Counting: Binary and Hexadecimal

November 19

Administrative Notes

Schedule for rest of semester:

- Final Exam Thursday, December 12, 6 - 8 pm, MEYR 030

A quick detour: Python Methods and Functions

(No, this won't be on the final :-)

```
list1 = [1,2,3,4]
new_list = list1.extend([5,6,7,8])
print(l1)
[1,2,3,4,5,6,7,8]
print new_list
```

WHAT??!!!

The python method returns None. Just like a function can return None. So new_list gets that value, None.

How does a “method” differ from a function?

A method is applied directly to an object and operates directly on that object. It can result in a modification to that object.

The method also returns a value, which can be None.

A function does not operate on an object - it doesn’t change the parameters passed to it. Instead it just returns one or more values. (Ignore those mutable variables behind the curtain)

Object - “a unique instance of a data structure defined by its class”

Why does list.extend() return None?

Because that’s the way the author of that code wanted it to work, and the rest of us use the code as written.

If you want to write your own version of list.extend() that returns a different value, Python lets you do it.

Now on to tonight’s lecture material

As humans, we tend to count things by powers of 10: 10, 100,1000,..

We have only ten symbols, or digits, used to count

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Why? Because most humans have ten digits - fingers, or toes - and that made it easy to keep track of things

While that’s the best way for humans to count, it’s not the best way for machines to count

Why not?

Most computers (and similar machines) are made up of electrical switches.

Electrical switches have two states: ***ON*** and ***OFF***

We can say that if something is ON it has the value 1 and if it is OFF it has the value 0.

This is called “binary” numbering. We only need two digits, 0 and 1, to represent all numbers, using powers of 2. So, there are only two binary digits

“***Bi***nary digi***t***” was a pain to say all the time, so it got shortened to … “bit”

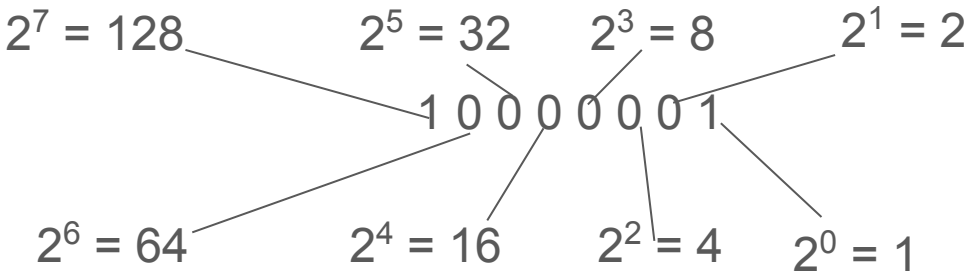
Binary math

A binary number

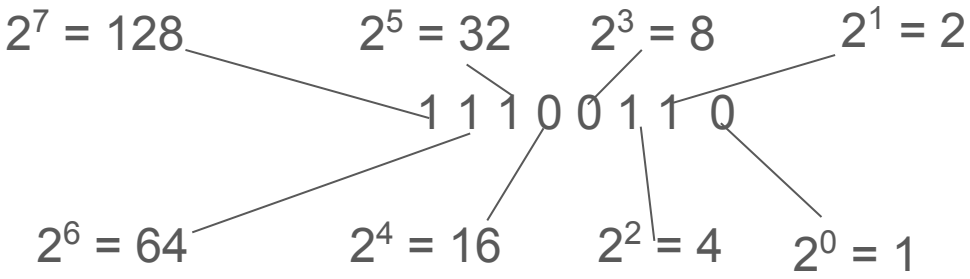
1 0 0 0 0 0 0 1

What's it's value in human-speak i.e. decimal?

Places mean the same as in decimal - the "least significant" is on the right and each place to the left is one power of two greater



Another example



Binary Math - just like decimal math (mostly)

Addition: start on the right; add the numbers in the column and carry when needed

$$\begin{array}{rr} & 1010 \\ + & 110 \\ \hline & 10000 \end{array}$$

Subtraction: start on the right; borrow from the next place over when needed

Multiplication: go column by column; add products

So how do you convert from decimal to binary?

Suppose I gave you the decimal number 10,000,001 and asked you to tell me what it is in binary?

You have to subtract powers of two from it until you get zero.

Huh?

What is the largest power of two that is less than 10,000,001?

n	2^n	n	2^n	n	2^n
1	2	6	64	22	4,194,304
2	4	10	1024	23	8,388,608
3	8	15	32768	24	16,777,216
4	16	20	1,048,576		
5	32	21	2,097,152		

Converting decimal to binary:

10,000,001 decimal: write down 1 in the 2^{23} column, subtract 8,388,608, from 10,000,001 = 1,611,393. The largest power of 2 that's less than that is 2^{20}. Write down zeroes in the next two columns, and then put a 1 in the 2^{20} column. Subtract 1,611,393 - 1,048,576 = 562,817.

Keep doing this, and you eventually wind up with:

100110001001011010000001

Awkward! That's big number and hard to deal with

Luckily, there's another numbering system: base 16, or "hexadecimal"

- "hex" for short

We need 16 different symbols to represent hexadecimal "digits"

We have 0,1,2,3,4,5,6,7,8,9 - that's 10. We need six more. So we use the first six letters of the alphabet.

A in hex = 10 in decimal; B = 11; C = 12; D = 13; E = 14 and F = 15. That's all we need, because 16 in decimal is 10 in hex. We go by powers of 16

Converting Decimal to Hex

The algorithm is similar to binary

Fortunately for us, there's a built-in function in Python3 that does the conversion for us

hex(int) takes a decimal integer value and returns its hexadecimal value

Converting binary to hexadecimal

This is actually easier - and more important

Four binary digits - four bits - together represent one hexadecimal digit, because four bits can represent exactly 16 different values

So you simply replace four bits with their hex equivalent

1001 1000 1001 0110 1000 0001

Start from the left - 1001 = 9 hex; 1000 = 8 hex; 1001 = 9 hex; 0110 = 6 hex; 1000 = 8 hex; 0001 = 1 hex. So the binary string above, converted into hex, is:

989681

Representation and examples

In python - and most other languages - hex numbers are prefaced with “0x” to indicate that they are NOT decimal numbers

Let’s use the built-in hex() function to do some conversions