# Theoretical Assignment 2 Solutions

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## 1 Problem 1. Solution

### 1.1 (a)

As we all know that $\Phi$ denotes an empty set. X is a random variable and $\Omega$ is the Sample space.

$$X^{-1}(\Phi) : \{\omega \in \Omega : X(\omega) \in \Phi\} = \Phi$$

We need to find a $\omega$ in $\Omega$ such that X($\omega$) belongs to an empty set. Since, no such $\omega$ exists. Therefore, its an empty set.

### 1.2 (b)

As we all know that by definition X takes values in $\mathbb{R}$ which implies that for $X(\omega) \in \mathbb{R}$ , $\omega \in \Omega$ as X is mapping them to $\mathbb{R}$.

$$X^{-1}(\mathbb{R}) = \{\omega \in \Omega : X(\omega) \in \mathbb{R}\} = \Omega$$

## 2 Problem 2. Solution

The Support of x , $S_x$ is defined as :- $S_x = \{x : D(x) > 0\}$ , where D(X) is the probability density function .
F(x) = $1 - e^{\lambda x}$ for x¿0
D(x) = $\frac{F(x)-F(x-h)}{h}$

$$\lim_{h \to 0} \frac{1 - e^{\lambda x} - (1 - e^{-\lambda(x-h)})}{h}$$

$$\lim_{h \to 0} \frac{e^{-\lambda(x-h)} - e^{-\lambda x}}{h}$$

$$\lim_{h \to 0} \lambda e^{-\lambda x} \times \frac{e^{-\lambda h} - 1}{\lambda h}$$

$$-\lambda e^{-\lambda x}$$

Since the support of X is the interval of non-negative real numbers which is $[0, \infty]$

## 3 Problem 3. Solution

We can write L2 norm as :-

$$L_2 = \sum_{i=1}^{N} (y_i^R - y_i^P)^2$$

where R stands for real and P stands for predicted value.

$$y_i^P = mx + c$$

$$\sum_{i=1}^{N} (y_i^R - mx_i - c)^2$$

Now take partial derivatives w.r.t m and c.
w.r.t m :-

$$\frac{\partial L_2}{\partial m} = -2 \sum_{i=1}^{N} (y_i^R - mx_i - c)x_i = 0$$

w.r.t c :-

$$\frac{\partial L_2}{\partial c} = -2 \sum_{i=1}^{N} (y_i^R - mx_i - c) = 0$$

Further solving these will result in the required solution.
Now, to check its indeed a minima we can use second partial derivative test but as we know that intuitively its true from partial derivatives also.