# DNA00

August 2, 2016

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## 2 Introduction

### 2.1 Dynamic Network Analysis of Enron Email Network Data

I use the Enron email network data from John Hopkins which has time, sender and receiver pair format data.

### 2.2 Data Preprocessing

From the JHU data, I have done the following in Excel: - The first column represents seconds elapsed since 1 January 1970, so I convert this in to days - I then add these days to the date to get time stamps for all nodes - From the timestamps, I extract the year field - The network can be partitioned by year in a cumulative manner for DNA

### 2.3 Key Assumption

The key assumptio in this analysis is that the nodes can be appended to the original network at time, t0 with the new nodes from time, t+1.

## 3 Import Libraries

```
In [1]: import pandas as pd
        import numpy as np
        import networkx as nx
        import seaborn as sns
        import matplotlib.pyplot as plt
        import scipy as sc
        %matplotlib inline
        sns.set(style="whitegrid", color_codes=True, context='paper')
        import random
        random.seed(1111111111111)
        plt.rc('axes', grid=False, titlesize='large', labelsize='medium',labelweigh
        plt.rc('lines', linewidth=4)
        plt.rc('font',family='serif',size=12, serif='Georgia')
        plt.rc('figure', figsize = (15,6),titlesize='large',titleweight='heavy')
        plt.rc('grid',linewidth=3)
        sns.set_palette('cubehelix')
        from scipy.signal import *
        from numpy.linalg import *
```

## 4 Import Data

```
In [2]: data = pd.read_excel("../Data/execs.email.linesnum.xlsx")

In [3]: data.head()

Out[3]:          sec  to  from                 date  year
        0  315522000  24   153 1979-12-31 21:00:00  1979
        1  315522000  24   153 1979-12-31 21:00:00  1979
        2  315522000  29    29 1979-12-31 21:00:00  1979
        3  315522000  29    29 1979-12-31 21:00:00  1979
        4  315522000  29    29 1979-12-31 21:00:00  1979

In [4]: data.min()
```

```
Out[4]: sec                315522000
        to                         0
        from                       0
        date     1979-12-31 21:00:00
        year                    1979
        dtype: object
```

```
In [5]: data.max()
```

```
Out[5]: sec               1024688419
        to                       183
        from                     183
        date     2002-06-21 19:40:19
        year                    2002
        dtype: object
```

# 5 Data Partition

```
In [6]: #year = data['year'].unique()
        year = sorted(set(data['year']))
        year
```

```
Out[6]: [1979, 1998, 1999, 2000, 2001, 2002]
```

```
In [7]: sorted(set(data['year']))
```

```
Out[7]: [1979, 1998, 1999, 2000, 2001, 2002]
```

```
In [8]: data.drop(["sec", "date"], axis=1,inplace=True)
```

```
In [9]: data.head()
```

```
Out[9]:    to  from  year
        0  24   153  1979
        1  24   153  1979
        2  29    29  1979
        3  29    29  1979
        4  29    29  1979
```

## 5.1 Break data into years

```
In [10]: G0 = data[data["year"]==year[0]]
         G1 = data[data["year"]==year[1]]
         G2 = data[data["year"]==year[2]]
         G3 = data[data["year"]==year[3]]
         G4 = data[data["year"]==year[4]]
         G5 = data[data["year"]==year[5]]
```

```
In [11]: G1.size,G1.shape
```

```
Out[11]: (246, (82, 3))

In [12]: G2.size,G1.shape

Out[12]: (11145, (82, 3))

In [13]: G3.size,G1.shape

Out[13]: (132177, (82, 3))

In [14]: G3.size,G1.shape

Out[14]: (132177, (82, 3))

In [15]: G4.size,G1.shape

Out[15]: (206664, (82, 3))

In [16]: G5.size,G1.shape

Out[16]: (25473, (82, 3))

In [17]: G1.head()

Out[17]:       to  from  year
         174  114   169  1998
         175  114   169  1998
         176  114   123  1998
         177  114   123  1998
         178  114   123  1998

In [18]: G1.tail()

Out[18]:       to  from  year
         251  112    65  1998
         252  112   114  1998
         253  112   114  1998
         254  112   145  1998
         255  112   145  1998

In [19]: G2.head()

Out[19]:       to  from  year
         256  114    65  1999
         257  114    65  1999
         258  114   169  1999
         259  114   169  1999
         260  114   112  1999

In [20]: G3.head()

Out[20]:        to  from  year
         3971   82    51  2000
         3972   82    51  2000
         3973   82    51  2000
         3974   82    51  2000
         3975   82    51  2000
```

## 5.2 Create networks at different timesteps

### 5.2.1 t = 0

```
In [21]: G0_ = np.asarray(G0.ix[:,:2])
         Gt0 = nx.Graph()
         Gt0= nx.from_edgelist(G0_)
```

### 5.2.2 t =1

```
In [22]: G1_ = G1.ix[:,:2]
         G1_ = np.concatenate((G1_,G0_), axis=0)
         Gt1 = nx.Graph()
         Gt1= nx.from_edgelist(G1_)
```

### 5.2.3 t = 2

```
In [23]: G2_ = G2.ix[:,:2]
         G2_ = np.concatenate((G2_,G1_), axis=0)
         Gt2 = nx.Graph()
         Gt2= nx.from_edgelist(G2_)
```

### 5.2.4 t = 3

```
In [24]: G3_ = G3.ix[:,:2]
         G3_ = np.concatenate((G3_,G2_), axis=0)
         Gt3 = nx.Graph()
         Gt3= nx.from_edgelist(G3_)
```

### 5.2.5 t = 4

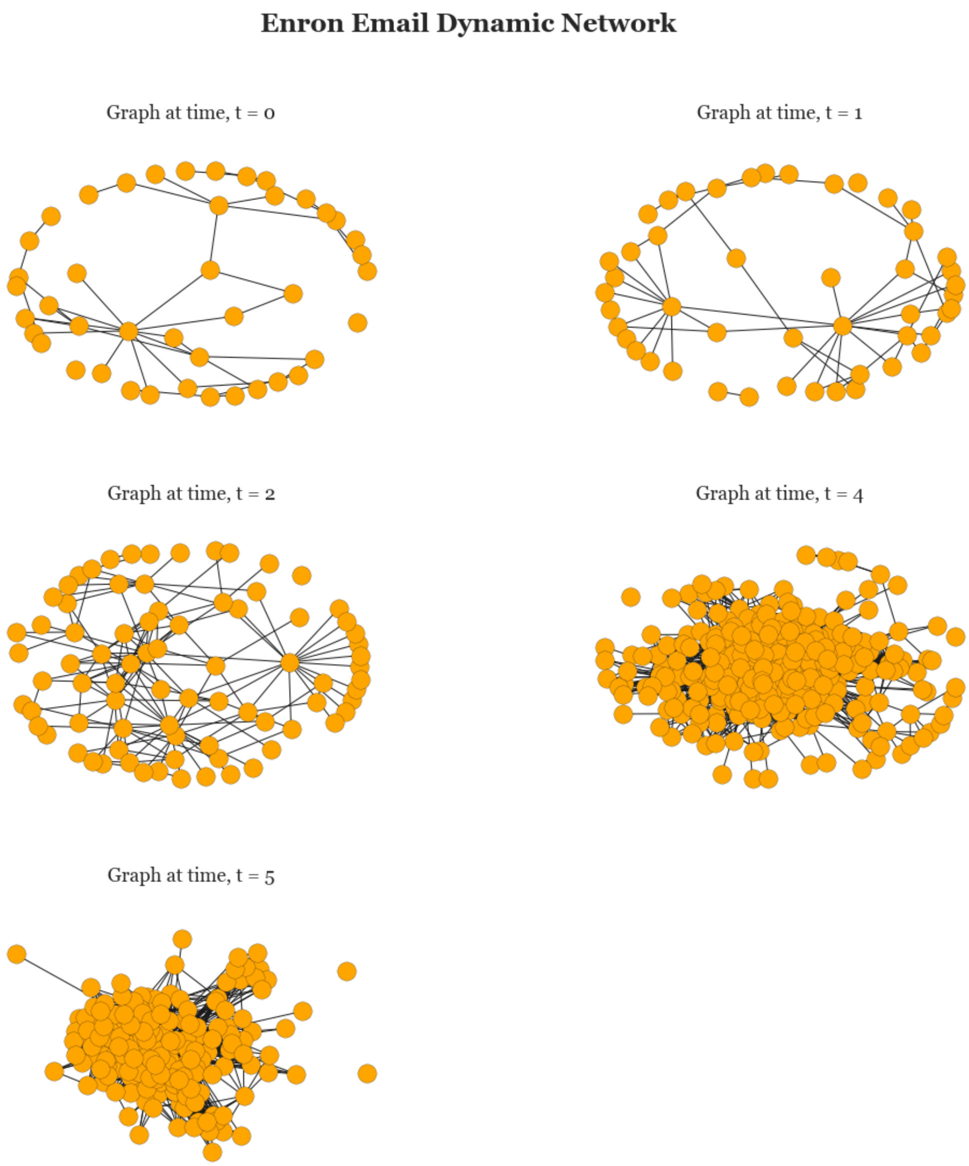
```
In [25]: G4_ = G4.ix[:,:2]
         G4_ = np.concatenate((G4_,G3_), axis=0)
         Gt4 = nx.Graph()
         Gt4= nx.from_edgelist(G4_)
```

### 5.2.6 t = 5

```
In [26]: G5_ = G5.ix[:,:2]
         G5_ = np.concatenate((G5_,G4_), axis=0)
         Gt5 = nx.Graph()
         Gt5= nx.from_edgelist(G5_)
```

```
In [27]: #Plot graphs together
         plt.figure(figsize=(18,18))
         plt.suptitle('Enron Email Dynamic Network', fontsize=24)
         plt.subplot(321)
         nx.draw_spring(Gt0, cmap=plt.cm.inferno, node_color='#FFA500')
         plt.title("Graph at time, t = 0", fontsize=18)
```

```
plt.subplot(322)
nx.draw_spring(Gt1, cmap=plt.cm.inferno, node_color='#FFA500')
plt.title("Graph at time, t = 1", fontsize=18)

plt.subplot(323)
nx.draw_spring(Gt2, cmap=plt.cm.inferno, node_color='#FFA500')
plt.title("Graph at time, t = 2", fontsize=18)

plt.subplot(324)
nx.draw_spring(Gt3, cmap=plt.cm.inferno, node_color='#FFA500')
plt.title("Graph at time, t = 3", fontsize=18)

plt.subplot(324)
nx.draw_spring(Gt4, cmap=plt.cm.inferno, node_color='#FFA500')
plt.title("Graph at time, t = 4", fontsize=18)

plt.subplot(325)
nx.draw_spring(Gt5, cmap=plt.cm.inferno, node_color='#FFA500')
plt.title("Graph at time, t = 5", fontsize=18)

plt.show()
```




# 6 Network Statistics

## 6.1 Centrality analysis without averaging

Define some helper functions here

```
In [28]: def get_cent(net):
             degC = nx.degree_centrality(net)
             cloC = nx.closeness_centrality(net)
             betC = nx.betweenness_centrality(net)
```

```
            eigC = nx.eigenvector_centrality_numpy(net)
            commCC = nx.communicability_centrality(net)
            katzC = nx.katz_centrality_numpy(net)
            loadC = nx.load_centrality(net)

            return [degC,cloC,betC,eigC,commCC,katzC, loadC]
```

```
In [29]: def get_val(val):
             return sorted(set(val.values()))
```

```
In [30]: def get_top_keys(dictionary, top):
             items = dictionary.items()
             items.sort(reverse=True, key=lambda x: x[1])
             return map(lambda x: x[0], items[:top])
```

```
In [31]: def fft_sig(att):
             return sc.fft(get_val(att))

         def hilbert_sig(att):
             return hilbert(get_val(att))
```

```
In [32]: def rms(a, axis=None):
             from numpy import mean, sqrt,square
             rms = sqrt(mean(square(a), axis=axis))
             return rms

         def nrms(a,b):
             nrms = rms(a-b)/(rms(a)+ rms(b))
             return nrms
```

```
In [33]: def cossim(x,y):
             from numpy import dot, sqrt
             csim = dot(x,y) / sqrt(dot(x,x))*sqrt(dot(y,y))
             return csim
```

```
In [34]: def pairwise_calc(df,func):
             val = []
             for x,y in df.iteritems():
                 for z,y in df.iteritems():
                     i = 0
                     #print(y[i], y[i+1])
                     val.append(func(y[i],y[i+1]))
                     i=i+1

             return val[:df.shape[0]]
```

```
In [35]: def avg_cent(cent):
             avg = sum(set(cent.values()))/len(cent)
             return avg
```

```
In [36]: def cal_stat(net):
             degC = nx.degree_centrality(net)
             cloC = nx.closeness_centrality(net)
             betC = nx.betweenness_centrality(net)
             eigC = nx.eigenvector_centrality_numpy(net)
             algC = nx.algebraic_connectivity(net)
             clustC = nx.average_clustering(net)
             commC = nx.communicability_centrality(net)
             katzC = nx.katz_centrality_numpy(net)
             loadC= nx.load_centrality(net)

             return [avg_cent(degC),avg_cent(cloC),\
                     avg_cent(betC),avg_cent(eigC),\
                     algC, clustC, avg_cent(commC),\
                     avg_cent(katzC), avg_cent(loadC)]
```

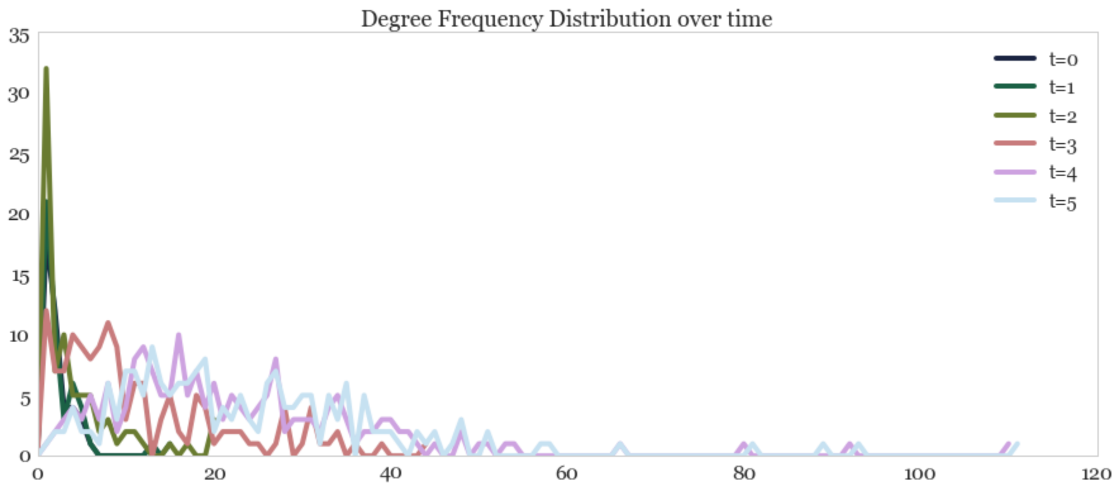
### 6.1.1 Calculate all centralities in one go

```
In [37]: degC0, cloC0, betC0, eigC0, commuC0, katzC0, loadC0 = get_cent(Gt0)
         degC1, cloC1, betC1, eigC1, commuC1, katzC1, loadC1 = get_cent(Gt1)
         degC2, cloC2, betC2, eigC2, commuC2, katzC2, loadC2 = get_cent(Gt2)
         degC3, cloC3, betC3, eigC3, commuC3, katzC3, loadC3 = get_cent(Gt3)
         degC4, cloC4, betC4, eigC4, commuC4, katzC4, loadC4 = get_cent(Gt4)
         degC5, cloC5, betC5, eigC5, commuC5, katzC5, loadC5 = get_cent(Gt5)
```

### 6.1.2 Degree Centrality

```
In [38]: plt.title("Degree Frequency Distribution over time", fontsize=18)
         plt.plot(nx.degree_histogram(Gt0), label='t=0')
         plt.plot(nx.degree_histogram(Gt1), label='t=1')
         plt.plot(nx.degree_histogram(Gt2), label='t=2')
         plt.plot(nx.degree_histogram(Gt3), label='t=3')
         plt.plot(nx.degree_histogram(Gt4), label='t=4')
         plt.plot(nx.degree_histogram(Gt5), label='t=5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

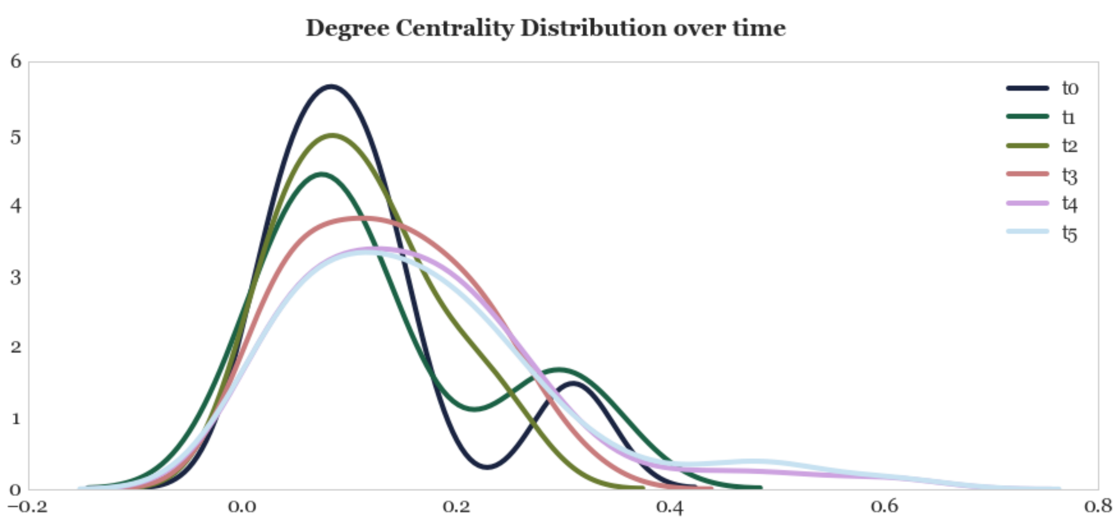
```
Out[38]: <matplotlib.legend.Legend at 0x21c1c6c8a20>
```




```
In [39]: plt.suptitle('Degree Centrality Distribution over time', fontsize=18)
         sns.distplot(get_val(degC0), hist=False, label='t0')
         sns.distplot(get_val(degC1), hist=False, label='t1')
         sns.distplot(get_val(degC2), hist=False, label='t2')
         sns.distplot(get_val(degC3), hist=False, label='t3')
         sns.distplot(get_val(degC4), hist=False, label='t4')
         sns.distplot(get_val(degC5), hist=False, label='t5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\statsmodels\nonparametric\kdetools.p
  y = X[:m/2+1] + np.r_[0,X[m/2+1:],0]*1j
```

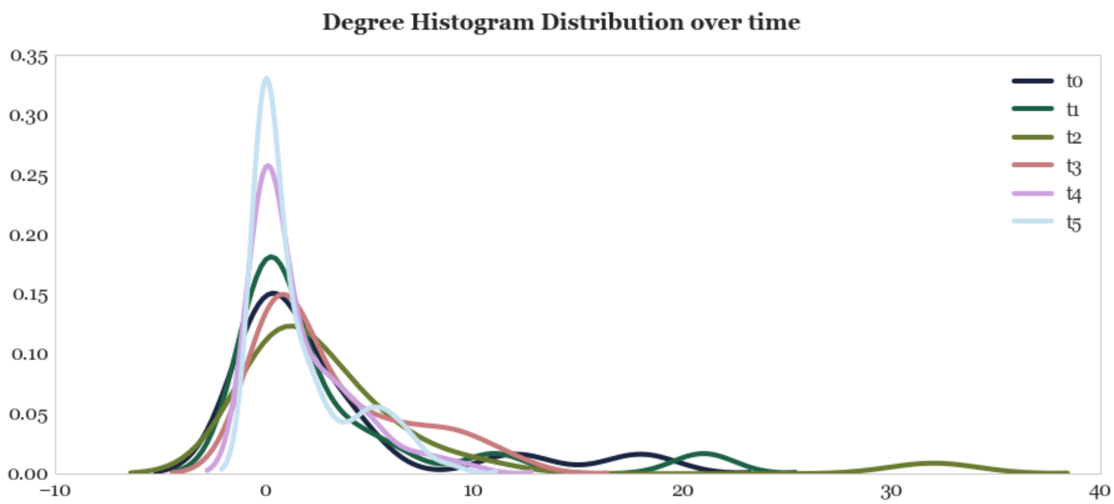
```
Out[39]: <matplotlib.legend.Legend at 0x21c1c8dba20>
```



```
In [40]: plt.suptitle('Degree Histogram Distribution over time', fontsize=18)
         sns.distplot(nx.degree_histogram(Gt0), hist=False, label='t0')
         sns.distplot(nx.degree_histogram(Gt1), hist=False, label='t1')
         sns.distplot(nx.degree_histogram(Gt2), hist=False, label='t2')
         sns.distplot(nx.degree_histogram(Gt3), hist=False, label='t3')
         sns.distplot(nx.degree_histogram(Gt4), hist=False, label='t4')
         sns.distplot(nx.degree_histogram(Gt5), hist=False, label='t5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\statsmodels\nonparametric\kdetools.p
  y = X[:m/2+1] + np.r_[0,X[m/2+1:],0]*1j
```

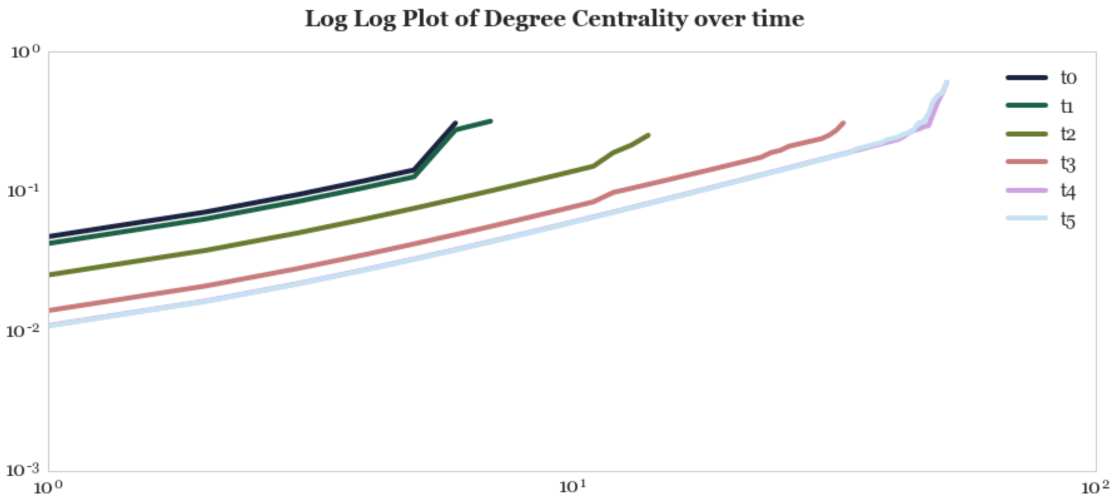
```
Out[40]: <matplotlib.legend.Legend at 0x21c1c95cb00>
```




```
In [41]: plt.suptitle('Log Log Plot of Degree Centrality over time', fontsize=18)

         plt.loglog(get_val(degC0), label='t0')
         plt.loglog(get_val(degC1), label='t1')
         plt.loglog(get_val(degC2), label='t2')
         plt.loglog(get_val(degC3), label='t3')
         plt.loglog(get_val(degC4), label='t4')
         plt.loglog(get_val(degC5), label='t5')

         plt.yticks(fontsize=16)
```

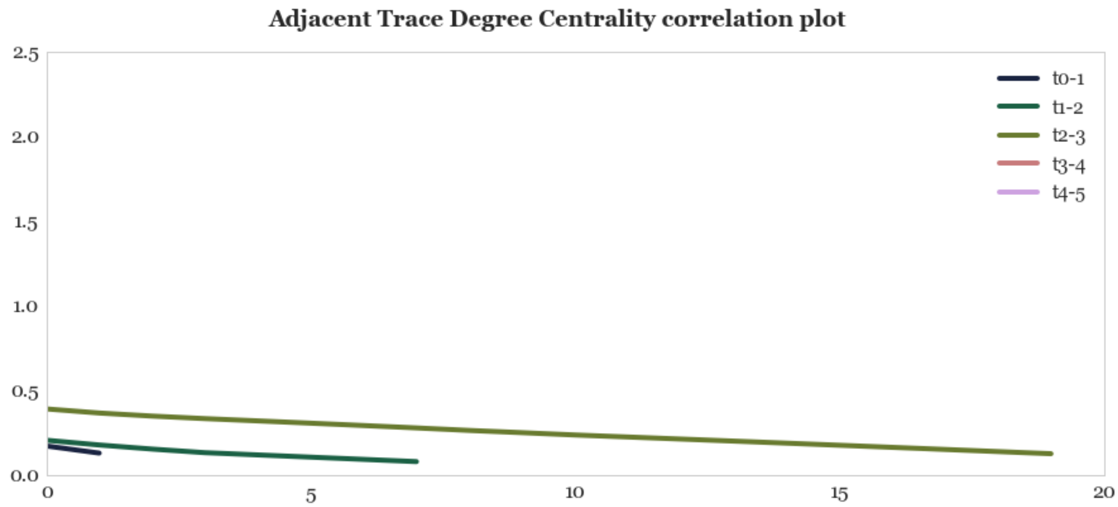
```
        plt.xticks(fontsize=16)
        plt.legend(loc=1, fontsize=15)
```

```
Out[41]: <matplotlib.legend.Legend at 0x21c1c62bdd8>
```

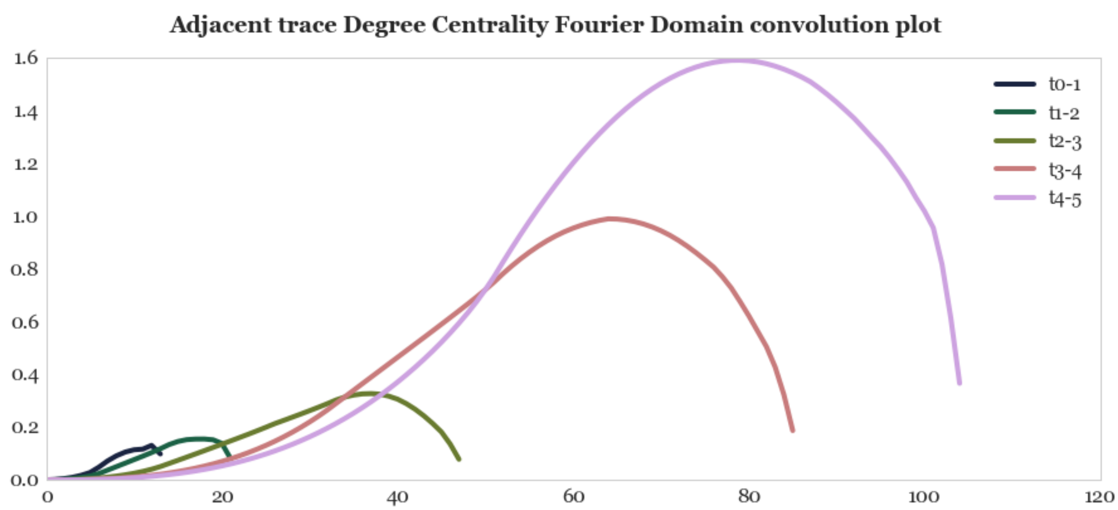



```
In [42]: plt.suptitle('Adjacent Trace Degree Centrality correlation plot', fontsize

         plt.plot(np.correlate(get_val(degC0),get_val(degC1)),label='t0-1')
         plt.plot(np.correlate(get_val(degC1),get_val(degC2)),label='t1-2')
         plt.plot(np.correlate(get_val(degC2),get_val(degC3)),label='t2-3')
         plt.plot(np.correlate(get_val(degC3),get_val(degC3)),label='t3-4')
         plt.plot(np.correlate(get_val(degC4),get_val(degC5)),label='t4-5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
Out[42]: <matplotlib.legend.Legend at 0x21c1ca23b38>
```



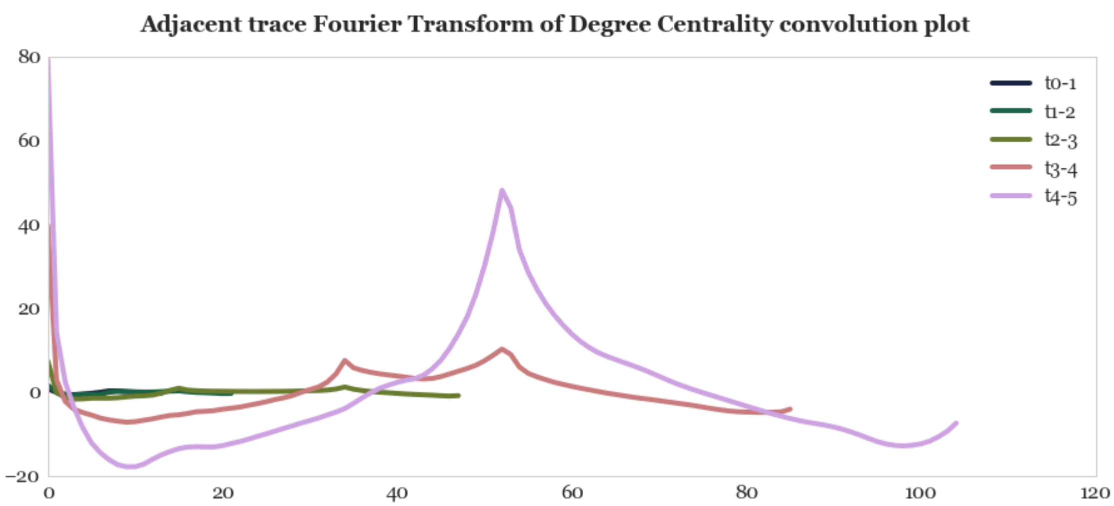

```
In [43]: plt.suptitle('Adjacent trace Degree Centrality Fourier Domain convolution

         plt.plot(fftconvolve(get_val(degC0),get_val(degC1)),label='t0-1')
         plt.plot(fftconvolve(get_val(degC1),get_val(degC2)),  label='t1-2')
         plt.plot(fftconvolve(get_val(degC2),get_val(degC3)), label='t2-3')
         plt.plot(fftconvolve(get_val(degC3),get_val(degC4)), label='t3-4')
         plt.plot(fftconvolve(get_val(degC4),get_val(degC5)),  label='t4-5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)

Out[43]: <matplotlib.legend.Legend at 0x21c1c243978>
```



```
In [44]: plt.suptitle('Adjacent trace Fourier Transform of Degree Centrality convol

         plt.plot(np.convolve(fft_sig(degC0),fft_sig(degC1)),label='t0-1')
         plt.plot(np.convolve(fft_sig(degC1),fft_sig(degC2)),  label='t1-2')
         plt.plot(np.convolve(fft_sig(degC2),fft_sig(degC3)), label='t2-3')
         plt.plot(np.convolve(fft_sig(degC3),fft_sig(degC4)), label='t3-4')
         plt.plot(np.convolve(fft_sig(degC4),fft_sig(degC5)),  label='t4-5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)
```

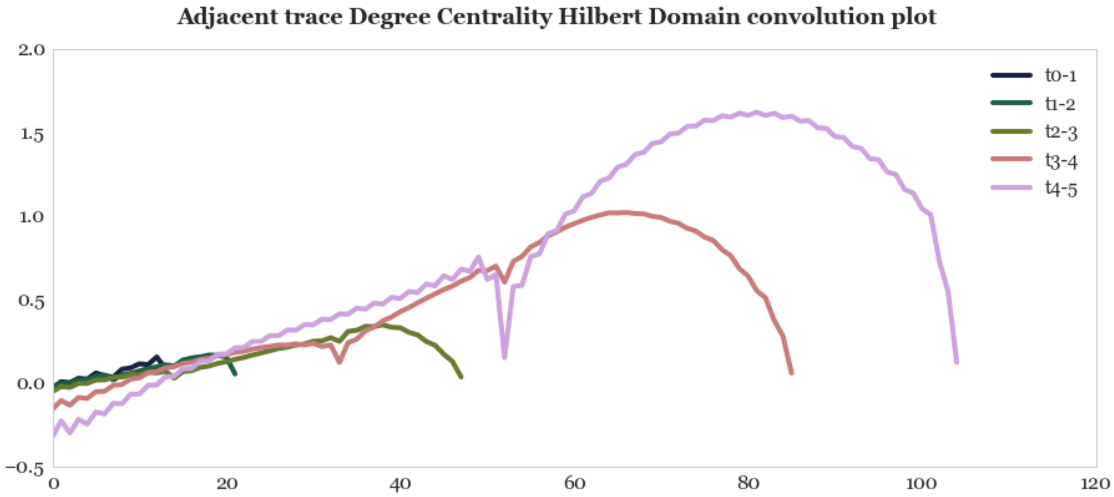
```
Out[44]: <matplotlib.legend.Legend at 0x21c1c554828>
```




```
In [45]: plt.suptitle('Adjacent trace Degree Centrality Hilbert Domain convolution

         plt.plot(np.convolve(hilbert(get_val(degC0)), hilbert(get_val(degC1))),   l
         plt.plot(np.convolve(hilbert(get_val(degC1)), hilbert(get_val(degC2))),   l
         plt.plot(np.convolve(hilbert(get_val(degC2)), hilbert(get_val(degC3))), la
         plt.plot(np.convolve(hilbert(get_val(degC3)), hilbert(get_val(degC4))), la
         plt.plot(np.convolve(hilbert(get_val(degC4)), hilbert(get_val(degC5))),   l

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)
```

```
Out[45]: <matplotlib.legend.Legend at 0x21c1c587b38>
```



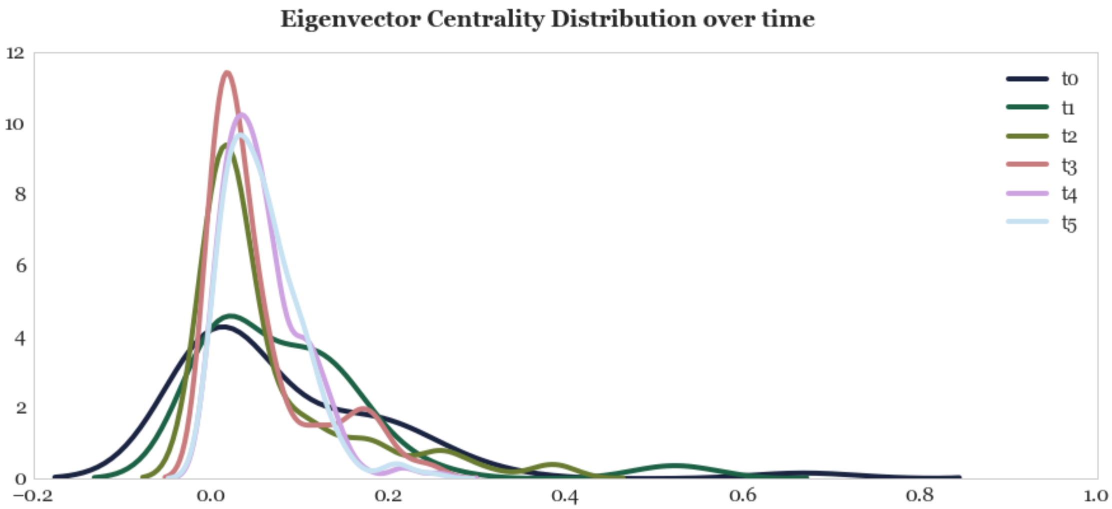

### 6.1.3 Eigenvector Centrality Histograms

**Plotting the Eigenvector Centrality for the different timesteps here. For the first plot it is difficult to discern the trends when all the 6 distributions are plotted together. So in the next series of plots I look at a few a time and its easier to see the change over time. The signal essentially becomes more spiked and squashed over time.**

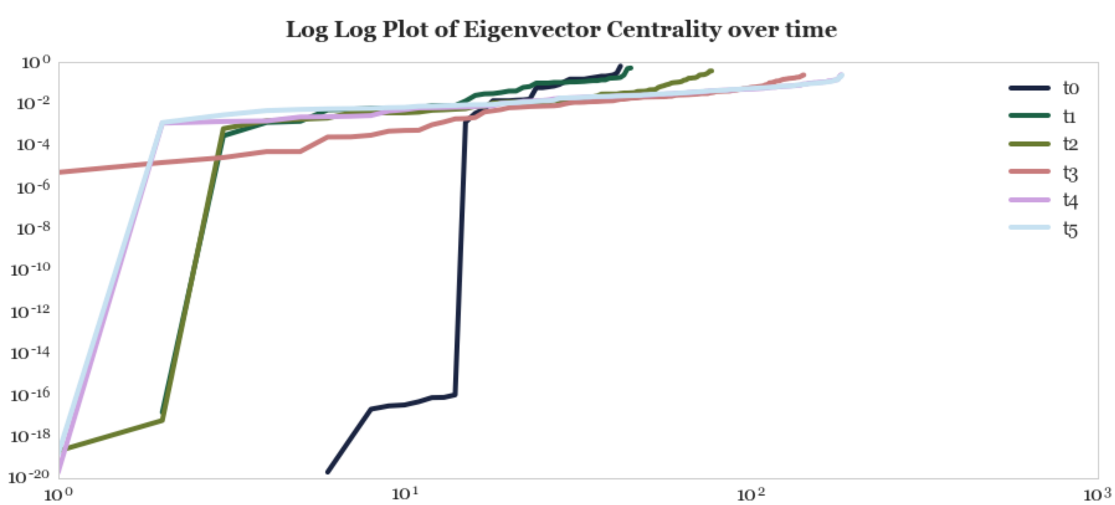
```
In [46]: plt.suptitle('Eigenvector Centrality Distribution over time', fontsize=18)
         sns.distplot(get_val(eigC0), hist=False, label='t0')
         sns.distplot(get_val(eigC1), hist=False, label='t1')
         sns.distplot(get_val(eigC2), hist=False, label='t2')
         sns.distplot(get_val(eigC3), hist=False, label='t3')
         sns.distplot(get_val(eigC4), hist=False, label='t4')
         sns.distplot(get_val(eigC5), hist=False, label='t5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\statsmodels\nonparametric\kdetools.p
  y = X[:m/2+1] + np.r_[0,X[m/2+1:],0]*1j
```

```
Out[46]: <matplotlib.legend.Legend at 0x21c1c117978>
```



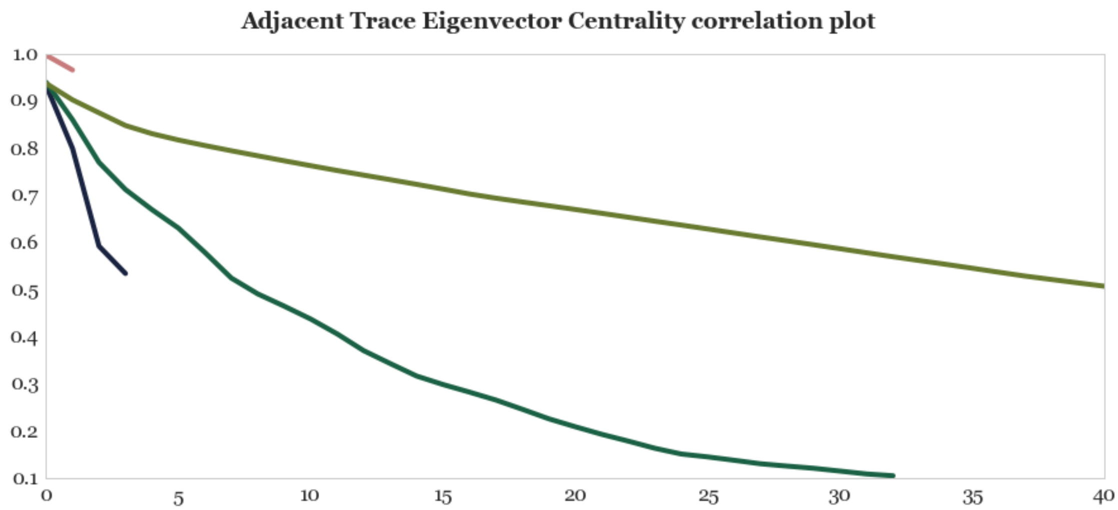

```
In [47]: plt.suptitle('Log Log Plot of Eigenvector Centrality over time', fontsize=

         plt.loglog(get_val(eigC0), label='t0')
         plt.loglog(get_val(eigC1), label='t1')
         plt.loglog(get_val(eigC2), label='t2')
         plt.loglog(get_val(eigC3), label='t3')
         plt.loglog(get_val(eigC4), label='t4')
         plt.loglog(get_val(eigC5), label='t5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)

Out[47]: <matplotlib.legend.Legend at 0x21c1cab2438>
```



```
In [48]: eigC0_a = np.asarray(get_val(eigC0))
         eigC1_a = np.asarray(get_val(eigC1))
```

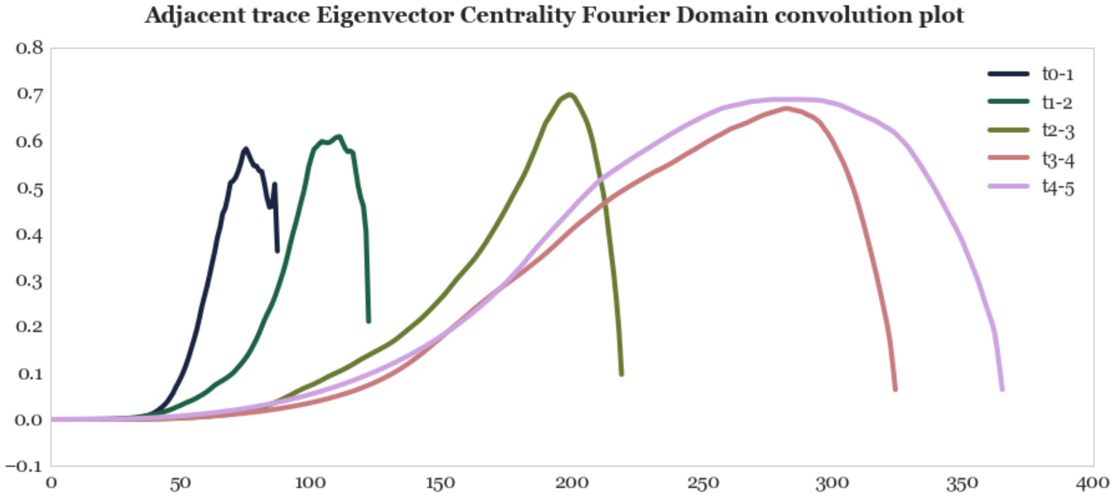
```
In [49]: plt.suptitle('Adjacent Trace Eigenvector Centrality correlation plot', for

         plt.plot(np.correlate(eigC0_a,eigC1_a))
         plt.plot(np.correlate(get_val(eigC1),get_val(eigC2)))
         plt.plot(np.correlate(get_val(eigC3),get_val(eigC4)))
         plt.plot(np.correlate(get_val(eigC4),get_val(eigC5)))

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\matplotlib\axes\_axes.py:519: UserWa
  warnings.warn("No labelled objects found. "
```

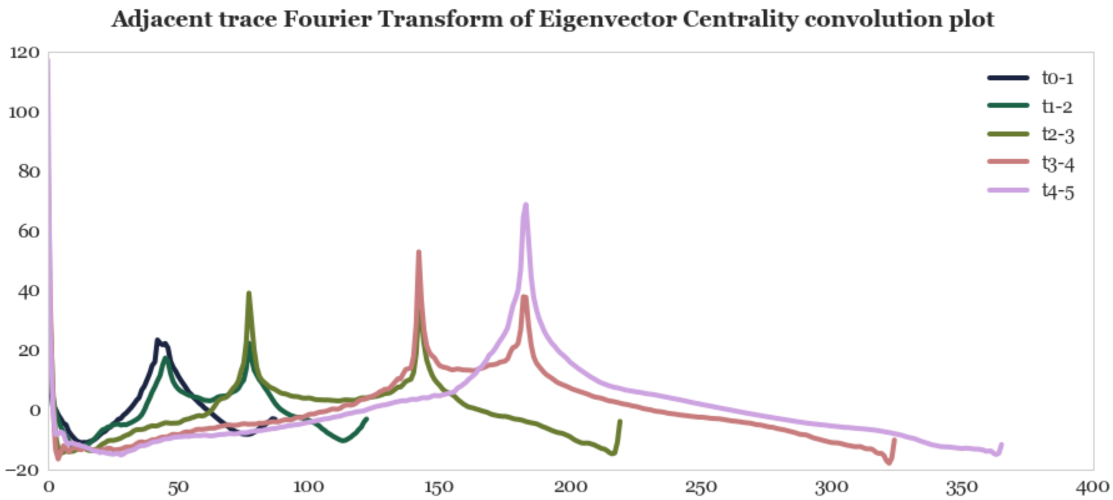



```
In [50]: plt.suptitle('Adjacent trace Eigenvector Centrality Fourier Domain convolu

         plt.plot(fftconvolve(get_val(eigC0),get_val(eigC1)),  label='t0-1')
         plt.plot(fftconvolve(get_val(eigC1),get_val(eigC2)),  label='t1-2')
         plt.plot(fftconvolve(get_val(eigC2),get_val(eigC3)), label='t2-3')
         plt.plot(fftconvolve(get_val(eigC3),get_val(eigC4)), label='t3-4')
         plt.plot(fftconvolve(get_val(eigC4),get_val(eigC5)),  label='t4-5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
Out[50]: <matplotlib.legend.Legend at 0x21c1e797518>
```



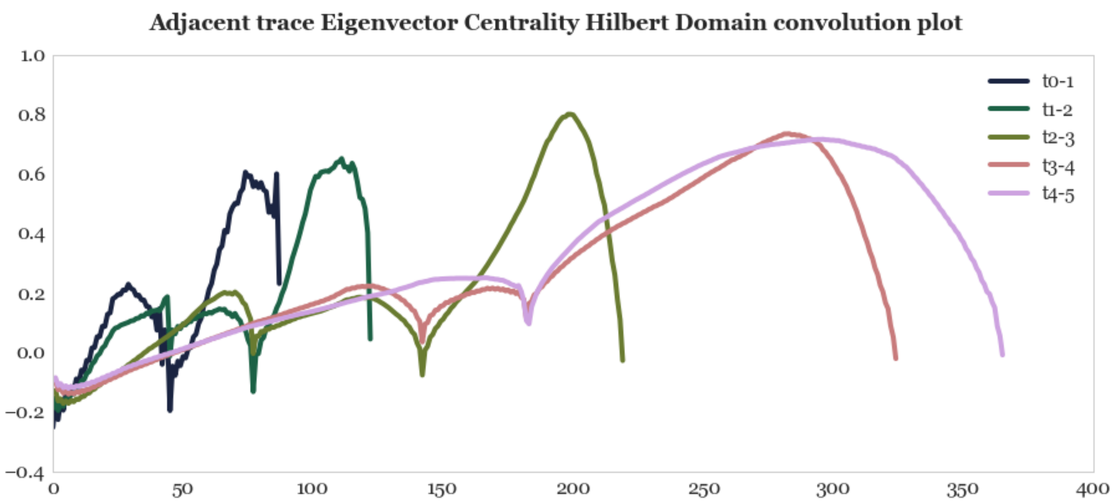

```
In [51]: plt.suptitle('Adjacent trace Fourier Transform of Eigenvector Centrality c

         plt.plot(np.convolve(fft_sig(eigC0),fft_sig(eigC1)),label='t0-1')
         plt.plot(np.convolve(fft_sig(eigC1),fft_sig(eigC2)),  label='t1-2')
         plt.plot(np.convolve(fft_sig(eigC2),fft_sig(eigC3)), label='t2-3')
         plt.plot(np.convolve(fft_sig(eigC3),fft_sig(eigC4)), label='t3-4')
         plt.plot(np.convolve(fft_sig(eigC4),fft_sig(eigC5)),  label='t4-5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)

C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)

Out[51]: <matplotlib.legend.Legend at 0x21c1f20ee48>
```




In [52]:
```
plt.suptitle('Adjacent trace Eigenvector Centrality Hilbert Domain convolu

plt.plot(np.convolve(hilbert(get_val(eigC0)), hilbert(get_val(eigC1))),  l
plt.plot(np.convolve(hilbert(get_val(eigC1)), hilbert(get_val(eigC2))),  l
plt.plot(np.convolve(hilbert(get_val(eigC2)), hilbert(get_val(eigC3))), la
plt.plot(np.convolve(hilbert(get_val(eigC3)), hilbert(get_val(eigC4))), la
plt.plot(np.convolve(hilbert(get_val(eigC4)), hilbert(get_val(eigC5))),  l

plt.yticks(fontsize=16)
plt.xticks(fontsize=16)
plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)
```

Out[52]: <matplotlib.legend.Legend at 0x21c1c1a87f0>



```
In [53]: print(np.correlate(eigC0_a,eigC1_a))
         print(np.correlate(eigC0_a,eigC1_a)/np.sqrt(np.correlate(eigC0_a,eigC1_a))
```

```
[ 0.934373    0.8003439   0.59281644  0.53482695]
[ 0.96662971  0.89461941  0.76994574  0.73131864]
```

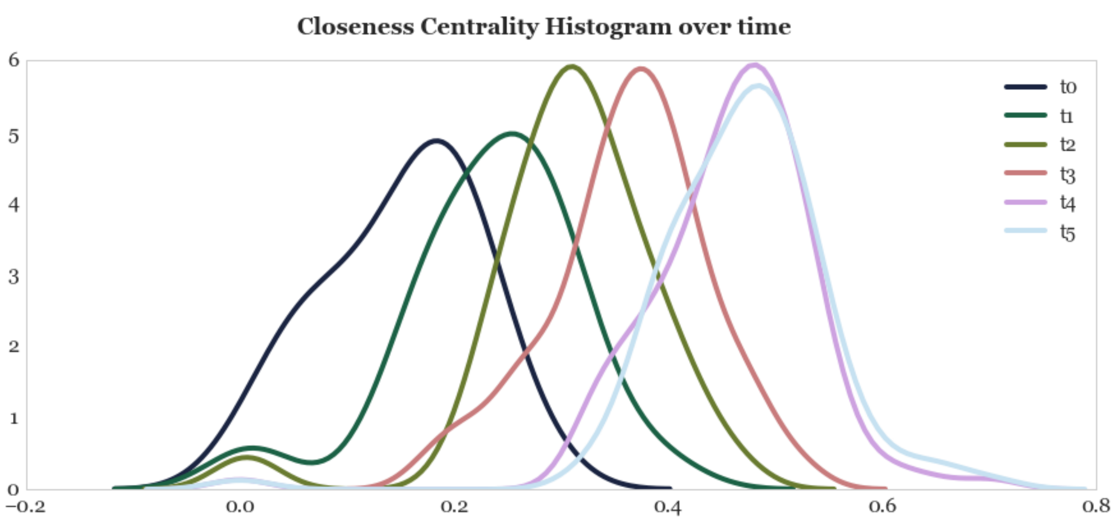
#### 6.1.4 Closeness Centrality Histograms

**The Closeness Centrality shows a much better evolution over time than the Eigenvector Centrality Histograms**

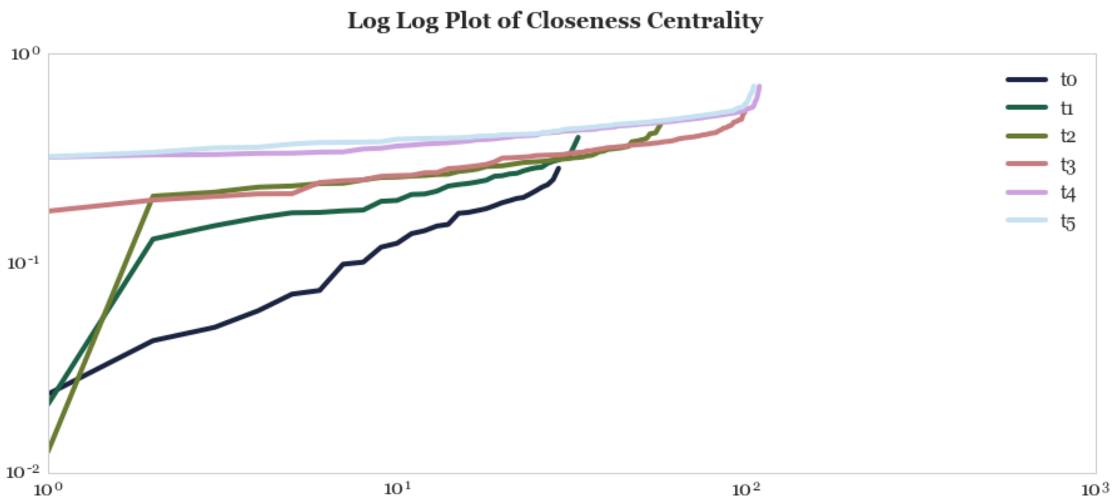
```
In [54]: plt.suptitle('Closeness Centrality Histogram over time', fontsize=18)
         sns.distplot(get_val(cloC0), hist=False, label='t0')
         sns.distplot(get_val(cloC1), hist=False, label='t1')
         sns.distplot(get_val(cloC2), hist=False, label='t2')
         sns.distplot(get_val(cloC3), hist=False, label='t3')
         sns.distplot(get_val(cloC4), hist=False, label='t4')
         sns.distplot(get_val(cloC5), hist=False, label='t5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\statsmodels\nonparametric\kdetools.p
  y = X[:m/2+1] + np.r_[0,X[m/2+1:],0]*1j
```

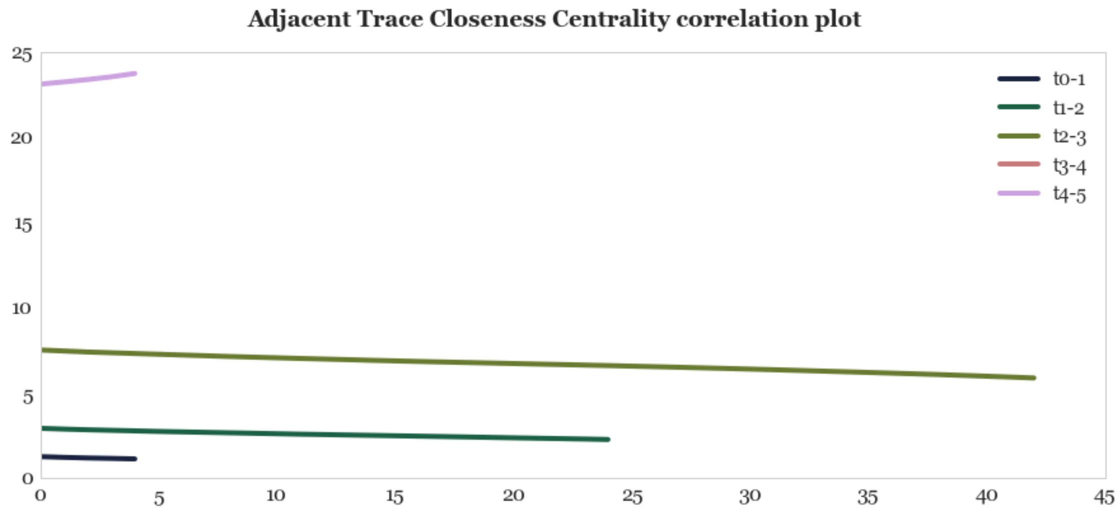
```
Out[54]: <matplotlib.legend.Legend at 0x21c1c382a58>
```



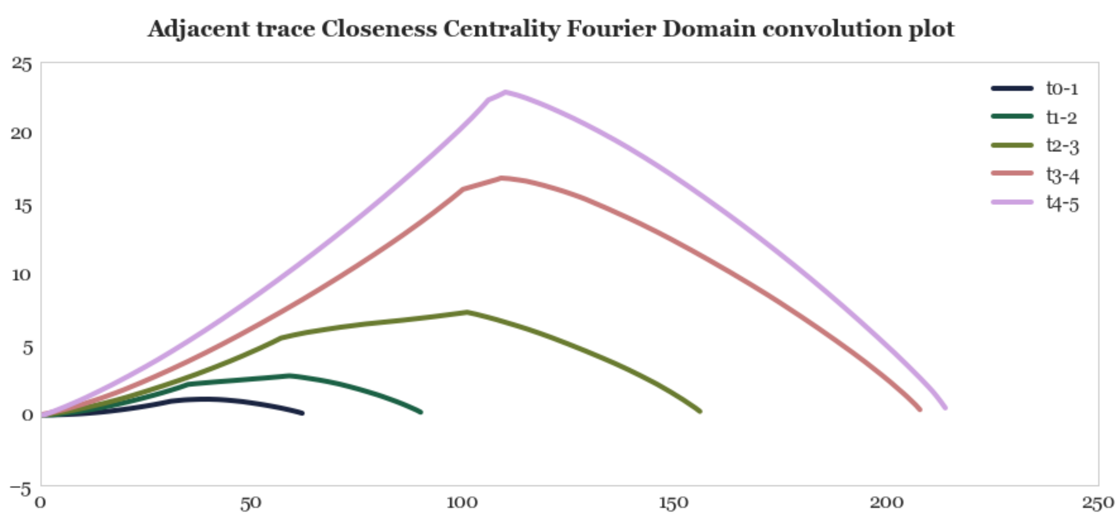
```
In [55]: plt.suptitle('Log Log Plot of Closeness Centrality', fontsize=18)

         plt.loglog(get_val(cloC0),label='t0')
         plt.loglog(get_val(cloC1),label='t1')
         plt.loglog(get_val(cloC2),label='t2')
         plt.loglog(get_val(cloC3),label='t3')
         plt.loglog(get_val(cloC4),label='t4')
         plt.loglog(get_val(cloC5),label='t5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)

Out[55]: <matplotlib.legend.Legend at 0x21c1e670be0>
```




```
In [56]: plt.suptitle('Adjacent Trace Closeness Centrality correlation plot', fonts

         plt.plot(np.correlate(get_val(cloC0),get_val(cloC1)),label='t0-1')
         plt.plot(np.correlate(get_val(cloC1),get_val(cloC2)),label='t1-2')
         plt.plot(np.correlate(get_val(cloC2),get_val(cloC3)),label='t2-3')
         plt.plot(np.correlate(get_val(cloC3),get_val(cloC3)),label='t3-4')
         plt.plot(np.correlate(get_val(cloC4),get_val(cloC5)),label='t4-5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)

Out[56]: <matplotlib.legend.Legend at 0x21c1f3ab470>
```



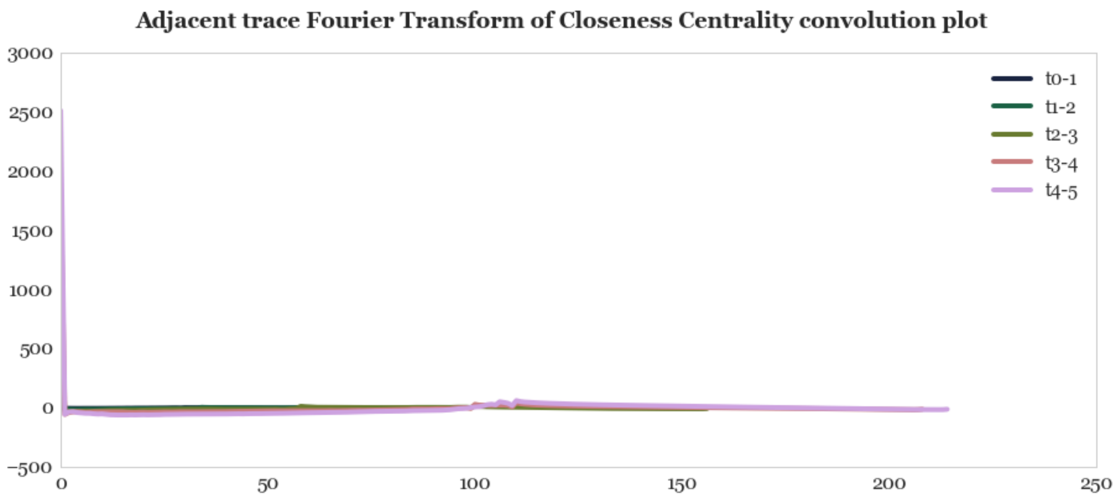

```
In [57]: plt.suptitle('Adjacent trace Closeness Centrality Fourier Domain convoluti

         plt.plot(fftconvolve(get_val(cloC0),get_val(cloC1)),label='t0-1')
         plt.plot(fftconvolve(get_val(cloC1),get_val(cloC2)),  label='t1-2')
         plt.plot(fftconvolve(get_val(cloC2),get_val(cloC3)), label='t2-3')
         plt.plot(fftconvolve(get_val(cloC3),get_val(cloC4)), label='t3-4')
         plt.plot(fftconvolve(get_val(cloC4),get_val(cloC5)),  label='t4-5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)

Out[57]: <matplotlib.legend.Legend at 0x21c1f38c6a0>
```



```
In [58]: plt.suptitle('Adjacent trace Fourier Transform of Closeness Centrality cor

         plt.plot(np.convolve(fft_sig(cloC0),fft_sig(cloC1)),label='t0-1')
         plt.plot(np.convolve(fft_sig(cloC1),fft_sig(cloC2)),label='t1-2')
         plt.plot(np.convolve(fft_sig(cloC2),fft_sig(cloC3)),label='t2-3')
         plt.plot(np.convolve(fft_sig(cloC3),fft_sig(cloC4)),label='t3-4')
         plt.plot(np.convolve(fft_sig(cloC4),fft_sig(cloC5)),label='t4-5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)
```

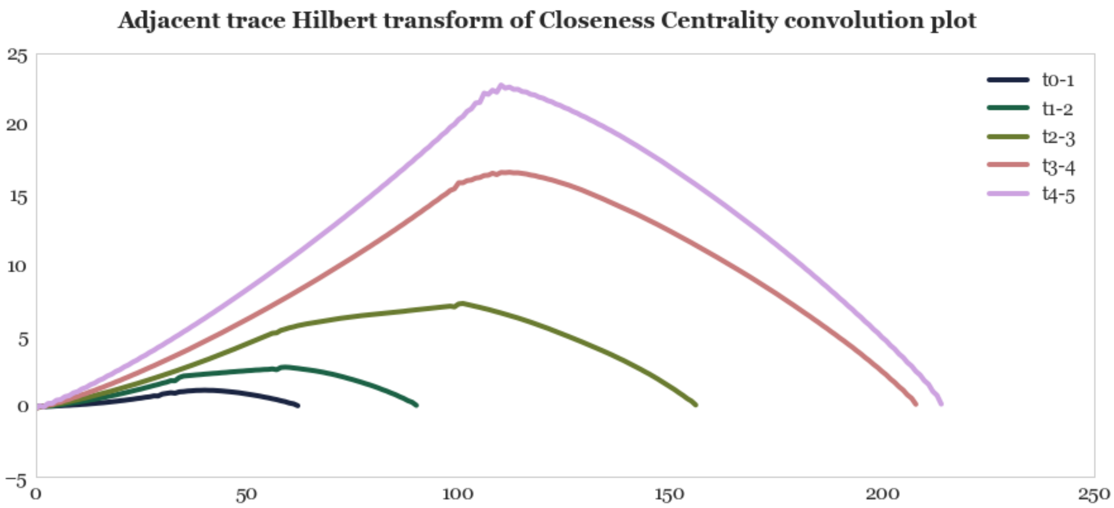
```
Out[58]: <matplotlib.legend.Legend at 0x21c215b4c18>
```




```
In [59]: plt.suptitle('Adjacent trace Hilbert transform of Closeness Centrality cor

         plt.plot(np.convolve(hilbert_sig(cloC0), hilbert_sig(cloC1)), label='t0-1'
         plt.plot(np.convolve(hilbert_sig(cloC1), hilbert_sig(cloC2)), label='t1-2'
         plt.plot(np.convolve(hilbert_sig(cloC2), hilbert_sig(cloC3)),label='t2-3')
         plt.plot(np.convolve(hilbert_sig(cloC3), hilbert_sig(cloC4)), label='t3-4'
         plt.plot(np.convolve(hilbert_sig(cloC4), hilbert_sig(cloC5)), label='t4-5'

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)
```

Out[59]: <matplotlib.legend.Legend at 0x21c21aece80>

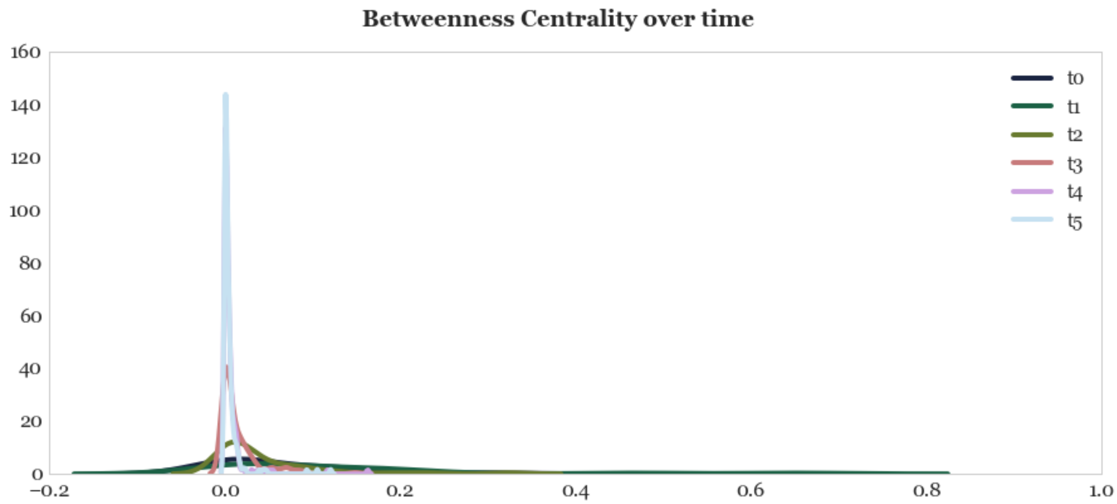



#### 6.1.5 Betweenness Centrality Histogram

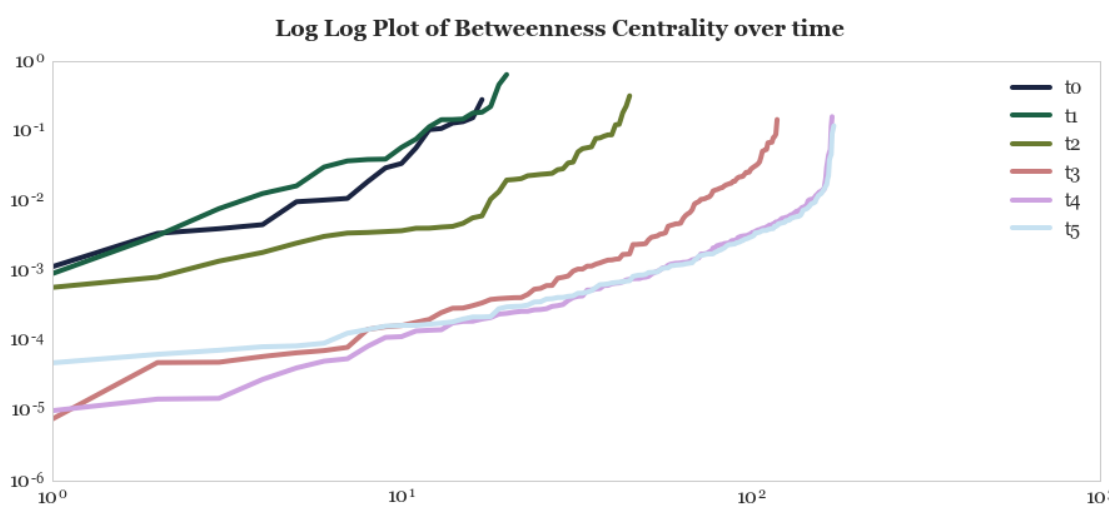
```
In [60]: plt.suptitle('Betweenness Centrality over time', fontsize=18)
         sns.distplot(get_val(betC0), hist=False, label='t0')
         sns.distplot(get_val(betC1), hist=False, label='t1')
         sns.distplot(get_val(betC2), hist=False, label='t2')
         sns.distplot(get_val(betC3), hist=False, label='t3')
         sns.distplot(get_val(betC4), hist=False, label='t4')
         sns.distplot(get_val(betC5), hist=False, label='t5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\statsmodels\nonparametric\kdetools.p
  y = X[:m/2+1] + np.r_[0,X[m/2+1:],0]*1j
```

Out[60]: <matplotlib.legend.Legend at 0x21c21d411d0>

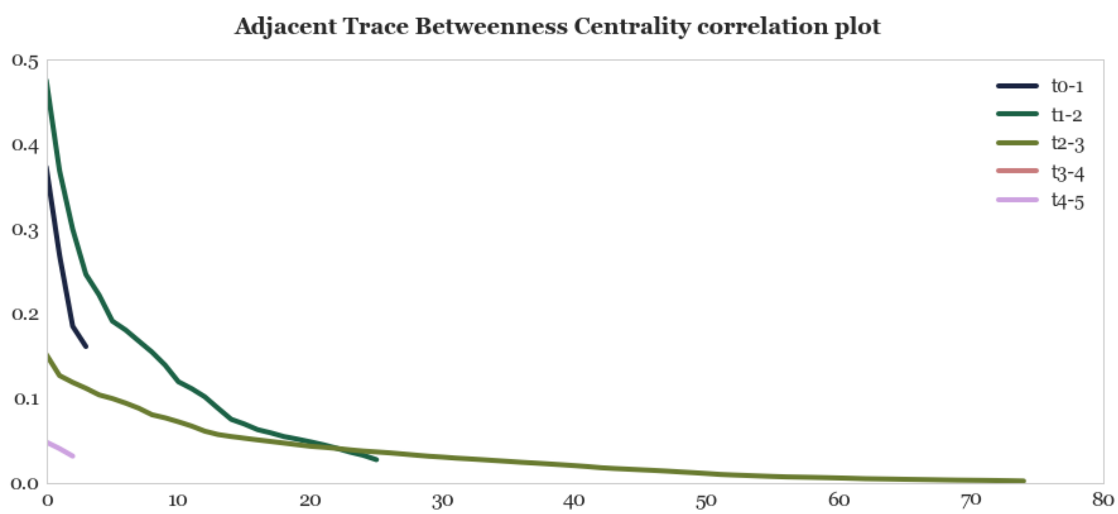



```
In [61]: plt.suptitle('Log Log Plot of Betweenness Centrality over time', fontsize=

         plt.loglog(get_val(betC0), label='t0')
         plt.loglog(get_val(betC1), label='t1')
         plt.loglog(get_val(betC2), label='t2')
         plt.loglog(get_val(betC3), label='t3')
         plt.loglog(get_val(betC4), label='t4')
         plt.loglog(get_val(betC5), label='t5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)

Out[61]: <matplotlib.legend.Legend at 0x21c21f1ce48>
```



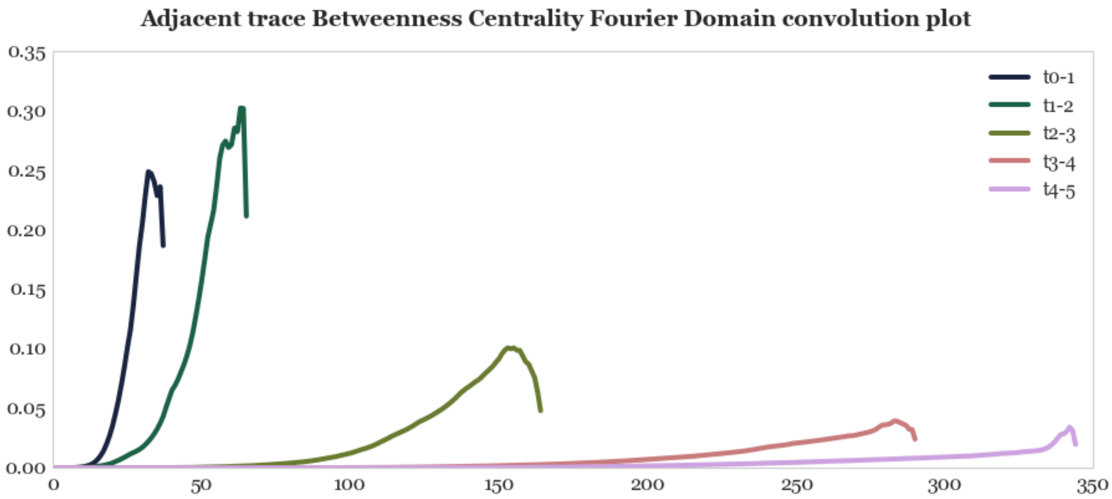
In [62]:
```
plt.suptitle('Adjacent Trace Betweenness Centrality correlation plot', for

plt.plot(np.correlate(get_val(betC0),get_val(betC1)),label='t0-1')
plt.plot(np.correlate(get_val(betC1),get_val(betC2)),label='t1-2')
plt.plot(np.correlate(get_val(betC2),get_val(betC3)),label='t2-3')
plt.plot(np.correlate(get_val(betC3),get_val(betC3)),label='t3-4')
plt.plot(np.correlate(get_val(betC4),get_val(betC5)),label='t4-5')

plt.yticks(fontsize=16)
plt.xticks(fontsize=16)
plt.legend(loc=1, fontsize=15)
```

Out[62]: <matplotlib.legend.Legend at 0x21c221e6fd0>

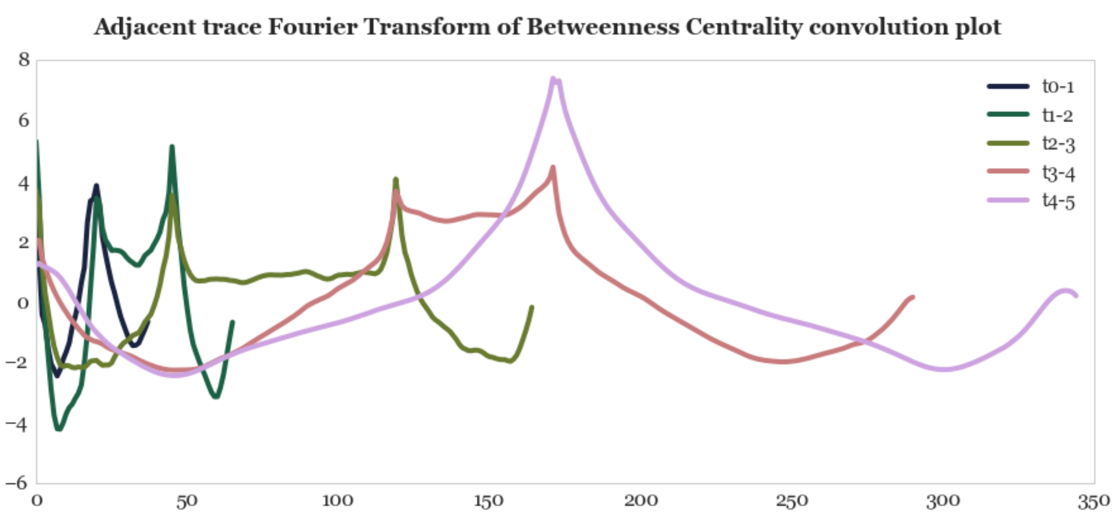



In [63]:
```
plt.suptitle('Adjacent trace Betweenness Centrality Fourier Domain convolu

plt.plot(fftconvolve(get_val(betC0),get_val(betC1)),label='t0-1')
plt.plot(fftconvolve(get_val(betC1),get_val(betC2)),  label='t1-2')
plt.plot(fftconvolve(get_val(betC2),get_val(betC3)), label='t2-3')
plt.plot(fftconvolve(get_val(betC3),get_val(betC4)), label='t3-4')
plt.plot(fftconvolve(get_val(betC4),get_val(betC5)),  label='t4-5')

plt.yticks(fontsize=16)
plt.xticks(fontsize=16)
plt.legend(loc=1, fontsize=15)
```

Out[63]: <matplotlib.legend.Legend at 0x21c1c3ce2b0>



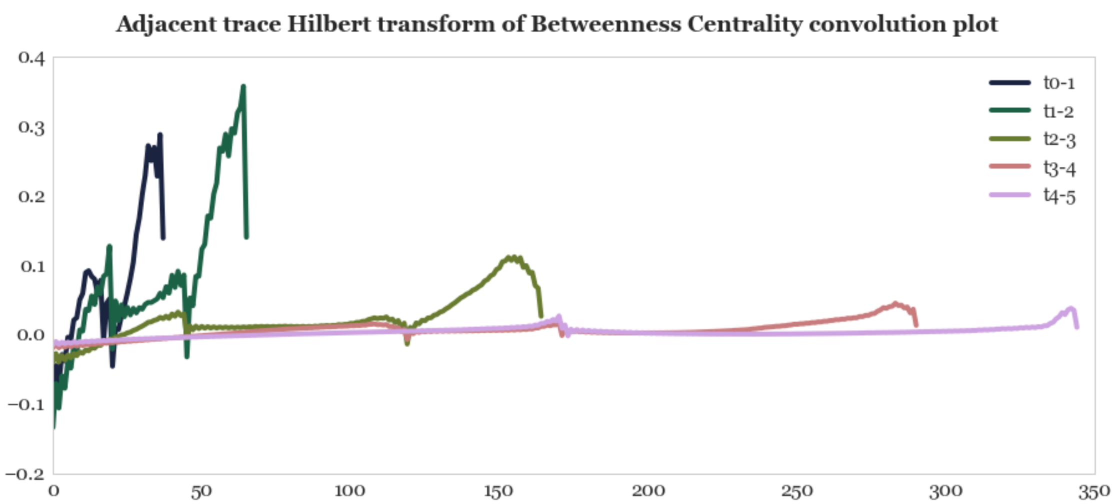

```
In [64]: plt.suptitle('Adjacent trace Fourier Transform of Betweenness Centrality c

         plt.plot(np.convolve(fft_sig(betC0),fft_sig(betC1)),label='t0-1')
         plt.plot(np.convolve(fft_sig(betC1),fft_sig(betC2)),label='t1-2')
         plt.plot(np.convolve(fft_sig(betC2),fft_sig(betC3)),label='t2-3')
         plt.plot(np.convolve(fft_sig(betC3),fft_sig(betC4)),label='t3-4')
         plt.plot(np.convolve(fft_sig(betC4),fft_sig(betC5)),label='t4-5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)

C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)

Out[64]: <matplotlib.legend.Legend at 0x21c2223ecc0>
```



```
In [65]: plt.suptitle('Adjacent trace Hilbert transform of Betweenness Centrality c

         plt.plot(np.convolve(hilbert_sig(betC0), hilbert_sig(betC1)), label='t0-1'
         plt.plot(np.convolve(hilbert_sig(betC1), hilbert_sig(betC2)), label='t1-2'
         plt.plot(np.convolve(hilbert_sig(betC2), hilbert_sig(betC3)),label='t2-3')
         plt.plot(np.convolve(hilbert_sig(betC3), hilbert_sig(betC4)), label='t3-4'
         plt.plot(np.convolve(hilbert_sig(betC4), hilbert_sig(betC5)), label='t4-5'

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)
```

```
Out[65]: <matplotlib.legend.Legend at 0x21c203d6b00>
```



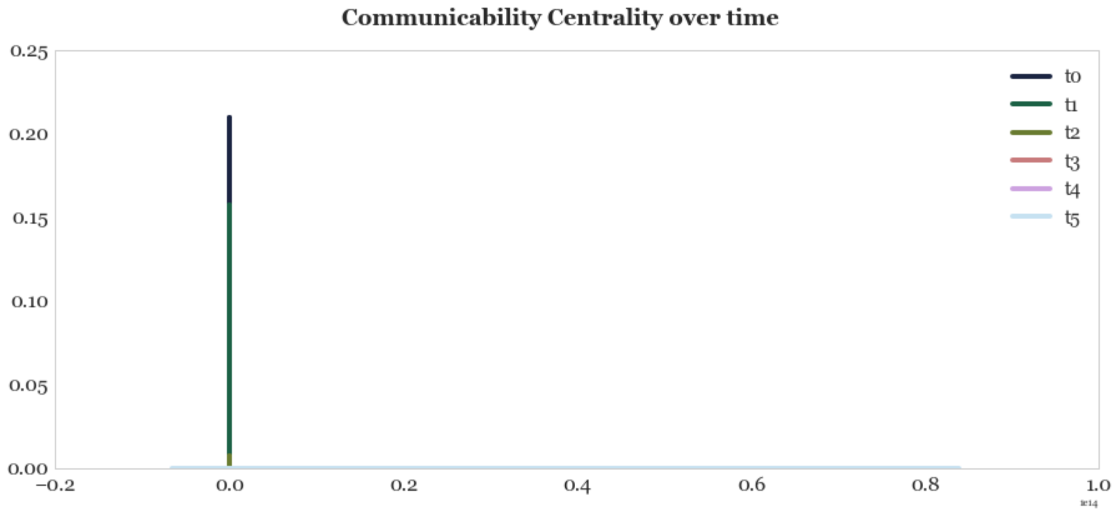

### 6.1.6 Communicability Centrality Histograms

```
In [66]: plt.suptitle('Communicability Centrality over time', fontsize=18)
         sns.distplot(get_val(commuC0), hist=False, label='t0')
         sns.distplot(get_val(commuC1), hist=False, label='t1')
         sns.distplot(get_val(commuC2), hist=False, label='t2')
         sns.distplot(get_val(commuC3), hist=False, label='t3')
         sns.distplot(get_val(commuC4), hist=False, label='t4')
         sns.distplot(get_val(commuC5), hist=False, label='t5')
```

```
         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

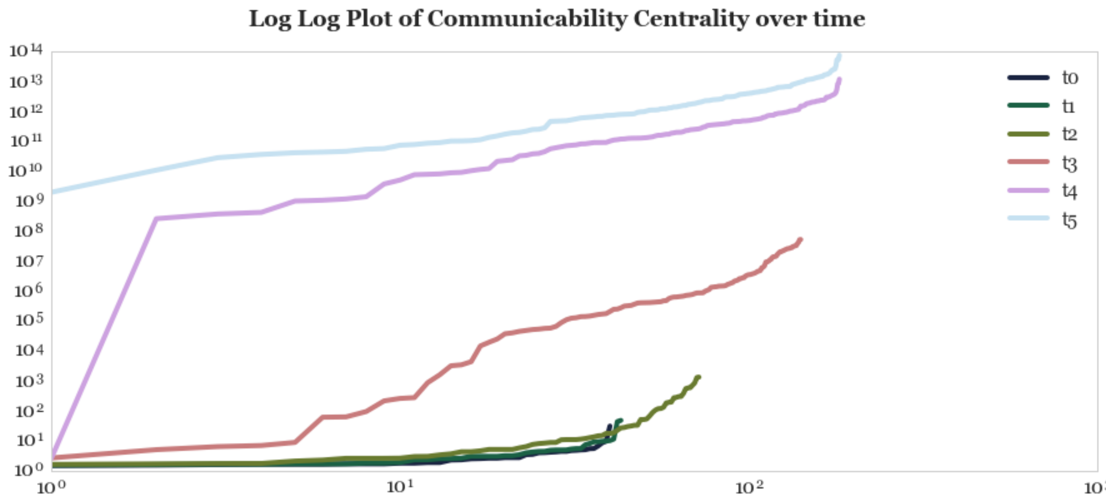
```
C:\Users\arsha_000\Anaconda3\lib\site-packages\statsmodels\nonparametric\kdetools.p
  y = X[:m/2+1] + np.r_[0,X[m/2+1:],0]*1j
```

```
Out[66]: <matplotlib.legend.Legend at 0x21c22590d68>
```

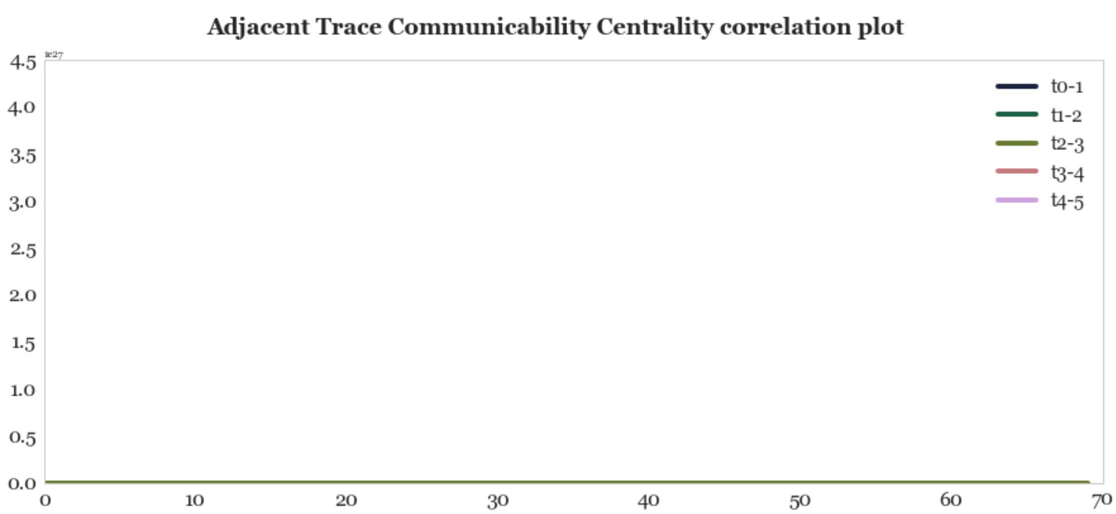



```
In [67]: plt.suptitle('Log Log Plot of Communicability Centrality over time', fonts

         plt.loglog(get_val(commuC0), label='t0')
         plt.loglog(get_val(commuC1), label='t1')
         plt.loglog(get_val(commuC2), label='t2')
         plt.loglog(get_val(commuC3), label='t3')
         plt.loglog(get_val(commuC4), label='t4')
         plt.loglog(get_val(commuC5), label='t5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
Out[67]: <matplotlib.legend.Legend at 0x21c225c5128>
```

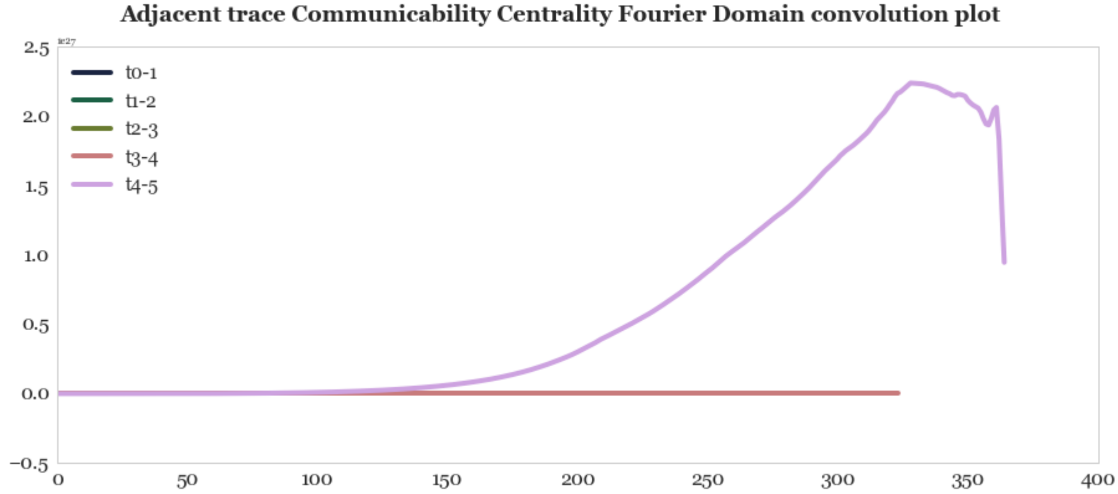



```
In [68]: plt.suptitle('Adjacent Trace Communicability Centrality correlation plot',

         plt.plot(np.correlate(get_val(commuC0),get_val(commuC1)),label='t0-1')
         plt.plot(np.correlate(get_val(commuC1),get_val(commuC2)),label='t1-2')
         plt.plot(np.correlate(get_val(commuC2),get_val(commuC3)),label='t2-3')
         plt.plot(np.correlate(get_val(commuC3),get_val(commuC3)),label='t3-4')
         plt.plot(np.correlate(get_val(commuC4),get_val(commuC5)),label='t4-5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)

Out[68]: <matplotlib.legend.Legend at 0x21c242ffd68>
```



In [69]:
```
plt.suptitle('Adjacent trace Communicability Centrality Fourier Domain con

plt.plot(fftconvolve(get_val(commuC0),get_val(commuC1)),label='t0-1')
plt.plot(fftconvolve(get_val(commuC1),get_val(commuC2)),  label='t1-2')
plt.plot(fftconvolve(get_val(commuC2),get_val(commuC3)), label='t2-3')
plt.plot(fftconvolve(get_val(commuC3),get_val(commuC4)), label='t3-4')
plt.plot(fftconvolve(get_val(commuC4),get_val(commuC5)),  label='t4-5')

plt.yticks(fontsize=16)
plt.xticks(fontsize=16)
plt.legend(loc=2, fontsize=15)
```

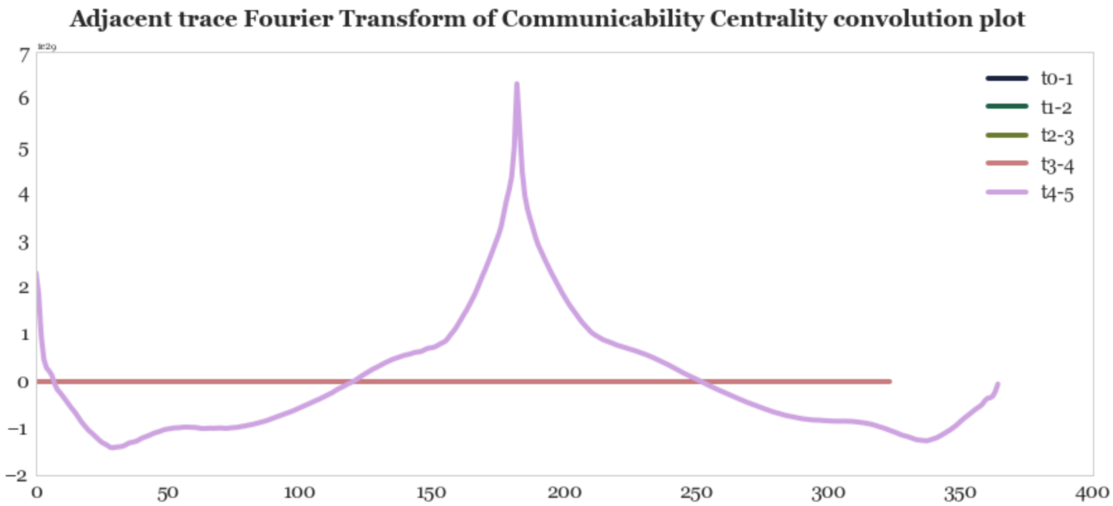
Out[69]: <matplotlib.legend.Legend at 0x21c244f2a90>




In [70]:
```
plt.suptitle('Adjacent trace Fourier Transform of Communicability Centrali

plt.plot(np.convolve(fft_sig(commuC0),fft_sig(commuC1)),label='t0-1')
plt.plot(np.convolve(fft_sig(commuC1),fft_sig(commuC2)),label='t1-2')
plt.plot(np.convolve(fft_sig(commuC2),fft_sig(commuC3)),label='t2-3')
plt.plot(np.convolve(fft_sig(commuC3),fft_sig(commuC4)),label='t3-4')
plt.plot(np.convolve(fft_sig(commuC4),fft_sig(commuC5)),label='t4-5')

plt.yticks(fontsize=16)
plt.xticks(fontsize=16)
plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)
```

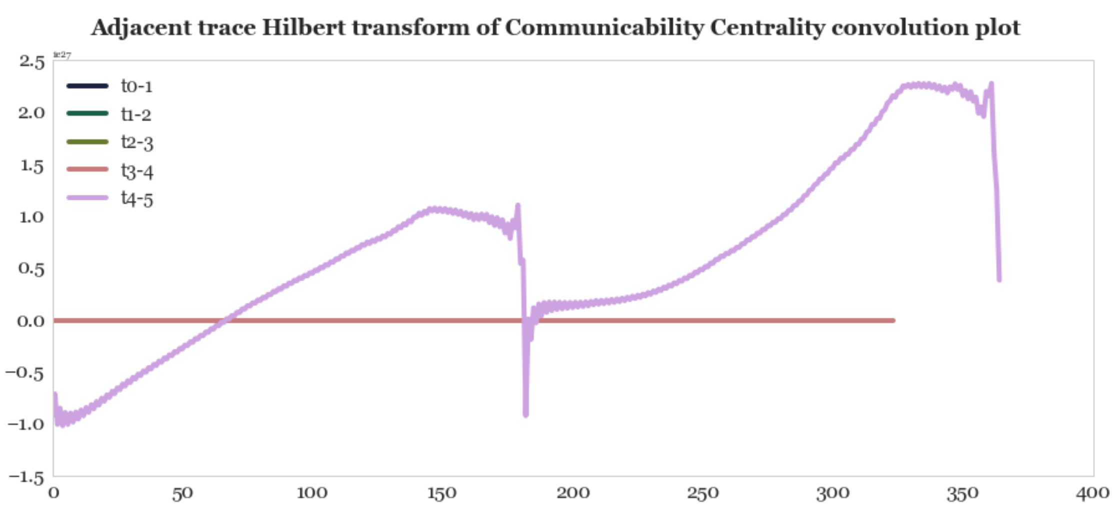
Out[70]: <matplotlib.legend.Legend at 0x21c246fbb38>




```
In [71]: plt.suptitle('Adjacent trace Hilbert transform of Communicability Centrali

         plt.plot(np.convolve(hilbert_sig(commuC0), hilbert_sig(commuC1)), label='t
         plt.plot(np.convolve(hilbert_sig(commuC1), hilbert_sig(commuC2)), label='t
         plt.plot(np.convolve(hilbert_sig(commuC2), hilbert_sig(commuC3)),label='t2
         plt.plot(np.convolve(hilbert_sig(commuC3), hilbert_sig(commuC4)), label='t
         plt.plot(np.convolve(hilbert_sig(commuC4), hilbert_sig(commuC5)), label='t

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=2, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)
```

```
Out[71]: <matplotlib.legend.Legend at 0x21c248f4550>
```

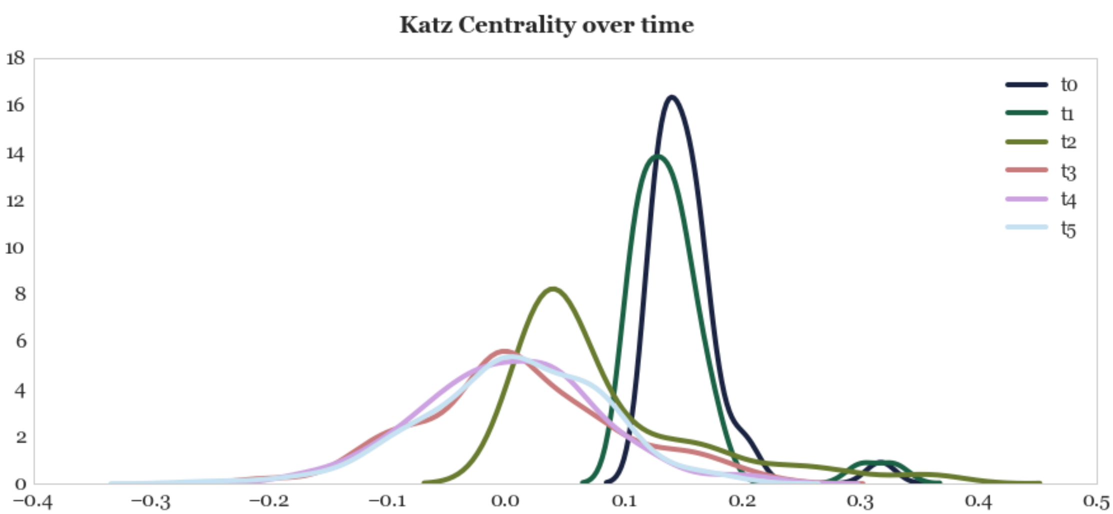


#### 6.1.7 Katz Centrality Histograms

```
In [72]: plt.suptitle('Katz Centrality over time', fontsize=18)
         sns.distplot(get_val(katzC0), hist=False, label='t0')
         sns.distplot(get_val(katzC1), hist=False, label='t1')
         sns.distplot(get_val(katzC2), hist=False, label='t2')
         sns.distplot(get_val(katzC3), hist=False, label='t3')
         sns.distplot(get_val(katzC4), hist=False, label='t4')
         sns.distplot(get_val(katzC5), hist=False, label='t5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\statsmodels\nonparametric\kdetools.p
  y = X[:m/2+1] + np.r_[0,X[m/2+1:],0]*1j
```

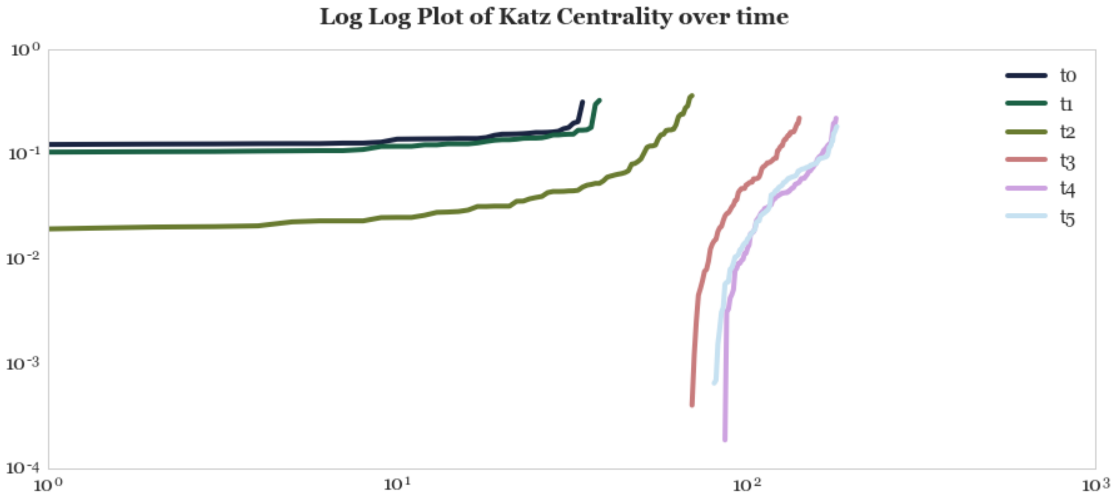
```
Out[72]: <matplotlib.legend.Legend at 0x21c221734a8>
```




```
In [73]: plt.suptitle('Log Log Plot of Katz Centrality over time', fontsize=18)

         plt.loglog(get_val(katzC0), label='t0')
         plt.loglog(get_val(katzC1), label='t1')
         plt.loglog(get_val(katzC2), label='t2')
         plt.loglog(get_val(katzC3), label='t3')
         plt.loglog(get_val(katzC4), label='t4')
```

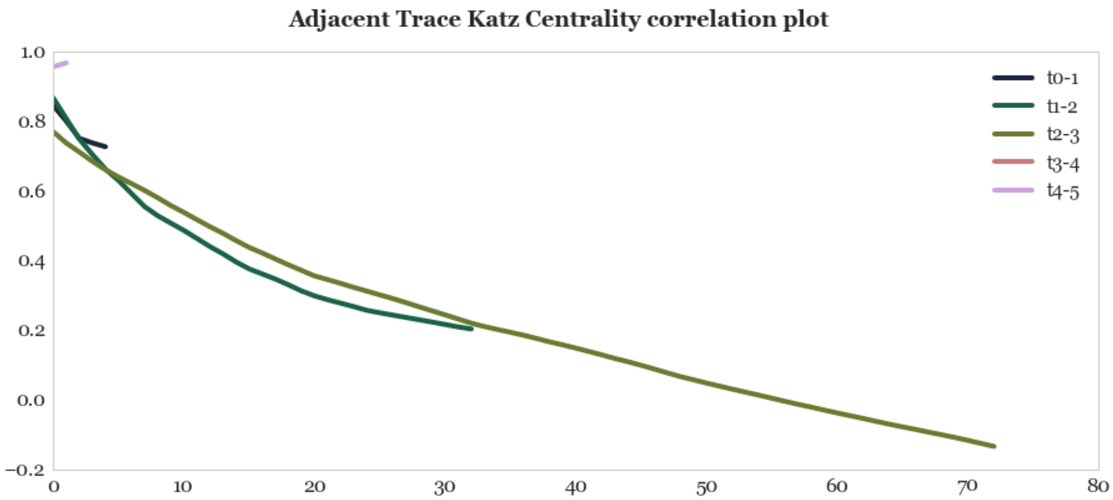
```
        plt.loglog(get_val(katzC5), label='t5')

        plt.yticks(fontsize=16)
        plt.xticks(fontsize=16)
        plt.legend(loc=1, fontsize=15)
```

Out[73]: <matplotlib.legend.Legend at 0x21c2429c438>



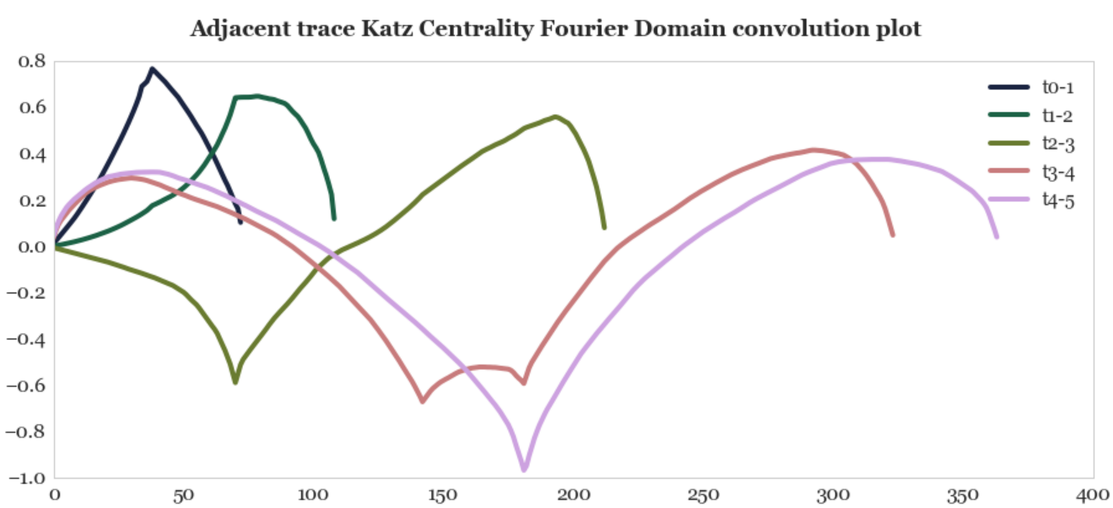

```
In [74]: plt.suptitle('Adjacent Trace Katz Centrality correlation plot', fontsize=1

        plt.plot(np.correlate(get_val(katzC0),get_val(katzC1)),label='t0-1')
        plt.plot(np.correlate(get_val(katzC1),get_val(katzC2)),label='t1-2')
        plt.plot(np.correlate(get_val(katzC2),get_val(katzC3)),label='t2-3')
        plt.plot(np.correlate(get_val(katzC3),get_val(katzC3)),label='t3-4')
        plt.plot(np.correlate(get_val(katzC4),get_val(katzC5)),label='t4-5')

        plt.yticks(fontsize=16)
        plt.xticks(fontsize=16)
        plt.legend(loc=1, fontsize=15)
```

Out[74]: <matplotlib.legend.Legend at 0x21c25062358>




In [75]:
```
plt.suptitle('Adjacent trace Katz Centrality Fourier Domain convolution pl

plt.plot(fftconvolve(get_val(katzC0),get_val(katzC1)),label='t0-1')
plt.plot(fftconvolve(get_val(katzC1),get_val(katzC2)),  label='t1-2')
plt.plot(fftconvolve(get_val(katzC2),get_val(katzC3)), label='t2-3')
plt.plot(fftconvolve(get_val(katzC3),get_val(katzC4)), label='t3-4')
plt.plot(fftconvolve(get_val(katzC4),get_val(katzC5)),  label='t4-5')

plt.yticks(fontsize=16)
plt.xticks(fontsize=16)
plt.legend(loc=1, fontsize=15)
```

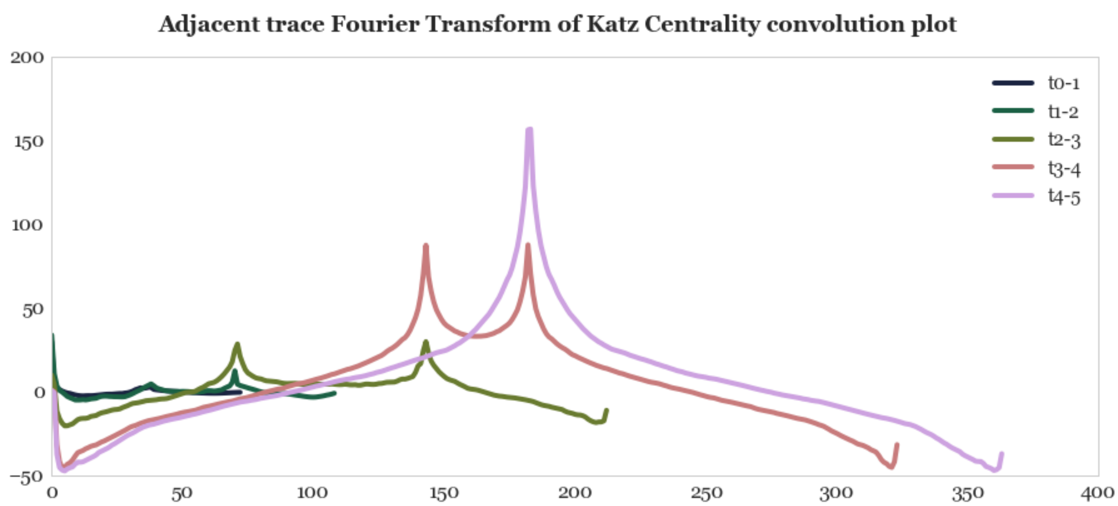
Out[75]: <matplotlib.legend.Legend at 0x21c2521d4e0>



In [76]:
```
plt.suptitle('Adjacent trace Fourier Transform of Katz Centrality convolut

plt.plot(np.convolve(fft_sig(katzC0),fft_sig(katzC1)),label='t0-1')
plt.plot(np.convolve(fft_sig(katzC1),fft_sig(katzC2)),label='t1-2')
plt.plot(np.convolve(fft_sig(katzC2),fft_sig(katzC3)),label='t2-3')
plt.plot(np.convolve(fft_sig(katzC3),fft_sig(katzC4)),label='t3-4')
plt.plot(np.convolve(fft_sig(katzC4),fft_sig(katzC5)),label='t4-5')

plt.yticks(fontsize=16)
plt.xticks(fontsize=16)
plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)
```

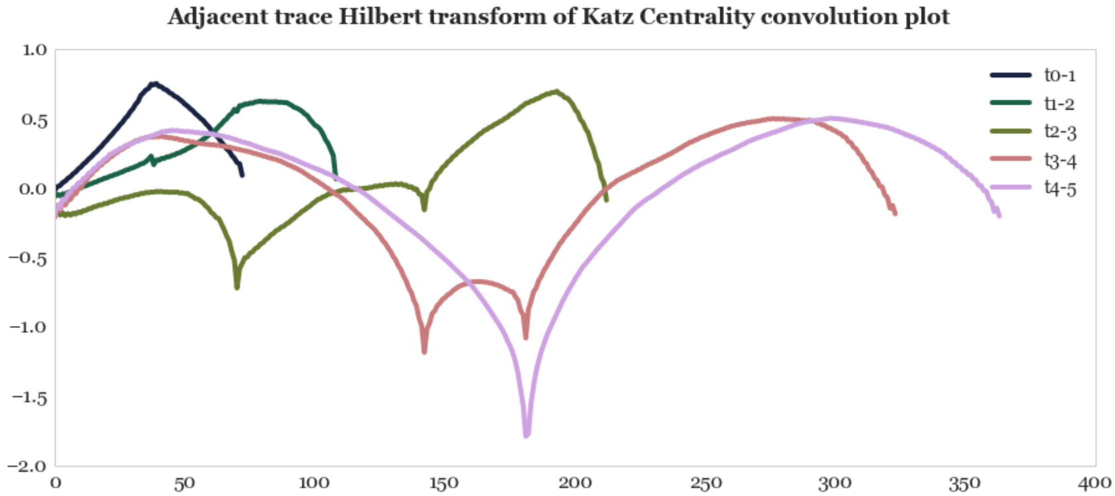
Out[76]: <matplotlib.legend.Legend at 0x21c22110be0>




In [77]:
```
plt.suptitle('Adjacent trace Hilbert transform of Katz Centrality convolut

plt.plot(np.convolve(hilbert_sig(katzC0), hilbert_sig(katzC1)), label='t0-
plt.plot(np.convolve(hilbert_sig(katzC1), hilbert_sig(katzC2)), label='t1-
plt.plot(np.convolve(hilbert_sig(katzC2), hilbert_sig(katzC3)),label='t2-3
plt.plot(np.convolve(hilbert_sig(katzC3), hilbert_sig(katzC4)), label='t3-
plt.plot(np.convolve(hilbert_sig(katzC4), hilbert_sig(katzC5)), label='t4-

plt.yticks(fontsize=16)
plt.xticks(fontsize=16)
plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)
```

```
Out[77]: <matplotlib.legend.Legend at 0x21c24276d30>
```

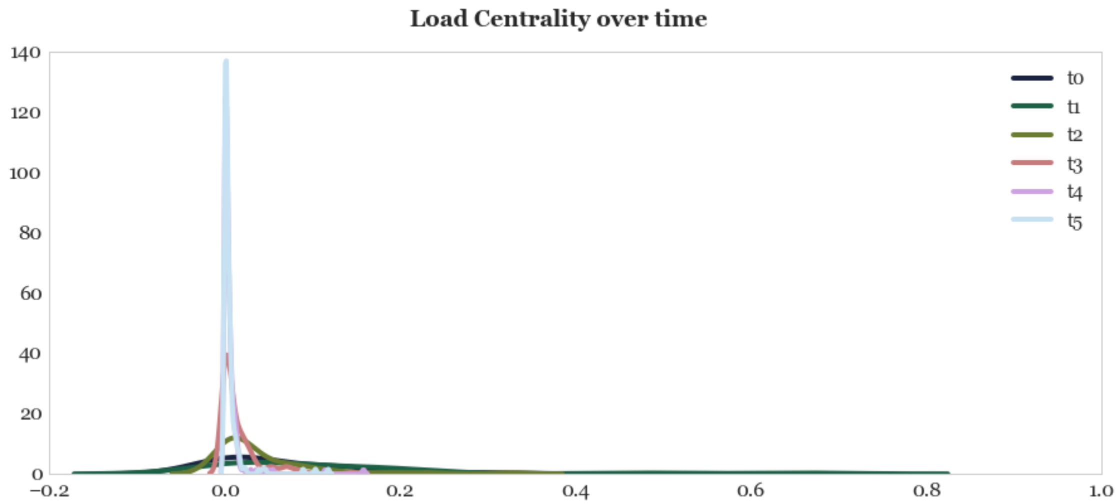



### 6.1.8 Load Centrality

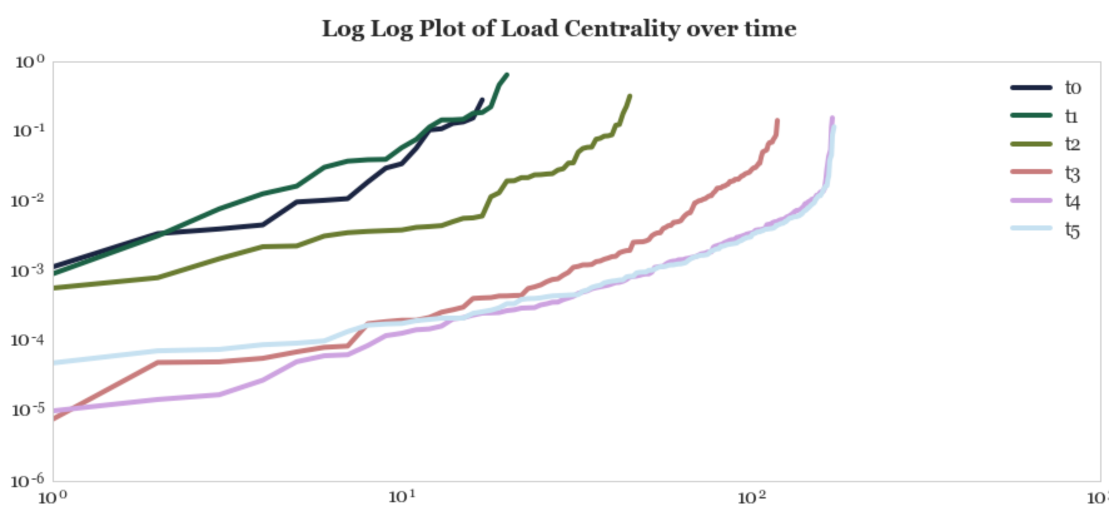
```
In [78]: plt.suptitle('Load Centrality over time', fontsize=18)

         sns.distplot(get_val(loadC0), hist=False, label='t0')
         sns.distplot(get_val(loadC1), hist=False, label='t1')
         sns.distplot(get_val(loadC2), hist=False, label='t2')
         sns.distplot(get_val(loadC3), hist=False, label='t3')
         sns.distplot(get_val(loadC4), hist=False, label='t4')
         sns.distplot(get_val(loadC5), hist=False, label='t5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\statsmodels\nonparametric\kdetools.p
  y = X[:m/2+1] + np.r_[0,X[m/2+1:],0]*1j
```

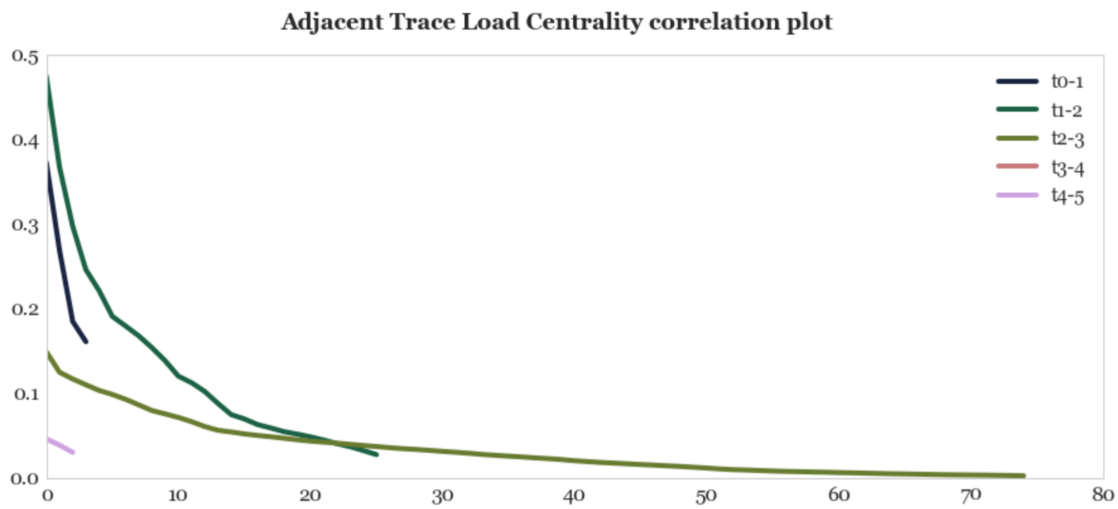
```
Out[78]: <matplotlib.legend.Legend at 0x21c24af4630>
```



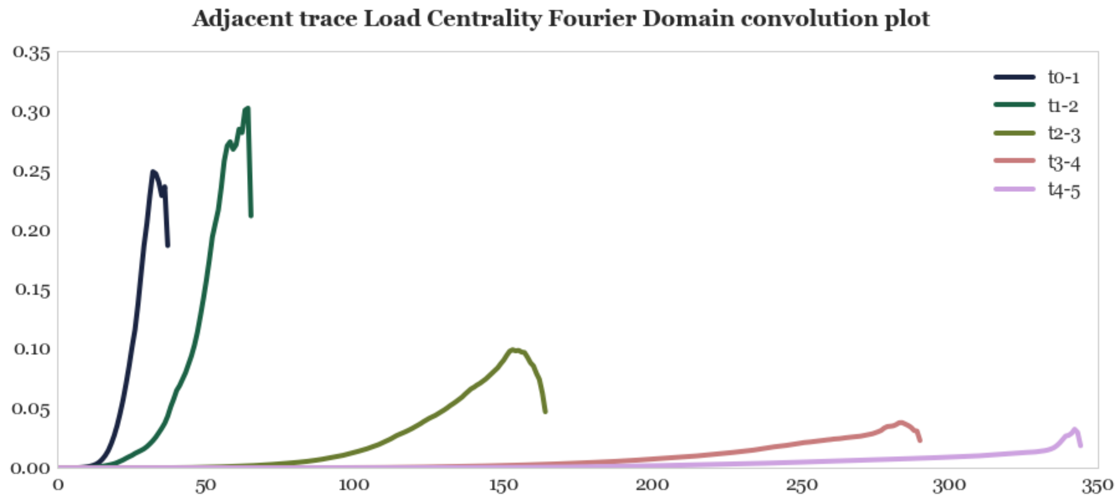

```
In [79]: plt.suptitle('Log Log Plot of Load Centrality over time', fontsize=18)

         plt.loglog(get_val(loadC0), label='t0')
         plt.loglog(get_val(loadC1), label='t1')
         plt.loglog(get_val(loadC2), label='t2')
         plt.loglog(get_val(loadC3), label='t3')
         plt.loglog(get_val(loadC4), label='t4')
         plt.loglog(get_val(loadC5), label='t5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)

Out[79]: <matplotlib.legend.Legend at 0x21c22e35358>
```



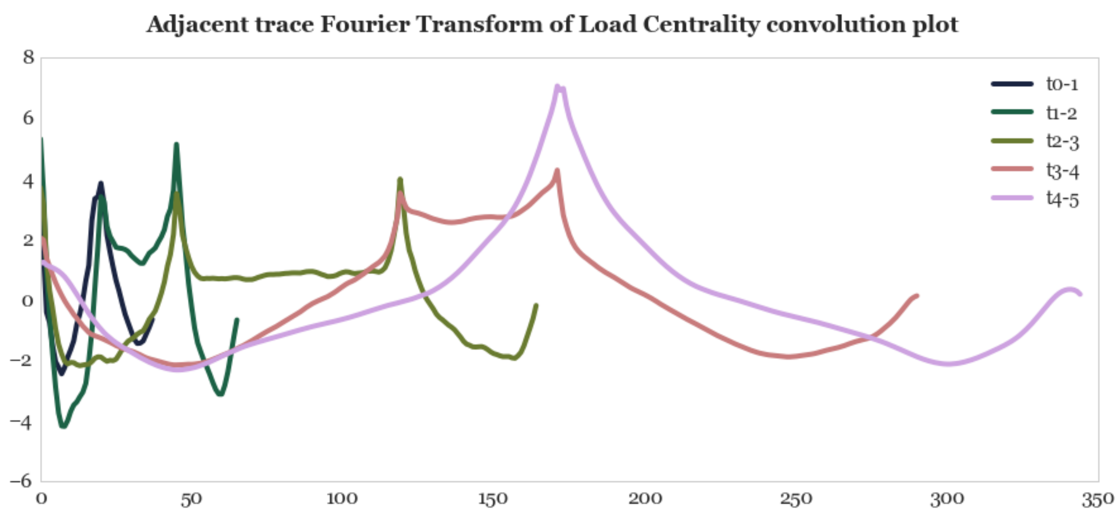
```
In [80]: plt.suptitle('Adjacent Trace Load Centrality correlation plot', fontsize=1

         plt.plot(np.correlate(get_val(loadC0),get_val(loadC1)),label='t0-1')
         plt.plot(np.correlate(get_val(loadC1),get_val(loadC2)),label='t1-2')
         plt.plot(np.correlate(get_val(loadC2),get_val(loadC3)),label='t2-3')
         plt.plot(np.correlate(get_val(loadC3),get_val(loadC3)),label='t3-4')
         plt.plot(np.correlate(get_val(loadC4),get_val(loadC5)),label='t4-5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)

Out[80]: <matplotlib.legend.Legend at 0x21c2500bef0>
```




```
In [81]: plt.suptitle('Adjacent trace Load Centrality Fourier Domain convolution pl

         plt.plot(fftconvolve(get_val(loadC0),get_val(loadC1)),label='t0-1')
         plt.plot(fftconvolve(get_val(loadC1),get_val(loadC2)),  label='t1-2')
         plt.plot(fftconvolve(get_val(loadC2),get_val(loadC3)), label='t2-3')
         plt.plot(fftconvolve(get_val(loadC3),get_val(loadC4)), label='t3-4')
         plt.plot(fftconvolve(get_val(loadC4),get_val(loadC5)),  label='t4-5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)

Out[81]: <matplotlib.legend.Legend at 0x21c2552fcc0>
```



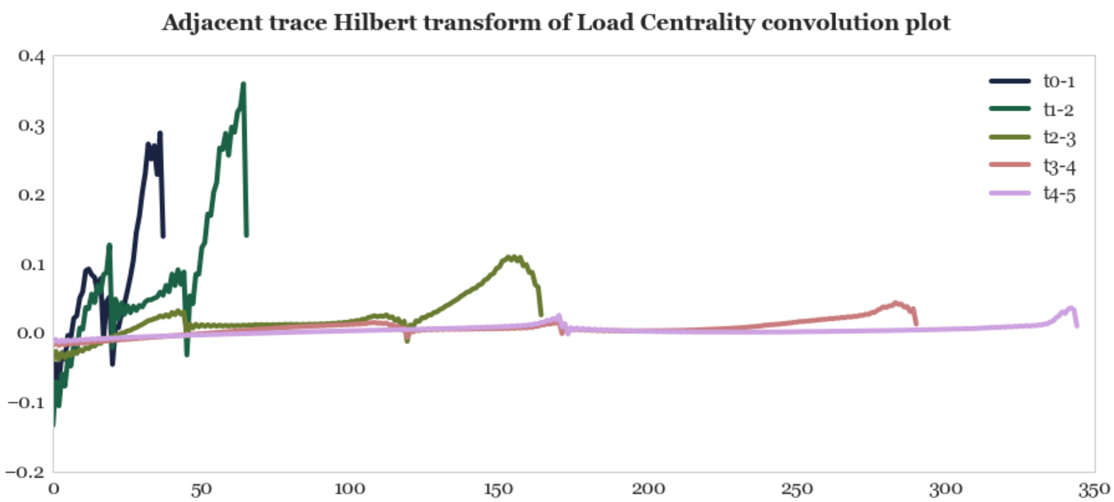

```
In [82]: plt.suptitle('Adjacent trace Fourier Transform of Load Centrality convolut

         plt.plot(np.convolve(fft_sig(loadC0),fft_sig(loadC1)),label='t0-1')
         plt.plot(np.convolve(fft_sig(loadC1),fft_sig(loadC2)),label='t1-2')
         plt.plot(np.convolve(fft_sig(loadC2),fft_sig(loadC3)),label='t2-3')
         plt.plot(np.convolve(fft_sig(loadC3),fft_sig(loadC4)),label='t3-4')
         plt.plot(np.convolve(fft_sig(loadC4),fft_sig(loadC5)),label='t4-5')

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)

C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)

Out[82]: <matplotlib.legend.Legend at 0x21c25416b70>
```



```
In [83]: plt.suptitle('Adjacent trace Hilbert transform of Load Centrality convolut

         plt.plot(np.convolve(hilbert_sig(loadC0), hilbert_sig(loadC1)), label='t0-
         plt.plot(np.convolve(hilbert_sig(loadC1), hilbert_sig(loadC2)), label='t1-
         plt.plot(np.convolve(hilbert_sig(loadC2), hilbert_sig(loadC3)),label='t2-3
         plt.plot(np.convolve(hilbert_sig(loadC3), hilbert_sig(loadC4)), label='t3-
         plt.plot(np.convolve(hilbert_sig(loadC4), hilbert_sig(loadC5)), label='t4-

         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(loc=1, fontsize=15)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:482: ComplexWa
  return array(a, dtype, copy=False, order=order)
```

```
Out[83]: <matplotlib.legend.Legend at 0x21c22f9ac18>
```

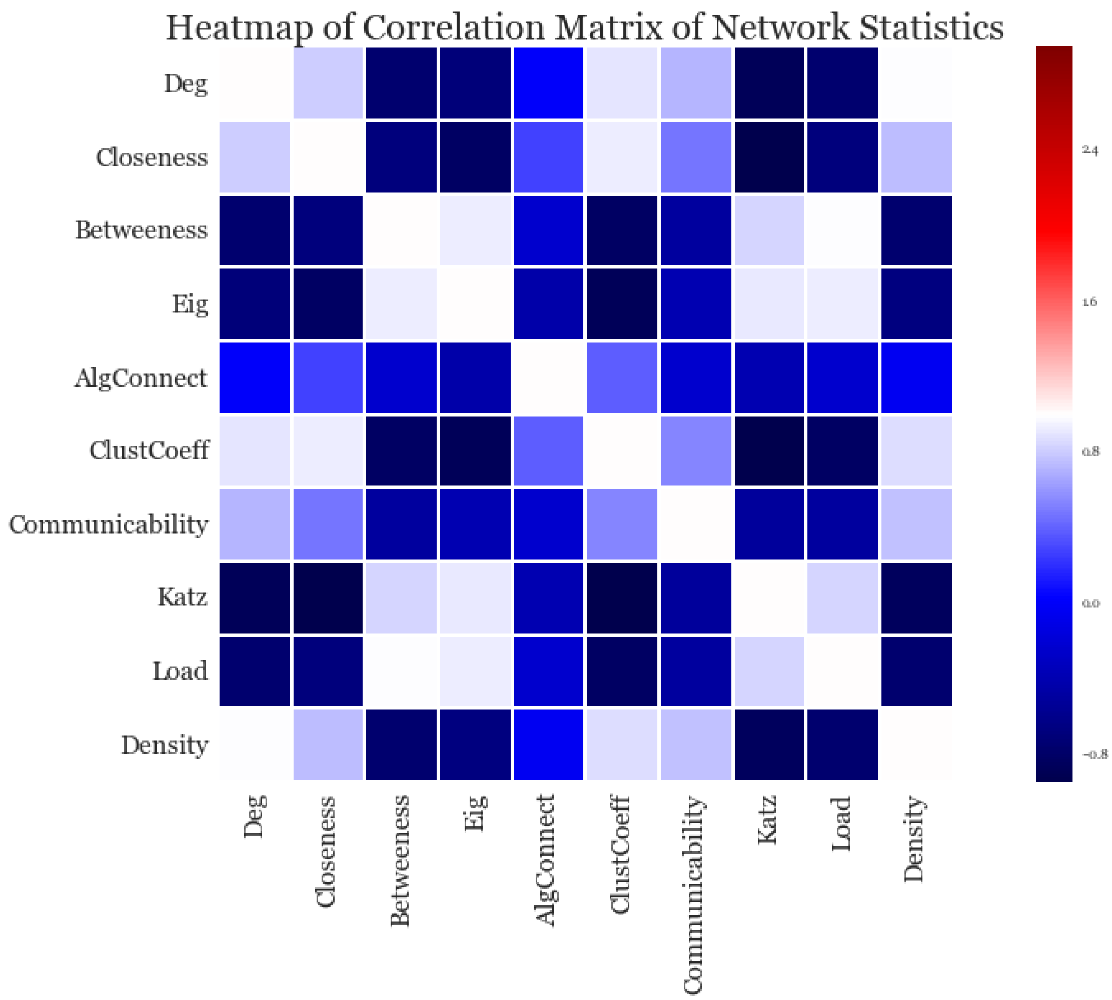



## 6.2 Centrality Analysis with averaging

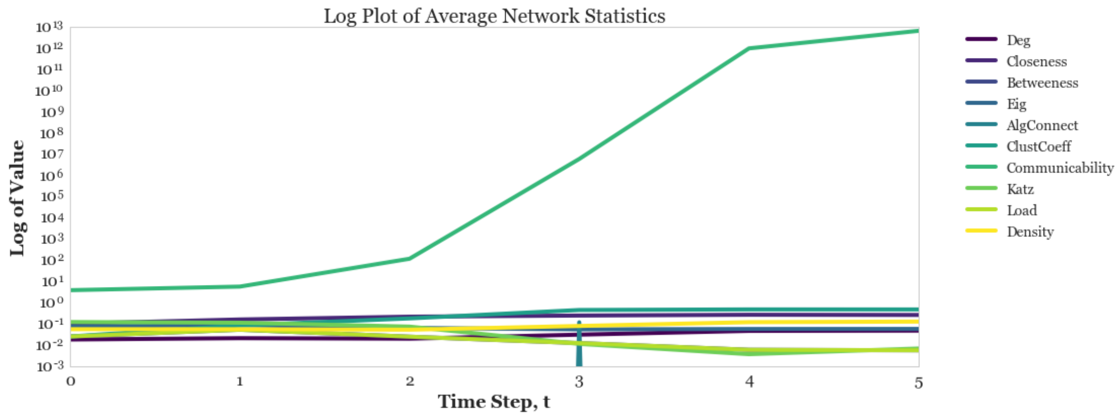
### 6.2.1 Calculate Centrality Statistics at different time steps

```
In [84]: deg0, clo0, bet0, eig0, alg0, clust0, commC0, katz0,load0 = cal_stat(Gt0)
         deg1, clo1, bet1, eig1, alg1, clust1, commC1, katz1,load1 = cal_stat(Gt1)
         deg2, clo2, bet2, eig2, alg2, clust2, commC2, katz2, load2 = cal_stat(Gt2)
         deg3, clo3, bet3, eig3, alg3, clust3, commC3, katz3,load3 = cal_stat(Gt3)
         deg4, clo4, bet4, eig4, alg4, clust4, commC4, katz4,load4 = cal_stat(Gt4)
         deg5, clo5, bet5, eig5, alg5, clust5, commC5, katz5, load5 = cal_stat(Gt5)
```

```
In [85]: stat_df = pd.DataFrame([deg0,deg1,deg2,deg3,deg4,deg5])

In [86]: #calculate density
         den0 = nx.density(Gt0)
         den1 = nx.density(Gt1)
         den2 = nx.density(Gt2)
         den3 = nx.density(Gt3)
         den4 = nx.density(Gt4)
         den5 = nx.density(Gt5)

In [87]: stat_df['Closeness'] = pd.DataFrame([clo0,clo1,clo2,clo3,clo4,clo5])
         stat_df['Betweeness'] = pd.DataFrame([bet0,bet1,bet2,bet3,bet4,bet5])
         stat_df['Eig'] = pd.DataFrame([eig0,eig1,eig2,eig3,eig4,eig5])
         stat_df['AlgConnect'] = pd.DataFrame([alg0,alg1,alg2,alg3,alg4,alg5])
         stat_df['ClustCoeff'] = pd.DataFrame([clust0,clust1,clust2,clust3,clust4,c
         stat_df['Communicability'] = pd.DataFrame([commC0,commC1,commC2,commC3,com
         stat_df['Katz'] = pd.DataFrame([katz0,katz1,katz2,katz3,katz4,katz5])
         stat_df['Load']=pd.DataFrame([load0,load1,load2,load3,load4,load5])
         stat_df['Density'] = pd.DataFrame([den0,den1,den2,den3,den4,den5])

In [88]: stat_df.columns.values[0]='Deg'

In [89]: stat_df.head()

Out[89]:         Deg  Closeness  Betweeness       Eig  AlgConnect  ClustCoeff  \
         0  0.018826   0.104661    0.025984  0.077739    0.000000    0.026004
         1  0.021720   0.165501    0.054367  0.089506    0.000000    0.087087
         2  0.020570   0.223937    0.025495  0.066241    0.000000    0.185696
         3  0.031813   0.253585    0.012739  0.056478    0.121915    0.462544
         4  0.047889   0.276022    0.006152  0.058781    0.000000    0.493717

            Communicability      Katz      Load   Density
         0     3.980933e+00  0.125240  0.025984  0.057586
         1     5.874870e+00  0.116543  0.054367  0.057624
         2     1.206113e+02  0.076152  0.025495  0.054430
         3     6.140791e+06  0.011703  0.012739  0.081651
         4     1.009380e+12  0.003815  0.006151  0.123401

In [90]: plt.figure(figsize=(18,8))
         sns.heatmap(stat_df.corr(), cmap='seismic', center=True, robust=True, fmt=
         plt.title('Heatmap of Correlation Matrix of Network Statistics', fontsize=
         plt.xticks(fontsize=16)
         plt.yticks(fontsize=16)

Out[90]: (array([ 0.5,  1.5,  2.5,  3.5,  4.5,  5.5,  6.5,  7.5,  8.5,  9.5]),
          <a list of 10 Text yticklabel objects>)
```




```
In [91]: stat_df.plot(colormap='viridis',logy=True, fontsize=14)
         plt.title('Log Plot of Average Network Statistics', fontsize=20)
         plt.yticks(fontsize=16)
         plt.xticks(fontsize=16)
         plt.legend(fontsize=14, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
         plt.xlabel("Time Step, t", fontsize=18)
         plt.ylabel("Log of Value", fontsize=18)

Out[91]: <matplotlib.text.Text at 0x21c2543c898>
```




# 7 Assortativity Analysis

```
In [92]: def cal_assort_att(net):
             dac = nx.degree_assortativity_coefficient(net)
             dpc = nx.degree_pearson_correlation_coefficient(net)
             avg_neigdeg = nx.average_neighbor_degree(net)
             avg_degconnect = nx.average_degree_connectivity(net)
             triangles = nx.triangles(net)

             return [dac,dpc,avg_neigdeg, avg_degconnect,triangles]
```

```
In [93]: dac0 = nx.degree_assortativity_coefficient(Gt0)
```

```
In [ ]: nx.degree_pearson_correlation_coefficient(Gt0)
```