Chapter 2 Quiz Practice Problems

Problem 1. For each statement below, circle `True` if the statement is known to be true, `False` if the statement is known to be false, and `Open` if the statement is not known to be either true or false.

1. `True` `False` `Open` Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two hypothesis classes with $d_{\mathrm{VC}}(H_1) > d_{\mathrm{VC}}(H_2)$. Let $g_1 \in H_1$ and $g_2 \in H_2$. Hoeffding's inequality predicts that $|E_{\mathrm{test}}(g_1) - E_{\mathrm{out}}(g_1)|$ is less than $|E_{\mathrm{test}}(g_2) - E_{\mathrm{out}}(g_2)|$.

2. `True` `False` `Open` Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two hypothesis classes with $d_{\mathrm{VC}}(H_1) > d_{\mathrm{VC}}(H_2)$. Then $\mathcal{H}_2 \subseteq \mathcal{H}_1$.

3. `True` `False` `Open` Let g be a hypothesis in the set of positive rays in 1 dimension. As the number of data points N goes to infinity, the generalization error is guaranteed to go to zero.

4. `True` `False` `Open` Let g be a hypothesis in the set of convex sets in 2 dimensions. As the number of data points N goes to infinity, the generalization error is guaranteed to go to zero.

5. `True` `False` `Open` Let
$$\mathcal{H}_{\mathrm{axis2}} = \left\{ \mathbf{x} \mapsto \sigma \operatorname{sign}(x_i) : \sigma \in \{+1, -1\}, i \in [d] \right\},$$
and
$$\mathcal{H}_{\mathrm{circles}} = \left\{ \mathbf{x} \mapsto [\![\|\mathbf{x}\|_2 \geq \alpha]\!] : \alpha \in \mathbb{R}^d \right\}.$$
For $d > 100$, we have that $d_{\mathrm{VC}}(\mathcal{H}_{\mathrm{axis2}}) > d_{\mathrm{VC}}(\mathcal{H}_{\mathrm{circles}})$.

6. `True` `False` `Open` Let $\mathcal{H}$ be a finite hypothesis class with size M. Then $d_{\mathrm{VC}}(\mathcal{H}) = \Theta(\log(M))$.

7. `True` `False` `Open` The VC dimension of finite hypothesis classes can never be ∞.

8. `True` `False` `Open` The VC dimension of infinite hypothesis classes can never be ∞.

9. `True` `False` `Open` Let $\mathcal{H}$ be a hypothesis class. If there exists a hypothesis $h \in \mathcal{H}$ such that $E_{\mathrm{out}}(h) = 0$, then the VC dimension of $\mathcal{H}$ must be finite.

10. True False Open Let $\mathcal{H}$ be a hypothesis class and $\mathcal{X}$ a dataset with N points. If $\mathcal{H}$ cannot shatter $\mathcal{X}$, then it must be the case that $d_{\mathrm{VC}}(\mathcal{H}) \leq N$.

11. True False Open Let $\mathcal{H}$ be a hypothesis class and $\mathcal{X}$ a dataset with N points. If $\mathcal{H}$ can shatter $\mathcal{X}$, then it must be the case that $d_{\mathrm{VC}}(\mathcal{H}) \geq N$.

12. True False Open Let $\mathcal{H}$ be a hypothesis class and $\mathcal{X}$ a dataset with N points. If $\mathcal{H}$ can shatter $\mathcal{X}$, then it must be the case that $m_{\mathcal{H}}(N) \geq N$.

13. True False Open Let $\mathcal{H}$ be a hypothesis class and $\mathcal{X}$ a dataset with N points. If $\mathcal{H}$ cannot shatter $\mathcal{X}$, then it must be the case that $m_{\mathcal{H}}(N) \leq N$.

14. True False Open Let $\mathcal{H}$ be a hypothesis class and $\mathcal{X}$ a dataset with N points. If $m_{\mathcal{H}}(N) = 2^N$, then it must be the case that $\mathcal{H}$ can shatter $\mathcal{X}$.

15. True False Open There exists some hypothesis class $\mathcal{H}$ with growth function $m_{\mathcal{H}}(N) = \Theta(2^{\sqrt{N}})$.

16. True False Open Let $\mathcal{H}$ be a hypothesis class with $m_{\mathcal{H}}(N) = 2^N$ for all N. Let $g \in \mathcal{H}$ be a hypothesis. Then as the number of training data points N goes to infinity, the generalization error of g goes to 0.

17. True False Open Let $\mathcal{H}$ be a hypothesis class with $m_{\mathcal{H}}(N) = \Theta(N^{20})$. Let $g \in \mathcal{H}$ be a hypothesis. Then as the number of training data points N goes to infinity, the generalization error of g goes to 0.

18. True False Open For every hypothesis class $\mathcal{H}$, $m_{\mathcal{H}}(N) = O(2^N)$.

19. True False Open For every hypothesis class $\mathcal{H}$, $m_{\mathcal{H}}(N) = \Omega(2^N)$.

20. True False Open If $m_{\mathcal{H}}(N) < 2^N$, then N is a breakpoint for $\mathcal{H}$.

21. True False Open If $m_{\mathcal{H}}(N) < 2^N$, then $N+1$ is a breakpoint for $\mathcal{H}$.