

Notes: maximum a posteriori estimation

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Problem 1. The editor for a school newspaper wants to predict the number of readers x who will “like” a given article on facebook. The poisson distribution is often used to model the number of times an event occurs, and so the editor chooses to model x as a poisson distributed random variable. That is,

$$p(x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}. \quad (1)$$

The editor is not satisfied with maximum likelihood estimation and prefers to use MAP estimation with a gamma prior on λ . The gamma distribution has density

$$p(\lambda|\alpha, \beta) = \frac{\beta^\alpha \lambda^{\alpha-1} \exp(-\beta\lambda)}{\Gamma(\alpha)}, \quad (2)$$

where $\beta \geq 0$ and $\alpha \geq 1$ are hyperparameters, and

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} \exp(-\alpha x) dx \quad (3)$$

is the gamma function. Calculate the MAP estimate of λ .