6) $\lim\limits_{x\to 4} \dfrac{2^x-16}{\sin \pi x} = \dfrac{0}{0} = \dfrac{2^x-16}{\pi x}$

$t = x-4, \quad x = t+4 \quad t\to 0$

$\dfrac{2^{t+4}-16}{\sin(\pi t+4\pi)} = 16 \lim\limits_{t\to 0} \dfrac{t(2^t-1)}{t(\sin \pi t \cdot \cos 4\pi + \cos}$

$= 16 \cdot \ln 2 \lim \dfrac{t \cdot \pi}{\pi \sin \pi t} \overset{=1}{=} \dfrac{16 \cdot \ln 2}{\pi}$

7) $\lim\limits_{x\to\pi} \dfrac{\cos\frac{x}{2}}{e^{\sin x} - e^{\sin 4x}} = \left(\dfrac{0}{0}\right) = \{ \sin 2\alpha = 2\sin\alpha \cdot \cos\alpha =$

$= \dfrac{\cos\frac{x}{2}}{e^{\sin x} - e^{4\sin x \cos x}} = \dfrac{\cos\frac{x}{2}}{e^{2\sin\frac{x}{2}\cos\frac{x}{2}} - e^{-8\cos\frac{x}{2}\sin\frac{x}{2}}} =$

$= \lim\limits_{x\to\pi} \dfrac{\cos\frac{x}{2}}{e^{2\cos\frac{x}{2}} - e^{-8\cos\frac{x}{2}}} = \lim\limits_{t\to 0} \dfrac{t}{e^{2t} - e^{-8t}} =$

$t = \cos\frac{x}{2} \quad t\to 0$

$= \dfrac{t}{e^{2t}(1-e^{-10t})} = -\lim\limits_{t\to 0} \dfrac{t}{e^{10t}-1} = -\dfrac{1}{10} \lim \dfrac{-10t}{e^{-10t}-1} =$

$= \dfrac{1}{10}$

8) $\lim\limits_{x \to 0} \dfrac{e^{2x} - e^{x}}{\sin 3x - \sin 5x} = \left(\dfrac{0}{0}\right) = \dfrac{e^{x}(e^{x} - 1)}{2\sin(-x)\cos \underset{=1}{4x}} =$

$= -\dfrac{1}{2} \lim\limits_{x \to 0} \dfrac{(e^{x}-1)}{\underset{\sim x}{\sin x}} \overset{=1}{=} -\dfrac{1}{2}$

9) $\lim\limits_{x \to 1} \dfrac{1-x}{\log_2 x} = \left(\dfrac{0}{0}\right) = \lim\limits_{t \to 0} \dfrac{1 - t - 1}{\log_2 (t+1)} = - \cancel{\log}$

$t = x - 1 \quad x = t + 1$

$= - \lim \dfrac{t}{\log_2 (t+1)} = - \ln 2$

$\left(\dfrac{1}{\ln 2}\right)^{-1}$

10) $\lim\limits_{x \to 0} (\cos x)^{\frac{1}{\ln(1+\sin^2 x)}} = 1^{\infty} = \left((1 + (\cos x - 1))^{\frac{1}{\cos x - 1}}\right)^{\frac{\cos x - 1}{\ln(1+\sin^2 x)}} =$

$= e^{\lim\limits_{x \to 0} \frac{\cos x - 1}{\ln(1+\sin^2 x)}} = \dfrac{-2\sin^2 \frac{x}{2}}{\ln(1+\sin^2 x)} \overset{=1}{\dfrac{\sin^2 x}{\sin^2 x}} =$

$= \cancel{2} \; e^{-2\lim\limits_{x \to 0} \frac{\sin^2 \frac{x}{2}}{\sin^2 x}} = e^{-\lim \frac{x^2}{4x^2}} = e^{-\frac{1}{2}} = \dfrac{1}{\sqrt{e}}$

$\sin \dfrac{x}{2} \sim \dfrac{x}{2}, \quad \sin x \sim x$

11) $\lim\limits_{x\to 0} (\sin(x+2))^{\frac{3}{3+x}} = 1 = \sin 2$

12) $\lim\limits_{x\to \frac{\pi}{2}} (\sin x)^{6\,tg\,x\,\cdot\,tg\,3x} = 1^{\infty} =$

Исп. ф. $\lim\limits_{x\to a} u(x)^{v(x)} = e^{\lim\limits_{x\to a}((u(x)-1)\cdot v(x))} =$

$= e^{\lim\limits_{x\to\frac{\pi}{2}} 6(\sin x-1)(tg\,x\cdot tg\,3x)} = \lim\limits_{x\to\pi/2} 6(\sin x-1)\,tg\,x\cdot tg\,3x =$

$= [0\cdot\infty] = 6\lim\limits_{x\to\pi/2} (\sin x-1)\underbrace{\frac{\sin x}{\cos x}}_{=+1}\underbrace{\frac{\sin 3x}{\cos 3x}}_{=-1} =$

$= -6\lim \frac{\sin x-1}{\cos x\cdot\cos 3x} = -6\lim\frac{\sin x-1}{\cos x(4\cos^3 x - 3\cos x)}$ $\overset{\shortparallel}{0}$

$= \overset{+2}{\cancel{-6}}\lim\limits_{x\to\pi/2}\frac{\sin x-1}{\cancel{\cos^2 x}\ \ 1-\sin^2 x} = -2\lim\frac{(1-\sin x)}{(1-\sin x)(1+\cancel{\cos x}\ \sin x)} =$ $\overset{\shortparallel}{1}$

$= -2\cdot\frac{1}{2} = -1 = e^{-1} = \frac{1}{e}$

13) $\lim\limits_{x\to 2}\left(\frac{\sqrt{x+2}-2}{x^2-4}\right)^{\frac{1}{x}} = \left(\frac{0}{0}\right) = \lim\limits_{x\to 2}\left(\frac{x+2-4}{(x^2-4)(\cancel{\sqrt{x^2-4}}\ \sqrt{x+2}+2)}\right)^{\frac{1}{x}} =$

$= \frac{(x-2)}{(x+2)(x-2)} = \left(\frac{x-2}{\frac{(x+2)\,4}{x}}\right)^{\frac{1}{2}} = = \frac{\cancel{-1}}{} = \left(\frac{1}{\cancel{4x+8}}\right)^{\frac{1}{2}}_{4\cdot 4} = = \sqrt{\frac{1}{16}} = \frac{1}{4}$

=

14) $$\lim_{x\to 0} \frac{\overbrace{\sqrt[3]{\overset{\to 0}{\operatorname{tg} x}}\ \operatorname{arctg}\frac{1}{x}}^{=0} + 3}{2 - \operatorname{tg}(1 + \underset{\to 0}{\sin x})} = \frac{3}{2 - \operatorname{tg} 1}$$

12.10.24

1) $$\lim_{n\to\infty} \frac{n\sqrt[3]{7n} - \sqrt[4]{81n^8-1}}{(n+4\sqrt{n})(\sqrt{n^2-5})} = \left(\frac{\infty-\infty}{\infty}\right) =$$

$$= \frac{\frac{1}{n}\cdot\sqrt{\frac{7}{n^5}} - \sqrt[4]{81-\frac{1}{n^8}}}{\left(\frac{1}{n}+1+\frac{4}{\sqrt{n}}\right)\left(\sqrt{1-\frac{5}{n^2}}\right)} = \frac{-3}{1\cdot 1} = -3$$

2) $$\lim_{n\to\infty} \frac{3^n-2^n}{3^{n-1}+2^n} = \left(\frac{\infty}{\infty}\right) = \Big|:3^n = \frac{1-\left(\frac{2}{3}\right)^n}{\frac{1}{3}+\left(\frac{2}{3}\right)^n} = 3$$

$\left(\frac{2}{3}\right)^n \to 0$

3) $$\lim_{x\to\frac{5}{2}} \frac{2x^2-9x+10}{2x-5} = \frac{0}{0} \Big| :(2x-5) = \lim_{x\to 5/2} x-2 = \frac{5}{2}-2 = 0{,}5$$

4) $$\lim_{x\to-2} \frac{\sqrt[3]{x-6}+2}{x+2} = \left(\frac{0}{0}\right) = \frac{(\sqrt[3]{x-6}+2)\left(\sqrt[3]{(x-6)^2}+2\sqrt[3]{x-6}+4\right)}{(x+2)\left(\sqrt[3]{(x-6)^2}+2\sqrt[3]{x-6}+4\right)} =$$

$$= \frac{x-6+8}{(x+2)(4+4+4)} = \frac{1}{12}\lim_{x\to-2}\frac{x+2}{x+2} = \frac{1}{12}$$

5) $$\lim_{x\to 0} \frac{\sqrt{4+x}-2}{3\,\mathrm{arctg}\, x} = \frac{0}{0} = \frac{1}{3}\lim \frac{\sqrt{4+x}-2}{x} =$$

$\mathrm{arctg}\, x \sim x$

$$= \frac{4+x-4}{x(\sqrt{4+x}+2)} = \frac{1}{3}\cdot\frac{1}{4} = \frac{1}{12}$$