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Homework 4

Problem 4.1

Solution:
Let's first transform the logic circuit to logical expression:
$(A \oplus B) \cdot \overline{B \oplus C} \cdot C$
Now let's make a truth table for this expression:

A	B	C	X
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	0
1	0	1	0
0	1	1	1
1	1	1	0

From the truth table we see that the combination of A = 0, and B = C = 1 is the only combination leading to X be 1.

Problem 4.2

Solution:
Let's start with writing the equation:
$\overline{(A \cdot B) + C} \oplus (\overline{A} \cdot (B + C))$
Truth table:

A	B	C	X
0	0	0	1
1	0	0	1
0	1	0	0
0	0	1	1
1	1	0	0
1	0	1	0
0	1	1	1
1	1	1	0

And the sum of products is :
$(\overline{A} \cdot \overline{B} \cdot \overline{C}) + (A \cdot \overline{B} \cdot \overline{C}) + (\overline{A} \cdot \overline{B} \cdot C) + (\overline{A} \cdot B \cdot C)$

Problem 4.3

Solution:
a) Convert 27 to binary number:
27/2 = 13 (1)
13/2 = 6 (1)
6/2 = 3 (0)
3/2 = 1 (1)
1/2 = 0 (1)
The binary representation is 11011. Since it is positive number, the 2's complement of 8 bit is (sign is +, so first digit is 0) 00011011

b) 66 to binary:
66/2 = 33 (0)
33/2 = 16 (1)
16/2 = 8 (0)
8/2 = 4 (0)
4/2 = 2 (0)
2/2 = 1 (0)

1/2 = 0 (1)
The sign is +, so first digit is 0. Binary: 1000010. 2’s complement 01000010.

c) -18
Convert 18 to binary:
18/2 = 9 (0)
9/2 = 4 (1)
4/2 = 2 (0)
2/2 = 1 (0)
1/2 = 0 (1)
Binary: 100010. The sign is -, so we need to invert the binary representation and add 1.
Invert:
00100010 $\xrightarrow{0}$ 0011101
Add 1:
00011101 + 1 = 00011110
Make it 8 bit, first digit is 1.
2’s complement: 10011110

d) 127 :
Positive number, so only convert to binary:
127/2 = 63 (1)
63/2 = 31 (1)
31/2 = 15 (1)
15/2 = 7 (1)
7/2 = 3 (1)
3/2 = 1 (1)
1/2 = 0 (1)
Binary: 1111111. 2’s complement : 01111111

e)-127. Convert to binary, invert, add one. We know that 127 is 1111111 from previous problem.
Invert:
10000000 Add one:
10000001
Negative, so first digit is 1.
2’s complement : 10000001

f)-128. Convert to binary, invert, add one. We can find binary representation by adding one to 127, which is 01111111 + 1 = 10000000.
Invert:
01111111
Add one:
10000000
2’s complement: 10000000

g)131
Binary:
131/2 = 65 (1)
65/2 = 32 (1)
32/2 = 16 (0)
16/2 = 8 (0)
8/2 = 4 (0)
4/2 = 2 (0)
2/2 = 1 (0)
1/2 = 0 (1)
Binary: 10000011.
It is out of the range, so we cannot represent it in 8-bit 2’s complement.

h) -7.
Convert 7 to binary, invert, and add 1.
Convert:
7/2 = 3 (1)
3/2 = 1 (1)
1/2 = 0 (1)
Binary:
00000111 Invert:
11111000

Add one:
11111001
First digit is 1,2's complement 11111001

Problem 4.4

Solution:
a) 00011000 :
First digit is 0, number is positive. Convert to decimal:
$2^4 + 2^3 = 16 + 8 = 24$

b) 11110101
Negative number:
Subtract one, invert, convert to decimal, add minus.
11110101 - 1 = 11110100
Invert:
00001011
Convert to decimal:
$2^3 + 2^1 + 2^0 = 8 + 2 + 1 = 11$
Decimal: -11
c) 01011011
Convert:

$01011011 = 2^6 + 2^4 + 2^3 + 2^1 + 2^0 = 64 + 16 + 8 + 2 + 1 = 91$
d)10110110
Negative number:
Subtract
10110110 - 1 = 10110101
Invert:
01001010
Convert:
$01001010 = 2^6 + 2^3 + 2^1 = 64 + 8 + 2 = 74$
Negate: -74

e) 11111111
Negative number:
Subtract:
11111111 - 1 = 11111110
Invert:
00000001
Convert:
$1 = 2^0 = 1$
Negate:
-1
f)01101111
Positive number:
Convert:
$01101111 = 2^6 + 2^5 + 2^3 + 2^2 + 2^1 + 2^0 = 64 + 32 + 8 + 4 + 2 + 1 = 111$

g) 10000001
Negative number:
Subtract:
10000001 - 1 = 10000000
Invert:
01111111
Convert: $01111111 = 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 = 64 + 32 + 16 + 8 + 4 + 2 + 1 = 127$
Negate:
-127

h) 10000000
Negative number:
Subtract:
10000000 - 1 = 01111111
Invert:
10000000
Convert: $10000000 = 2^7 = 128$
Negate:

-128

Problem 4.5

Solution:
a) 27 + 36
Convert each digit to binary numbers:
27 = 0010 0111
36 = 0011 0110

	0010	0111	
+	0011	0110	
=	0101	1101	Add 6 to right sum
+	0000	0110	
=	0110	0011	

b) 73 + 29
73 = 0111 0011
29 = 0010 1001

	0111	0011	
+	0010	1001	
	1001	1100	More than nine, add 6 to last digit
+	0000	0110	
	1010	0010	More than nine, add 6 to first digit
+	0110	0010	
=	0001 0000	0010	

Problem 4.6

Solution:
a) 00000000 is smallest number and it is equal to 0. Biggest number would be 11111111. Convert to decimal:
$11111111_2 = 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 = 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 = 256.$
Range 0-255.
b) 1 bit for sign, and 7 bits left, so we have 2^7 numbers. If it is 2's compliment, then we have one more negative number, than positive. Therefore range is $[-2^7, 2^7 - 1]$ c)Same as a, but $[0, 2^{11}]$ d)Same as b, but $[-2^{10}, 2^{10} - 1]$ e) Same as b, but $[-2^{15}, 2^{15} - 1]$