Assignment 1

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Problem 1.a

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \; X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$A \cdot X = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

$$\frac{\partial}{\partial X}(A \cdot X) = \begin{bmatrix} \frac{\partial}{\partial x1}(a_{11}x_1 + \cdots + a_{1n}x_n) & \cdots & \frac{\partial}{\partial xn}(a_{11}x_1 + \cdots + a_{1n}x_n) \\ \frac{\partial}{\partial x1}(a_{21}x_1 + \cdots + a_{2n}x_n) & \cdots & \frac{\partial}{\partial xn}(a_{21}x_1 + \cdots + a_{2n}x_n) \\ \vdots & \ddots & \vdots \\ \frac{\partial}{\partial x1}(a_{m1}x_1 + \cdots + a_{mn}x_n) & \cdots & \frac{\partial}{\partial xn}(a_{m1}x_1 + \cdots + a_{mn}x_n) \end{bmatrix}$$

$$\frac{\partial}{\partial X}(A \cdot X) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = A$$

Problem 1.b

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \; X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$A \cdot X = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

$$X^T \cdot A \cdot X = \begin{bmatrix} (a_{11}x_1^2 + \cdots + a_{1n}x_nx_1) + (a_{21}x_1x_2 + a_{22}x_2^2 + \cdots + a_{2n}x_nx_2) + \cdots + (a_{m1}x_1x_n + \cdots + a_{mn}x_n^2) \end{bmatrix}$$

Let $Z = (a_{11}x_1^2 + \cdots + a_{1n}x_nx_1) + (a_{21}x_1x_2 + a_{22}x_2^2 + \cdots + a_{2n}x_nx_2) + \cdots + (a_{m1}x_1x_n + \cdots + a_{mn}x_n^2)$

$$\frac{\partial}{\partial X}(X^T \cdot A \cdot X) = \begin{bmatrix} \frac{\partial}{\partial x1}(Z) & \frac{\partial}{\partial x2}(Z) & \cdots & \frac{\partial}{\partial xn}(Z) \end{bmatrix}$$

$$\frac{\partial}{\partial X}(X^T \cdot A \cdot X) = \begin{bmatrix}(2a_{11}x_1 + (a_{12}+a_{21})x_2 + \cdots (a_{n1}+a_{1n})x_n) & \cdots & ((a_{1n}+a_{n1})x_1 + (a_{2n}+a_{n2})x_2 + \cdots + a_{nn}x_n)\end{bmatrix}$$

$$\frac{\partial}{\partial X}(X^T \cdot A \cdot X) = \begin{bmatrix}\sum_{i=1}^{n}(a_{1i}+a_{i1})x_i & \sum_{i=1}^{n}(a_{2i}+a_{21})x_i & \cdots & \sum_{i=1}^{n}(a_{ni}+a_{in})x_i\end{bmatrix}$$

Comparing with $X^T(A^T + A)$, we get:

$$X^T(A+A^T) = \begin{bmatrix}x_1 & x_2 & \cdots & x_n\end{bmatrix}\begin{bmatrix}2a_{11} & a_{12}+a_{21} & \cdots & a_{1n}+a_{n1} \\ a_{21}+a_{12} & 2a_{22} & \cdots & a_{2n}+a_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}+a_{1n} & a_{n2}+a_{2n} & \cdots & 2a_{nn}\end{bmatrix}$$

$$X^T(A+A^T) = \begin{bmatrix}\sum_{i=1}^{n}(a_{1i}+a_{i1})x_i & \sum_{i=1}^{n}(a_{2i}+a_{21})x_i & \cdots & \sum_{i=1}^{n}(a_{ni}+a_{in})x_i\end{bmatrix}$$

So, $\frac{\partial}{\partial X}(X^T \cdot A \cdot X) = X^T(A^T + A)$

Problem 2

Extending from the differentiation of vector with respect to another vector where each entry in a vector is differentiated by each entry of the other vector, in the differentiation of a matrix w.r.t a vector, each entry in the matrix is differentiated with each entry of the vector. So, the resulting matrix will have dimensions m * n * k.

Problem 3.a

$$\frac{d\begin{bmatrix}2sin^2(x)cos(y) \\ x^2+3e^y\end{bmatrix}}{d\begin{bmatrix}x \\ y\end{bmatrix}} = \begin{bmatrix}\frac{d(2sin^2(x)cos(y))/}{dx} & \frac{d(2sin^2(x)cos(y))}{dy} \\ \frac{d(x^2+3e^y)}{dx} & \frac{d(x^2+3e^y)}{dy}\end{bmatrix} = \begin{bmatrix}2sin(2x)cos(y) & -2sin^2(x)sin(y) \\ 2x & 3e^y\end{bmatrix}$$

Problem 3.b

$$\frac{d\begin{bmatrix}3x^2y + xyzw \\ sin(x^2+yw-z)\end{bmatrix}}{\begin{bmatrix}x \\ y \\ z \\ w\end{bmatrix}} = \begin{bmatrix}6xy + yzw & 3x^2 + xzw & xyw & xyz \\ 2xcos(x^2+yw-z) & wcos(x^2+yw-z) & -cos(x^2+yw-z) & ycos(x^2+yw-z)\end{bmatrix}$$

Problem 4

$$\beta = \begin{bmatrix}a_1 \\ a_2 \\ \vdots \\ a_n\end{bmatrix} X = \begin{bmatrix}x_1 \\ x_2 \\ \vdots \\ x_n\end{bmatrix}$$

$$\beta^T \cdot X = \begin{bmatrix}\sum_{i=1}^{n} a_i x_i\end{bmatrix}$$

$$\frac{d}{dX}e^{\left[\sum_{i=1}^{n} a_i x_i\right]} = e^{\left[\sum_{i=1}^{n} a_i x_i\right]}\begin{bmatrix}\frac{\partial}{\partial x1}\sum_{i=1}^{n} a_i x_i & \frac{\partial}{\partial x2}\sum_{i=1}^{n} a_i x_i & \cdots \frac{\partial}{\partial xn}\sum_{i=1}^{n} a_i x_i\end{bmatrix}$$

$$\frac{d}{dX}e^{\left[\sum_{i=1}^{n} a_i x_i\right]} = e^{\left[\sum_{i=1}^{n} a_i x_i\right]}\begin{bmatrix}a_1 & a_2 & \cdots & a_n\end{bmatrix} = B^T e^{B^T X}$$