Damped Oscillations in Fluids

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Lab Section 8, Group 3

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Abstract

The aim of this report is to analyze the relationship between fluid viscosity and oscillation damping constant using a spring-mass system. We hypothesized that, if a mass on a spring oscillates in a fluid, the damping constant will decrease proportionally as the viscosity of the fluid increases. This is based on the knowledge that a fluid of higher viscosity will lead to more resistance against objects that flow through it. The mass in this experiment is a small cylindrical weight. An Arduino set-up with an ultrasonic sensor was used to record the displacement of the mass. The ultrasonic sensor sent out pulses, which were reflected off of a piece of cardboard at the top of the spring system. The fluids used include the control variable air, water, olive oil, and hand sanitizer gel. A best-fit function was created using least-squares in Python to represent oscillation for each data set. The resulting equations model sinusoidal motion with decreasing amplitude through each cycle. The average damping constant of each fluid was found using this function. The average of three trials was calculated for each data set to account for error. Comparing the viscosities of each fluid in Pascal-seconds, obtained from external sources, and their respective damping constants, we determined that the damping constant for a spring-mass oscillating system will decrease as viscosity increases.

Introduction

This experiment utilizes the idea of damped harmonic motion. Additionally, this experiment takes inspiration from a previous report. The objective of that project was to determine the effect of salt levels in water on a mass oscillating in the water. For this experiment, we calculated damped oscillations when liquids of different viscosities are used as mediums. To start, the idea of damped harmonic motion comes from simple harmonic motion. Specifically, in an ideal scenario, oscillating motion in SMH does not die out. However, without a continuous force, there is friction that acts to dampen the motion. This phenomenon is known as damped harmonic motion. Relevant equations include the following: [1] $y = Acos(\omega t + \phi)$; [2] $\tau = \frac{t}{ln(\frac{y(0)}{y(t)})}$; [3]

$y = Ae^{-\frac{t}{\tau}} cos(\omega t + \phi)$. The first equation is the function for simple harmonic motion; the second equation describes the damping constant, τ; and the last equation describes damped harmonic motion function. Where A is the amplitude, t is for time, ω is the angular frequency, and ϕ is the phase shift.

The damping constant, τ, is a measurement of the system's time as it returns to equilibrium as the force dissipates its energy[5]. Depending on the value, the system can be underdamped, critically damped, and overdamped. Underdamped means the oscillating motion's "amplitude of the motion decays exponentially"[6] and the damping constant is small. Overdamped is when equilibrium does not approach equilibrium until a long period of time where there is no oscillation and the damping constant is large[6]. Critically damped is where there are no oscillations and the system rapidly approaches equilibrium[6]. The experiment looks into the undamping of a system, since it is expected that the more viscous a liquid, the lower the tau. Viscosity is the "quantity that describes a fluid's resistance to flow"[7].

To reiterate, the hypothesis is that if a mass on a spring oscillates in a fluid, the damping constant will decrease proportionally as the viscosity of the fluid increases. What is expected of this project is to measure the damping constant to prove the hypothesis that an increase in viscosity of a liquid will decrease this constant. The set-up of the experiment included a mass-spring system, with a container to hold the liquid that the mass goes in and out of when it oscillates. Once the data is collected with Arduino components from the three liquids (water, hand sanitizer, and oil) and air, it is then analyzed using Python to create graphs like position over time. Data gathering is done using an ultrasonic sensor which is transferred to the computer using a bluetooth module.

Methods

Equipment

- Ultrasonic sensor HC-SR04
- Uno R3 board
- Expansion module Prototype Shield v.5
- Battery pack
- Bluetooth module HC-05
- Jumper wires
- String
- Support stand
- Clamp and metal rod
- Spring
- Tape
- Plastic bottle
- Water
- Olive oil
- Hand sanitizer gel

This experiment aims to utilize a spring-mass system to test how resistance due to different viscosities impacts the damping of oscillation. Specifically, we will be comparing the damping constant for the best-fit functions representing oscillation with the viscosity of fluids used in each trial. Figure 1 shows the Arduino components used to collect data. The movement of the spring was measured using an ultrasonic sensor and a static reference surface. As shown in Figure 1, we utilized a bluetooth module and a battery pack to run the Arduino components without a wire connection to a laptop, which could potentially disrupt the motion of the oscillations.

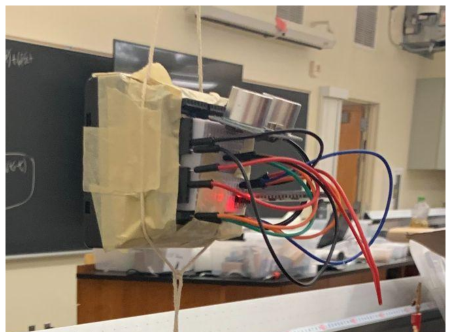


Figure 1

To perform this experiment we attached a spring to a stand, from which we suspended the Arduino components. We attached the mass that is submerged in the fluid to the Arduino, using a long enough string to avoid fluid from being splashed on the electronic components. We made the decision to attach a flat surface to the top of the stand to give the ultrasonic sensor a point of reference. If pointing directly downwards, the container of fluid may have caused interference,making the table an unreliable point of reference. As shown in the concept drawing in Figure 2, we initially intended to cut a hole into a piece of cardboard for the spring to pass through, giving the ultrasonic sensor a close surface, minimizing noise. However, since the pulses may have passed through this opening, we eliminated this source of error by placing the cardboard above the spring as shown in Figure 3. For the purpose of this experiment, we made the assumption that the motion of the spring-mass system was simply vertical, that no pendulum motion affected the data recorded by the ultrasonic sensor.

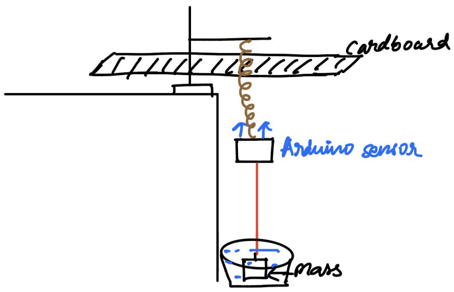


Figure 2

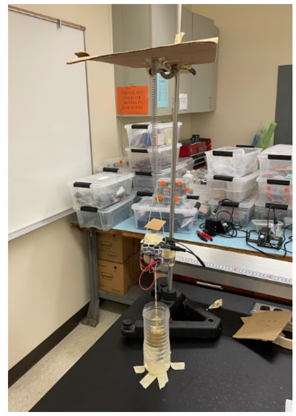


Figure 3

We chose a total of four fluids to run tests on: air, water, olive oil, and hand sanitizer. Air[3] was our control variable, and was anticipated to result in the least damping as it had the lowest viscosity of 1.789 × 10-5 Pa*s. Water[1] has a viscosity of 0.001 Pa*s and olive oil[2] has a viscosity of 0.0341 Pa*s. While running experiments on pure hand sanitizer, we found it was too viscous to collect reliable data; we could not get the mass to oscillate. So, we diluted the sanitizer gel[4] with water for a mixture with a viscosity of 66.67 Pa*s.

The code controlling the function of the ultrasonic sensor is shown in Figure 4. The ultrasonic sensor measures distance by sending and receiving a series of pulses. The code sends a pulse by setting trigPin to “HIGH”. The code includes a delay of 10 milliseconds between each sample. For each loop, “trigPin” is initially set to “LOW” for two microseconds, then set to “HIGH” for ten microseconds, then set back to “LOW”. The ultrasonic sensor reads in the pulses reflected off of a surface, and records the time taken for the pulse to travel there and back. These data points are displayed on the monitor through the Ardino serial port.

```
//------- UltraSound -------
digitalWrite(trigPin, LOW);
delayMicroseconds(2);
digitalWrite(trigPin, HIGH); //Send pulse
delayMicroseconds(10);
digitalWrite(trigPin, LOW);
duration = pulseIn(echoPin, HIGH); // Recieve pulse
cm = (duration/2)*0.0343;
delay(1);
mySerial.print(millis());
mySerial.print(';');
mySerial.println(cm);
delay(10); // Delay in between samples
```

Figure 4

We ran each experiment by displacing the mass to the table, ensuring the initial displacement of each trial was the same. We collected data for fifteen oscillations, assuming this would allow enough data points to be collected to demonstrate damping. We ran three trials for each of the four fluids.

Analysis

The first step in the analysis part involves reading in the raw data from the ultrasonic sensor. The plot for the raw position vs time is shown in Figure 5 below.

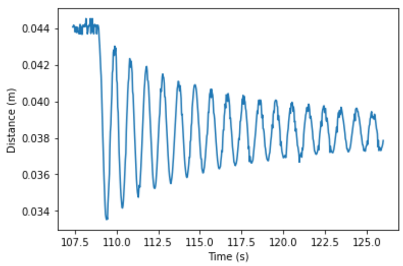


Figure 5

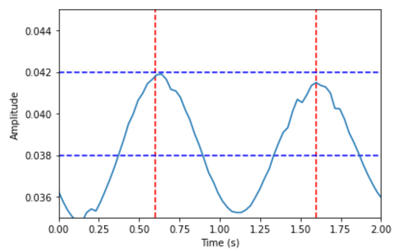


Figure 6

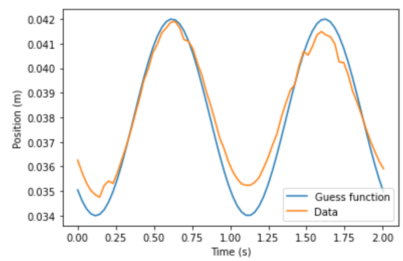


Figure 7

In order to guess the parameters better, we focus on a small part of the domain. Therefore, we clip the data. Figure 6 highlights the desired curve. Next, we guess the parameters for our cosine fit function, which are amplitude, omega, offset and phi. Then, we plot our guess function along with the curve obtained in Figure 6, shown in Figure 7.

Next, we find the best parameters for the cosine curve using the function `best_parameters_water_1 = res_lsq_water_1['x'],` where `res_lsq_water_1` uses the least_squares function. This particular plot is shown in Figure 8.

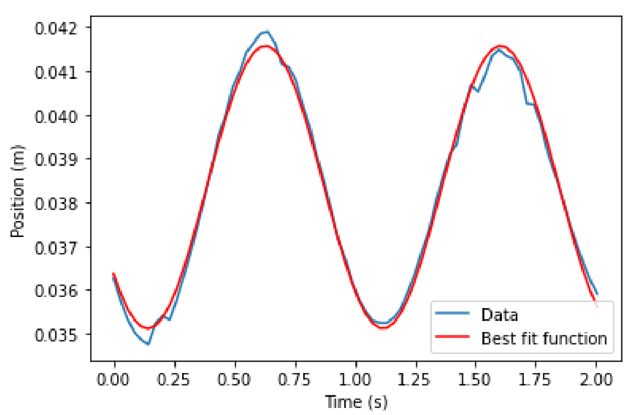


Figure 8

Observe that the best fit function overlaps with the raw data plot quite well. Therefore, we proceed with applying the same best fit function on the entire raw dataset and not just a segment. The result of this procedure is shown in figure 9.

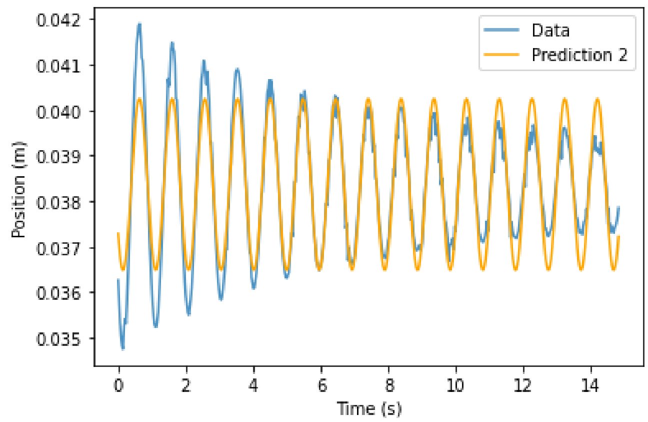


Figure 9

We notice that the best fit function doesn't fit the **entire** raw data. The reason for this is that we did not take into account the damping effect caused by the fluid. Next, we include the damping factor as well.

First, we need to guess the damping constant. Solving the equations for y(0) and y(t), we get the equation $y(t)/\ y(0)\ =\ e^{-t/\tau}$. Taking log on both sides and solving for τ gives us the equation

$$\tau\ =\ t/ln(y(0)/y(t)) \qquad [1]$$

We use this mathematical relationship to guess the value of tau (τ) or the damping constant from the graph.

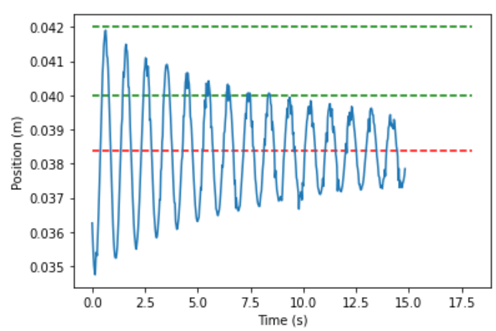


Figure 10

We draw two horizontal lines to note the values of the function at two different intervals, y(0) and y(t). Then, we use the function `find_tau(t, y_t, y_0)` , which uses the relationship [1] to get the value of tau (τ). Next, we append this value to the array of guess parameters. In the final step, we perform best curve fitting using the `get_residuals` function, which gives the final graph as shown in figure 11 below. Observe that the best fit curve fits the raw data very well. We note down the best value of tau for further analysis.

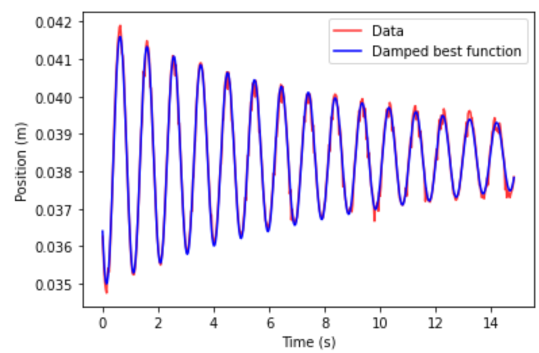
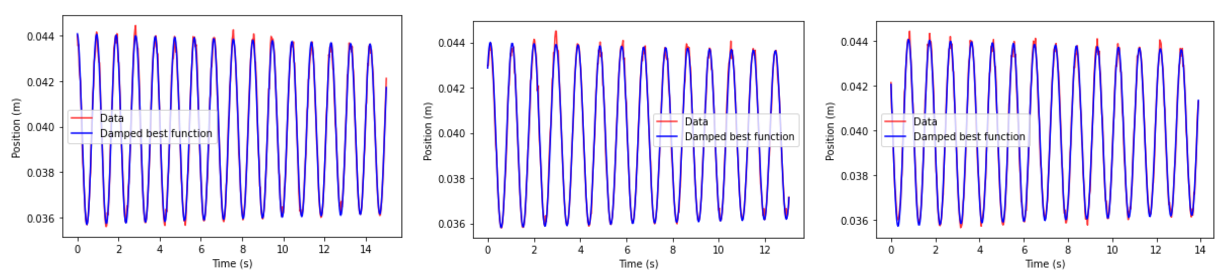


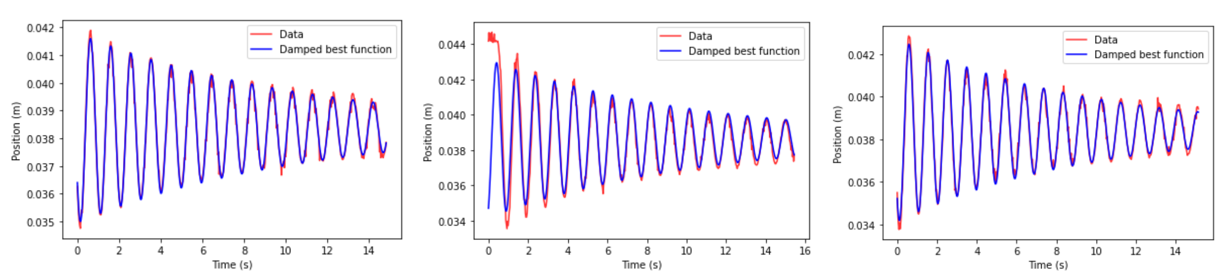
Figure 11.

The analysis method described above is used for different fluids and the values of tau are noted. The final graphs for different fluids are shown in Figure 12.

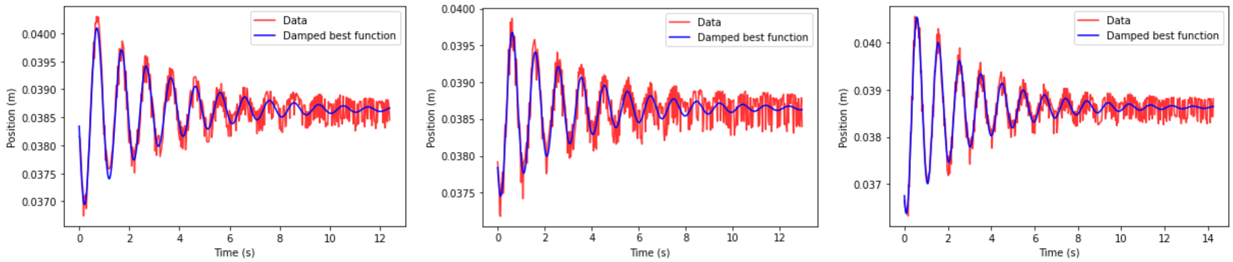
AIR:



WATER:



OIL:



SANITIZER:

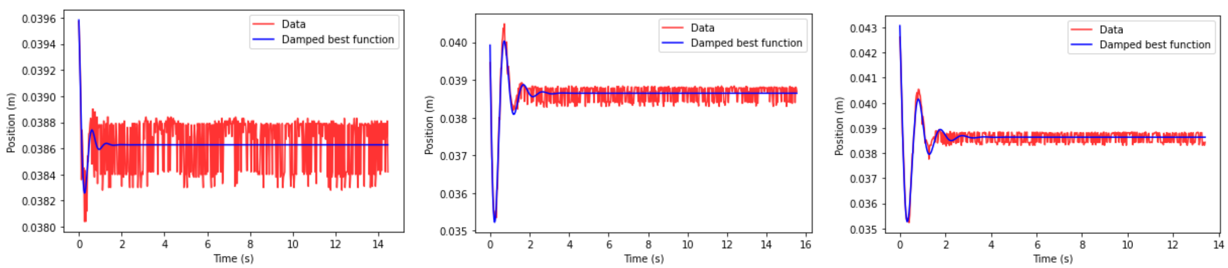


Figure 12

The tau value or the damping constant for each of these experiments are summarized in the table below. To account for the uncertainty in the values for tau, we calculate the standard deviations for the three trials for each fluid.

Fluid	Trial 1	Trial 2	Trial 3	Standard Deviation	**Average Tau (s)**
Air	122.410	107.396	130.053	9.411	119.953 +/- 9.411
Water	10.914	9.385	10.564	0.654	10.288 +/- 0.654
Oil	2.975	3.117	3.117	0.067	3.069 +/- 0.067
Sanitizer	0.288	0.603	0.534	0.136	0.471 +/- 0.136

Table 1

The plot of the average tau values gives the plot in Figure 13.

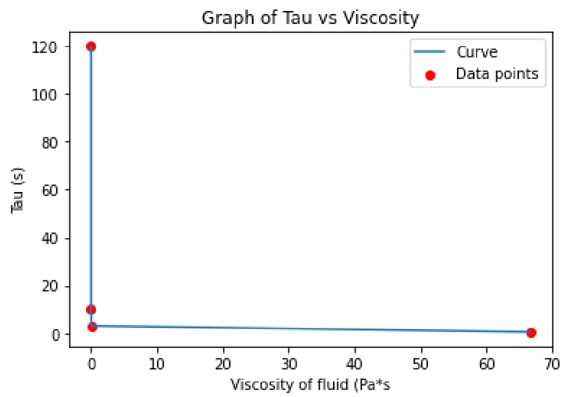


Figure 13

The plot above supports our hypothesis that the value of tau decreases as viscosity of fluid increases.

Conclusion

We initially hypothesized that there would be an inverse relationship between viscosity and the damping constant obtained via analysis in Python. Our results demonstrate such a relationship, confirming our hypothesis is correct. The damped best-fit curves generated for each data set closely fit the oscillation recorded by the ultrasonic sensor. As predicted, air had the largest damping constant, an average of 199.953. The effects of this large constant can be observed by looking at the graph representing oscillation in air in Figure 12. The damping constant is so large, for the first fifteen oscillations, the graph almost resembles simple harmonic motion. On the other hand, the damping constant of hand sanitizer gel is so small, 0.471, we were unable to obtain more than a few oscillations. Water had an average damping constant of 10.288. Oil had a damping constant of 3.069. The results, as graphed in Figure 13, demonstrate an inverse relationship between viscosity and damping constant.

While we have obtained viable evidence to support our ideas, there are numerous potential sources of error to be addressed. We made the assumption that the spring-mass system solely moved vertically; however, the unbalanced masses attached to the spring may have caused a slight pendulum motion, altering the sinusoidal motion recorded by the ultrasonic sensor. Error in our data is evidenced by the fact that none of the best-fit curves are a completely perfect match, especially for trails run using fluids of higher viscosity. Comparing the fitted curves of water and water in Figure 12, it is clear that there is more noise in the last few oscillations of the oil data due to its higher viscosity. In addition, for each trial run testing hand sanitizer, we failed to achieve fifteen oscillations to model motion. For other data samples, when running analysis in Python, we were able to isolate the first few peaks to resemble simple harmonic motion and attain initial parameters. We were unable to do this for hand sanitizer as each data sample resulted in less than three oscillations. Thus, the tau value obtained for sanitizer may be less reliable than the other damping constants.

We addressed error by calculating the standard deviation of tau values for each fluid. Although there was slight variation in the tau values from each data set, the values, as shown in table 1, were similar across all three trials for each fluid. For water and oil, the standard deviation is small; both are less than one. The standard deviation for water is within 6.35% of the average tau value. The standard deviation for oil is within 2.18% of the average. The standard deviation for air, 9.411, is larger, but the average tau value of air, 119.953, is much larger than that of the other fluids; so, a larger error value is acceptable. It is within 7.84% of the average. The standard deviation calculated for hand sanitizer gel is 28.87% of the average value. Because the standard deviation is almost a third of the average tau, the results obtained from the hand sanitizer gel are not as reliable as the damping constants of the other fluids. This is possibly due to having less oscillations to analyze in each data set. The results for air, water, and olive oil are reliable.

This experiment could be expanded upon by performing the same methods to fluids of different viscosities. This experiment only used four different fluids, thus the graph shown in Figure 13 only shows the relationship for four data points. Having a plethora of data points to generate a best fit curve would allow us to better assess the relationship between fluid viscosity and damping. Along with using different fluids, we could use masses of different surface areas. Because the object oscillating in the fluid experiences resistance, a larger surface area would predictably result in more resistance, affecting the resulting damping constant.

References

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