

Theoretical ML Assignment 1

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Solution 1. Since \mathbf{A} is scalar matrix, Apply product rule

$$\frac{d(\mathbf{Ax})}{d\mathbf{x}} = \mathbf{A} \cdot \mathbf{I} + 0$$

As the differentiation of \mathbf{x} vector with respect to itself is Identity matrix

$$\frac{d\mathbf{x}}{d\mathbf{x}} = \mathbf{I}$$

Solution b). Let \mathbf{A} be an $n \times n$ matrix and \mathbf{x} be an $n \times 1$ vector.

$$\frac{d}{d\mathbf{x}}(\mathbf{x}^T \mathbf{Ax}) = \mathbf{x}^T (\mathbf{A} + \mathbf{A}^T)$$

Using the product rule, we have:

$$\begin{aligned} \frac{d}{d\mathbf{x}}(\mathbf{x}^T \mathbf{Ax}) &= \frac{d}{d\mathbf{x}}(\mathbf{x}^T) \mathbf{Ax} + \mathbf{x}^T \frac{d}{d\mathbf{x}}(\mathbf{Ax}) \\ &= \frac{d}{d\mathbf{x}}(\mathbf{x}^T) \mathbf{Ax} + \mathbf{x}^T \mathbf{A} \end{aligned}$$

The derivative of \mathbf{x}^T with respect to \mathbf{x} is \mathbf{x}^T .

$$\begin{aligned} &= \mathbf{x}^T \mathbf{Ax} + \mathbf{x}^T \mathbf{A} \\ &= \mathbf{x}^T (\mathbf{A} + \mathbf{A}^T) \end{aligned}$$

Solution 2.) The dimension would be $m \times n \times k$

Solution 3.)

$$\mathbf{J} = \begin{pmatrix} 4\sin(x)\cos(y)\cos(x) & -2\sin^2(x)\sin(y) \\ 2x & 2e^y \end{pmatrix}$$

Solution 3b)

$$\mathbf{J} = \begin{pmatrix} 6xy + yzw & 3x^2 + xzw & xyw & xyz \\ 2x \cos(x^2 + yw - z) & w \cos(x^2 + yw - z) & -\cos(x^2 + yw - z) & y \cos(x^2 + yw - z) \end{pmatrix}$$