Assignment 1

Tejas S

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1 Solution 1. (a)

To Prove : $\frac{d}{dX}[\text{AX}] = \text{A}$

Proof :

here,

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

and

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

So,

$$\mathbf{AX} = \begin{bmatrix} \sum_{i=1}^{n} a_{1i}x_i \\ \sum_{i=1}^{n} a_{2i}x_i \\ \vdots \\ \sum_{i=1}^{n} a_{mi}x_i \end{bmatrix}$$

$$\frac{\mathbf{d}}{\mathbf{dX}}[\mathbf{AX}] = \begin{bmatrix} \frac{d}{dX}\sum_{i=1}^{n} a_{1i}x_i \\ \frac{d}{dX}\sum_{i=1}^{n} a_{2i}x_i \\ \vdots \\ \frac{d}{dX}\sum_{i=1}^{n} a_{mi}x_i \end{bmatrix}$$

$$= \begin{bmatrix} \frac{d}{dx_1}\sum_{i=1}^{n} a_{1j}x_i & \cdots & \frac{d}{dx_n}\sum_{i=1}^{n} a_{1i}x_i \\ \frac{d}{dx_1}\sum_{i=1}^{n} a_{2j}x_i & \cdots & \frac{d}{dx_n}\sum_{i=1}^{n} a_{2i}x_i \\ \vdots & \vdots & \ddots & \vdots \\ \frac{d}{dx_1}\sum_{i=1}^{n} a_{mi}x_i & \cdots & \frac{d}{dx_n}\sum_{i=1}^{n} a_{mi}x_i \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Hence, $\frac{d}{dX}[AX] =$ A

2 Solution 1. (b)

To Prove : $\frac{d}{dX}[X^TAX] = X^T(A + A^T)$

Proof : Since $A + A^T$ exists A should be a square matix. So, let dim(A) = (n,n). Then dim(X) will be (n, 1)
So,

$$\mathbf{AX} = \begin{bmatrix} \sum_{i=1}^{n} a_{1i}x_i \\ \sum_{i=1}^{n} a_{2i}x_i \\ \vdots \\ \sum_{i=1}^{n} a_{ni}x_i \end{bmatrix}$$

$$\mathbf{X^TAX} = \sum_{j=1}^{n}\sum_{i=1}^{n} a_{ji}x_ix_j$$

$$\frac{\mathbf{d}}{\mathbf{dX}}(\mathbf{X^TAX}) = \begin{bmatrix} \frac{d}{dx_1}\sum_{j=1}^{n}\sum_{i=1}^{n} a_{ji}x_ix_j & \cdots & \cdots & \frac{d}{dx_n}\sum_{j=1}^{n}\sum_{i=1}^{n} a_{ji}x_ix_j \end{bmatrix}$$

Simplifying We get,

$$\mathbf{\frac{d}{dX}(X^T A X)} = \left[\sum_{i=1}^{n}(a_{i1}+a_{1i})x_i \quad \sum_{i=1}^{n}(a_{i2}+a_{2i})x_i \quad \cdots \quad \sum_{i=1}^{n}(a_{in}+a_{ni})x_i\right]$$

Clearly, $\frac{d}{dX}(X^T AX) = X^T(A + A^T)$

Hence Proved.

3 Solution 2.

In a m*n matrix there are m*n scalars and when we differentiate it with a n dimensional vector, corresponding to each scalar we get a vector of dimension n*1. So, the dimension of final result would be m*n*k.

4 Solution 3.(a)

$$\frac{d\begin{bmatrix}2sin^2(x)cos(y)\\ x^2+3e^y\end{bmatrix}}{d\begin{bmatrix}x\\ y\end{bmatrix}} = \begin{bmatrix}\frac{d(2sin^2(x)cos(y))}{dx} & \frac{d(2sin^2(x)cos(y))}{dy}\\ \frac{d(x^2+3e^y)}{dx} & \frac{d(x^2+3e^y)}{dy}\end{bmatrix} = \begin{bmatrix}2sin(2x)cos(y) & -2sin^2(x)sin(y)\\ 2x & 3e^y\end{bmatrix}$$

5 Solution 3. (b)

$$\frac{d\begin{bmatrix}3x^2y+xywz\\ sin(x^2+yw-z)\end{bmatrix}}{d\begin{bmatrix}x\\ y\\ z\\ w\end{bmatrix}} = \begin{bmatrix}\frac{d(3x^2y+xywz)}{dx} & \frac{d(3x^2y+xywz)}{dy} & \frac{d(3x^2y+xywz)}{dz} & \frac{d(3x^2y+xywz)}{dw}\\ \frac{d(sin(x^2+yw-z))}{dx} & \frac{d(sin(x^2+yw-z))}{dy} & \frac{d(sin(x^2)+yw-z)}{dz} & \frac{d(sin(x^2)+yw-z)}{dw}\end{bmatrix}$$

$$= \begin{bmatrix}6xy+ywz & 3x^2+xwz & xyw & xyz\\ 2xcos(x^2+yw-z) & wcos(x^2+yw-z) & -cos(x^2+yw-z) & ycos(x^2+yw-z)\end{bmatrix}$$

6 Solution 4.

$$\beta^{\mathbf{T}}\mathbf{X} = \sum_{i=1}^{n} \beta_i x_i$$

$$\mathbf{e}^{\beta^{\mathbf{T}}\mathbf{X}} = e^{\sum_{i=1}^{n} \beta_i x_i}$$

$$\frac{\mathbf{d}}{\mathbf{dX}}\mathbf{e}^{\beta^{\mathbf{T}}\mathbf{X}} = \begin{bmatrix} \frac{d}{dx_1} e^{\sum_{i=1}^{n} \beta_i x_i} & \frac{d}{dx_2} e^{\sum_{i=1}^{n} \beta_i x_i} & \cdots & \frac{d}{dx_n} e^{\sum_{i=1}^{n} \beta_i x_i} \end{bmatrix}$$

$$\frac{\mathbf{d}}{\mathbf{dX}}\mathbf{e}^{\beta^{\mathbf{T}}\mathbf{X}} = \begin{bmatrix} \beta_1 e^{\sum_{i=1}^{n} \beta_i x_i} & \beta_2 e^{\sum_{i=1}^{n} \beta_i x_i} & \cdots & \beta_n e^{\sum_{i=1}^{n} \beta_i x_i} \end{bmatrix}$$

$$\frac{\mathbf{d}}{\mathbf{dX}}\mathbf{e}^{\beta^{\mathbf{T}}\mathbf{X}} = \begin{bmatrix} \beta_1 & \beta_2 & \cdots & \beta_n \end{bmatrix} e^{\sum_{i=1}^{n} \beta_i x_i}$$

$$\frac{\mathbf{d}}{\mathbf{dX}}\mathbf{e}^{\beta^{\mathbf{T}}\mathbf{X}} = \beta^T e^{\beta^T X}$$