# Non-Base 10 numbering systems: Binary and Hexadecimal

April 27, 2020

# Administrative Announcements

Lab 10 released tonight - due Thursday night

Project 3 will be released on Wednesday (April 29)

Project 2 due tonight - submit cmsc201 PROJ2

Homework 6 due tonight - submit cmsc201 HW6

# Now on to today’s lecture material

As humans, we tend to count things by powers of 10: 10, 100,1000,..

We have only ten symbols, or digits, used to count

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Why? Because most humans have ten digits - fingers, or toes - and that made it easy to keep track of things

While that’s the best way for humans to count, it’s not the best way for machines to count

# Why not?

Most computers (and similar machines) are made up of electrical switches.

Electrical switches have two states: ***ON*** and ***OFF***

We can say that if something is ON it has the value 1 and if it is OFF it has the value 0.

This is called “binary” numbering. We only need two digits, 0 and 1, to represent all numbers, using powers of 2. So, there are only two binary digits

“***Bi***nary digi***t***” was a pain to say all the time, so it got shortened to … “bit”

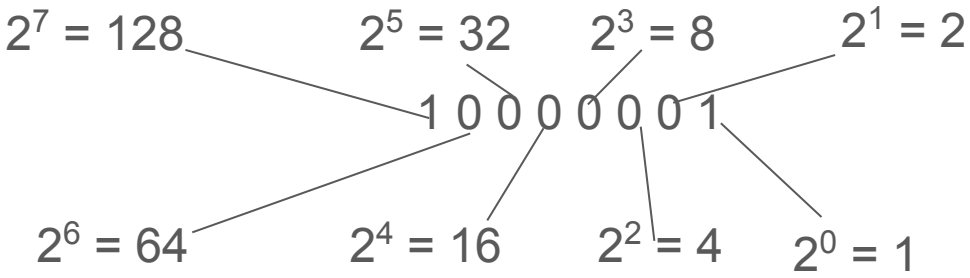
***There are 10 types of people in the world - those who understand binary and those who don't.***

A binary number

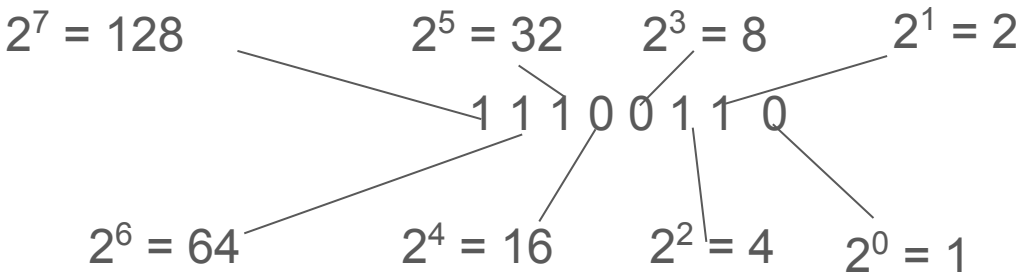
1 0 0 0 0 0 0 1

What's it's value in human-speak i.e. decimal?

Places mean the same as in decimal - the "least significant" is on the right and each place to the left is one power of two greater



# Another example



# Binary Math - just like decimal math (mostly)

Addition: start on the right; add the numbers in the column and carry when needed

$$\begin{array}{rr} & 1010 \\ + & 110 \\ \hline & 10000 \end{array}$$

Subtraction: start on the right; borrow from the next place over when needed

1010

- 101

101

Multiplication: go column by column; add products

# So how do you convert from decimal to binary?

Suppose I gave you the decimal number 10,000,001 and asked you to tell me what it is in binary?

You have to subtract powers of two from it until you get zero.

Huh?

What is the largest power of two that is less than 10,000,001?

| n | $2^n$ | n | $2^n$ | n | $2^n$ |
|---|---|---|---|---|---|
| 1 | 2 | 6 | 64 | 22 | 4,194,304 |
| 2 | 4 | 10 | 1024 | 23 | 8,388,608 |
| 3 | 8 | 15 | 32768 | 24 | 16,777,216 |
| 4 | 16 | 20 | 1,048,576 | | |
| 5 | 32 | 21 | 2,097,152 | | |

# Converting decimal to binary:

10,000,001 decimal: write down 1 in the $2^{23}$ column, subtract 8,388,608, from 10,000,001 = 1,611,393. The largest power of 2 that's less than that is $2^{20}$. Write down zeroes in the next two columns, and then put a 1 in the $2^{20}$ column. Subtract 1,611,393 - 1,048,576 = 562,817.

Keep doing this, and you eventually wind up with:

100110001001011010000001

# Fortunately, Python makes this somewhat easier

To indicate that a number represents a binary number, preface it with “0b”

```
x = 0b110111011
```

Tells python you mean this to be the binary number, NOT the decimal number 110,111,011

To convert decimal numbers to Binary, use the built-in Python function “bin”.

```
i = 54

bin_i = bin(i)
```

As before, we can convert (almost) anything to an integer - base 10 - by using int()

# Awkward!  That's big number and hard to deal with

Luckily, there's another numbering system: base 16, or "hexadecimal"

- "hex" for short

We need 16 different symbols to represent hexadecimal "digits"

We have 0,1,2,3,4,5,6,7,8,9 - that's 10. We need six more.  So we use the first six letters of the alphabet.

A in hex = 10 in decimal; B = 11; C = 12; D = 13; E = 14 and F = 15.  That's all we need, because 16 in decimal is 10 in hex. We go by powers of 16

# Converting Decimal to Hex

The algorithm is similar to binary

Fortunately for us, there's a built-in function in Python3 that does the conversion for us

hex(int) takes a decimal integer value and returns its hexadecimal value

# Converting binary to hexadecimal

This is actually easier - and more important

Four binary digits - four bits - together represent one hexadecimal digit, because four bits can represent exactly 16 different values

So you simply replace four bits with their hex equivalent

1001  1000  1001 0110  1000  0001

Start from the left - 1001 = 9 hex; 1000 = 8 hex; 1001 = 9 hex; 0110 = 6 hex; 1000 = 8 hex; 0001 = 1 hex.  So the binary string above, converted into hex, is:

0x989681

0000 =  0

0001 = 1

0010 = 2

1010 = A

1011 = B

1100 = C

1101 = D

1110 = E  1111 = F

# Representation and examples

In python - and most other languages - hex numbers are prefaced with “0x” to indicate that they are NOT decimal numbers

Let’s use the built-in hex() function to do some conversions