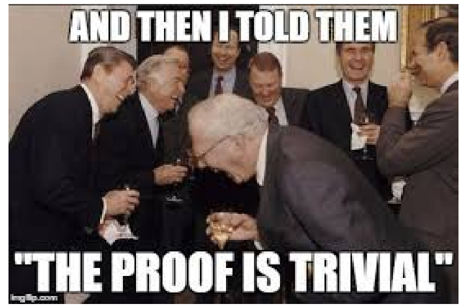
Notes: Pagerank II

1 Conceptual Understanding



The "trivial" problems in the previous notes packet were designed to help you understand "what" the definitions say. The following problems are designed to help you understand "why" the definition of pagerank is defined the way it is. We do this by exploring the strengths and weaknesses both of pagerank and other possible methods of ranking nodes.

The problems below are "less trivial" than the problems in the previous notes packet, but are still "fairly trivial".

Problem 1. Recall that the purpose of the pagerank vector $\boldsymbol{\pi}$ is to provide a ranking of of how important a node is. There are many alternative ways to provide such a ranking. One simple alternative is to rank nodes by their in-degree. For "typical" graphs, the in-degree ranking and the pagerank ranking will be similar, but there are graphs for which the two rankings can be arbitrarily different from each other.

Draw a graph such that the top ranked node according to pagerank is the bottom ranked node according to in-degree.

Problem 2. Either prove or give a counterexample to the following claims.

1. The node with the largest out-degree can never have the highest pagerank.

2. Two nodes in a graph cannot have the same pagerank value.

3. Assume that all nodes in the graph have an in-degree of 2. Then all nodes will have the same pagerank.

2 Runtimes

Problem 3. In this question you will calculate the runtime of the power method for computing pagerank. Assume that P is a sparse matrix and that $\boldsymbol{\pi}$ is dense.

1. Equation 5.1 shows the power method iteration for solving for $\boldsymbol{\pi}$. It is reproduced below

$$\mathbf{x}^{(k)T} = \alpha \mathbf{x}^{(k-1)T}\mathbf{P} + \big(\alpha \mathbf{x}^{(k-1)T}\mathbf{a} + (1-\alpha)\big)\mathbf{v}^T. \tag{1}$$

   What is the runtime of computing $\mathbf{x}^{(k)}$ from $\mathbf{x}^{(k-1)}$?

2. Given only $\mathbf{x}^{(0)}$, what is the runtime of computing $\mathbf{x}^{(K)}$ by iterating Equation (1) K times?

3. When computing pagerank, we typically do not know the final number of iterations K in advance. Instead, we continue our computation until the following condition is met:

$$\|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\|_2 \leq \epsilon, \tag{2}$$

where ϵ is a "small" number that controls how accurate we want our solution to be. The expression $\|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\|_2$ is often called the *residual*.

Compute a formula for the number of iterations K required to achieve a residual less than ϵ.

HINT: See the discussion on page 346.

4. Substitute your answer for part 3 into your answer for part 2 to get a formula for the overall runtime in terms of the final desired residual ϵ.

Problem 4. Repeat Problem 3 assuming P is stored as a dense matrix instead of a sparse matrix.

Problem 5. How do the results from problems 3 and 4 above compare to the LAPACK runtimes for computing the top-eigenvalue of a matrix?

3 Implementation Details

Problem 6. Why does it never make sense to store $\boldsymbol{\pi}$ as a sparse vector?

Problem 7. Why is the following inequality "almost always" true:

$$\|\mathbf{x}^{(k)}\|_2 < \|\mathbf{x}^{(k-1)}\|_2. \tag{3}$$

Based on Inequality (3) above, how should we adjust our implementation of the power method to ensure numerical stability?