Assignment 2

AI1110: Probability and Random Variables
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Question 12.13.6.7 :
A die is thrown again and again until 3 sixes are obtained. Find the probability of obtaining third six in the sixth throw of the die.

Solution:

Let X be a random variable representing a an outcome of 6 in a throw of die and Y be the random variable of the number of sixes in first 5 throws

$$X = \begin{cases} 1 & \text{if outcome is 6} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$\therefore \Pr(X = 1) = \frac{1}{6}$, $\Pr(X = 0) = \frac{5}{6}$

Y is binomial random variable with n=5 ,p=1/6

Clearly,

$$\Pr(Y = k) = {}^{n}C_k \times p^k \times (1-p)^{n-k} \tag{2}$$
$$\implies \Pr(Y = k) = {}^{5}C_k \times (1/6)^k \times (5/6)^{5-k} \tag{3}$$

Since, the sixth throw is needed to be a 6, exactly 2 throws out of first 5 should show '6'
Let E be the event that first 5 throws will result in exactly 2 throws

$$\implies \Pr(E) = \Pr(Y = 2) \tag{4}$$
$$\Pr(E) = {}^{5}C_2 \times (1/6)^2 \times (5/6)^3 \tag{5}$$
$$\Pr(E) = 10 \times \frac{(5)^3}{(6)^5} \tag{6}$$

Let F be the random variable that third 6 comes in the sixth throw

$$\Pr(F) = \Pr(E) \times P(X = 1) \tag{7}$$
$$\Pr(F) = 10 \times \frac{(5)^3}{(6)^5} \times \frac{1}{6} \tag{8}$$
$$\Pr(F) = \frac{625}{23328} \tag{9}$$

(Assumed that all the throws are mutually independent)

$\therefore$ The required probability = 625/23328