Assignment 1

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15th May 2024

1 My Attempt

Solution 1

(a) Prove:

Let A be an $m \times n$ matrix and x be an $n \times 1$ vector. We want to show that $\frac{d}{dx}(Ax) = A$.

Let $y = Ax$. Then, y is a vector of dimension $m \times 1$.

The i-th component of y can be written as:

$$y_i = \sum_{j=1}^{n} A_{ij} x_j$$

Taking the derivative of y_i with respect to x_k gives:

$$\frac{dy_i}{dx_k} = \frac{d}{dx_k}\left(\sum_{j=1}^{n} A_{ij} x_j\right) = A_{ik}$$

So, $\frac{d}{dx}(Ax)$ is a matrix whose i, j (as j=k) entry is A_{ij}, i.e., A.

Hence Proved, $\frac{d}{dx}(Ax) = A$.

(b) Prove:

Let A be an $n \times n$ matrix and x be an $n \times 1$ vector. We want to show that $\frac{d}{dx}(x^T Ax) = x^T(A + A^T)$.

First, let's compute the derivative of $x^T Ax$:

$$\frac{d}{dx}(x^T Ax) = \frac{d}{dx}(x^T(Ax))$$

Using the result from part (a), $\frac{d}{dx}(Ax) = A\frac{d}{dx}x$, we have:

$$= x^T A + (Ax)^T = x^T A + x^T A^T$$

$$= x^T(A + A^T)$$

Solution 2

Suppose we have a matrix of dimension $m \times n$ and we differentiate it with respect to a vector of dimension $k \times 1$.

The dimension of the final result is determined by the dimensions of the resulting partial derivatives. Since differentiation results in a derivative matrix, the dimension of the final result will be $m \times n \times k$.

Solution 3

(a) Solving: $\frac{d}{d[x,y]^T}[2\sin^2(x)\cos(y), x^2+3e^y]^T$

$$\frac{d}{d[x,y]^T}[2\sin^2(x)\cos(y), x^2+3e^y]^T = \begin{bmatrix} 4\sin(x)\cos(x)\cos(y) & -2\sin^2(x)\sin(y) \\ 2x & 3e^y \end{bmatrix}$$

(b) Solving: $\frac{d}{d[x,y,z,w]^T}[3x^2y + xyzw, \sin(x^2+yw-z)]^T$

$$\frac{d}{d[x,y,z,w]^T}[3x^2y+xyzw, \sin(x^2+yw-z)]^T = \begin{bmatrix} 6xy+yzw & 2x\cos(x^2+yw-z) \\ 3x^2+xzw & w\cos(x^2+yw-z) \\ xyw & -\cos(x^2+yw-z) \\ xyz & y\cos(x^2+yw-z) \end{bmatrix}$$

Solution 4

Given β as an $n \times 1$ vector and x as an $n \times 1$ vector, we want to solve for $\frac{d}{dx}[e^{\beta^T x}]$.

$$\frac{d}{dx}[e^{\beta^T x}] = \frac{d}{dx}[e^{\beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n}]$$

$$\frac{d}{dx}[e^{\beta^T x}] = \begin{bmatrix} \frac{\partial}{\partial x_1}(e^{\beta^T x}) \\ \frac{\partial}{\partial x_2}(e^{\beta^T x}) \\ \vdots \\ \frac{\partial}{\partial x_n}(e^{\beta^T x}) \end{bmatrix} = \begin{bmatrix} \beta_1 e^{\beta^T x} \\ \beta_2 e^{\beta^T x} \\ \vdots \\ \beta_n e^{\beta^T x} \end{bmatrix}$$