Problem 1

(a) To prove:

$$\frac{d}{d\mathbf{x}}[\mathbf{A}\mathbf{x}] = \mathbf{A}$$

Given that $\mathbf{A}$ is an $m \times n$ matrix and $\mathbf{x}$ is an $n \times 1$ vector.

Solution:

$$\begin{aligned}
\frac{d}{d\mathbf{x}}[\mathbf{A}\mathbf{x}] &= \frac{d}{d\mathbf{x}}\left[\sum_{i=1}^{n}(\mathbf{A}_{i\cdot}\mathbf{x}_i)\right] \\
&= \frac{d}{d\mathbf{x}}\left[\begin{pmatrix} \mathbf{A}_{1\cdot}\mathbf{x} \\ \mathbf{A}_{2\cdot}\mathbf{x} \\ \vdots \\ \mathbf{A}_{m\cdot}\mathbf{x} \end{pmatrix}\right] \\
&= \begin{pmatrix} \frac{d}{d\mathbf{x}}(\mathbf{A}_{1\cdot}\mathbf{x}) \\ \frac{d}{d\mathbf{x}}(\mathbf{A}_{2\cdot}\mathbf{x}) \\ \vdots \\ \frac{d}{d\mathbf{x}}(\mathbf{A}_{m\cdot}\mathbf{x}) \end{pmatrix} \\
&= \begin{pmatrix} \mathbf{A}_{1\cdot} \\ \mathbf{A}_{2\cdot} \\ \vdots \\ \mathbf{A}_{m\cdot} \end{pmatrix} \\
&= \mathbf{A}
\end{aligned}$$

(b) To prove:

$$\begin{aligned}
\frac{d}{d\mathbf{x}}[\mathbf{x}^T\mathbf{A}\mathbf{x}] &= \frac{d}{d\mathbf{x}}\left[\mathbf{x}^T \begin{pmatrix} \mathbf{A}_{1\cdot} \\ \mathbf{A}_{2\cdot} \\ \vdots \\ \mathbf{A}_{n\cdot} \end{pmatrix} \mathbf{x}\right] \\
&= \frac{d}{d\mathbf{x}}\left[\sum_{i=1}^{n} \mathbf{x}^T\mathbf{A}_{i\cdot}\mathbf{x}_i\right] \\
&= \frac{d}{d\mathbf{x}}\left[\sum_{i=1}^{n} \mathbf{x}_i(\mathbf{A}_{i\cdot}\mathbf{x})\right] \\
&= \sum_{i=1}^{n} \frac{d}{d\mathbf{x}}[\mathbf{x}_i(\mathbf{A}_{i\cdot}\mathbf{x})] \\
&= \sum_{i=1}^{n} \mathbf{x}_i(\mathbf{A}_{i\cdot} + \mathbf{A}_{i\cdot}^T) \\
&= \mathbf{x}^T(\mathbf{A} + \mathbf{A}^T)
\end{aligned}$$

Problem 2

If differentiating a matrix of dimension $m \times n$ with respect to a $k \times 1$ vector, the final result would have dimensions $m \times n$.

Problem 3

(a) Solution: For this we will get a $2\times\ 2 matrix\cdot$

$$\begin{bmatrix} \frac{d[2\sin^2(\mathbf{x})\cos(\mathbf{y})]}{d(\mathbf{x})} & \frac{d[2\sin^2(\mathbf{x})\cos(\mathbf{y})]}{d(\mathbf{y})} \\ \frac{d[x^2+3e^y]}{d(\mathbf{x})} & \frac{d[x^2+3e^y]}{d(\mathbf{y})} \end{bmatrix}$$

$$\begin{bmatrix} 4sin(\mathbf{x})cos\mathbf{x}cos(\mathbf{y}) & \text{-}2sin^2(\mathbf{x})cos(\mathbf{y}) \\ 2\mathbf{x} & 3e^{\mathbf{y}} \end{bmatrix}$$

(b) Solution: For this we will get a $2\times\ 4 matrix\cdot$

$$\begin{bmatrix} \frac{d[3x^2yw+xyzw]}{d(\mathbf{x})} & \frac{d[3x^2yw+xyzw]}{d(\mathbf{y})} & \frac{d[3x^2yw+xyzw]}{d(\mathbf{z})} & \frac{d[3x^2yw+xyzw]}{d(\mathbf{w})} \\ \frac{d[sin(x^2+yw-z]}{d(\mathbf{x})} & \frac{d[sin(x^2+yw-z]}{d(\mathbf{y})} & \frac{d[sin(x^2+yw-z]}{d(\mathbf{z})} & \frac{d[sin(x^2+yw-z]}{d(\mathbf{w})} \end{bmatrix}$$

$$\begin{bmatrix} 6xyw + yzw & 3x^2w + xzw) & xyw & 3x^2y + xyz) \\ 2xcos(x^2 + yw - z) & wcos(x^2 + yw - z) & -cos(x^2 + yw - z) & cos(x^2 + yw - z) \end{bmatrix}$$