Question 1:

(a) X^{-1} maps probabilty space to sample space. Since, probability space is ϕ so sample space should also be ϕ. Hence, $X^{-1}(\phi) = \phi$

(b) Here, probability space is $\mathbb{R}$ so this will include the entire sample space ,i.e., Ω. Hence, $X^{-1}(\mathbb{R}) = \Omega$

Question 2:

The range of $F_X(x)$ is $[0, 1]$. Hence, the support of X is the entire probability space, i.e., $R_X = \{x : x >= 0\}$

Question 3:

Let, $L = \sum_{i=1}^{N} (y_i - mx_i - c)^2$

Therefore for L to be minimum ,

$$\frac{\partial L}{\partial m} = \sum_{i=1}^{N} 2 * (y_i - mx_i - c) * (-x_i) = 0$$

$$\sum_{i=1}^{N} (y_i x_i - m{x_i}^2 - cx_i) = 0$$

$$m = \frac{\sum_{i=1}^{N} y_i x_i - c \sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} {x_i}^2} \tag{1}$$

$$\frac{\partial L}{\partial c} = \sum_{i=1}^{N} 2 * (y_i - mx_i - c) * (-1) = 0$$

$$c = \frac{\sum_{i=1}^{N} y_i - m \sum_{i=1}^{N} x_i}{N} \tag{2}$$

Solving eqn. (1) and (2), we get,

$$m = \frac{N(\sum_{i=1}^{N} xy) - (\sum_{i=1}^{N} x)(\sum_{i=1}^{N} y)}{N(\sum_{i=1}^{N} x^2) - (\sum_{i=1}^{N} x)^2}$$

and

$$c = \frac{(\sum_{i=1}^{N} y)(\sum_{i=1}^{N} x^2) - (\sum_{i=1}^{N} x)(\sum_{i=1}^{N} xy)}{N(\sum_{i=1}^{N} x^2) - (\sum_{i=1}^{N} x)^2}$$