent of the project
- To avoid redundancy
 - "Syntactic sugar"
 - Not a new language construct!
 - Simplified notation equivalent to existing language construct

Standard case for conditional expressions

- One case must always apply
- Can be achieved by condition `true`
- Disadvantage: unclear whether "standard case" is meant or expression random/false `true`
- Syntactic sugar: Keyword `else`

```
(define (note points)
```

```
(cond
```

```
  [(>= points 90) 1]
```

```
  [(>= points 80) 2]
```

```
  [(>= points 70) 3]
```

```
  [(>= points 60) 4]
```

```
  [(>= points 50) 5]
```

```
  [else 6]))
```

Standard case for conditional expressions

- Advantage of else
 - Intention of the programmer explicit
 - Can be used by programming language
 - Standard case is only used if all other cases are not fulfilled
 - else can only be used in the latter case

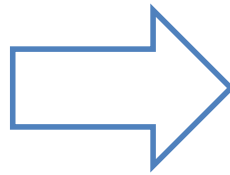
```
>(cond [(> 3 2) 5]  
        [else 7]  
        [(< 2 1) 13])
```

*cond: found an else clause that isn't the last clause
in its cond expression*

Transformation for others

- Syntactic sugar adds nothing new
- Abbreviation only
- Therefore no reduction rules, but transformation

(cond [e_0 e]₁
 [e_2 e]₃
 ...
 [else e_n])



(cond [e_0 e]₁
 [e_2 e]₃
 ...
 [true e_n])

else becomes
true.

Only one condition

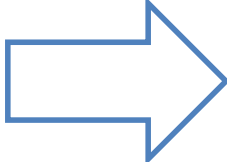
- With cond expressions, the evaluation depends on any number of conditions
- Often only one condition is relevant ("either one or the other")
- Syntactic sugar: if
- Advantage: better readability

```
(define (frost? temperature)
  (if (< temperature 0) true false))

> (frost? -5)
true
```

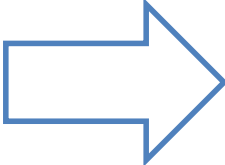
if-expressions

- Transformation

$(\text{if } e_{\text{Condition}} \ e \ e)_{\text{Then Else}}$  $(\text{cond } [e_{\text{Condition}} \ e]_{\text{Then}} [e_{\text{Else}} \ e_{\text{Else}}])$

if and cond

- if and cond are equivalent
- This means that every cond expression can also be transformed into an if expression

<pre>(cond [e₀ e₁] [e₂ e₃] ... [e_{n-2} e_{n-1}] [else e_n])</pre>		<pre>(if e₀ e₁ (if e₂ e₃ (... (if e_{n-2} e_{n-1} e_n) ...)))</pre>
--	---	--

Several conditions lead to nested if expressions. A cond expression is easier to read here.

Definition of constants

- Previously: user-defined functions
- We can also define constants

```
> (define B 42)
```

```
> B
```

```
42
```

From here on, B can be used as an expression and corresponds to the value 42.

- Constant definitions do not result in a value
- However, may be specified in the interaction window

Constant definitions

(define ConstantName BodyExpression)

- define
 - Keyword introduces constant and function definition
- ConstantName
 - Name via which the constant can be used
- BodyExpression
 - Expression that determines the value of the constant
 - Evaluated as soon as the constant is defined

Constant definitions

- When reading in the constant definition, DrRacket immediately evaluates the body expression
- This value is used later for all occurrences of the constants
- Is that allowed?

```
(define A (+ B 1))
```

```
(define B 42)
```

```
> A
```

Constant definitions


- When reading in the constant definition, DrRacket immediately evaluates the body expression
- This value is used later for all occurrences of the constants
- Is that allowed?

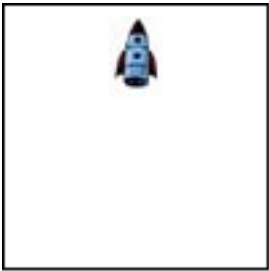
```
(define A (+ B 1))  
(define B 42)  
> A
```

No!

B is used here before its definition

Magic Numbers

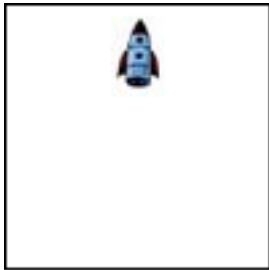
- `>(place-image  20 (empty-scene 100 100))`



Why 50? Does it
always have to be
50?

Magic Numbers

- `>(place-image  20 (empty-scene 100 100))`




Why 50? Does it
always have to be
50?

The number 50 has a certain
meaning here!
The horizontal center of the scene


- The meaning of a number cannot be seen
- But it is there, which is why we speak of "magic numbers"

Magic Numbers

- `>(place-image  20 (empty-scene 150 100))`

Width is changed. What does this mean?


Magic Numbers

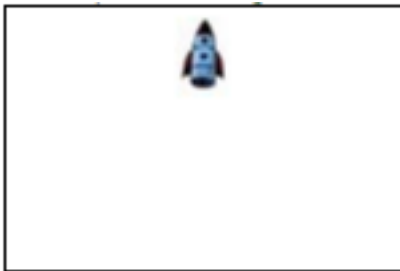
- `>(place-image  20 (empty-scene 150 100))`

The horizontal center depends on the width.

Width is changed. What does this mean?

Magic Numbers

- Avoid magic numbers with constants
- (define WIDTH 150)
- (define CENTER (/ WIDTH 2))
- >(place-image  CENTER 20 (empty-scene WIDTH 100))



Meaning of function and constant definitions

- Function definition
 - Function names, parameters and body expression are registered
 - **No evaluation**
- Constant definition
 - Body expression is **evaluated**
 - Constant name and value of the body expression are registered

Reduction with constants

- Reduction of an expression e
 1. If e has the form $(f\ v_1 \dots v_n)$, f is a **"primitive"** function and the application of f to $v_1 \dots v_n$ has the value v ,
(PRIM) then $(f\ e_1 \dots e_n) \rightarrow v$ applies.
 2. If e has the form $(f\ v_1 \dots v_n)$, f is **not a primitive** function and the "context" contains the function definition of f :
(FUN) $(\text{define } (f\ x_1 \dots x_n)\ e_{\text{Body}})$,
then $(f\ v_1 \dots v_n) \rightarrow_{e_{\text{NewBody}}}$ applies, whereby e_{NewBody} is created from e_{Body} by replacing all x_i with v_i ($i = 1 \dots n$).
 3. If e has the form $(\text{cond } [\text{false } e_1] [e_2\ e_3] \dots [e_{n-1}\ e_n])$,
(COND-false) then $(\text{cond } [\text{false } e_1] [e_2\ e_3] \dots [e_{n-1}\ e_n]) \rightarrow (\text{cond } [e_2\ e_3] \dots [e_{n-1}\ e_n])$ applies.
 4. If e has the form $(\text{cond } [\text{true } e_1] [e_2\ e_3] \dots [e_{n-1}\ e_n])$,
(COND-true) then $(\text{cond } [\text{true } e_1] [e_2\ e_3] \dots [e_{n-1}\ e_n]) \rightarrow e_1$
 5. If e is the symbol c and the "context" contains the constant definition of
(CONST) c :
 $(\text{define } c\ e_{\text{Body}})$ with $e_{\text{Body}} \rightarrow^* v$,
then $e \rightarrow v$
 6. If e has a sub-expression e_1 in an evaluation position with $e_1 \rightarrow e_1'$,
(KONG) then $e \rightarrow e'$ applies, whereby e' is generated from e by replacing e_1 with e_1' .

Meaning of a program

- Program

- Sequence of expressions, constant definitions and function definitions ("program elements")
- Any order

According to convention:
Definitions before expressions

- Context

- Regarding a program element
- Set of all function and constant definitions up to the program element

Meaning of a program

- Program is evaluated from left to right and from top to bottom

If the next program element is ...

- ... an expression, it is evaluated according to the reduction rules.
- ... a function definition, then this is added to the context
- ... a constant definition, the body expression is evaluated according to the reduction rules and the constant with this value is added to the context

No statement about the order of reductions but about the evaluation of expressions in the sequence.

Tasks

What is the result of the following programs?

1.

```
(define (g x) (* F (f x)))  
(define (f x) (+ x 1))  
(define F 4)  
(g 2)
```

- a) 12
- b) 5
- c) A mistake

2.

```
(define A 42)  
A
```

- a) 42
- b) A mistake

3.

```
(define A (* B 2))  
(define B 3)  
(A)
```

- a) 6
- b) 0
- c) A mistake