Assignment 2

Question 1

Let X be a random variable and Ω be the sample space.

(a) $X^{-1}(\emptyset) = \emptyset$

By definition,

$$X^{-1}(\emptyset) = \{\omega \in \Omega \mid X(\omega) \in \emptyset\}$$

Thus,

$$X^{-1}(\emptyset) = \emptyset$$

(b) $X^{-1}(R) = \Omega$

By definition,

$$X^{-1}(R) = \{\omega \in \Omega \mid X(\omega) \in R\}$$

Thus,

$$X^{-1}(R) = \Omega$$

Question2

$$F_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 - e^{-\lambda x} & \text{for } x \geq 0 \end{cases}$$

let f be the probability density function.

For x¡0,

$$f_X(x) = 0$$

For x$\geq$ 0,

$$f_X(x) = \frac{d(1 - e^{-\lambda x})}{dx}$$

$$f_X(x) = e^{-\lambda x}$$

$$0$$

the probability density function $f_X(x)$ is positive for all x 0. Hence,

$$\text{Support of } X = [0, \infty)$$

Question3

To find the slope m and intercept c that minimize the L2 loss function in linear regression, we start with the model:

$$y = mx + c$$

To minimize the loss function:

$$L(m, c) = \sum_{i=1}^{N} (y_i - (mx_i + c))^2$$

take the partial derivatives and set them to zero:

$$\frac{\partial L}{\partial c} = -2 \sum_{i=1}^{N} (y_i - mx_i - c) = 0 \Rightarrow \sum_{i=1}^{N} y_i = m \sum_{i=1}^{N} x_i + Nc$$

$$c = \frac{\sum_{i=1}^{N} y_i - m \sum_{i=1}^{N} x_i}{N}$$

$$\frac{\partial L}{\partial m} = -2 \sum_{i=1}^{N} x_i(y_i - mx_i - c) = 0 \Rightarrow \sum_{i=1}^{N} x_i y_i = m \sum_{i=1}^{N} x_i^2 + c \sum_{i=1}^{N} x_i$$

Substituting c:

$$\sum_{i=1}^{N} x_i y_i = m \sum_{i=1}^{N} x_i^2 + \frac{\sum_{i=1}^{N} y_i \sum_{i=1}^{N} x_i - m(\sum_{i=1}^{N} x_i)^2}{N}$$

Solving for m:

$$m = \frac{N(\sum xy) - (\sum x)(\sum y)}{N(\sum x^2) - (\sum x)^2}$$

Substituting m back into the equation for c:

$$c = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{N(\sum x^2) - (\sum x)^2}$$