Assignment 2 Theoretical ML

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Question 1

Part 1

Since $\varnothing$ is the empty set, there are no elements in $\varnothing$. Therefore, there are no $\omega \in \Omega$ such that $X(\omega) \in \varnothing$. Hence:

$$X(\omega) \in \varnothing \text{ is always false for any } \omega \in \Omega.$$

Consequently, the set of all such ω is empty. Thus:

$$X^{-1}(\varnothing) = \varnothing$$

Part 2

Since X is a random variable, by definition, it takes values in $\mathbb{R}$. That means for every $\omega \in \Omega$, $X(\omega) \in \mathbb{R}$ must be true because X maps elements of the sample space Ω to real numbers. Therefore, every $\omega \in \Omega$ satisfies $X(\omega) \in \mathbb{R}$. Hence:

$$X^{-1}(\mathbb{R}) = \{\omega \in \Omega \mid X(\omega) \in \mathbb{R}\} = \Omega$$

Question 2

From the definition of support of a random variable we know that it is the values of x which gives strictly positive values of the distribution function. So, for the given distribution function if $\lambda \leq 0$ then support of the function is $\varnothing$ which in the other case the support is all positive real numbers.

Question 3

Part 1

The loss function (L2 norm) to minimize is $L(m, c) = \sum_{i=1}^{N}(y_i - (mx_i + c))^2$. So we have two equations $\frac{\partial L(m,c)}{\partial m} = 0$ and $\frac{\partial L(m,c)}{\partial c} = 0$ which on solving gives

$$-2\sum_{i=1}^{N} x_i(y_i - mx_i - c) = 0$$

$$\sum_{i=1}^{N}(y_i - mx_i - c) = 0$$

. Solving these 2 linear equations of 2 variables m and c we will get the required values. We can use elimination method by multiplying the first equation with N and second equation with $\sum x_i$ and then subtract both of them.