ML Assignment-1

May 16, 2024

1 Problem 1:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$$

$$A\vec{x} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 \cdots + a_{mn}x_n \end{bmatrix}_{m\times 1}$$

$$\frac{d[A\vec{x}]}{d\vec{x}} = \begin{bmatrix} \frac{d(a_{11}x_1+a_{12}x_2\cdots+a_{1n}x_n)}{d\vec{x}} \\ \frac{d(a_{21}x_1+a_{22}x_2\cdots+a_{2n}x_n)}{d\vec{x}} \\ \vdots \\ \frac{d(a_{m1}x_1+a_{m2}x_2\cdots+a_{mn}x_n)}{d\vec{x}} \end{bmatrix}_{m\times 1}$$

$$\frac{d[A\vec{x}]}{d\vec{x}} = \begin{bmatrix} \frac{d(a_{11}x_1+a_{12}x_2\cdots+a_{1n}x_n)}{dx_1} & \frac{d(a_{11}x_1+a_{12}x_2\cdots+a_{1n}x_n)}{dx_2} & \cdots & \frac{d(a_{11}x_1+a_{12}x_2\cdots+a_{1n}x_n)}{dx_n} \\ \frac{d(a_{21}x_1+a_{22}x_2\cdots+a_{2n}x_n)}{dx_1} & \frac{d(a_{21}x_1+a_{22}x_2\cdots+a_{2n}x_n)}{dx_2} & \cdots & \frac{d(a_{21}x_1+a_{22}x_2\cdots+a_{2n}x_n)}{dx_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{d(a_{m1}x_1+a_{m2}x_2\cdots+a_{mn}x_n)}{dx_1} & \frac{d(a_{m1}x_1+a_{m2}x_2\cdots+a_{mn}x_n)}{dx_2} & \cdots & \frac{d(a_{m1}x_1+a_{m2}x_2\cdots+a_{mn}x_n)}{dx_n} \end{bmatrix}_{m\times n}$$

$$\frac{d[A\vec{x}]}{d\vec{x}} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

$$\frac{d[A\vec{x}]}{d\vec{x}} = A$$

b) Let,

$$A = [a_{ij}]_{n\times n}\vec{x} = [x_i]_{n\times 1}\vec{x}^T = [x_j]^T_{1\times n}$$

then,

$$A\vec{x} = \begin{bmatrix} \sum_{k=1}^n a_{1k}x_k \\ \sum_{k=1}^n a_{2k}x_k \\ \vdots \\ \sum_{k=1}^n a_{nk}x_k \end{bmatrix}_{n\times 1}$$

$$\vec{x}^T A\vec{x} = \left[x_1 \sum_{k=1}^n a_{1k}x_k + x_2 \sum_{k=1}^n a_{2k}x_k + \cdots + x_n \sum_{k=1}^n a_{nk}x_k\right]_{1\times 1}$$

$$\frac{d(\vec{x}^T A\vec{x})}{d\vec{x}} = \begin{bmatrix} \sum_{k=1}^n a_{1k}x_k + x_1a_{11} + x_2a_{21} + \cdots + x_na_{n1} \\ x_1a_{12} + \sum_{k=1}^n a_{2k}x_k + x_2a_{22} + \cdots + x_na_{n2} \\ \vdots \\ x_1a_{1n} + x_2a_{2n} + \cdots + x_na_{nn} + \sum_{k=1}^n a_{nk}x_k \end{bmatrix}^T$$

$$\frac{d(\vec{x}^T A\vec{x})}{d\vec{x}} = \begin{bmatrix} \sum_{k=1}^n a_{1k}x_k + \sum_{k=1}^n a_{k1}x_k \\ \sum_{k=1}^n a_{2k}x_k + \sum_{k=1}^n a_{k2}x_k \\ \vdots \\ \sum_{k=1}^n a_{1nk}x_k + \sum_{k=1}^n a_{kn}x_k \end{bmatrix}^T_{1\times n}$$

and

$$\vec{x}^T A = \begin{bmatrix} \sum_{k=1}^n a_{k1}x_k \\ \sum_{k=1}^n a_{k2}x_k \\ \vdots \\ \sum_{k=1}^n a_{kn}x_k \end{bmatrix}^T_{1\times n}, \vec{x}^T A^T = \begin{bmatrix} \sum_{k=1}^n a_{1k}x_k \\ \sum_{k=1}^n a_{2k}x_k \\ \vdots \\ \sum_{k=1}^n a_{1nk}x_k \end{bmatrix}^T_{1\times n}$$

Thus,

$$\frac{d(\vec{x}^T A\vec{x})}{d\vec{x}} = \vec{x}^T A^T + \vec{x}^T A = \vec{x}^T(A + A^T)$$

Hence, proved.

2 Problem 2:

The dimension of the final matrix would be m $\times nk$

3 Problem 3:

a)

$$ans = \begin{bmatrix} \frac{d(2sin^2(x)cos(y))}{dx} & \frac{d(2sin^2(x)cos(y))}{dy} \\ \frac{d(x^2+3e^y)}{dx} & \frac{d(x^2+3e^y)}{dy} \end{bmatrix} = \begin{bmatrix} 2sin(2x)cos(y) & -2sin^2(x)sin(y) \\ 2x & 3e^y \end{bmatrix}$$

b) ans= $\begin{bmatrix} \frac{d(3x^2y+xyzw)}{dx} & \frac{d(3x^2y+xyzw)}{dy} & \frac{d(3x^2y+xyzw)}{dz} & \frac{d(3x^2y+xyzw)}{dw} \\ \frac{d(sin(x^2+yw-z))}{dx} & \frac{d(sin(x^2+yw-z))}{dy} & \frac{d(sin(x^2+yw-z))}{dz} & \frac{d(sin(x^2+yw-z))}{dw} \end{bmatrix}$

$= \begin{bmatrix} 6xy+yzw & 3x^2+xzw & xyw & xyz \\ cos(x^2+yw+z)(2x) & cos(x^2+yw+z)(w) & cos(x^2+yw+z)(-1) & cos(x^2+yw+z)(y) \end{bmatrix}$

4 Problem 4:

Let,

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}_{n\times 1}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$$

then,

$$\beta^T\vec{x} = \begin{bmatrix} \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n \end{bmatrix}_{1\times 1}$$

$$\frac{d(e^{\beta^T\vec{x}})}{d\vec{x}} = e^{\beta^T\vec{x}}\frac{d(\beta^T\vec{x})}{d\vec{x}} = e^{\beta^T\vec{x}} \begin{bmatrix} \frac{d(\beta_1x_1+\beta_2x_2+\cdots+\beta_nx_n)}{dx_1} \\ \frac{d(\beta_1x_1+\beta_2x_2+\cdots+\beta_nx_n)}{dx_2} \\ \vdots \\ \frac{d(\beta_1x_1+\beta_2x_2+\cdots+\beta_nx_n)}{dx_n} \end{bmatrix}^T_{1\times n}$$

$$\frac{d(e^{\beta^T\vec{x}})}{d\vec{x}} = e^{\beta^T\vec{x}} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}^T_{1\times n}$$

$$\frac{d(e^{\beta^T\vec{x}})}{d\vec{x}} = e^{\beta^T\vec{x}}\beta^T$$