Computer Architecture and Programming Languages
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Homework 2

Problem 2.1

Solution:

a) 777 (base 8). We just increment the last digit by one. It hits the maximum, so we set to 0 and increment the more significant one. Repeat it until we have incremented everything properly if we still need to increment then add digit from left.
777 (base 8) + 1 (base 8) $\rightarrow 77(7+1) \rightarrow 7(7+1)0 \rightarrow (7+1)00 \rightarrow = 1000$(base 8)

b) 888(base 16)+1(base 16)= 889(base 16)

c) 32007(base 8)+1(base 8)= 32008 $\rightarrow$ 32010(base 8)

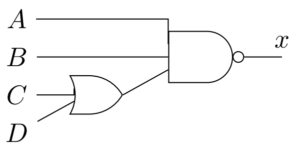
d) 32108(base 16)+1(base 16) = 31209(base 16)

e) $8BFF$(base 16)+1(base 16)= $8BF(F+1) \rightarrow 8B(F+1)0 \rightarrow 8(B+1)00 \rightarrow = 8C00$(base 16)

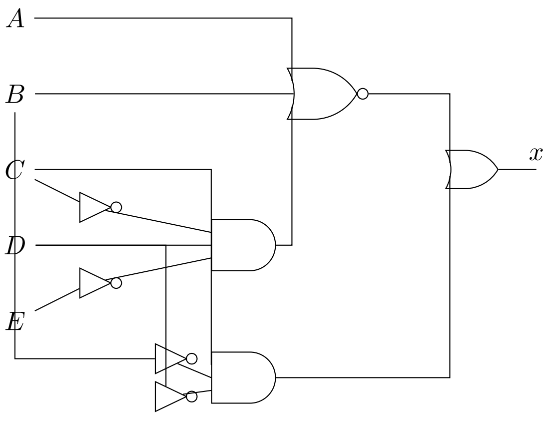
f) 1219(base 16)+1(base 16)= $121A$(base 16)

Problem 2.2

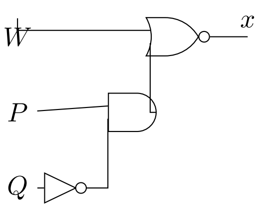
Solution:



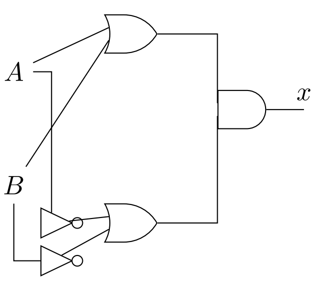
a)



b)



c)



d)

Problem 2.3

Solution:

Truth table:

M	N	Q	R
0	0	0	0
0	0	1	0
0	1	0	0
1	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0
1	1	1	1

To find the sum-of-products expression we should find DNF:

$$(\overline{M} \cdot N \cdot Q) + (M \cdot \overline{N} \cdot Q) + (M \cdot N \cdot Q) \tag{1}$$

We see common factor Q, and using Distributive rule, we can take it out:

$$Q \cdot (\overline{M} \cdot N + M \cdot \overline{N} + M \cdot N) \tag{2}$$

Now let's use it once more inside of bracket:

$$Q \cdot (\overline{M} \cdot N + M \cdot (\overline{N} + N)) \tag{3}$$

Now using compliment law, we get rid of second N in bracket.

$$Q \cdot (\overline{M} \cdot N + M) \tag{4}$$

Again using distributive equation:

$$Q \cdot ((\overline{M} + M) \cdot (M + N)) \tag{5}$$

Again, complement, and $1 * w = w$ formula brings us to

$$Q \cdot (N + M) \tag{6}$$

Check the truth table, and they correspond, so the answer is correct.

Problem 2.4

Solution:

a) First, left part:

$$X + (\overline{X} \cdot Y) \tag{7}$$

X	Y	Q
0	0	0
0	1	1
1	0	1
1	1	1

Now, right part:

$$X + Y \tag{8}$$

X	Y	Q
0	0	0
0	1	1
1	0	1
1	1	1

We can see, that they are same.

b) First, left part:

$$\overline{X} + (X \cdot Y) \tag{9}$$

X	Y	Q
0	0	1
0	1	1
1	0	0
1	1	1

Now, right part:

$$\overline{X} + Y \tag{10}$$

X	Y	Q
0	0	1
0	1	1
1	0	0
1	1	1

Problem 2.5

Solution:

a)

$$A + 1 = 1 \tag{11}$$

b)

$$A \cdot A = A \tag{12}$$

c)

$$B \cdot \overline{B} = 0 \tag{13}$$

d)

$$C + C = C \tag{14}$$

e)

$$x \cdot 0 = 0 \tag{15}$$

f)

$$D \cdot 1 = D \tag{16}$$

g)

$$D + 0 = D \tag{17}$$

h)

$$C + \overline{C} = 1 \tag{18}$$

i)

$$G + G \cdot F = G \cdot (1 + F) = G \tag{19}$$

j)

$$y + \overline{w} \cdot y = y \cdot (1 + \overline{w}) = y \tag{20}$$

Problem 2.6

Solution:

DeMorgan's Theorem:

$$\overline{(A + B)} = \overline{A} \cdot \overline{B} \tag{21}$$

Proof: First, left part:

$$\overline{(A + B)} \tag{22}$$

A	B	R
0	0	1
0	1	0
1	0	0
1	1	0

Now, right part:

$$\overline{A} \cdot \overline{B} \tag{23}$$

A	B	Q
0	0	1
0	1	0
1	0	0
1	1	0

Next theorem:

$$\overline{(A \cdot B)} = \overline{A} + \overline{B} \tag{24}$$

Proof: First, left part:

$$\overline{(A \cdot B)} \tag{25}$$

A	B	R
0	0	1
0	1	1
1	0	1
1	1	0

Now, right part:

$$\overline{A} + \overline{B} \tag{26}$$

A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0

Problem 2.7

Solution:
The sum of products expression is

$$\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot D+\overline{A}\cdot\overline{B}\cdot C\cdot D+\overline{A}\cdot B\cdot\overline{C}\cdot D+\overline{A}\cdot B\cdot C\cdot D+A\cdot\overline{B}\cdot\overline{C}\cdot\overline{D}+A\cdot B\cdot\overline{C}\cdot D+A\cdot B\cdot C\cdot D \tag{27}$$

Simplify: Applying associative and distributive rules:

$$\overline{A}\cdot(\overline{B}\cdot\overline{C}\cdot D+\overline{B}\cdot C\cdot D+B\cdot\overline{C}\cdot D+B\cdot C\cdot D)+A\cdot(\overline{B}\cdot\overline{C}\cdot\overline{D}+B\cdot\overline{C}\cdot D+B\cdot C\cdot D) \tag{28}$$

Distributive rule:

$$\overline{A}\cdot(\overline{B}\cdot(\overline{C}\cdot D+C\cdot D)+B\cdot(\overline{C}\cdot D+C\cdot D))+A\cdot(\overline{B}\cdot\overline{C}\cdot\overline{D}+B\cdot(\overline{C}\cdot D+C\cdot D)) \tag{29}$$

Distributive rule:

$$\overline{A}\cdot((\overline{B}+B)\cdot(\overline{C}\cdot D+C\cdot D))+A\cdot(\overline{B}\cdot\overline{C}\cdot\overline{D}+B\cdot(D\cdot(\overline{C}+C))) \tag{30}$$

Complement rule:

$$\overline{A}\cdot((1)\cdot(\overline{C}\cdot D+C\cdot D))+A\cdot(\overline{B}\cdot\overline{C}\cdot\overline{D}+B\cdot(D\cdot(1))) \tag{31}$$

De Morgan law and distributive rule:

$$\overline{A}\cdot((\overline{C}+C)\cdot D)+A\cdot(\overline{B}\cdot\overline{C}\cdot\overline{D}+B\cdot D) \tag{32}$$

Complement rule and de Morgan law:

$$\overline{A}\cdot D+A\cdot(\overline{B}\cdot\overline{C}\cdot\overline{D}+B\cdot D) \tag{33}$$

Since, they have identical truth tables, we did no mistake and found the simplified version of equation.

Problem 2.8

Solution:

First we generate the table for 4 variables. We fill it with combinations from sums of products.

	$\overline{C} \cdot \overline{D}$	$\overline{C} \cdot D$	$C \cdot D$	$C \cdot \overline{D}$
$\overline{A} \cdot \overline{B}$	0	1	1	0
$\overline{A} \cdot B$	0	1	1	0
$A \cdot B$	0	1	1	0
$A \cdot \overline{B}$	1	0	0	0

We can see one isolated case: $A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D}$ Now, we have to loop through other groups and eliminate bad options. Then, second we loop through groups of 4. There are 2 of them. One containing squares 2,3,6,7 and the other one containing 6,7,10,11. That give us next two elements: $\overline{A} \cdot D$ and $B \cdot D$. Therefore the final equation is $A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D} + \overline{A} \cdot D + B \cdot D$