Theoretical Machine Learning

Niket Dhar Diwan

May 2024

1 Problem 1 part (a)

$$X^{-1}(\varnothing) = \{a \in \Omega \mid X(a) \in \varnothing\}$$

Clearly, there are no such elements that consequently will satisfy both the conditions. Hence,

$$X^{-1}(\varnothing) = \varnothing.$$

2 Problem 1 part (b)

$$X^{-1}(\mathbb{R}) = \{a \in \Omega \mid X(a) \in \mathbb{R}\}$$

Since $X(\Omega)$ maps the entire set of $\mathbb{R}$, every $a \in \Omega$ satisfies the above relation. Hence,

$$X^{-1}(\mathbb{R}) = \Omega.$$

3 Problem 2

The support of a function is defined as $P(x) = 0$. Now, $P(x) = \frac{\mathrm{d}F_X(x)}{\mathrm{d}x}$. Therefore, $P(x)$ is defined as

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \lambda e^{-\lambda x} & \text{if } x \geq 0 \end{cases}$$

Now, if $\lambda = 0$, the entire set of $\mathbb{R}$ is the support. Otherwise, $x < 0$ is the support.

4 Problem 3

Given, that

$$y = mx + c$$

we have to find the best fit line using loss function

$$S = \sum_{i=1}^{N} (y_i - (mx_i + c))^2$$

Now, to find the values of m and c,

$$\frac{\partial S}{\partial m} = \frac{\partial S}{\partial c} = 0$$

Upon solving we will get the required result.