HW #1 - Due Friday, August 30, at 9am

This homework is a brief review of the mathematical concepts that you will need for this class, which you should have learned in high school or a class like MT260. These will not be significantly re-taught in this course; starting next week, I will assume that you understand them completely. If you find yourself unable to solve some of these problems, it is important that you seek help early. If you find that you don't know how to approach any of these problems at all, you may be lacking some of the prerequisite math skills needed for this course, and we should discuss your math background.

Remember, where relevant, you must show your work. When solving any equations in this course, you must solve algebraically *before* plugging in numbers (if you don't know what this means, ask). Numbers that should have units must have those units for your answer to be correct.

1. Consider the following equation, in which each letter is an algebraic variable:

$$\frac{\left(q^{-3}\right)(B+A)}{R(gC+Dg)} = \frac{\frac{1}{R}}{\frac{q}{(A+B)}}.$$

   Solve this equation for C. Your final answer must be fully simplified.

2. Consider the two right triangles show in Figure **??**, which share a side of length b. The variable a refers to the combined length of the bottom sides, and θ is the indicated angle. The figure is **not** drawn to scale.

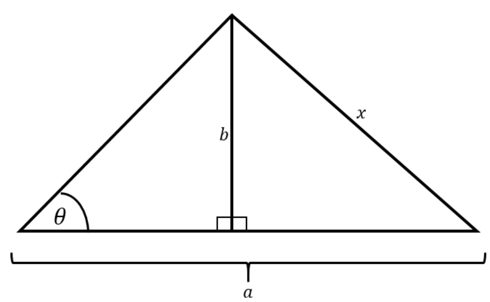

Figure 1: Two right triangles that share a side.

   (a) Solve algebraically for the unknown length x in terms of the known quantities θ, a, and b only. (It may help you solve the problem to assign algebraic names to triangle segments that I haven't named, however, your final result should only contain the variables indicated.)

   (b) If $a = 20$, $b = 12$, and $\theta = 40°$, what is x?

3. Units of measurement are very important in science. Equations that we use to describe reality must have consistent units (i.e., each added or subtracted term must work out to have the same units). For example, a length can never equal a time, and you can't add a mass to a speed. Just like algebraic variables, however, units can be multiplied and cancelled. Consider the simple equation $q = wy^2$. Assume that q is a speed in meters per second (i.e., m/s, a distance unit divided by a time unit). Also assume that y has units of seconds squared (i.e., s^2, a time unit squared). What units must w have for the equation to make sense? Explain and/or show how you know.

4. Similarly, converting between units can be very important, and it can be done by canceling units. For example, suppose that I want to convert 66 feet/second into miles per hour. If I know that there are 5280 feet in a mile, 60 seconds in a minute, and 60 minutes in an hour, then I can convert the quantity by multiplying it by those conversion factors in such a way that they cancel:

$$\left(\frac{66\ \text{ft}}{1\ \text{s}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right)\left(\frac{60\ \text{min}}{1\ \text{hr}}\right)\left(\frac{1\ \text{mile}}{5280\ \text{ft}}\right) = 45\ \text{mile/hr} \tag{1}$$

I can do this because each conversion factor is equal to one, and multiplying by one does not change anything. Notice that if you cancel all the units that you can, you are left with the desired units. The numbers are evaluated with normal arithmetic.

Now you try, except I'm going to make up some fictional units for you to work with: There are 12 grees in a zool. There are 5 grees in a blob. There are 2 grees in 5 klopps. There are 4 glints in a filn.

(a) Convert the quantity 7 glint/klopp (i.e., 7 glints-per-klopp) into filn/zool (i.e., filns-per-zool). As always, show your work.

(b) Often we use prefixes for scientific units. For example, there are one-hundred **centi**meters in a meter. If you need to review these prefixes, there is a handy chart on Wikipedia: `https://en.wikipedia.org/wiki/Metric_prefix`. The prefix **kilo-** means a factor of 10^3. For example, a **kilo**meter is 10^3 meters or 1000 meters. Convert your answer from part (a) into kilofiln/zool (i.e., kilofilns-per-zool). (Hint: Check your answer for plausibility. Should the number of kilofilns-per-zool be bigger or smaller than the number of filns-per-zool? If someone is going 1 foot-per-hour, are they going more or less feet-per-hour compared to miles-per-hour?)

5. Consider the following equation:

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

You don't yet need to fully understand this equation, but you will by the end of the semester. Roughly speaking: The left-hand side is the expression for a centripetal force, i.e., the force needed to keep something moving in circular motion (m is the mass of the moving object, v is its speed, and r is the radius of the circle it traces). The right-hand side is the expression for the strength of a gravitational force (G is a constant and M is the mass of another object). I've set up the equation this way because this describes the situation of an *orbit*. An orbit is the situation in which one object is moving in a circle around another object, and the force of gravity is what is making that circular motion happen. Put anopther way, an orbit is what you get when the centripetal force is equal to the graviational force (which is all this equation says!). Let's use this to figure out the speed of an object in an orbit.

(a) Solve algebraically for speed v.

(b) If $G = 6.67 \times 10^{-11}\ \text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}$, $M = 5.972 \times 10^{24}$ kg, and $r = 6787$ km, then what is the value of v (including units)? **Express your answer in scientific notation.** This is roughly the situation for the International Space Station orbiting Earth! (Hint: Remember that units must be consistent in any equation. Which given value should you convert?)

(c) Look at your answers and think about what they mean. Does the size or mass of the ISS affect its orbit? Explain how you know.

6. The following equation defines a particular line:

$$y = \frac{2x - 6}{3} \tag{2}$$

(a) By looking at this equation, what are the values of the slope and the y-intercept?

(b) Sketch a graph of the line without using a calculator (or any other device) to help you. Your sketch should have labeled x- and y-axes, and their scales should be clearly indicated with labeled tick marks. Make your sketch large enough to show the points where the line hits both the x- and y-axes.