Theoretical Machine Learning Assignment 1

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1 Answer 1(a)

$$\text{Let } A_{mxn} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \text{ and } \vec{x_{nx1}} = \begin{pmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{n1} \end{pmatrix}$$

Now, $A\vec{x} = \begin{pmatrix} \sum_{i=1}^{n} a_{1i}x_{i1} \\ \sum_{i=1}^{n} a_{2i}x_{i1} \\ \vdots \\ \sum_{i=1}^{n} a_{mi}x_{i1} \end{pmatrix}$

Therefore, $\frac{\mathrm{d}A\vec{x}}{\mathrm{d}x} = \begin{pmatrix} \frac{\mathrm{d}\sum_{i=1}^{n} a_{1i}x_{i1}}{\mathrm{d}x_{11}} & \frac{\mathrm{d}\sum_{i=1}^{n} a_{1i}x_{i1}}{\mathrm{d}x_{21}} & \cdots & \frac{\mathrm{d}\sum_{i=1}^{n} a_{1i}x_{i1}}{\mathrm{d}x_{n1}} \\ \frac{\mathrm{d}\sum_{i=1}^{n} a_{2i}x_{i1}}{\mathrm{d}x_{11}} & \frac{\mathrm{d}\sum_{i=1}^{n} a_{2i}x_{i1}}{\mathrm{d}x_{21}} & \cdots & \frac{\mathrm{d}\sum_{i=1}^{n} a_{2i}x_{i1}}{\mathrm{d}x_{n1}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\mathrm{d}\sum_{i=1}^{n} a_{mi}x_{i1}}{\mathrm{d}x_{11}} & \frac{\mathrm{d}\sum_{i=1}^{n} a_{mi}x_{i1}}{\mathrm{d}x_{21}} & \cdots & \frac{\mathrm{d}\sum_{i=1}^{n} a_{mi}x_{i1}}{\mathrm{d}x_{n1}} \end{pmatrix}$

Clearly, upon differentiating we will get respective terms to make A_{mxn}.

2 Answer 1(b)

$$\text{Let } A_{nxn} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \text{ and } \vec{x_{nx1}} = \begin{pmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{n1} \end{pmatrix}$$

Now, $A\vec{x} = \begin{pmatrix} \sum_{i=1}^{n} a_{1i}x_{i1} \\ \sum_{i=1}^{n} a_{2i}x_{i1} \\ \vdots \\ \sum_{i=1}^{n} a_{ni}x_{i1} \end{pmatrix}$ **and** $\vec{x^T} = \begin{pmatrix} x_{11} & x_{21} & \cdots & x_{n1} \end{pmatrix}$

So, $\vec{x^T}A\vec{x} = \left(x_{11}\sum_{i=1}^{n} a_{1i}x_{i1} + x_{21}\sum_{i=1}^{n} a_{2i}x_{i1} ..... + x_{n1}\sum_{i=1}^{n} a_{ni}x_{i1}\right)$

Now, $\frac{d\vec{x^T}A\vec{x}}{d\vec{x}} = \begin{pmatrix}\sum_{i=1}^{n}(a_{1i}+a_{i1})x_{i1} & \sum_{i=1}^{n}(a_{2i}+a_{i2})x_{i1} & \cdots & \sum_{i=1}^{n}(a_{ni}+a_{in})x_{i1}\end{pmatrix}$ ....(1)

Also, $\vec{x^T}(A{+}A^T) = \begin{pmatrix}x_{11} & x_{21} & \cdots & x_{n1}\end{pmatrix}\begin{pmatrix}2a_{11} & a_{12}+a_{21} & \cdots & a_{1n}+a_{n1}\\ a_{21}+a12 & 2a_{22} & \cdots & a_{2n}+a_{n2}\\ \vdots & \vdots & \ddots & \vdots\\ a_{n1}+a_{1n} & a_{n2}+a_{2n} & \cdots & 2a_{nn}\end{pmatrix}$

$= \begin{pmatrix}\sum_{i=1}^{n}(a_{1i}+a_{i1})x_{i1} & \sum_{i=1}^{n}(a_{2i}+a_{i2})x_{i1} & \cdots & \sum_{i=1}^{n}(a_{ni}+a_{in})x_{i1}\end{pmatrix}$ ....(2)

Therefore, from (1) and (2), $\frac{d\vec{x^T}A\vec{x}}{d\vec{x}} = \vec{x^T}(A+A^T)$

Hence, proved.

3 Answer 2

The dimension of the matrix is m x n x k.

4 Answer 3(a)

$$\begin{pmatrix}4\text{sin(x)cos(x)cos(y)} & -2sin^2(x)sin(y)\\ 2x & 3e^y\\ & \end{pmatrix}$$

5 Answer 3(b)

$$\begin{pmatrix}\text{6xy + yzw} & 3x^2+xzw & xyw & xyz & \\ 2xcos(x^2+yw-z) & wcos(x^2+yw-z) & -cos(x^2+yw-z) & ycos(x^2+yw-z) & \end{pmatrix}$$

6 Answer 4

$\frac{de^{\beta^T\vec{x}}}{d\vec{x}} = e^{\beta^T\vec{x}}\frac{d\beta^T\vec{x}}{d\vec{x}} = e^{\beta^T\vec{x}}\beta^T$, using the identity $\frac{dA\vec{x}}{d\vec{x}} = A$.