Practice Quiz: Computational Linear Algebra

Note 1. Your real quiz will have 4 problems following the format below. You will be allowed to use any hand written or electronic reference material that you would like, including websites like wolfram alpha and chatgpt. The only restriction is that you will not be able to communicate with other students.

HINT: Ensure that you pay careful attention to the formal definitions of asymptotic notation in your responses.

Problem 1. For each statement below, circle `True` if the statement is known to be true, `False` if the statement is known to be false, and `Open` if the statement reduces to an open problem. You will receive +1 point for each correct answer, -1 point for each incorrect anwser, and 0 points for each blank answer.

1. `True` `False` `Open` Let $f(n) = 1/(1+n)$. Then $f = \Omega(n^{-2})$.

2. `True` `False` `Open` Let $f(n) = n^3 + 1/n$. Then $f = \Omega(n^3)$.

3. `True` `False` `Open` Let $f(n) = 1/n$. Then $f = \Omega(n^{-2})$.

4. `True` `False` `Open` Let $f(n) = 2^n$. Then $f = \Theta(3^n)$.

5. `True` `False` `Open` Let $f(a, b) = 5a^2 + 3ab$. Then $f = O(a^2 b)$.

6. `True` `False` `Open` Let $f(a, b) = 5a^2 + 3ab$. Then $f = O(a^2 + ab)$.

7. `True` `False` `Open` Let A and B be $n \times n$ matrices. The fastest algorithm for computing the matrix product AB has runtime $O(n^4)$.

8. `True` `False` `Open` Let A and B be $n \times n$ matrices. The fastest algorithm for computing the matrix product AB has runtime $\Theta(n^2 \log n)$.

9. `True` `False` `Open` Let A and B be $n \times n$ matrices. The fastest algorithm for computing the matrix product AB has runtime $\Omega(n^{2.1})$.

10. `True` `False` `Open` Let A and B be $n \times n$ matrices. The fastest algorithm for computing the matrix product AB has runtime $\Omega(n^2)$.

11. `True` **`False`** `Open` Let A and B be $n \times n$ matrices. The fastest algorithm for computing the matrix product AB has runtime $\Omega(n^{2.5})$.

12. `True` `False` **`Open`** The matrix chain ordering problem can be solved in time $\Theta(n)$.

13. **`True`** `False` `Open` Computing $\mathbf{x}^T\mathbf{x}$ is faster than computing $\mathbf{x}\mathbf{x}^T$.

14. **`True`** `False` `Open` There exists a family of $n \times n$ matrices A and B with $\text{nnz}(A) = O(1)$ and $\text{nnz}(B) = O(1)$ such that the product AB satisfies $\text{nnz}(AB) = O(1)$.

15. `True` **`False`** `Open` There exists a family of $n \times n$ matrices A and B with $\text{nnz}(A) = O(1)$ and $\text{nnz}(B) = O(1)$ such that the product AB satisfies $\text{nnz}(AB) = \Omega(n)$.

16. **`True`** `False` `Open` Let A and B be arbitrary $n \times n$ matrices satisfying $\text{nnz}(A) = O(1)$ and $\text{nnz}(B) = O(1)$. Then the product AB must satisfy $\text{nnz}(AB) = O(1)$.

17. **`True`** `False` `Open` Let A and B be arbitrary $n \times n$ matrices satisfying $\text{nnz}(A) = \Omega(n)$ and $\text{nnz}(B) = \Omega(1)$. Then the product AB must satisfy $\text{nnz}(AB) = O(n^2)$.

18. `True` **`False`** `Open` Let A be an $n \times n$ matrix. Then $\text{nnz}(A) = \Omega(n^3)$.

19. **`True`** `False` `Open` Let A be an $n \times n$ matrix. Then $\text{nnz}(A) = O(n^3)$.

20. **`True`** `False` `Open` Let $A : n \times n$ and $\mathbf{x} : n$, then the best possible runtime of computing $(AA^T)^{-1}\mathbf{x}$ is $\Omega(n^2)$.

21. **`True`** `False` `Open` Let $\mathbf{x} : n$, then the best possible runtime of computing $\mathbf{x}\mathbf{x}^T\mathbf{x}\mathbf{x}^T$ is $O(n^2)$.

22. `True` **`False`** `Open` Let $\mathbf{x} : n$, then the best possible runtime of computing $\mathbf{x}\mathbf{x}^T\mathbf{x}\mathbf{x}^T$ is $\Theta(n)$.

23. `True` **`False`** `Open` Let $A : a \times b$ and $\mathbf{x} : b$, then the best possible runtime of computing $\|A\mathbf{x}\|_F^2$ is $\Omega(n^3)$.