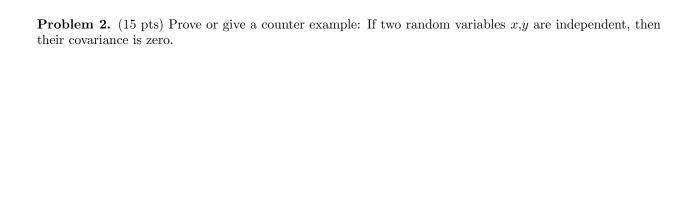
Homework 0: review of probability, statistics, linear algebra, and calculus $${\rm CSCI145/MATH166,\,Mike\,Izbicki}$$

DUE: Thursday, 12 September at the beginning of class

Name:
Problem 1. (10 pts) Section 1.2 of Bishop defines the following terms. Reproduce their definitions below1. sum rule
2. product rule
3. Bayes Theorem
4. expectation
5. linearity of expectation (not formally defined in Bishop, but is an immediate consequence of the definition of expectation) if x, y are random variables, then $\mathbb{E}(x+y) = \mathbb{E}x + \mathbb{E}y$

6.	law of large numbers (Eq. 1.35)
7.	conditional expectation
8.	variance
9.	covariance
10.	standard deviation
11.	mode
need to prior tribute	vill use the following terms frequently in class, so you need to know what they mean, but you do not so be able to formally define them: marginal distribution, conditional distribution, joint distribution, distribution, posterior distribution, normalization, probability density function (PDF), cumulative disting function (CDF), probability mass function (PMF), independent and identically distributed, bias, tting, frequentist statistics versus bayesian statistics, likelihood function, error function, maximum like-

 $lihood\ (ML),\ maximum\ a\ posteriori\ (MAP),\ hyperparameters,\ measure\ theory.$



Problem 3. (15 pts) If two random variables x,y have zero covariance, then they are independent.

Problem 4. (10pts) The variance can be defined as either

$$Var(x) = \mathbb{E}(x - \mathbb{E}(x))^2 \tag{1}$$

or

$$Var(x) = \mathbb{E}(x^2) - (\mathbb{E}x)^2.$$
 (2)

Show that these two definitions are equivalent. HINT: The problem requires applying the definition of expectation and algebraic manipulations.

Problem 5. (5 pts) Section 9.5 of *The Matrix Cookbook* defines orthogonal matrices and lists some of their properties. Reproduce this definition and properties below. (You do not need to reproduce subsections 9.5.1, 9.5.2, or 9.5.3, just the main information under 9.5.)

Problem 6. (5 pts) Sections 9.6.1-9.6.5 of *The Matrix Cookbook* defines positive (semi-)definite matrices and lists some of their properties. Reproduce this definition and properties below.

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3.												
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Problem 8. (15 pts) The log-loss is defined to be

$$\ell(\mathbf{x}; \mathbf{w}) = \log(1 + \exp(-\mathbf{x}^T \mathbf{w})). \tag{3}$$

Calculate the first and second derivatives of (3) with respect to \mathbf{w} . Note that because \mathbf{w} is a vector, the loss is a scalar, the derivative is a vector, and the second derivative is a matrix. See Section 2 of *The Matrix Cookbook* for a review of vector derivatives.

Problem 9. (15 pts) Calculate

$$\underset{\mathbf{w}}{\operatorname{arg\,min}} \left(\|\mathbf{y} - X\mathbf{w}\|^2 + \lambda \|\mathbf{w}\|^2 \right) \tag{4}$$

where **w** is a vector of dimension d, **y** is a vector of dimension n, and X is a matrix with shape $n \times d$. HINT: Recall that the arg min is defined as

$$\underset{\mathbf{w}}{\arg\min} f(\mathbf{w}) \triangleq \{\mathbf{w} : f(\mathbf{w}) = \min_{\mathbf{w}'} f(\mathbf{w}')\}.$$
 (5)

That is, the arg min returns the set of values that minimize the function f. To calculate the arg min, take the derivative of f with respect to \mathbf{w} , set it equal to zero, and solve for \mathbf{w} .

Problem 10. (2pts extra credit) If you complete this assignment in LATEX, then you will receive +2 pts extra credit. The source files are available at:

https://github.com/mikeizbicki/cmc-csci145-math166/tree/master/hw_00.