Probability density function of the energy spectrum of fission neutrons

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14 November 2019

Monte Carlo Methods and Simulations
in Nuclear Technology (SH2704)
KTH Royal Institute of Technology

Problem definition

Find the analytical equation giving the probability density function (pdf) of the energy spectrum of fission neutrons.

Calculate using a deterministic method:

1. the most probable energy value,
2. the mean value,
3. the variance,
4. the standard deviation of the energy of the fission neutrons.

Watt Fission Spectrum

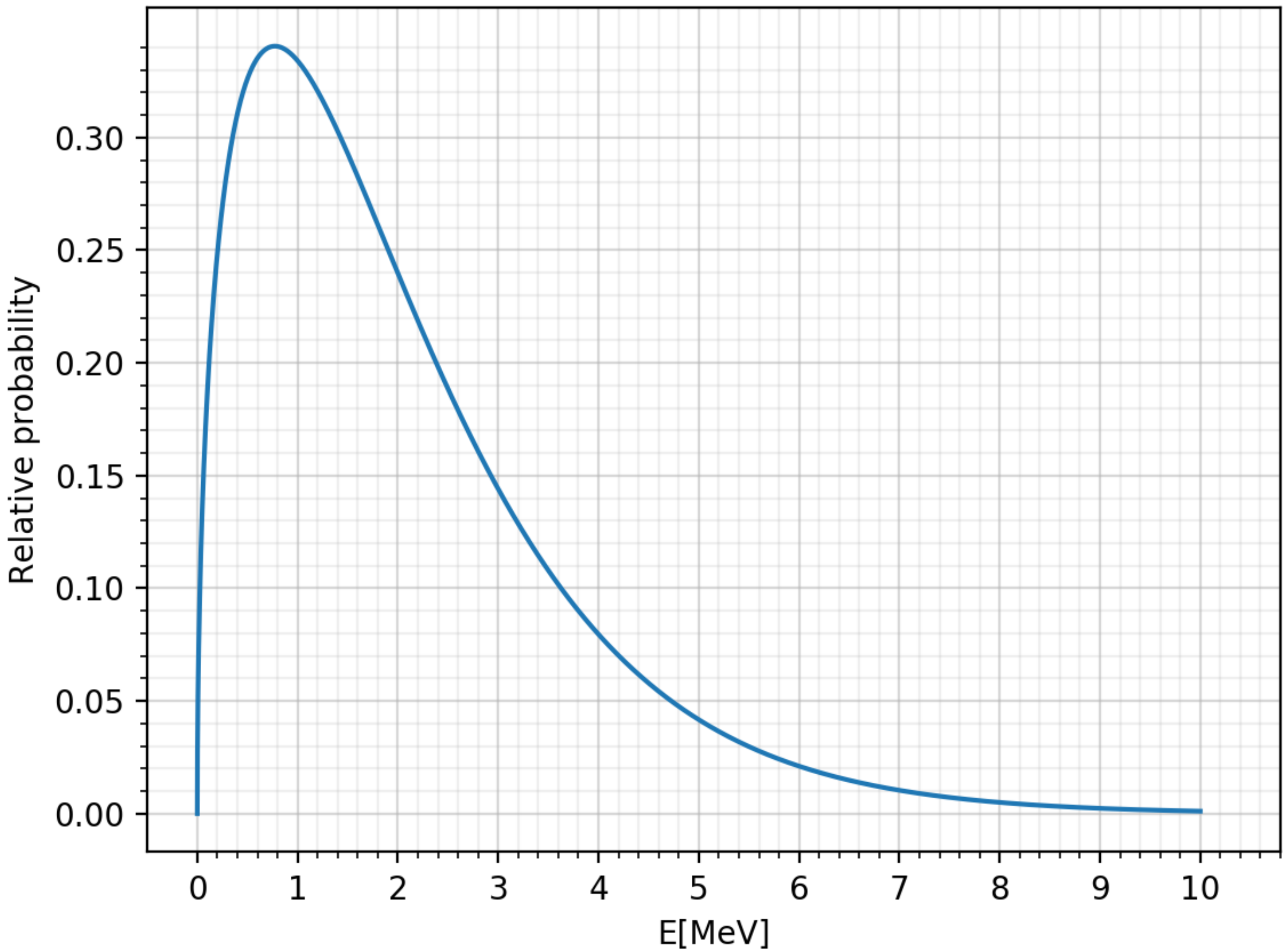
The analytical expression of the Watt Fission Spectrum is:

$$\chi(E) = C\, e^{-E/a} \sinh(\sqrt{bE}),$$

Where a and b are constants that depend on the nuclide and C is the normalization constant. This study was performed using the constants corresponding to U-233 for thermal fissions which are[1]:

$$a = 0.977;\, b = 2.546.$$

Watt Fission Spectrum



Solution approach

1. The most probable energy corresponds to:

$$Max[\chi(E)] \rightarrow \frac{d\chi(E)}{dE} = 0,$$

where the derivative can be expressed as:

$$\frac{d\chi(E)}{dE} = C\,\frac{e^{-E/a}\left[ab\cosh\left(\sqrt{bE}\right) - \sqrt{bE}\sinh\left(\sqrt{bE}\right)\right]}{2a\sqrt{bE}}.$$

2. The mean value can be obtained as:

$$\mathrm{E}[E] = m_E = \int_0^{\infty} E\,\chi(E)\,dE.$$

Solution approach

3. The variance can be calculated as:

$$\mathrm{Var}[E] = \mathrm{E}[E^2] - m_E^2 = \int_0^\infty E^2 \chi(E)\, dE \; - m_E^2.$$

4. The standard deviation can be obtained as:

$$\sigma = \sqrt{\mathrm{Var}[E]}.$$

Solution implementation

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# Functions definition

watt = lambda var,paramA,paramB,paramC: paramC * np.exp(-var/paramA) * np.sinh(np.sqrt(paramB*var))

x = lambda var: var

x2 = lambda var: var**2

funOp = lambda var,paramA,paramB,paramC,fun1,fun2: fun1(var,paramA,paramB,paramC) * fun2(var)

normF = lambda paramA,paramB,paramC,fun : paramC/quad(fun, 0, np.inf, args=(paramA,paramB,paramC))[0]
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# Main

c = normF(a,b,c,watt) #Normalization

maxE = opt.fminbound(lambda x: -watt(x,a,b,c), 0, 20) # Most probable energy

meanE = quad(funOp, 0, np.inf, args=(a,b,c,watt,x))[0] # Mean energy

varE = quad(funOp, 0, np.inf, args=(a,b,c,watt,x2))[0] - meanE**2 # Variance

stdDvE = np.sqrt(varE) # Standard deviation
```

Results

Parameter	Value
Normalization constant C	0.39320
Most probable energy	0.77376 MeV
Mean value	2.07306 MeV
Variance	2.61896 MeV^2
Standard deviation	1.61832 MeV

References

[1] X-5 Monte Carlo Team. “MCNP — A General Monte CarloN-Particle Transport Code, Version 5”. Volume I: Overview and Theory. 2003.

Thank you for your attention

Tack för er uppmärksamhet