Assignment 2

AI1110: Probability and Random Variables
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Question 12.13.6.7 :
A die is thrown again and again until 3 sixes are obtained. Find the probability of obtaining third six in the sixth throw of the die.

Solution:

Let X_i be a random variable representing a an outcome of 6 in i^{th} throw of die and Y be the random variable of the number of sixes in first 5 throws
Y is binomial random variable with n=5 ,p=1/6

Parameter	Value	Description
X_i	1	An outcome of 6
X_i	0	Not an outcome of 6
Y	$\{0,1,2,3,4,5\}$	Number of 6's in 5 throws of die

TABLE 1
X,Y ARE RANDOM VARIABLES

$\therefore \Pr(X_i = 1)=\frac{1}{6}$, $\Pr(X_i = 0)=\frac{5}{6}$

Clearly,

$$\Pr(Y = k) = {}^{n}C_k \times p^k \times (1-p)^{n-k} \tag{1}$$
$$\implies \Pr(Y = k) = {}^{5}C_k \times (1/6)^k \times (5/6)^{5-k} \tag{2}$$

Since, the sixth throw is needed to be a 6, exactly 2 throws out of first 5 should show '6', i.e, Y=2

$$\implies \Pr(Y = 2) = {}^{5}C_2 \times (1/6)^2 \times (5/6)^3 \tag{3}$$
$$\Pr(Y = 2) = 10 \times \frac{(5)^3}{(6)^5} \tag{4}$$

The sixth throw is needed to be a 6 as given in the question
$\therefore$ For the sixth throw, $X_6 = 1$

$$\therefore \text{Required Probability} = \Pr(Y = 2) \times P(X_6 = 1) \tag{5}$$
$$= 10 \times \frac{(5)^3}{(6)^5} \times \frac{1}{6} \tag{6}$$
$$= \frac{625}{23328} \tag{7}$$

(Assumed that all the throws are mutually independent)

$\therefore$ The required probability = 625/23328