Theoretical Machine Learning

ASSIGNMENT 2

Theoretical

Problem 1. Prove the following, where X is a Random Variable, and Ω is the Sample Space:

(a) $X^{-1}(\Phi) = \Phi$

(b) $X^{-1}(\mathbb{R}) = \Omega$

Problem 2. We have a Random Variable X from the Exponential Distribution. Distribution Function of Exp(λ) is as -

$$F_X(x) = \begin{cases} 0 & \text{x} < 0 \\ 1 - e^{-\lambda x} & \text{x} \geq 0 \end{cases}$$

Find the support of X.

Problem 3. You were given the expression of m and c is a linear regression in 2 variable case with L2 - Norm as -

$m = \frac{N(\sum xy) - (\sum x)(\sum y)}{N(\sum x^2) - (\sum x)^2}$

$c = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{N(\sum x^2) - (\sum x)^2}$

Show how you get these values, and that it is indeed a minima for the loss function for L2 Norm.

Programming

Problem 1. Uniformly draw 50 samples from 0 to 10, and get that as your x, then get y by 3x + 5 + ϵ, where for each x, ϵ is drawn from N(0, 2).

Plot this sample of x and y, and find the regression line, which you will plot on top of the plot of x and y.