DNA01

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2 Introduction

Here using the Enron Email datawith timestamps from John Hopkins, I will attempt to recreatea dynamic network at monthly level of granularity. Previously I conducted this analysis at the yearly level.

For this analysis we use some typical attributes in addition to novel ones.

2.1 The traditional attributes used are:

- Degree Centrality
- Closeness Centrality
- Betweenness Centrality

- Eigenvector Centrality
- Katz Centrality
- Load Centrality
- Density
- Clustering Coefficient

2.2 Novel attributes introduced in this analysis:

- Instantaneous Phase
- Instantaneous Amplitude
- Instantatneous Frequency
- Gaussian Curvature
- Energy Envelope
- First Derivative of Energy Envelope
- Second Derivative of Energy Envelope
- Kernel PCA 3 Component Ratio Average Correlation

2.3 Attributes implemented from other authors:

- Persistence & Emergence (ref: CMU)
- Resistance Distance (ref: Klein 93)

2.4 Novel Visualisation suggested in this analysis:

- Frequency vs Wavenumber Plots
- Radon Domain plots

3 Import Libraries

In [488]:
```
import pandas as pd
import numpy as np
import networkx as nx
import seaborn as sns
import matplotlib.pyplot as plt
import scipy as sc
%matplotlib inline
sns.set(style="whitegrid", color_codes=True, context='paper')
import random
random.seed(1111111111111)
plt.rc('axes', grid=False, titlesize='large', labelsize='medium',labelwei
plt.rc('lines', linewidth=4)
plt.rc('font',family='serif',size=12, serif='Georgia')
plt.rc('figure', figsize = (15,6),titlesize='large',titleweight='heavy')
plt.rc('grid',linewidth=3)
sns.set_palette('cubehelix')
from scipy.signal import *
from numpy.linalg import *
```

4 Helper Functions

```
In [489]: def get_val(val):
              return sorted(set(val.values()))

In [490]: def avg_cent(cent):
              avg = sum(set(cent.values()))/len(cent)
              return avg

In [491]: def get_cent(net):
              degC = nx.degree_centrality(net)
              cloC = nx.closeness_centrality(net)
              betC = nx.betweenness_centrality(net)
              eigC = nx.eigenvector_centrality_numpy(net)
              katzC = nx.katz_centrality_numpy(net)
              loadC = nx.load_centrality(net)

              return [degC,cloC,betC,eigC,katzC, loadC]

In [492]: def stationarity_ratio(G):
              #stationarity ratio with laplian
              L = nx.laplacian_matrix(G).todense()
              U = nx.laplacian_spectrum(G)
              C =np.cov(L)
              CF = np.dot(L,np.dot(np.dot(U.T,C),U))
              r = np.linalg.norm(np.diag(CF))/np.linalg.norm(CF)

              return [r]

In [493]: #cite:`klein1993resistance`
          def resistance_distance(net):
              M = nx.laplacian_matrix(net).todense()
              pseudo = pinv(M)
              N = M.shape[0]
              d = np.diag(pseudo)
              rd  = np.kron(d,np.ones((N,1))).T+np.kron(d,np.ones((N,1))).T - pseud

              return [rd, rd.mean()]

In [494]: def calc_seisatt(net):
              M = nx.laplacian_matrix(net).todense()
              Ht = hilbert(M)
              IA = np.real(np.nan_to_num(np.sqrt(np.dot(M,M)+np.dot(Ht,Ht))))
              IP = np.real(np.nan_to_num(np.arctan(Ht/M)))
              IF,_ = np.real(np.nan_to_num(np.asarray(np.gradient(IP))))
              E = np.real(np.sqrt(np.dot(M,M)+np.dot(Ht,Ht)))
              dE, _ = np.nan_to_num(np.asarray(np.gradient(E)))
              dEe, _ = np.nan_to_num(np.asarray(np.gradient(dE)))
```

```
        att_globalval = pd.DataFrame([IA.mean(),IP.mean(),IF.mean(),\
                                      E.mean(),dE.mean(),dEe.mean()]).T
        att_globalval.columns =['InstAmp','InstPhase','InstFreq.','EnergyEnv'

        return [IA,IP,IF,E, dE, dEe, att_globalval]
```

In [746]:
```
def curvature(net):
    from skimage.feature import hessian_matrix, hessian_matrix_det, hessi
    M = nx.laplacian_matrix(net).todense()
    M = np.float64(M)
    fx, fy = np.gradient(M)
    Hxx, Hxy, Hyy = hessian_matrix(M)
    K = np.divide((np.dot(Hxx,Hxy)-np.dot(Hxy,Hxy)),\
                  (1+np.dot(fx,fx)+np.dot(fy,fy)))

    He1,_ = hessian_matrix_eigvals(Hxx,Hxy,Hyy)
    mean_curv = np.trace(He1)
    s, a = np.linalg.slogdet(He1)
    conc = s * np.exp(a)
    Pmax = np.max(He1)
    Pmin = np.min(He1)

    return [K,mean_curv,conc]
```

In [521]:
```
def cal_avgstat(net):
    #calculate all attributes from previously defined functions here
    degC,cloC,betC,eigC,katzC, loadC = get_cent(net)
    _,meanK,_ = curvature(net)
    IA,IP,IF,E, dE, dEe, att_globalval = calc_seisatt(net)
    _,norm_rd = resistance_distance(net)
    r = stationarity_ratio(net)
    den = nx.density(net)
    clustcof = nx.clustering(net)

    #create attribute volume here
    stat_df = pd.DataFrame([avg_cent(degC),avg_cent(cloC), avg_cent(betC)
            avg_cent(katzC), avg_cent(loadC), meanK, den, avg_cent(clustc
    stat_df.columns= ['AvgDeg','AvgCloseness','AvgBet','AvgEig','AvgKatz'
    stat_df = stat_df.join(att_globalval)
    stat_df['MeanResistanceDist']= norm_rd
    stat_df['StatRat']=r


    return stat_df
```

In [839]:
```
def get_top_keys(dictionary, top):
    items = dictionary.items()
```

```
            items = sorted(items,reverse=True, key=lambda x: x[1])
            obj = map(lambda x: x[0], items[:top])
            for i in obj:
                print(i)
            #return map(lambda x: x[0], items[:top])
```

```
In [523]: def std_klpca_ratio(net):
            from sklearn.decomposition import KernelPCA
            M = nx.laplacian_matrix(net)
            kpca =KernelPCA(n_components=3,kernel='rbf')
            eigv = kpca.fit_transform(M)
            pc1_std = eigv[0] - eigv[0].mean() /eigv[0].std()
            pc2_std = eigv[1] - eigv[1].mean() /eigv[1].std()
            pc3_std = eigv[2] - eigv[2].mean() /eigv[2].std()
            klpca_ratio_std = pc1_std - pc3_std/pc1_std - pc2_std

            return klpca_ratio_std
```

5 Data Analysis Monthly Aggregation

5.1 Load Preprocessed Data

```
In [524]: data = pd.read_excel("../Data/data 03.2.xlsx")
```

```
In [525]: data.head()
```

Out[525]:

	timestamp	to	from	year	month
0	1979-12-31 21:00:00	24	153	1979	12
1	1979-12-31 21:00:00	24	153	1979	12
2	1979-12-31 21:00:00	29	29	1979	12
3	1979-12-31 21:00:00	29	29	1979	12
4	1979-12-31 21:00:00	29	29	1979	12

5.2 Check Year Labels

```
In [526]: set(data.year)
```

```
Out[526]: {1979, 1998, 1999, 2000, 2001, 2002}
```

```
In [527]: data[data.year==1979].count()
```

```
Out[527]: timestamp    174
          to           174
          from         174
          year         174
          month        174
          dtype: int64
```

```
In [528]: data.shape
```

```
Out[528]: (125409, 5)

In [529]: #total % of mislabelled 1979 entries
          (data[data.year==1979].count()/data.shape[0]) * 100

Out[529]: timestamp    0.138746
          to           0.138746
          from         0.138746
          year         0.138746
          month        0.138746
          dtype: float64
```

5.3 Split data into yearly slices

The entries labelled 1979 are mislabelled hence they will be excluded from analysis. As we see that they are a tiny fraction of the dataset anyway

```
In [530]: data = data[data.year!= 1979]

In [531]: years = sorted(set(data.year))

In [532]: years

Out[532]: [1998, 1999, 2000, 2001, 2002]

In [533]: df_98 = data[data.year==years[0]]
          df_99 = data[data.year==years[1]]
          df_2k = data[data.year==years[2]]
          df_2k1 = data[data.year==years[3]]
          df_2k2 = data[data.year==years[4]]

In [534]: df_98.head()

Out[534]:                timestamp  to   from  year  month
          174 1998-11-13 09:07:00  114   169  1998     11
          175 1998-11-13 09:07:00  114   169  1998     11
          176 1998-11-19 12:19:00  114   123  1998     11
          177 1998-11-19 12:19:00  114   123  1998     11
          178 1998-11-19 13:24:00  114   123  1998     11

In [535]: df_98.describe()

Out[535]:                to         from    year      month
          count   82.000000   82.000000    82.0  82.000000
          mean   114.292683  119.000000  1998.0  11.634146
          std      2.051725   48.393449     0.0   0.484633
          min    112.000000   11.000000  1998.0  11.000000
          25%    114.000000  110.000000  1998.0  11.000000
          50%    114.000000  123.000000  1998.0  12.000000
          75%    114.000000  169.000000  1998.0  12.000000
          max    123.000000  169.000000  1998.0  12.000000
```

```
In [536]: df_99.head()
```

Out[536]:

	timestamp	to	from	year	month
256	1999-01-04 07:21:00	114	65	1999	1
257	1999-01-04 07:21:00	114	65	1999	1
258	1999-01-04 09:11:00	114	169	1999	1
259	1999-01-04 09:11:00	114	169	1999	1
260	1999-01-07 13:42:00	114	112	1999	1

```
In [537]: df_2k.head()
```

Out[537]:

	timestamp	to	from	year	month
3971	2000-01-03 06:47:00	82	51	2000	1
3972	2000-01-03 06:47:00	82	51	2000	1
3973	2000-01-03 06:47:00	82	51	2000	1
3974	2000-01-03 06:47:00	82	51	2000	1
3975	2000-01-03 06:47:00	82	51	2000	1

```
In [538]: df_99.describe()
```

Out[538]:

	to	from	year	month
count	3715.000000	3715.000000	3715.0	3715.000000
mean	116.191386	115.949933	1999.0	9.725707
std	56.692443	43.626945	0.0	2.648675
min	11.000000	2.000000	1999.0	1.000000
25%	65.000000	88.000000	1999.0	8.000000
50%	145.000000	114.000000	1999.0	10.000000
75%	169.000000	156.000000	1999.0	12.000000
max	178.000000	178.000000	1999.0	12.000000

```
In [539]: df_2k1.head()
```

Out[539]:

	timestamp	to	from	year	month
48030	2001-01-01 13:36:00	78	82	2001	1
48031	2001-01-01 13:36:00	78	82	2001	1
48032	2001-01-01 13:36:00	78	82	2001	1
48033	2001-01-01 13:55:00	78	127	2001	1
48034	2001-01-01 13:55:00	78	127	2001	1

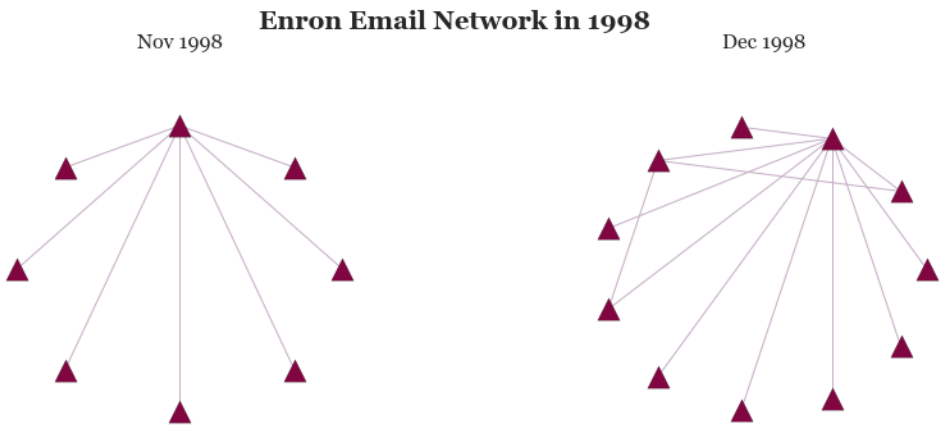
```
In [540]: df_2k2.head()
```

Out[540]:

	timestamp	to	from	year	month
116918	2002-01-01 17:27:27	0	9	2002	1
116919	2002-01-01 17:27:27	0	48	2002	1
116920	2002-01-01 20:12:31	0	20	2002	1
116921	2002-01-01 21:27:27	0	9	2002	1
116922	2002-01-01 21:27:27	0	48	2002	1

6 Network Year 1998

```
In [541]: nov_98 = df_98[df_98.month==11]
          dec_98= df_98[df_98.month==12]

In [542]: def create_graph(df):
              tmp = df.values[:,1:3]
              G= nx.Graph()
              G = nx.from_edgelist(tmp)

              return G

In [543]: G_nov98 = create_graph(nov_98)
          G_dec98 = create_graph(dec_98)

In [632]: plt.figure()
          plt.suptitle("Enron Email Network in 1998", fontsize=20)
          plt.subplot(121)
          plt.title("Nov 1998", fontsize=16)
          nx.draw_circular(G_nov98, node_color='#810541',node_shape='^',edge_color
          plt.subplot(122)
          plt.title("Dec 1998", fontsize=16)
          nx.draw_circular(G_dec98, node_color='#810541',node_shape='^',edge_color
```



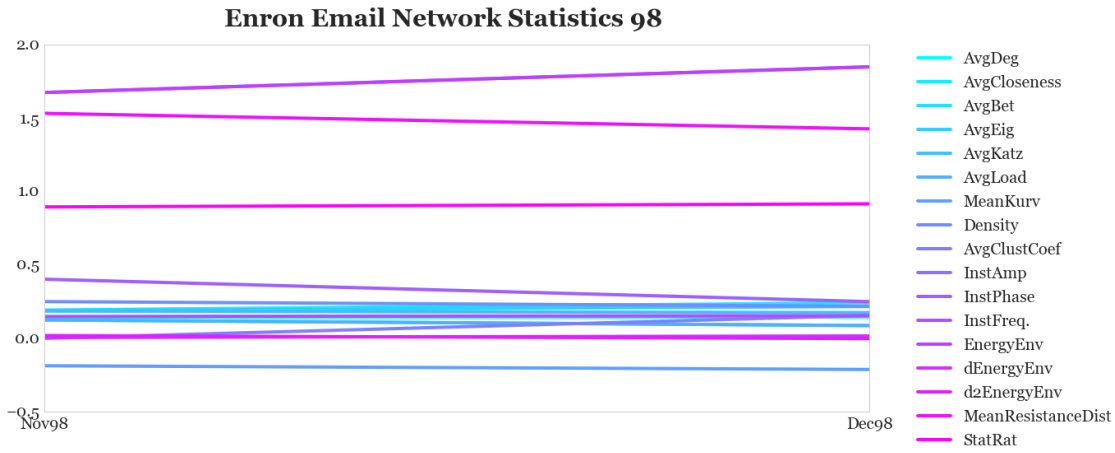

```
In [747]: degC_nov98,cloC_nov98,betC_nov98,eigC_nov98,katzC_nov98, loadC_nov98 = ge

In [843]: print("Top 5 Degree Centrality Nodes")
          degC_nov98_10 = get_top_keys(degC_nov98,5)
          print("Top 5 Closeness Centrality Nodes")
          cloC_nov98_10 = get_top_keys(cloC_nov98,5)
          print("Top 5 Betweeness Centrality Nodes")
```

```
betC_nov98_10 = get_top_keys(betC_nov98,5)
print("Top 5 Eigenvector Centrality Nodes")
eigC_nov98_10 = get_top_keys(eigC_nov98,5)
print("Top 5 Katz Centrality Nodes")
katzC_nov98_10 = get_top_keys(katzC_nov98,5)
print("Top 5 Load Centrality Nodes")
loadC_nov98_10 = get_top_keys(loadC_nov98,5)
```

```
Top 5 Degree Centrality Nodes
114
112
65
123
38
Top 5 Closeness Centrality Nodes
114
112
65
123
38
Top 5 Betweeness Centrality Nodes
114
112
65
123
38
Top 5 Eigenvector Centrality Nodes
114
65
123
169
155
Top 5 Katz Centrality Nodes
114
38
112
65
169
Top 5 Load Centrality Nodes
114
112
65
123
38
```

```
In [545]: stat_nov98 = cal_avgstat(G_nov98)
          stat_dec98 = cal_avgstat(G_dec98)
```

```
In [546]: stat_nov98

Out[546]:      AvgDeg  AvgCloseness  AvgBet    AvgEig   AvgKatz  AvgLoad  MeanKurv
          0  0.142857      0.192308   0.125  0.188611  0.185435    0.125 -0.186746

             Density  AvgClustCoef   InstAmp  InstPhase  InstFreq.  EnergyEnv  \
          0     0.25           0.0  1.673053   0.402325   0.148795   1.673053

             dEnergyEnv  d2EnergyEnv  MeanResistanceDist   StatRat
          0    0.005621     0.018756             1.53125  0.894427

In [547]: stat_dec98

Out[547]:      AvgDeg  AvgCloseness    AvgBet   AvgEig   AvgKatz   AvgLoad  MeanKur
          0  0.145455      0.242737  0.086869  0.21638  0.175111  0.086869 -0.21192

              Density  AvgClustCoef  InstAmp  InstPhase  InstFreq.  EnergyEnv  \
          0  0.218182      0.155556  1.84759   0.249824   0.154485    1.84759

             dEnergyEnv  d2EnergyEnv  MeanResistanceDist   StatRat
          0    0.016204    -0.001828             1.42562  0.915335

In [549]: stat98= stat_nov98.append(stat_dec98).T

          stat98.columns = ['Nov98','Dec98']

In [550]: stat98.T.plot(fontsize=20, figsize=(18,8),cmap='cool')
          plt.suptitle("Enron Email Network Statistics 98", fontsize=30)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

Out[550]: <matplotlib.legend.Legend at 0x1a94f526940>
```



7 Network Year 1999

In [551]:
```
df_99.describe()
```

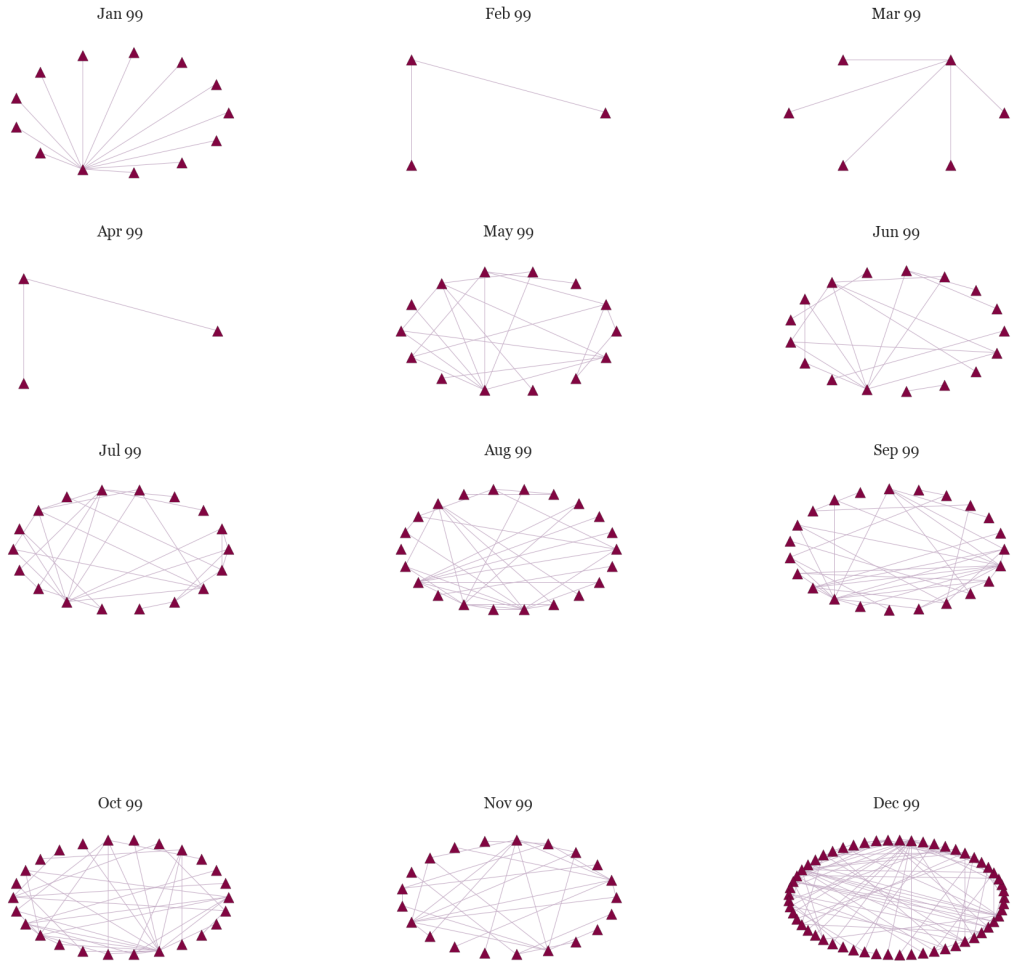
Out[551]:

	to	from	year	month
count	3715.000000	3715.000000	3715.0	3715.000000
mean	116.191386	115.949933	1999.0	9.725707
std	56.692443	43.626945	0.0	2.648675
min	11.000000	2.000000	1999.0	1.000000
25%	65.000000	88.000000	1999.0	8.000000
50%	145.000000	114.000000	1999.0	10.000000
75%	169.000000	156.000000	1999.0	12.000000
max	178.000000	178.000000	1999.0	12.000000

In [552]:
```
jan_99=df_99[df_99.month==1]
feb_99=df_99[df_99.month==2]
mar_99=df_99[df_99.month==3]
apr_99=df_99[df_99.month==4]
may_99=df_99[df_99.month==5]
jun_99=df_99[df_99.month==6]
jul_99=df_99[df_99.month==7]
aug_99=df_99[df_99.month==8]
sep_99=df_99[df_99.month==9]
oct_99=df_99[df_99.month==10]
nov_99=df_99[df_99.month==11]
dec_99=df_99[df_99.month==12]


G_jan_99=create_graph(jan_99)
G_feb_99=create_graph(feb_99)
G_mar_99=create_graph(mar_99)
G_apr_99=create_graph(apr_99)
G_may_99=create_graph(may_99)
G_jun_99=create_graph(jun_99)
G_jul_99=create_graph(jul_99)
G_aug_99=create_graph(aug_99)
G_sep_99=create_graph(sep_99)
G_oct_99=create_graph(oct_99)
G_nov_99=create_graph(nov_99)
G_dec_99=create_graph(dec_99)
```

In [633]:
```
plt.figure(figsize=(32,18))
plt.suptitle("Enron Email Network in 99", fontsize=40)

plt.subplot(331)
plt.title("Jan 99", fontsize=25)
nx.draw_circular(G_jan_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(332)
```

```
plt.title("Feb 99", fontsize=25)
nx.draw_circular(G_feb_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(333)
plt.title("Mar 99", fontsize=25)
nx.draw_circular(G_mar_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(334)
plt.title("Apr 99", fontsize=25)
nx.draw_circular(G_apr_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(335)
plt.title("May 99", fontsize=25)
nx.draw_circular(G_may_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(336)
plt.title("Jun 99", fontsize=25)
nx.draw_circular(G_jun_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(337)
plt.title("Jul 99", fontsize=25)
nx.draw_circular(G_jul_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(338)
plt.title("Aug 99", fontsize=25)
nx.draw_circular(G_aug_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(339)
plt.title("Sep 99", fontsize=25)
nx.draw_circular(G_sep_99, node_color='#810541',node_shape='^',edge_color

plt.figure(figsize=(32,5))
#plt.suptitle("Enron Email Network in 99", fontsize=40)

plt.subplot(131)
plt.title("Oct 99", fontsize=25)
nx.draw_circular(G_oct_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(132)
plt.title("Nov 99", fontsize=25)
nx.draw_circular(G_nov_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(133)
plt.title("Dec 99", fontsize=25)
nx.draw_circular(G_dec_99, node_color='#810541',node_shape='^',edge_color
```

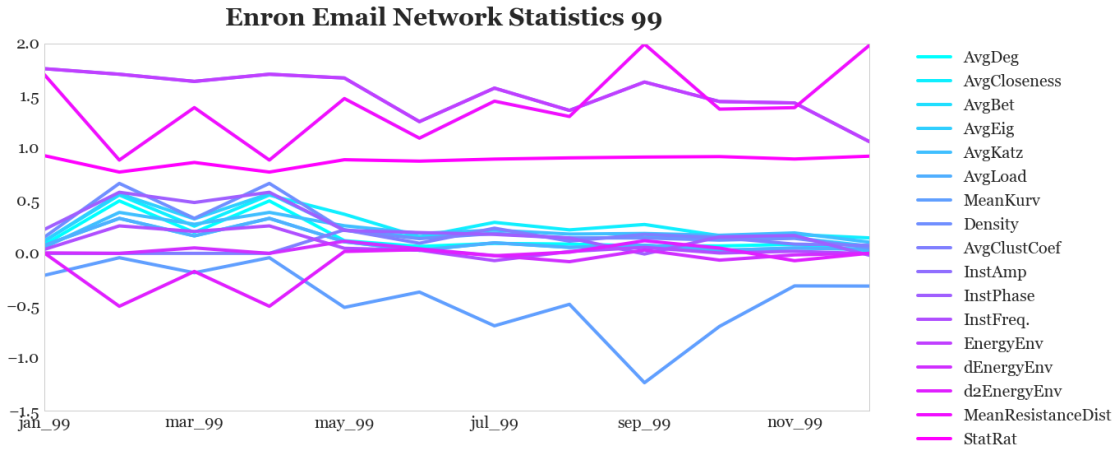



```
In [555]: stat_jan_99=cal_avgstat(G_jan_99)
          stat_feb_99=cal_avgstat(G_feb_99)
          stat_mar_99=cal_avgstat(G_mar_99)
          stat_apr_99=cal_avgstat(G_apr_99)
          stat_may_99=cal_avgstat(G_may_99)
          stat_jun_99=cal_avgstat(G_jun_99)
          stat_jul_99=cal_avgstat(G_jul_99)
          stat_aug_99=cal_avgstat(G_aug_99)
          stat_sep_99=cal_avgstat(G_sep_99)
          stat_oct_99=cal_avgstat(G_oct_99)
          stat_nov_99=cal_avgstat(G_nov_99)
          stat_dec_99=cal_avgstat(G_dec_99)

          stat_99 = stat_jan_99.append(stat_feb_99).append(stat_mar_99).append(stat
          stat_99 = stat_99.append(stat_jun_99).append(stat_jul_99).append(stat_aug
```

```
stat_99 = stat_99.append(stat_oct_99).append(stat_nov_99).append(stat_dec
stat_99.columns = ['jan_99','feb_99','mar_99','apr_99','may_99','jun_99',
```

In [556]:
```
#'YlOrRd'
stat_99.T.plot(fontsize=20, figsize=(18,8), cmap='cool')
plt.suptitle("Enron Email Network Statistics 99", fontsize=30)
plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
```

Out[556]:
```
<matplotlib.legend.Legend at 0x1a9523505f8>
```



8 Network Year 2000

In [557]:
```
df_2k.month.describe()
```

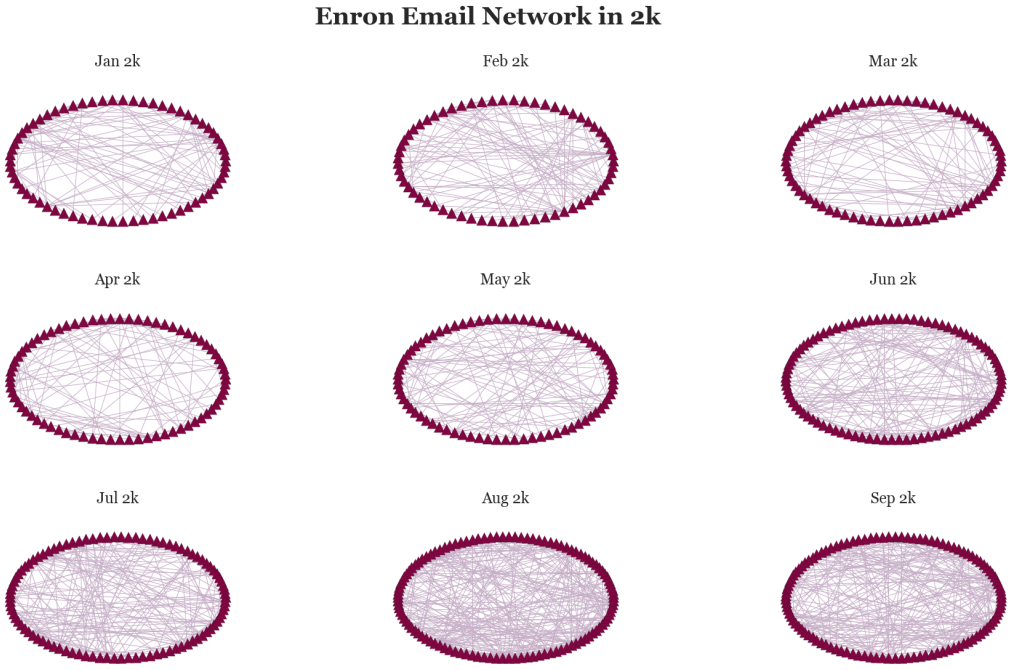
Out[557]:
```
count    44059.000000
mean         8.183163
std          3.169912
min          1.000000
25%          6.000000
50%          9.000000
75%         11.000000
max         12.000000
Name: month, dtype: float64
```

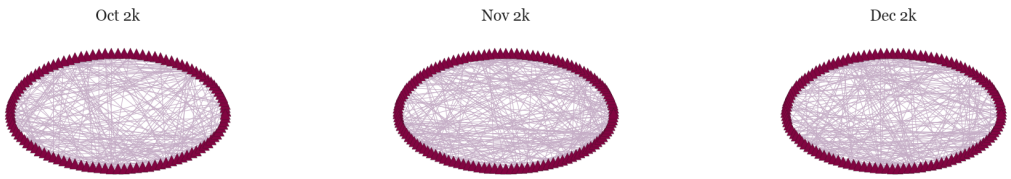
In [558]:
```
jan_2k=df_2k[df_2k.month==1]
feb_2k=df_2k[df_2k.month==2]
mar_2k=df_2k[df_2k.month==3]
apr_2k=df_2k[df_2k.month==4]
may_2k=df_2k[df_2k.month==5]
jun_2k=df_2k[df_2k.month==6]
jul_2k=df_2k[df_2k.month==7]
```

```
aug_2k=df_2k[df_2k.month==8]
sep_2k=df_2k[df_2k.month==9]
oct_2k=df_2k[df_2k.month==10]
nov_2k=df_2k[df_2k.month==11]
dec_2k=df_2k[df_2k.month==12]


G_jan_2k=create_graph(jan_2k)
G_feb_2k=create_graph(feb_2k)
G_mar_2k=create_graph(mar_2k)
G_apr_2k=create_graph(apr_2k)
G_may_2k=create_graph(may_2k)
G_jun_2k=create_graph(jun_2k)
G_jul_2k=create_graph(jul_2k)
G_aug_2k=create_graph(aug_2k)
G_sep_2k=create_graph(sep_2k)
G_oct_2k=create_graph(oct_2k)
G_nov_2k=create_graph(nov_2k)
G_dec_2k=create_graph(dec_2k)
```

In [634]:
```
plt.figure(figsize=(32,18))
plt.suptitle("Enron Email Network in 2k", fontsize=40)

plt.subplot(331)
plt.title("Jan 2k", fontsize=25)
nx.draw_circular(G_jan_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(332)
plt.title("Feb 2k", fontsize=25)
nx.draw_circular(G_feb_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(333)
plt.title("Mar 2k", fontsize=25)
nx.draw_circular(G_mar_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(334)
plt.title("Apr 2k", fontsize=25)
nx.draw_circular(G_apr_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(335)
plt.title("May 2k", fontsize=25)
nx.draw_circular(G_may_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(336)
plt.title("Jun 2k", fontsize=25)
nx.draw_circular(G_jun_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(337)
```

```
plt.title("Jul 2k", fontsize=25)
nx.draw_circular(G_jul_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(338)
plt.title("Aug 2k", fontsize=25)
nx.draw_circular(G_aug_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(339)
plt.title("Sep 2k", fontsize=25)
nx.draw_circular(G_sep_2k, node_color='#810541',node_shape='^',edge_color

plt.figure(figsize=(32,5))
#plt.suptitle("Enron Email Network in 2k", fontsize=40)

plt.subplot(131)
plt.title("Oct 2k", fontsize=25)
nx.draw_circular(G_oct_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(132)
plt.title("Nov 2k", fontsize=25)
nx.draw_circular(G_nov_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(133)
plt.title("Dec 2k", fontsize=25)
nx.draw_circular(G_dec_2k, node_color='#810541',node_shape='^',edge_color
```



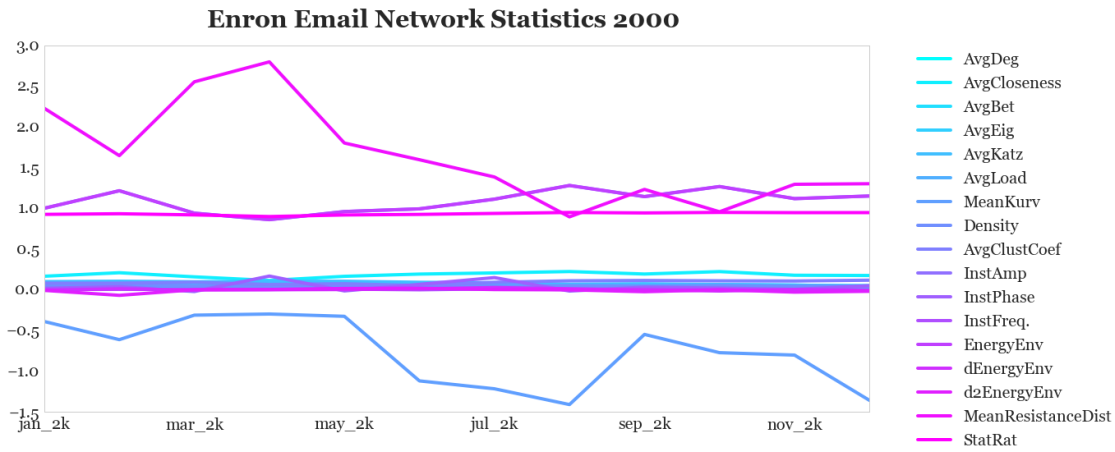



```
In [561]: stat_jan_2k=cal_avgstat(G_jan_2k)
          stat_feb_2k=cal_avgstat(G_feb_2k)
          stat_mar_2k=cal_avgstat(G_mar_2k)
          stat_apr_2k=cal_avgstat(G_apr_2k)
          stat_may_2k=cal_avgstat(G_may_2k)
          stat_jun_2k=cal_avgstat(G_jun_2k)
          stat_jul_2k=cal_avgstat(G_jul_2k)
          stat_aug_2k=cal_avgstat(G_aug_2k)
          stat_sep_2k=cal_avgstat(G_sep_2k)
          stat_oct_2k=cal_avgstat(G_oct_2k)
          stat_nov_2k=cal_avgstat(G_nov_2k)
          stat_dec_2k=cal_avgstat(G_dec_2k)

          stat_2k = stat_jan_2k.append(stat_feb_2k).append(stat_mar_2k).append(stat
          stat_2k = stat_2k.append(stat_jun_2k).append(stat_jul_2k).append(stat_aug
          stat_2k = stat_2k.append(stat_oct_2k).append(stat_nov_2k).append(stat_dec
          stat_2k.columns = ['jan_2k','feb_2k','mar_2k','apr_2k','may_2k','jun_2k',

In [562]: stat_2k.head()

Out[562]:
```

	jan_2k	feb_2k	mar_2k	apr_2k	may_2k	jun_2k
AvgDeg	0.022378	0.026626	0.014809	0.010464	0.014035	0.010867
AvgCloseness	0.165628	0.208593	0.157985	0.113773	0.164460	0.192022
AvgBet	0.034948	0.035326	0.046531	0.042264	0.033091	0.027877
AvgEig	0.068894	0.083170	0.065712	0.053698	0.072435	0.068502
AvgKatz	0.099556	0.104652	0.097354	0.103359	0.104188	0.091210

	jul_2k	aug_2k	sep_2k	oct_2k	nov_2k	dec_2k
AvgDeg	0.012697	0.016514	0.012147	0.018086	0.010884	0.013887
AvgCloseness	0.206035	0.223131	0.192691	0.222388	0.177589	0.174585
AvgBet	0.026787	0.017815	0.019417	0.019777	0.017789	0.019199
AvgEig	0.068311	0.063214	0.061838	0.062537	0.054661	0.051451
AvgKatz	0.086094	0.068437	0.073747	0.064164	0.058620	0.038943

```
In [845]: stat_2k.T.plot(fontsize=20, figsize=(18,8), cmap='cool')
          plt.suptitle("Enron Email Network Statistics 2000", fontsize=30)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

Out[845]: <matplotlib.legend.Legend at 0x1a97b888780>
```

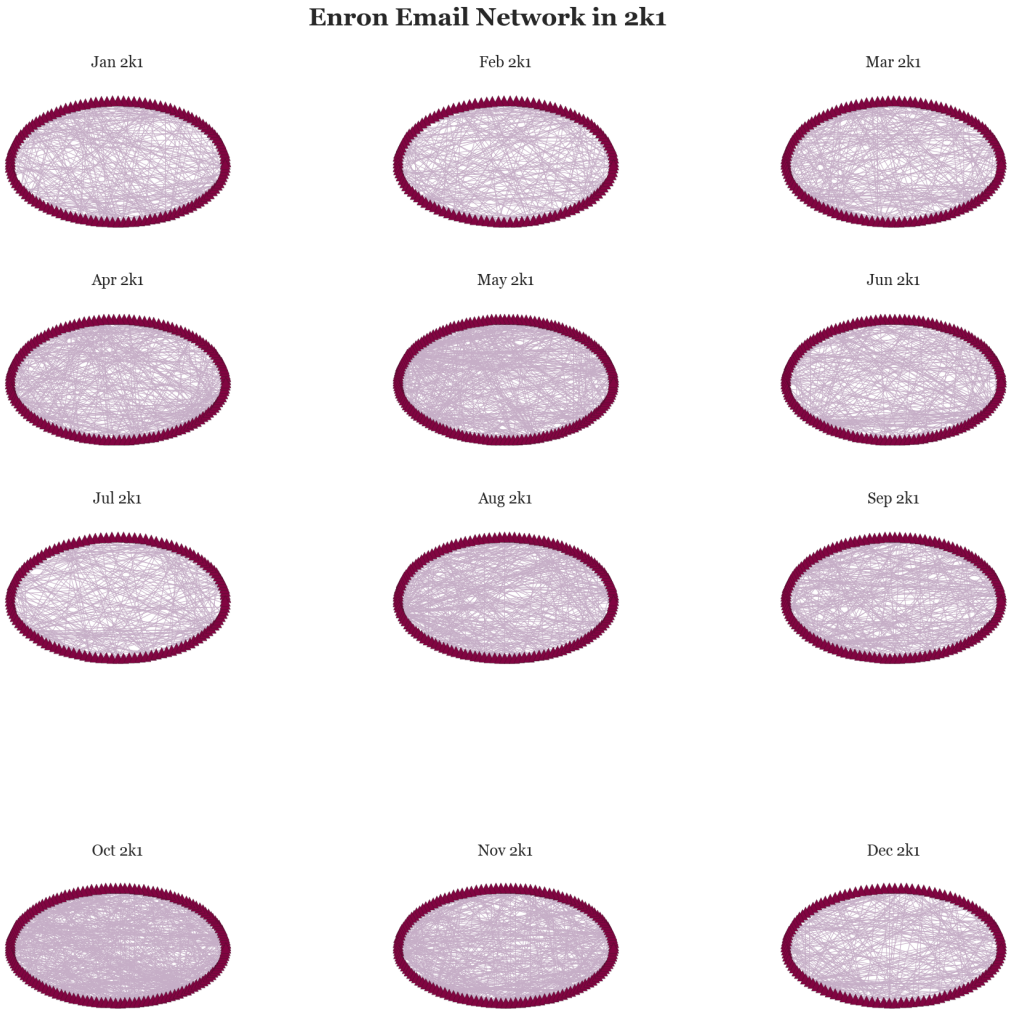


9 Network Year 2k1

```
In [564]: df_2k1.month.describe()

Out[564]: count    68888.000000
          mean         6.125073
          std          3.537309
          min          1.000000
          25%          3.000000
          50%          5.000000
          75%         10.000000
          max         12.000000
          Name: month, dtype: float64

In [635]: jan_2k1=df_2k1[df_2k1.month==1]
          feb_2k1=df_2k1[df_2k1.month==2]
          mar_2k1=df_2k1[df_2k1.month==3]
          apr_2k1=df_2k1[df_2k1.month==4]
          may_2k1=df_2k1[df_2k1.month==5]
          jun_2k1=df_2k1[df_2k1.month==6]
          jul_2k1=df_2k1[df_2k1.month==7]
          aug_2k1=df_2k1[df_2k1.month==8]
          sep_2k1=df_2k1[df_2k1.month==9]
          oct_2k1=df_2k1[df_2k1.month==10]
          nov_2k1=df_2k1[df_2k1.month==11]
          dec_2k1=df_2k1[df_2k1.month==12]


          G_jan_2k1=create_graph(jan_2k1)
          G_feb_2k1=create_graph(feb_2k1)
          G_mar_2k1=create_graph(mar_2k1)
```

```
G_apr_2k1=create_graph(apr_2k1)
G_may_2k1=create_graph(may_2k1)
G_jun_2k1=create_graph(jun_2k1)
G_jul_2k1=create_graph(jul_2k1)
G_aug_2k1=create_graph(aug_2k1)
G_sep_2k1=create_graph(sep_2k1)
G_oct_2k1=create_graph(oct_2k1)
G_nov_2k1=create_graph(nov_2k1)
G_dec_2k1=create_graph(dec_2k1)

plt.figure(figsize=(32,18))
plt.suptitle("Enron Email Network in 2k1", fontsize=40)

plt.subplot(331)
plt.title("Jan 2k1", fontsize=25)
nx.draw_circular(G_jan_2k1, node_color='#810541',node_shape='^',edge_colo

plt.subplot(332)
plt.title("Feb 2k1", fontsize=25)
nx.draw_circular(G_feb_2k1, node_color='#810541',node_shape='^',edge_colo

plt.subplot(333)
plt.title("Mar 2k1", fontsize=25)
nx.draw_circular(G_mar_2k1, node_color='#810541',node_shape='^',edge_colo

plt.subplot(334)
plt.title("Apr 2k1", fontsize=25)
nx.draw_circular(G_apr_2k1, node_color='#810541',node_shape='^',edge_colo

plt.subplot(335)
plt.title("May 2k1", fontsize=25)
nx.draw_circular(G_may_2k1, node_color='#810541',node_shape='^',edge_colo

plt.subplot(336)
plt.title("Jun 2k1", fontsize=25)
nx.draw_circular(G_jun_2k1, node_color='#810541',node_shape='^',edge_colo

plt.subplot(337)
plt.title("Jul 2k1", fontsize=25)
nx.draw_circular(G_jul_2k1, node_color='#810541',node_shape='^',edge_colo

plt.subplot(338)
plt.title("Aug 2k1", fontsize=25)
nx.draw_circular(G_aug_2k1, node_color='#810541',node_shape='^',edge_colo

plt.subplot(339)
plt.title("Sep 2k1", fontsize=25)
nx.draw_circular(G_sep_2k1, node_color='#810541',node_shape='^',edge_colo
```

```
plt.figure(figsize=(32,5))
#plt.suptitle("Enron Email Network in 2k1", fontsize=40)

plt.subplot(131)
plt.title("Oct 2k1", fontsize=25)
nx.draw_circular(G_oct_2k1, node_color='#810541',node_shape='^',edge_colo

plt.subplot(132)
plt.title("Nov 2k1", fontsize=25)
nx.draw_circular(G_nov_2k1, node_color='#810541',node_shape='^',edge_colo

plt.subplot(133)
plt.title("Dec 2k1", fontsize=25)
nx.draw_circular(G_dec_2k1, node_color='#810541',node_shape='^',edge_colo
```



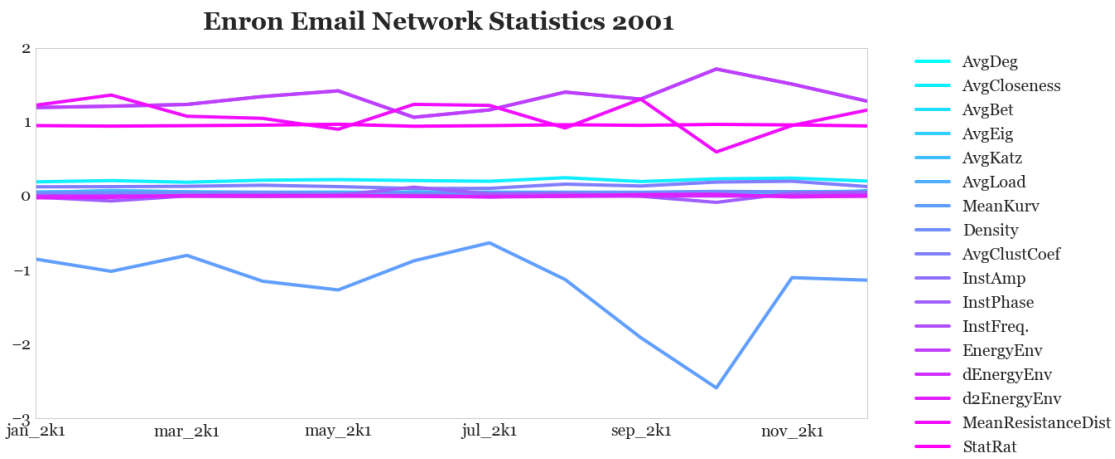
```
In [566]: stat_jan_2k1=cal_avgstat(G_jan_2k1)
          stat_feb_2k1=cal_avgstat(G_feb_2k1)
          stat_mar_2k1=cal_avgstat(G_mar_2k1)
          stat_apr_2k1=cal_avgstat(G_apr_2k1)
          stat_may_2k1=cal_avgstat(G_may_2k1)
          stat_jun_2k1=cal_avgstat(G_jun_2k1)
          stat_jul_2k1=cal_avgstat(G_jul_2k1)
          stat_aug_2k1=cal_avgstat(G_aug_2k1)
          stat_sep_2k1=cal_avgstat(G_sep_2k1)
          stat_oct_2k1=cal_avgstat(G_oct_2k1)
          stat_nov_2k1=cal_avgstat(G_nov_2k1)
          stat_dec_2k1=cal_avgstat(G_dec_2k1)

          stat_2k1 = stat_jan_2k1.append(stat_feb_2k1).append(stat_mar_2k1).append
          stat_2k1 = stat_2k1.append(stat_jun_2k1).append(stat_jul_2k1).append(stat
          stat_2k1 = stat_2k1.append(stat_oct_2k1).append(stat_nov_2k1).append(stat
          stat_2k1.columns = ['jan_2k1','feb_2k1','mar_2k1','apr_2k1','may_2k1','ju

In [567]: stat_2k1.head()
```

Out[567]:

	jan_2k1	feb_2k1	mar_2k1	apr_2k1	may_2k1	jun_2k1
AvgDeg	0.013179	0.012052	0.012982	0.015607	0.012401	0.008815
AvgCloseness	0.191559	0.208446	0.186307	0.214056	0.221824	0.209019
AvgBet	0.017710	0.021867	0.017525	0.017928	0.011548	0.019936
AvgEig	0.054981	0.064686	0.058794	0.051914	0.053268	0.048962
AvgKatz	0.048244	0.074620	0.057339	0.002995	0.022108	0.059015

	jul_2k1	aug_2k1	sep_2k1	oct_2k1	nov_2k1	dec_2k1
AvgDeg	0.016036	0.016639	0.012930	0.019379	0.016612	0.011789
AvgCloseness	0.199306	0.247621	0.195912	0.232163	0.241737	0.202481
AvgBet	0.020539	0.015186	0.021407	0.012743	0.017048	0.022688
AvgEig	0.053660	0.051803	0.051611	0.058752	0.056969	0.065293
AvgKatz	0.046579	0.000057	0.021531	0.003949	0.002392	0.074214

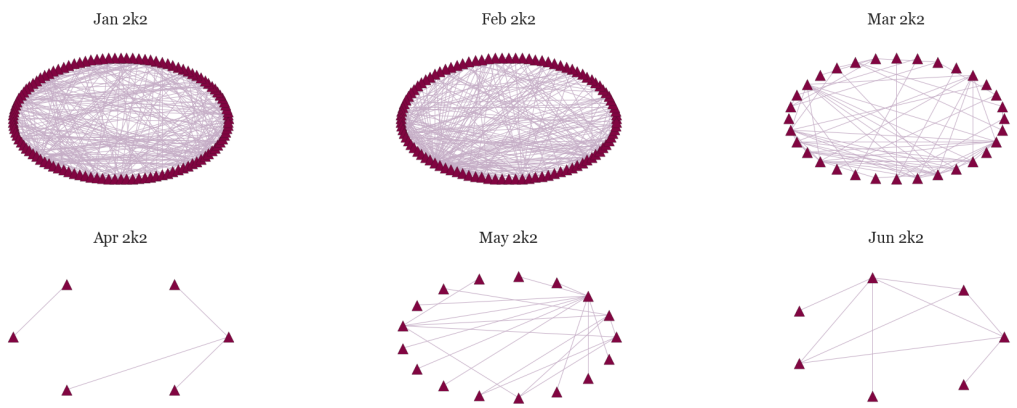
```
In [568]: stat_2k1.T.plot(fontsize=20, figsize=(18,8), cmap='cool')
          plt.suptitle("Enron Email Network Statistics 2001", fontsize=30)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

Out[568]: <matplotlib.legend.Legend at 0x1a94921ccf8>
```



10 Network Year 2k2

```
In [569]: df_2k2.month.describe()

Out[569]: count    8491.000000
          mean        1.758921
          std         0.807467
          min         1.000000
          25%         1.000000
          50%         2.000000
          75%         2.000000
          max         6.000000
          Name: month, dtype: float64

In [636]: jan_2k2=df_2k2[df_2k2.month==1]
          feb_2k2=df_2k2[df_2k2.month==2]
          mar_2k2=df_2k2[df_2k2.month==3]
          apr_2k2=df_2k2[df_2k2.month==4]
          may_2k2=df_2k2[df_2k2.month==5]
          jun_2k2=df_2k2[df_2k2.month==6]
          jul_2k2=df_2k2[df_2k2.month==7]
          aug_2k2=df_2k2[df_2k2.month==8]
          sep_2k2=df_2k2[df_2k2.month==9]
          oct_2k2=df_2k2[df_2k2.month==10]
          nov_2k2=df_2k2[df_2k2.month==11]
          dec_2k2=df_2k2[df_2k2.month==12]


          G_jan_2k2=create_graph(jan_2k2)
          G_feb_2k2=create_graph(feb_2k2)
          G_mar_2k2=create_graph(mar_2k2)
```

```
G_apr_2k2=create_graph(apr_2k2)
G_may_2k2=create_graph(may_2k2)
G_jun_2k2=create_graph(jun_2k2)
G_jul_2k2=create_graph(jul_2k2)
G_aug_2k2=create_graph(aug_2k2)
G_sep_2k2=create_graph(sep_2k2)
G_oct_2k2=create_graph(oct_2k2)
G_nov_2k2=create_graph(nov_2k2)
G_dec_2k2=create_graph(dec_2k2)

plt.figure(figsize=(32,18))
plt.suptitle("Enron Email Network in 2k2", fontsize=40)

plt.subplot(331)
plt.title("Jan 2k2", fontsize=25)
nx.draw_circular(G_jan_2k2, node_color='#810541',node_shape='^',edge_colo

plt.subplot(332)
plt.title("Feb 2k2", fontsize=25)
nx.draw_circular(G_feb_2k2, node_color='#810541',node_shape='^',edge_colo

plt.subplot(333)
plt.title("Mar 2k2", fontsize=25)
nx.draw_circular(G_mar_2k2, node_color='#810541',node_shape='^',edge_colo

plt.subplot(334)
plt.title("Apr 2k2", fontsize=25)
nx.draw_circular(G_apr_2k2, node_color='#810541',node_shape='^',edge_colo

plt.subplot(335)
plt.title("May 2k2", fontsize=25)
nx.draw_circular(G_may_2k2, node_color='#810541',node_shape='^',edge_colo

plt.subplot(336)
plt.title("Jun 2k2", fontsize=25)
nx.draw_circular(G_jun_2k2, node_color='#810541',node_shape='^',edge_colo
```

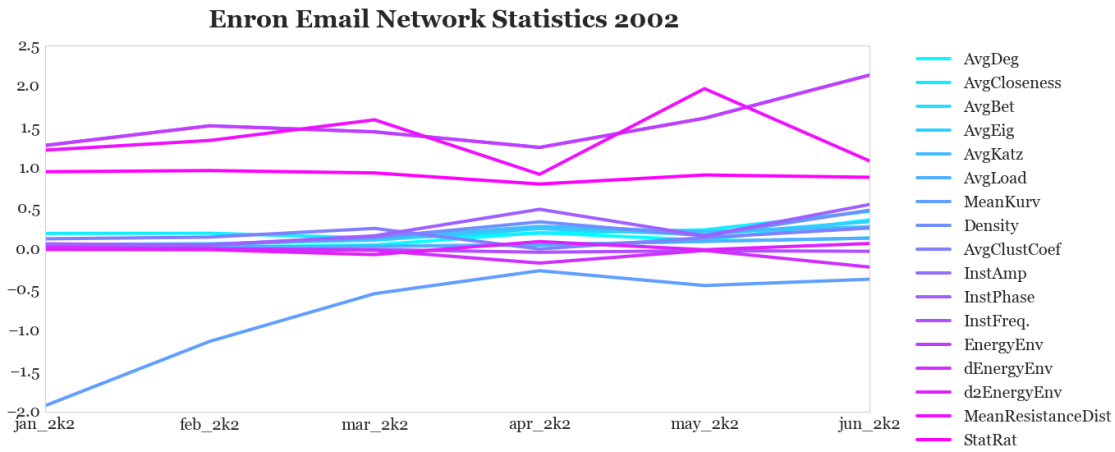



```
In [571]: stat_jan_2k2=cal_avgstat(G_jan_2k2)
          stat_feb_2k2=cal_avgstat(G_feb_2k2)
          stat_mar_2k2=cal_avgstat(G_mar_2k2)
          stat_apr_2k2=cal_avgstat(G_apr_2k2)
          stat_may_2k2=cal_avgstat(G_may_2k2)
          stat_jun_2k2=cal_avgstat(G_jun_2k2)

          stat_2k2 = stat_jan_2k2.append(stat_feb_2k2).append(stat_mar_2k2).append
          stat_2k2.columns = ['jan_2k2','feb_2k2','mar_2k2','apr_2k2','may_2k2','ju

In [572]: stat_2k2.head()
```

Out[572]:

	jan_2k2	feb_2k2	mar_2k2	apr_2k2	may_2k2	jun_2k2
AvgDeg	0.018018	0.021818	0.049395	0.200000	0.106618	0.357143
AvgCloseness	0.191514	0.193146	0.133727	0.193333	0.234243	0.462193
AvgBet	0.022664	0.022488	0.033737	0.050000	0.094118	0.133333
AvgEig	0.055595	0.064133	0.108396	0.248878	0.176454	0.336275
AvgKatz	0.042359	0.046586	0.131887	0.277666	0.212198	0.273203

```
In [573]: stat_2k2.T.plot(fontsize=20, figsize=(18,8), cmap='cool')
          plt.suptitle("Enron Email Network Statistics 2002", fontsize=30)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

Out[573]: <matplotlib.legend.Legend at 0x1a959ac7b00>
```

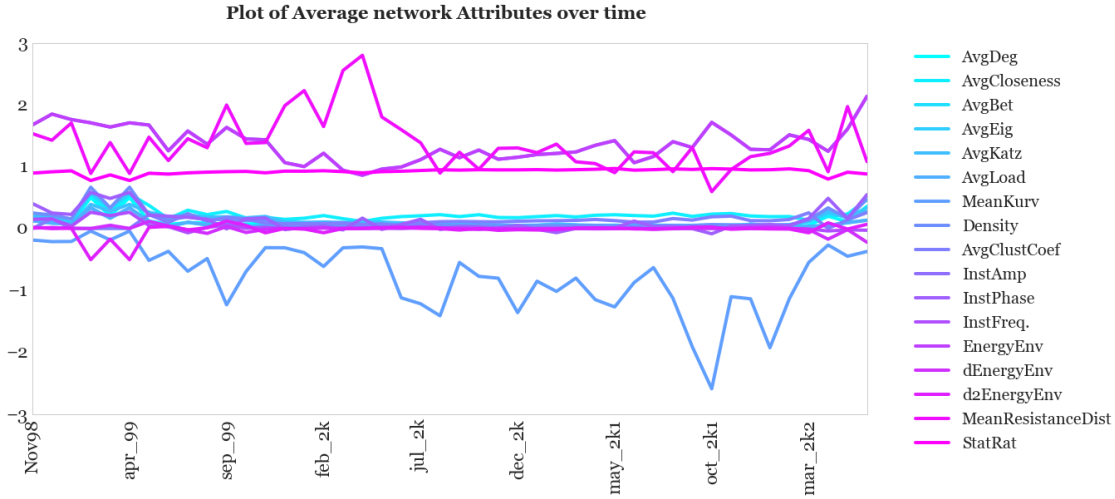


Network Attributes over full time range

```
In [574]: stat_all = stat98.join(stat_99).join(stat_2k).join(stat_2k1).join(stat_2k
```

```
In [869]: stat_all.plot(figsize=(18,8), fontsize=22, rot=90,cmap='cool')
          plt.suptitle("Plot of Average network Attributes over time", fontsize=22)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
```

```
Out[869]: <matplotlib.legend.Legend at 0x1a90804eb70>
```



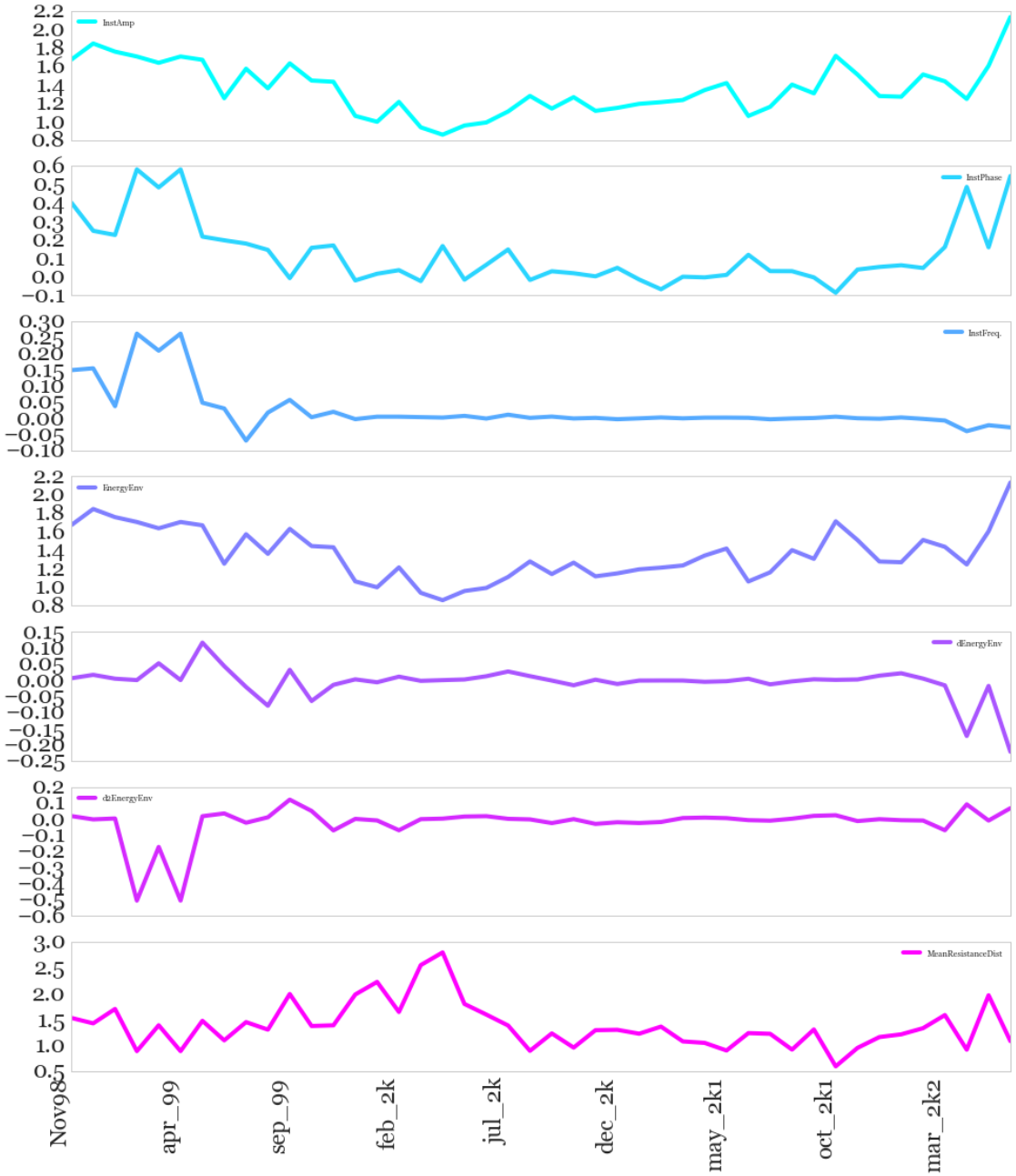

```
In [852]: stat_all.columns[6:]
```

```
Out[852]: Index(['MeanKurv', 'Density', 'AvgClustCoef', 'InstAmp', 'InstPhase',
                 'InstFreq.', 'EnergyEnv', 'dEnergyEnv', 'd2EnergyEnv',
                 'MeanResistanceDist', 'StatRat'],
                dtype='object')
```

```
In [867]: stat_all.iloc[:,9:16].plot(figsize=(16,20), fontsize=22, rot=90,cmap='coo
          plt.suptitle("Plot of Selected Attributes over time", fontsize=22)
          #plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0
```

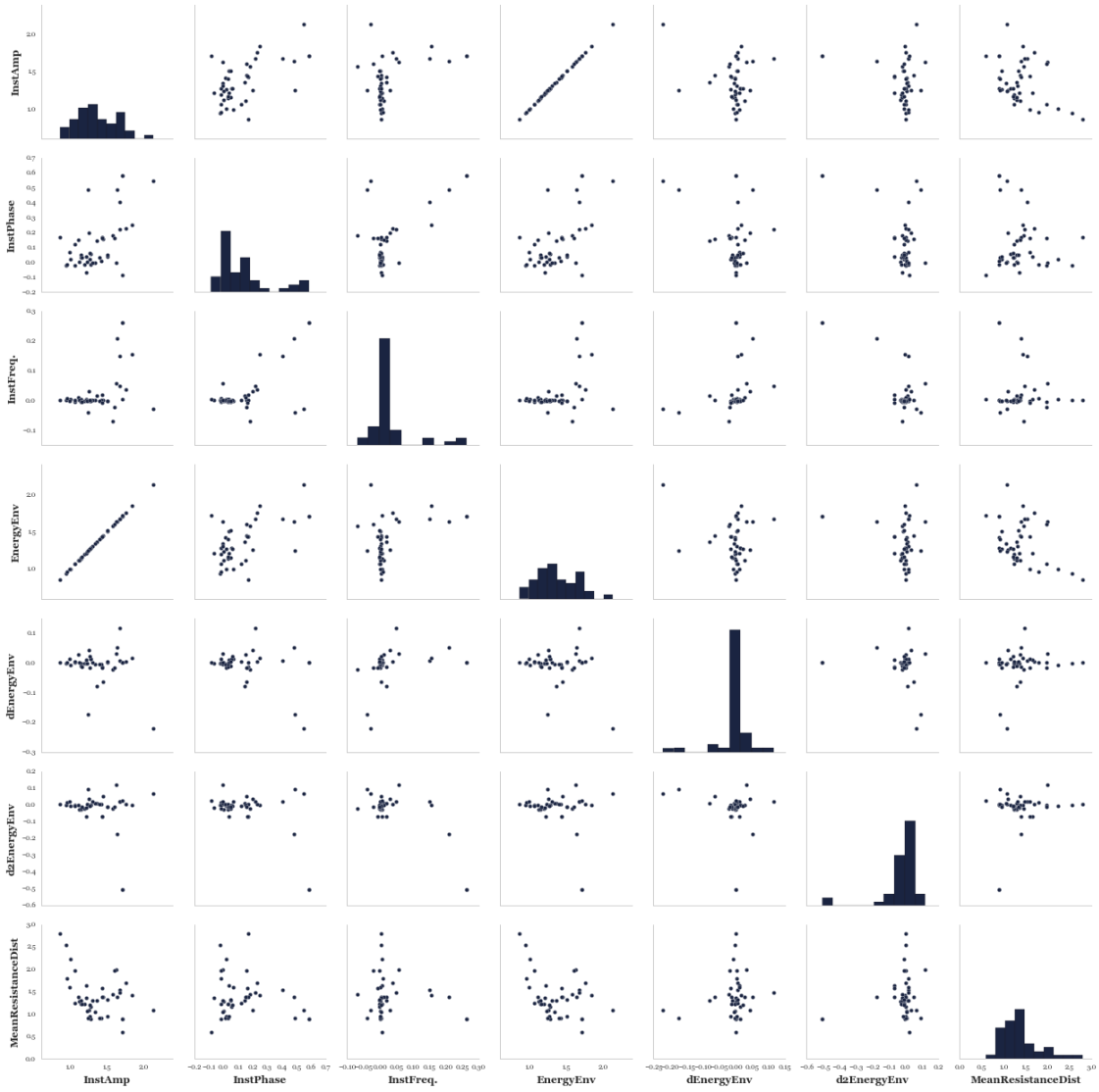
```
Out[867]: <matplotlib.text.Text at 0x1a90a5ed7b8>
```

Plot of Selected Attributes over time



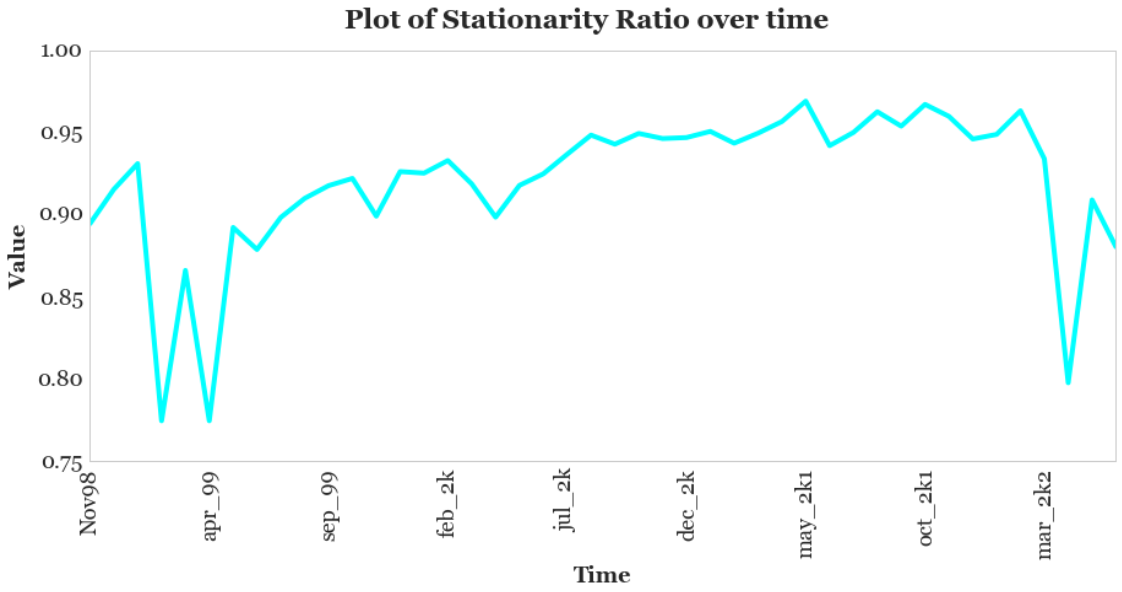
```
In [865]: sns.pairplot(stat_all.iloc[:,9:16])
```

```
Out[865]: <seaborn.axisgrid.PairGrid at 0x1a97cc97860>
```



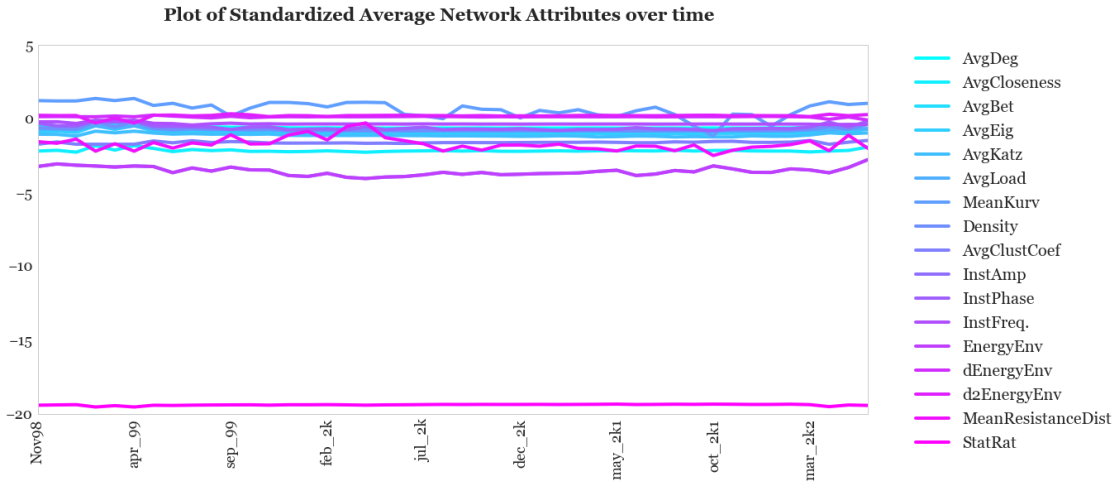

```
In [637]: stat_all.StatRat.plot(fontsize=18, rot=90, cmap='cool')
          plt.suptitle('Plot of Stationarity Ratio over time', fontsize=22)
          plt.xlabel("Time", fontsize=18)
          plt.ylabel("Value", fontsize=18)
```

```
Out[637]: <matplotlib.text.Text at 0x1a94b9db7b8>
```




```
In [577]: stat_all_std = stat_all - stat_all.mean()/ stat_all.std()

In [578]: stat_all_std.head()

Out[578]:          AvgDeg  AvgCloseness    AvgBet    AvgEig   AvgKatz   AvgLoad  \
          Nov98  -0.501559     -2.190459 -0.683674 -0.861368 -1.049629 -0.683703
          Dec98  -0.498961     -2.140030 -0.721805 -0.833599 -1.059953 -0.721834
          jan_99 -0.561083     -2.265710 -0.731751 -0.932779 -1.177372 -0.731780
          feb_99 -0.144416     -1.827211 -0.475340 -0.480943 -0.844841 -0.475370
          mar_99 -0.444416     -2.123508 -0.642007 -0.721309 -0.957380 -0.642036

                  MeanKurv   Density  AvgClustCoef   InstAmp  InstPhase  InstFreq.
          Nov98   1.223986 -0.614423     -1.728109 -3.234486  -0.322693  -0.213067
          Dec98   1.198805 -0.646242     -1.572554 -3.059949  -0.475194  -0.207377
          jan_99  1.201116 -0.710577     -1.728109 -3.148094  -0.497733  -0.324490
          feb_99  1.369452 -0.197757     -1.728109 -3.200872  -0.142786  -0.100063
          mar_99  1.225887 -0.531090     -1.728109 -3.269003  -0.240465  -0.152871

                  EnergyEnv  dEnergyEnv  d2EnergyEnv  MeanResistanceDist     StatRat
          Nov98   -3.234486    0.136924     0.231110           -1.554605 -19.387492
          Dec98   -3.059949    0.147507     0.210526           -1.660235 -19.366584
          jan_99  -3.148094    0.135652     0.216238           -1.381713 -19.350969
          feb_99  -3.200872    0.131303    -0.291969           -2.196966 -19.507322
          mar_99  -3.269003    0.183316     0.040216           -1.696966 -19.415893

In [579]: stat_all_std.plot(figsize=(18,8), cmap='cool',fontsize=18, rot=90)
          plt.suptitle("Plot of Standardized Average Network Attributes over time",
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
```

Out[579]: <matplotlib.legend.Legend at 0x1a959a3da20>

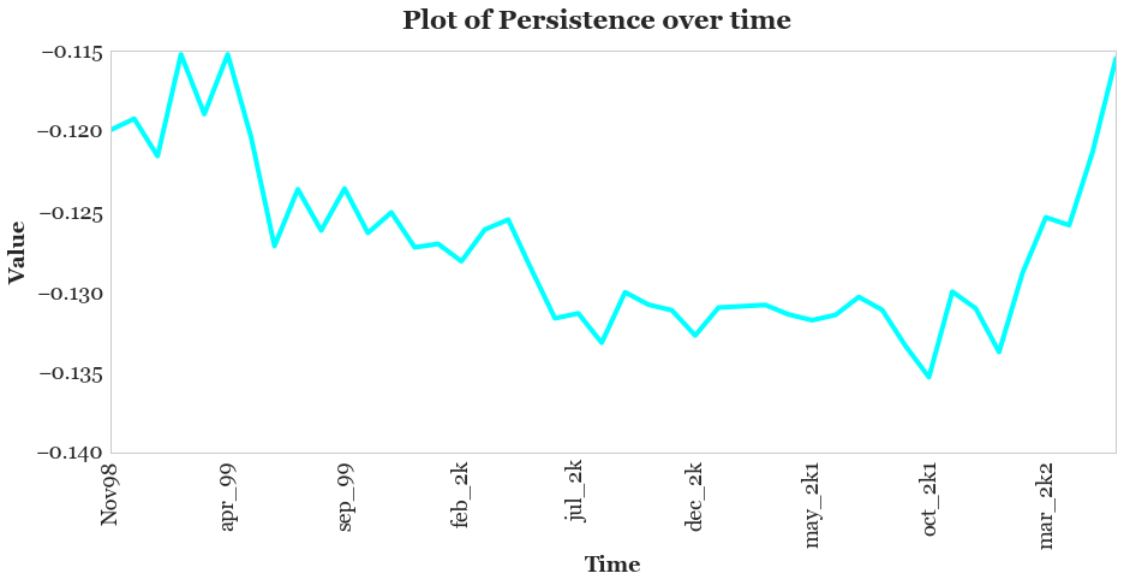


11 Persistence and Emergence

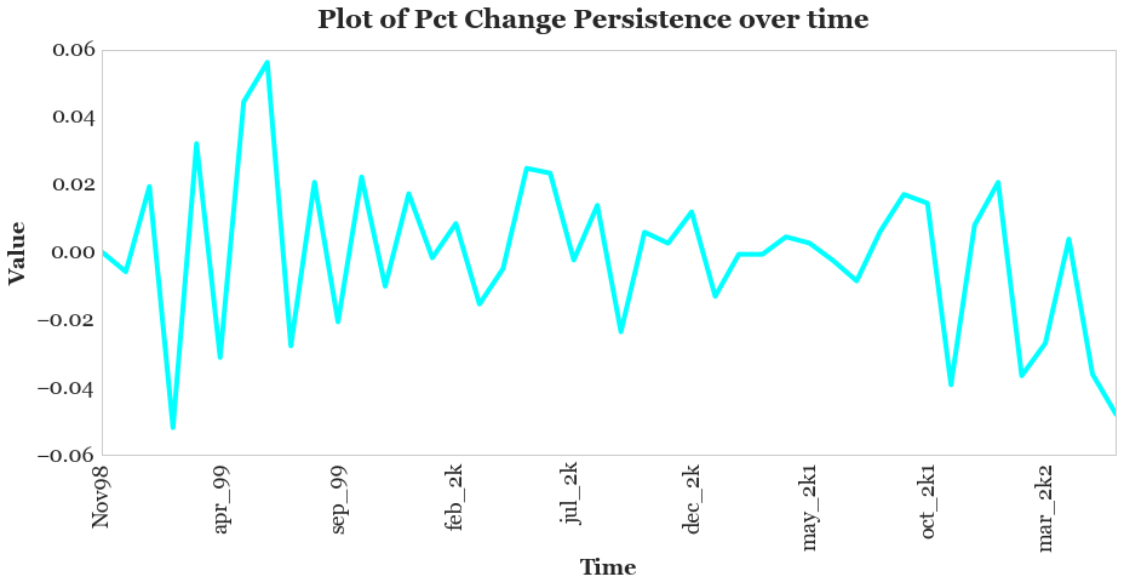
In [580]:
```
persistence = stat_all_std.T.mean()/stat_all_std.T.shape[0]
```

In [581]:
```
persistence.plot(fontsize=18, rot=90, cmap='cool')
plt.suptitle("Plot of Persistence over time", fontsize=22)
plt.xlabel("Time", fontsize=18)
plt.ylabel("Value", fontsize=18)
```

Out[581]: <matplotlib.text.Text at 0x1a959adafd0>

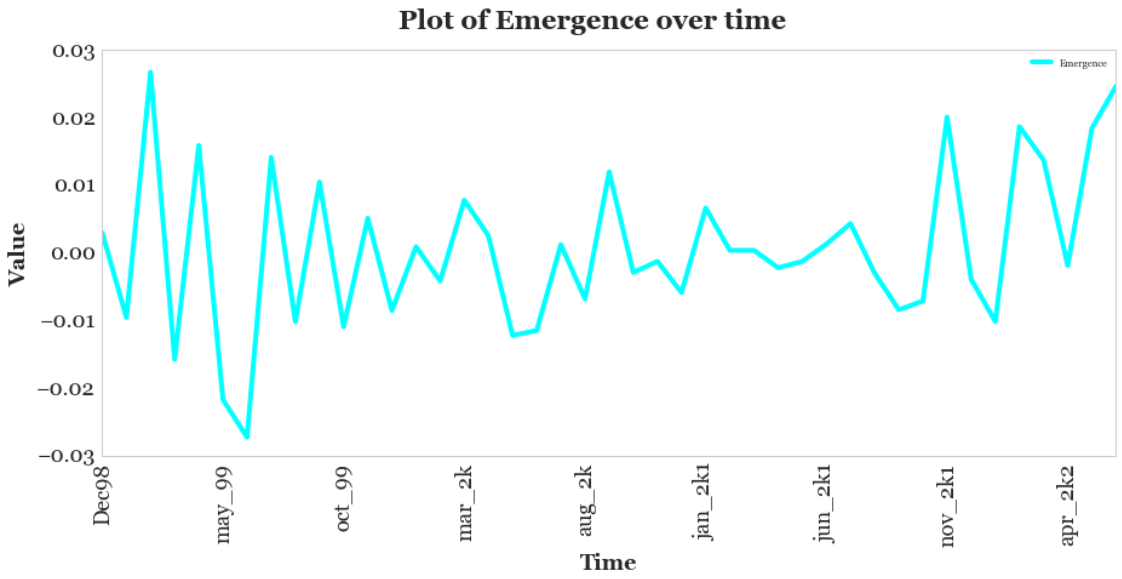


```
In [582]: persistence.pct_change().fillna(0).plot(fontsize=18, rot=90, cmap='cool')
          plt.suptitle("Plot of Pct Change Persistence over time", fontsize=22)
          plt.xlabel("Time", fontsize=18)
          plt.ylabel("Value", fontsize=18)

Out[582]: <matplotlib.text.Text at 0x1a95632d1d0>
```



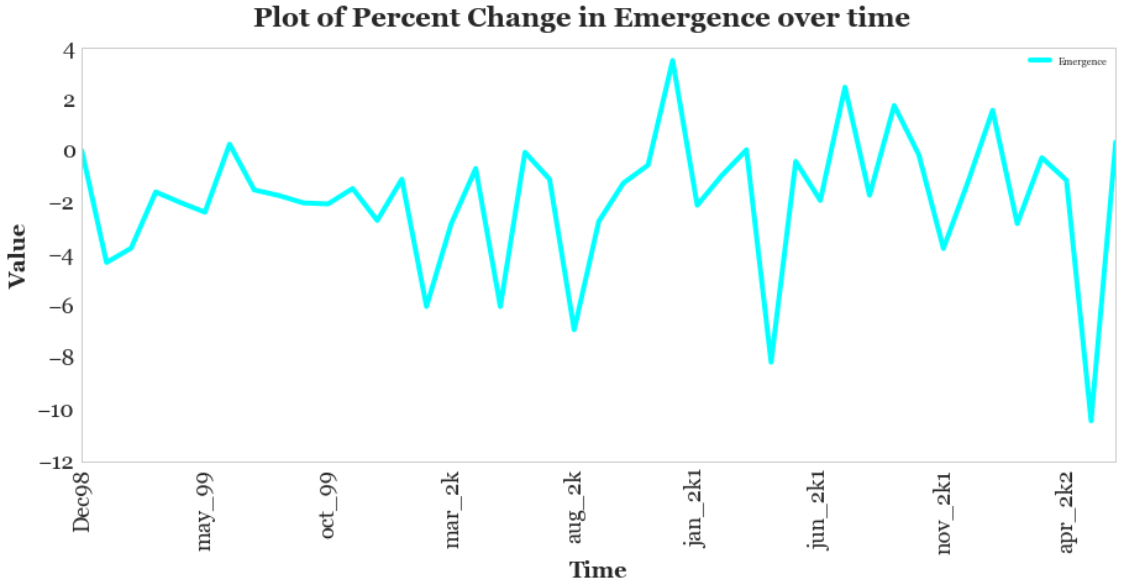

```
In [583]: def emergence(a,b):
              e = (a-b)/ (np.linalg.norm(a)+np.linalg.norm(b))
              return e

In [584]: emerg = []
          for i in range(0,persistence.shape[0]-1):
              x = int(i)
              y = x +1
              emerg.append(emergence(persistence.values[y],persistence.values[x]))

In [585]: emerg_df = pd.DataFrame(emerg).T
          emerg_df.columns = list(stat_all.T.columns)[1:]
          emerg_df = emerg_df.T
          emerg_df.columns = ['Emergence']

In [586]: emerg_df.plot(fontsize=18, rot=90, cmap='cool')
          plt.suptitle("Plot of Emergence over time", fontsize=22)
          plt.xlabel("Time", fontsize=18)
          plt.ylabel("Value", fontsize=18)
```

Out[586]: <matplotlib.text.Text at 0x1a959347358>

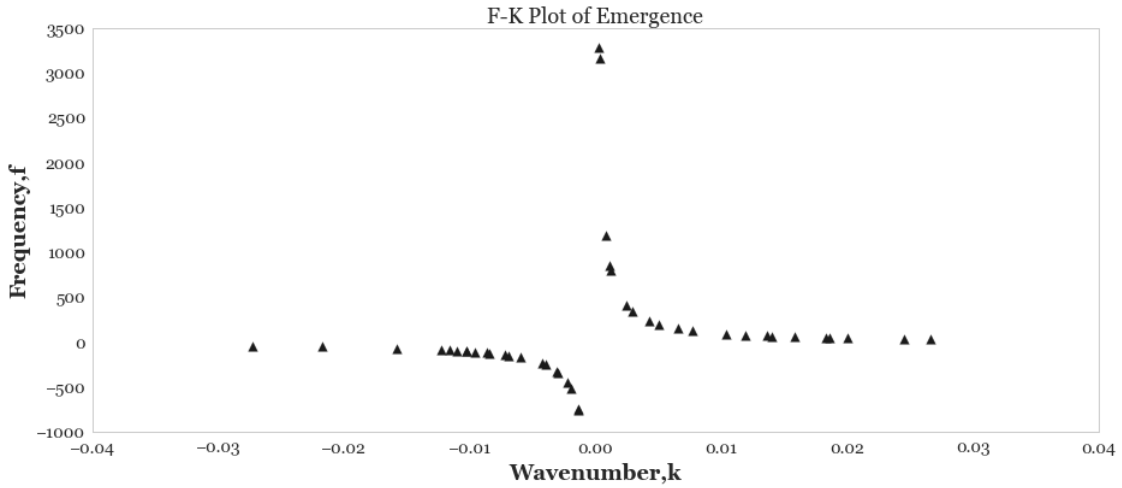


In [587]:
```
emerg_df.pct_change().fillna(0).plot(fontsize=18, rot=90, cmap='cool')
plt.suptitle("Plot of Percent Change in Emergence over time", fontsize=22
plt.xlabel("Time", fontsize=18)
plt.ylabel("Value", fontsize=18)
```

Out[587]: <matplotlib.text.Text at 0x1a9562804e0>



```
In [913]: def fk_plot(f,name):
              freq = sc.fft(f)
              wavnum = 1/freq

              plt.scatter(freq, wavnum, s=60, marker='^', c='k')
              plt.title("F-K Plot of "+str(name), fontsize=18)
              plt.xticks(fontsize=14)
              plt.yticks(fontsize=14)
              plt.xlabel("Wavenumber,k", fontsize=18)
              plt.ylabel("Frequency,f", fontsize=18)

              plt.show()

              return [freq,wavnum]

In [914]: freq,k = fk_plot(emerg_df,'Emergence')

C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:533: ComplexWa
  return array(a, dtype, copy=False, order=order, subok=True)
```

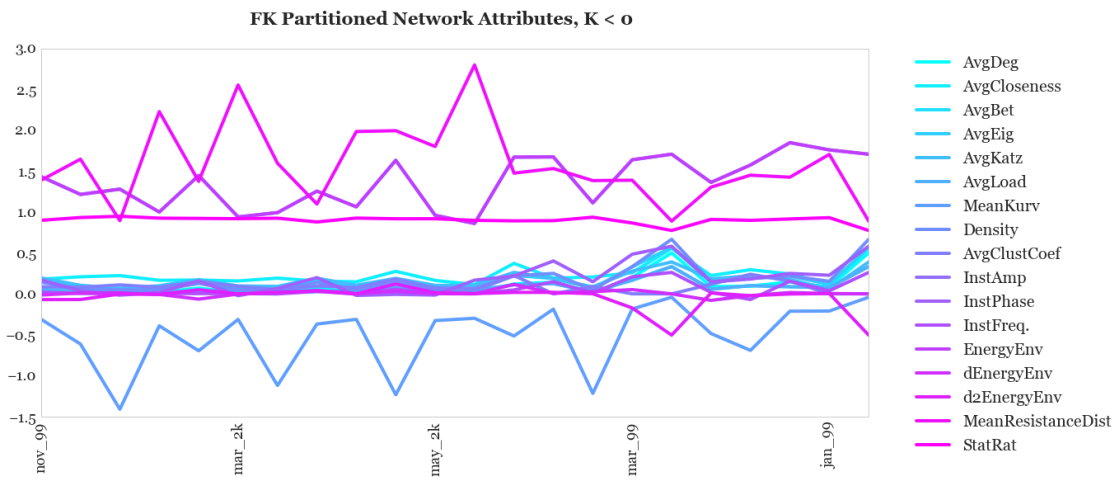


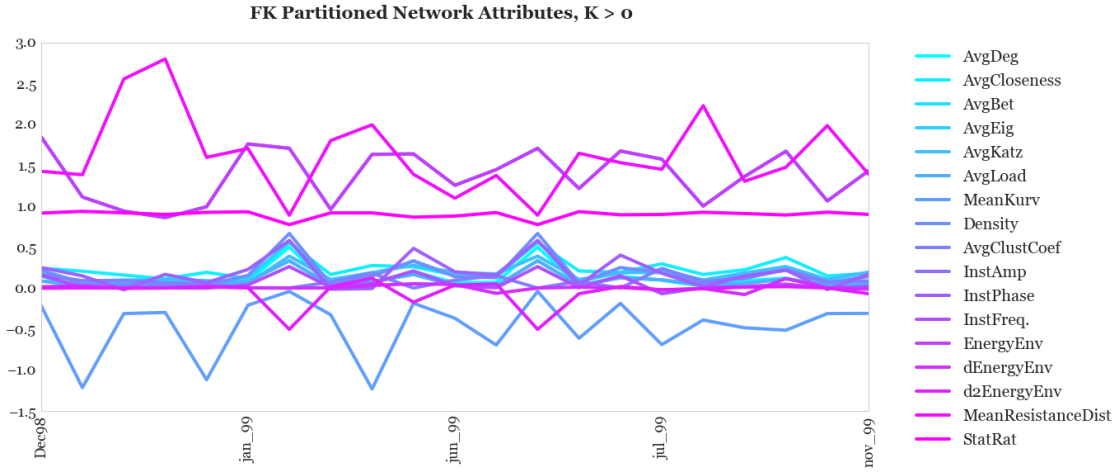

```
In [590]: print("Maximum Frequency :\n", emerg_df.iloc[np.argmax(freq)])
          print("-----------------\n")
          print("Minimum Frequency :\n", emerg_df.iloc[np.argmin(freq)])

Maximum Frequency :
 Emergence    0.026634
Name: feb_99, dtype: float64
-----------------

Minimum Frequency :
```

```
 Emergence   -0.027314
Name: jun_99, dtype: float64
```

11.1 Partition Network Attributes based on F > 0 and F < 0

```
In [956]: idx_f_less0 = np.argsort(k[k < 0])
          idx_f_more0 = np.argsort(k[k > 0])

In [957]: emerg_time = emerg_df.index

In [960]: print("Times with Emergence K < 0 \n\n", emerg_time[idx_f_less0])
          print("\n")
          print("Times with Emergence K > 0 \n\n", emerg_time[idx_f_more0])

Times with Emergence K < 0

 Index(['dec_99', 'mar_2k', 'sep_2k', 'feb_2k', 'nov_99', 'apr_2k', 'jul_2k',
        'jul_99', 'jan_2k', 'oct_99', 'jun_2k', 'may_2k', 'jun_99', 'Dec98',
        'aug_2k', 'apr_99', 'may_99', 'sep_99', 'aug_99', 'jan_99', 'feb_99',
        'mar_99'],
       dtype='object')


Times with Emergence K > 0

 Index(['jan_99', 'aug_2k', 'apr_2k', 'may_2k', 'jul_2k', 'feb_99', 'mar_99',
        'jun_2k', 'oct_99', 'apr_99', 'jul_99', 'nov_99', 'may_99', 'mar_2k',
        'Dec98', 'aug_99', 'feb_2k', 'sep_99', 'jun_99', 'jan_2k', 'dec_99'],
       dtype='object')


In [965]: stat_all.ix[idx_f_less0].plot(figsize=(18,8), cmap='cool',fontsize=18, rc
          plt.suptitle("FK Partitioned Network Attributes, K < 0", fontsize=22)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

          stat_all.ix[idx_f_more0].plot(figsize=(18,8), cmap='cool',fontsize=18, rc
          plt.suptitle("FK Partitioned Network Attributes, K > 0", fontsize=22)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

Out[965]: <matplotlib.legend.Legend at 0x1a915d95da0>
```





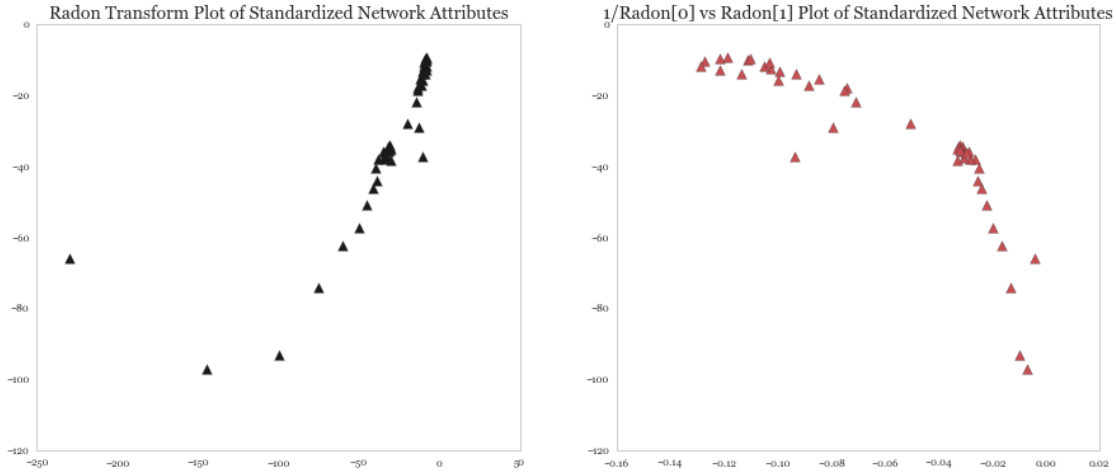
In [ ]:

In [593]:
```
def plot_radon(m, name):
    from skimage.transform import radon
    theta = np.linspace(0., 180., max(m.shape), endpoint=False)
    sinogram = radon(m, theta=theta, circle=True)

    fig, ax = plt.subplots(1,2)

    ax[0].scatter(sinogram[0], sinogram[1], s=60, marker='^', c='k')
    ax[0].set_title("Radon Transform Plot of "+str(name), fontsize=14)


    ax[1].scatter(1/sinogram[0], sinogram[1], s=60, marker='^', c='r')
```

```
            ax[1].set_title("1/Radon[0] vs Radon[1] Plot of "+str(name), fontsize

            plt.show()

            return sinogram
```

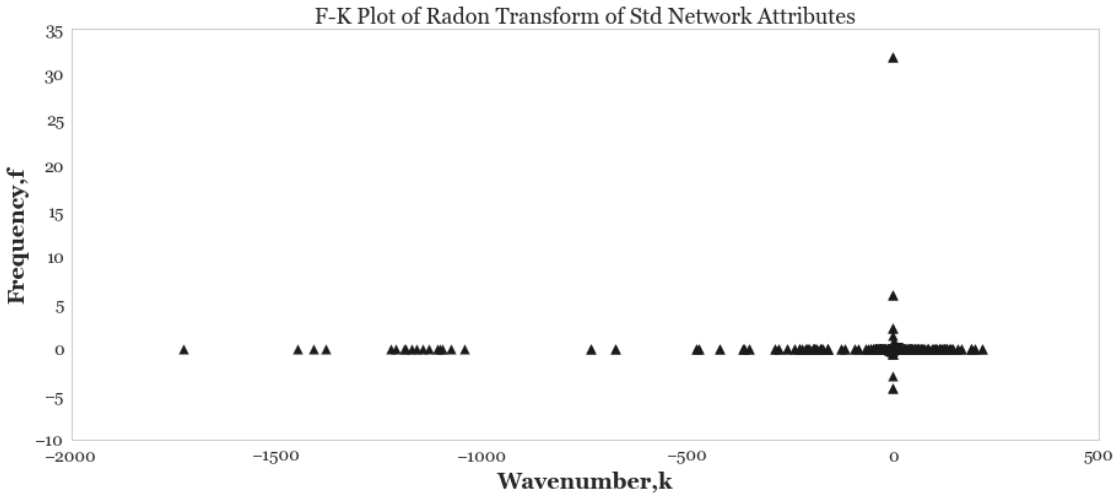
```
In [594]: radon_att_std = plot_radon(stat_all_std.values,'Standardized Network Attr
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\skimage\transform\radon_transform.py
  warn('Radon transform: image must be zero outside the '
```

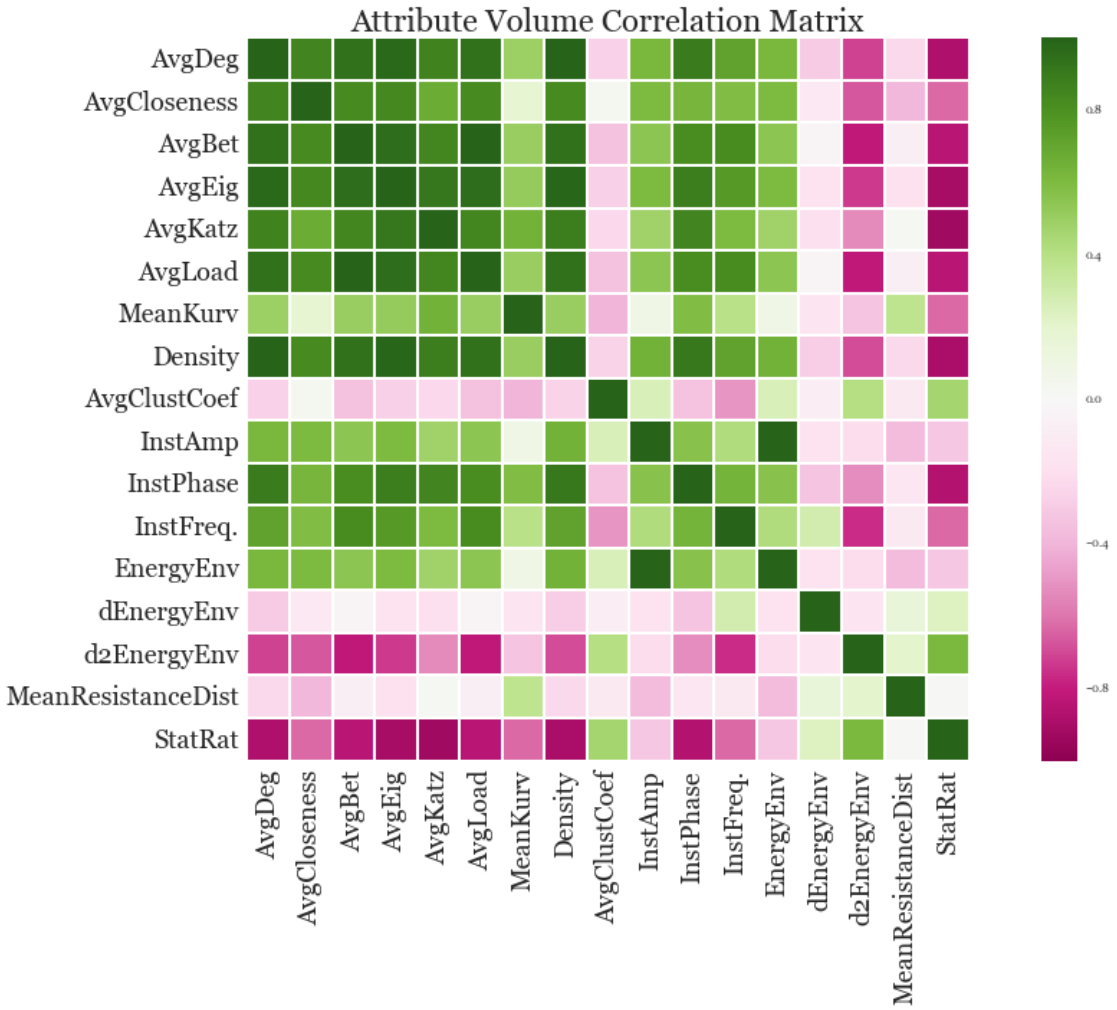



```
In [595]: f_rad,k_rad = fk_plot(radon_att_std,'Radon Transform of Std Network Attri
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:533: ComplexWa
  return array(a, dtype, copy=False, order=order, subok=True)
```



```
In [596]: print('Radon max',np.argmax(radon_att_std[0]),np.argmax(radon_att_std[1]))
          print('Radon min',np.argmin(radon_att_std[0]),np.argmin(radon_att_std[1]))

Radon max 15 12
Radon min 43 42


In [597]: print("Maximum Radon Transform [0]:\n", emerg_df.iloc[15])
          print("Maximum Radon Transform [1]:\n",emerg_df.iloc[12])
          print("-----------------\n")
          print("Minimum Radon Transform :\n", emerg_df.iloc[42])

Maximum Radon Transform [0]:
 Emergence    0.007737
Name: mar_2k, dtype: float64
Maximum Radon Transform [1]:
 Emergence   -0.008585
Name: dec_99, dtype: float64
-----------------

Minimum Radon Transform :
 Emergence    0.02456
Name: jun_2k2, dtype: float64


In [598]: plt.figure(figsize=(18,9))
          sns.heatmap(stat_all_std.corr(), cmap='PiYG',robust=True, fmt='d', linewi
          plt.title("Attribute Volume Correlation Matrix", fontsize=22)
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)

Out[598]: (array([  0.5,   1.5,   2.5,   3.5,   4.5,   5.5,   6.5,   7.5,   8.5,
                    9.5,  10.5,  11.5,  12.5,  13.5,  14.5,  15.5,  16.5]),
           <a list of 17 Text yticklabel objects>)
```



12 Subgraph Stationarity

```
In [599]: def pad_shape(x,ref,offset=0):
              result = np.zeros_like(ref)
              result[0:x.shape[0]+0,0:x.shape[1]+0] = x

              return result
```

```
In [600]: all_graphs = tuple([G_nov98,G_dec98,G_jan_99,G_feb_99,G_mar_99,G_apr_99,G
                          G_nov_99,G_dec_99,G_jan_2k,G_feb_2k,G_mar_2k,G_apr_2k,G_may
                          G_oct_2k,G_nov_2k,G_dec_2k,G_jan_2k1,G_feb_2k1,G_mar_2k1,G_
                          G_aug_2k1,G_sep_2k1,G_oct_2k1,G_nov_2k1,G_dec_2k1,G_jan_2k2
```

```
In [601]: all_graphs[:5]
```

```
Out[601]: (<networkx.classes.graph.Graph at 0x1a93f2a2dd8>,
           <networkx.classes.graph.Graph at 0x1a93f2a2668>,
```

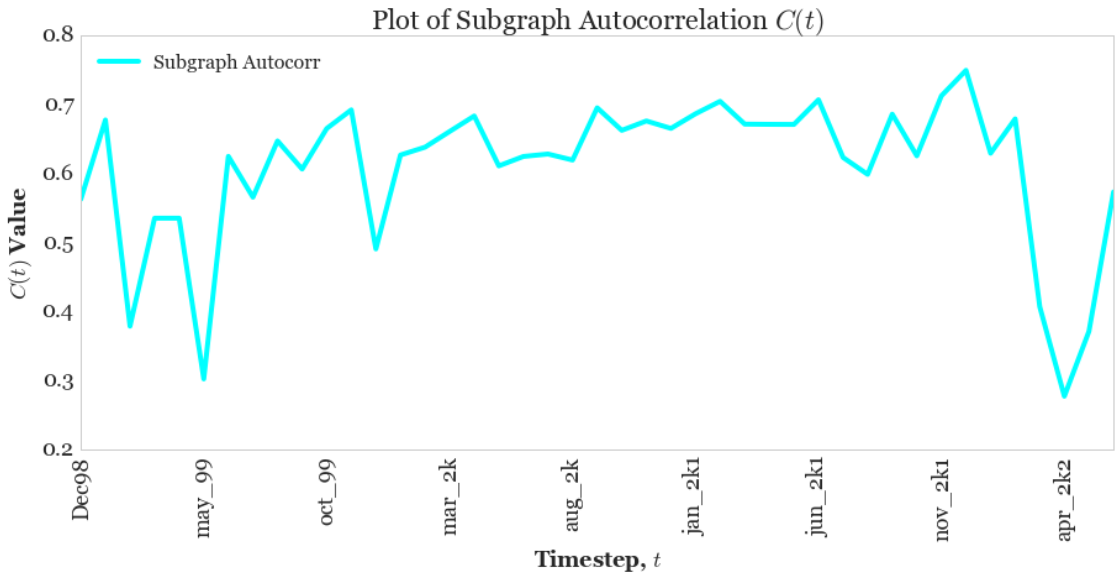
```
 <networkx.classes.graph.Graph at 0x1a9413f6668>,
 <networkx.classes.graph.Graph at 0x1a94c59d7f0>,
 <networkx.classes.graph.Graph at 0x1a94c59d780>)
```

In [602]:
```
def subgraph_stat(net1,net2):
    net1_int_net2 = net1.copy()
    net1_int_net2.remove_nodes_from(n for n in net1 if n not in net2)
    net1_u_net2 = nx.disjoint_union(net1, net2)
    int_adjmat = nx.adjacency_matrix(net1_int_net2).todense()
    uni_adjmat = nx.adjacency_matrix(net1_u_net2).todense()
    int_adjmat_pad = pad_shape(int_adjmat,uni_adjmat)

    Ct = np.divide(norm(int_adjmat_pad),norm(uni_adjmat))

    return Ct
```

In [603]:
```
Ct = []
for i in range(0,len(all_graphs)-1):
    x = int(i)
    y = x +1
    Ct.append(subgraph_stat(all_graphs[x],all_graphs[y]))
```

In [604]:
```
Ct_df = pd.DataFrame(Ct, columns=['Subgraph Autocorr']).T
Ct_df.columns = list(stat_all.T.columns)[1:]
Ct_df = Ct_df.T

Ct_df.plot(fontsize=18, cmap='cool', rot = 90)
plt.title("Plot of Subgraph Autocorrelation $C(t)$", fontsize=22)
plt.xlabel("Timestep, $t$", fontsize=18)
plt.ylabel("$C(t)$ Value", fontsize=18)
plt.legend(fontsize=16, loc=2)
plt.xticks(fontsize=18)
plt.yticks(fontsize=18)
```

Out[604]:
```
(array([ 0.2,  0.3,  0.4,  0.5,  0.6,  0.7,  0.8]),
 <a list of 7 Text yticklabel objects>)
```

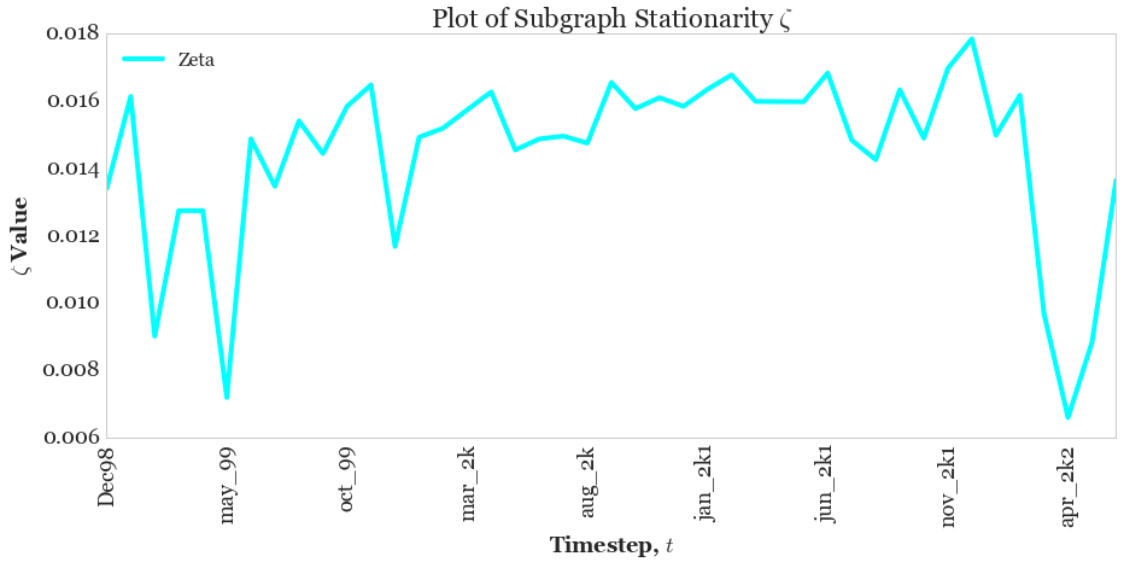



```
In [605]: zeta = Ct_df.cumsum(axis=1)/(Ct_df.shape[0]-1)
          zeta =  zeta.T
          zeta.columns = list(stat_all.T.columns)[1:]
          zeta = zeta.T
          zeta.columns = ['Zeta']
          zeta.head()
```

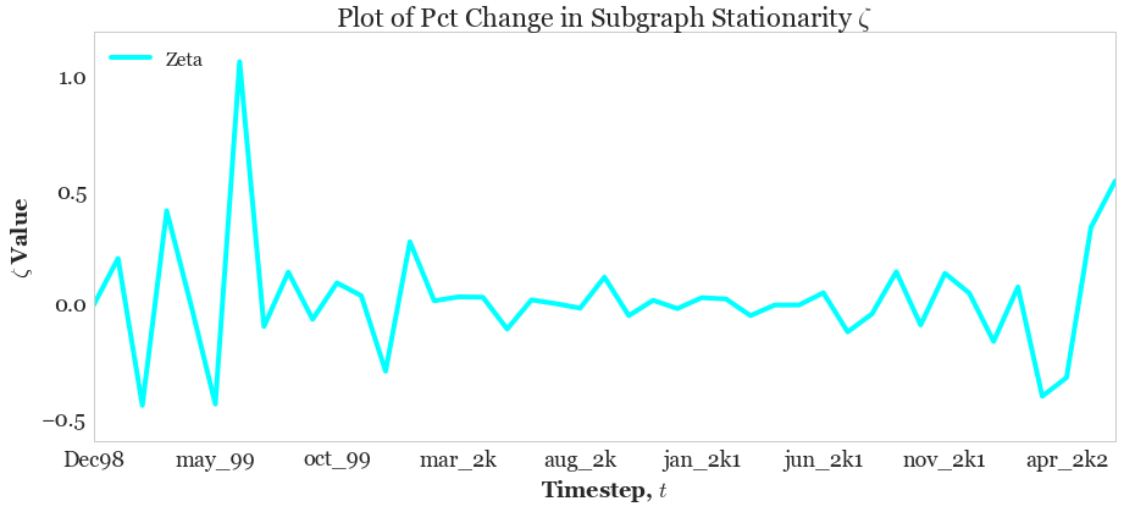
```
Out[605]:            Zeta
          Dec98   0.013380
          jan_99  0.016119
          feb_99  0.008999
          mar_99  0.012727
          apr_99  0.012727
```

```
In [606]: zeta.plot(fontsize=18, cmap='cool', rot =90)
          plt.title("Plot of Subgraph Stationarity $\zeta$", fontsize=22)
          plt.xlabel("Timestep, $t$", fontsize=18)
          plt.ylabel("$\zeta$ Value", fontsize=18)
          plt.legend(fontsize=16, loc=2)
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)
```

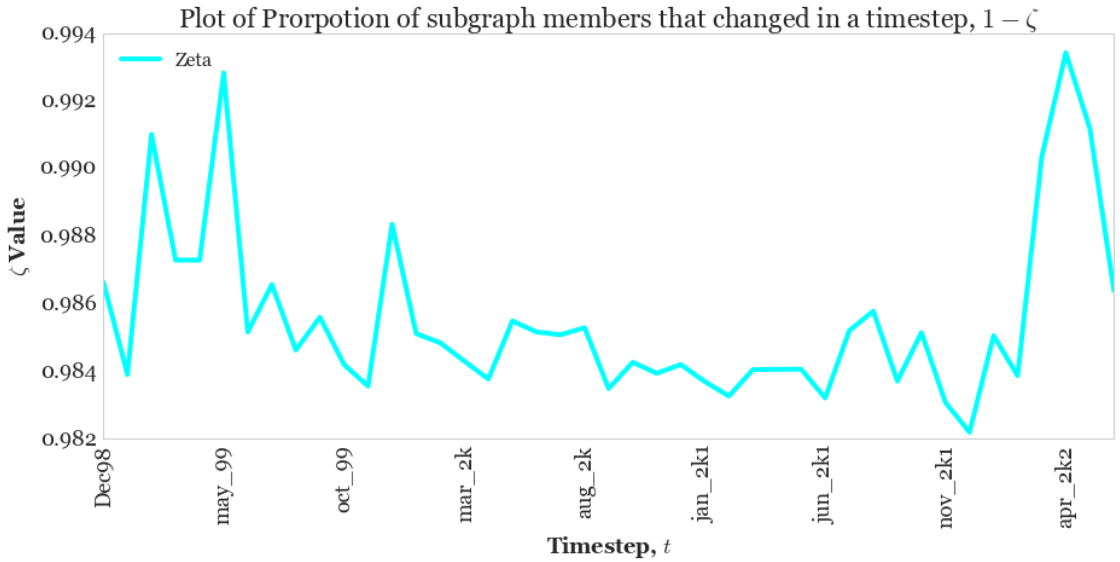
```
Out[606]: (array([ 0.006,  0.008,  0.01 ,  0.012,  0.014,  0.016,  0.018,  0.02 ]),
           <a list of 8 Text yticklabel objects>)
```



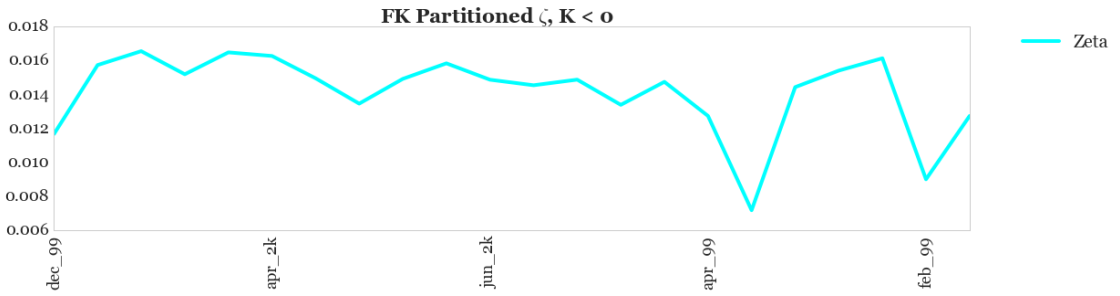

```
In [607]: zeta.pct_change().fillna(0).plot(fontsize=18, cmap='cool')
          plt.title("Plot of Pct Change in Subgraph Stationarity $\zeta$", fontsize
          plt.xlabel("Timestep, $t$", fontsize=18)
          plt.ylabel("$\zeta$ Value", fontsize=18)
          plt.legend(fontsize=16, loc=2)
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)

Out[607]: (array([-1. , -0.5,  0. ,  0.5,  1. ,  1.5]),
           <a list of 6 Text yticklabel objects>)
```

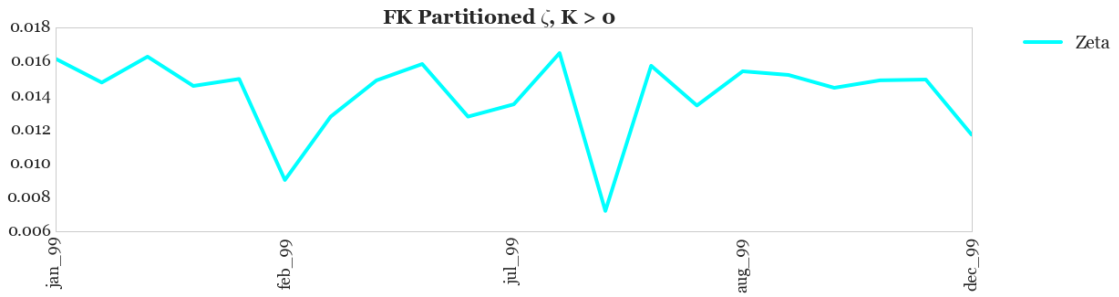


```
In [608]: (1-zeta).plot(fontsize=18, cmap='cool', rot =90)
          plt.title("Plot of Proportion of subgraph members that changed in a times
          plt.xlabel("Timestep, $t$", fontsize=18)
          plt.ylabel("$\zeta$ Value", fontsize=18)
          plt.legend(fontsize=16, loc=2)
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)

Out[608]: (array([ 0.98 ,  0.982,  0.984,  0.986,  0.988,  0.99 ,  0.992,  0.994]),
           <a list of 8 Text yticklabel objects>)
```



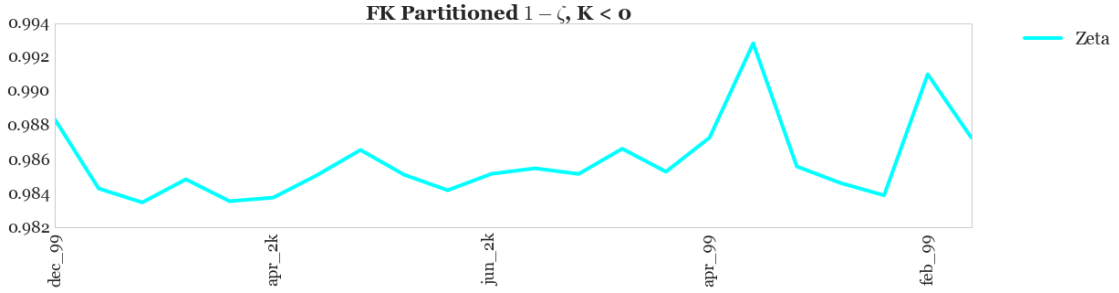

```
In [968]: zeta.ix[idx_f_less0].plot(figsize=(18,4), cmap='cool',fontsize=18, rot=90
          plt.suptitle("FK Partitioned $\zeta$, K < 0", fontsize=22)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

          zeta.ix[idx_f_more0].plot(figsize=(18,4), cmap='cool',fontsize=18, rot=90
          plt.suptitle("FK Partitioned $\zeta$, K > 0", fontsize=22)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

Out[968]: <matplotlib.legend.Legend at 0x1a91927bc18>
```

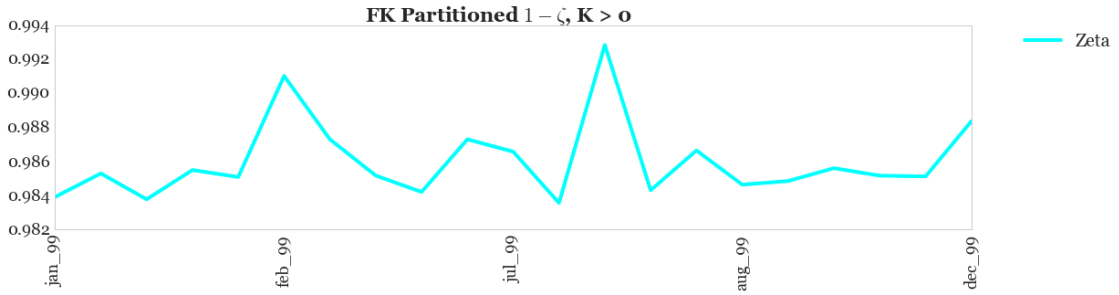




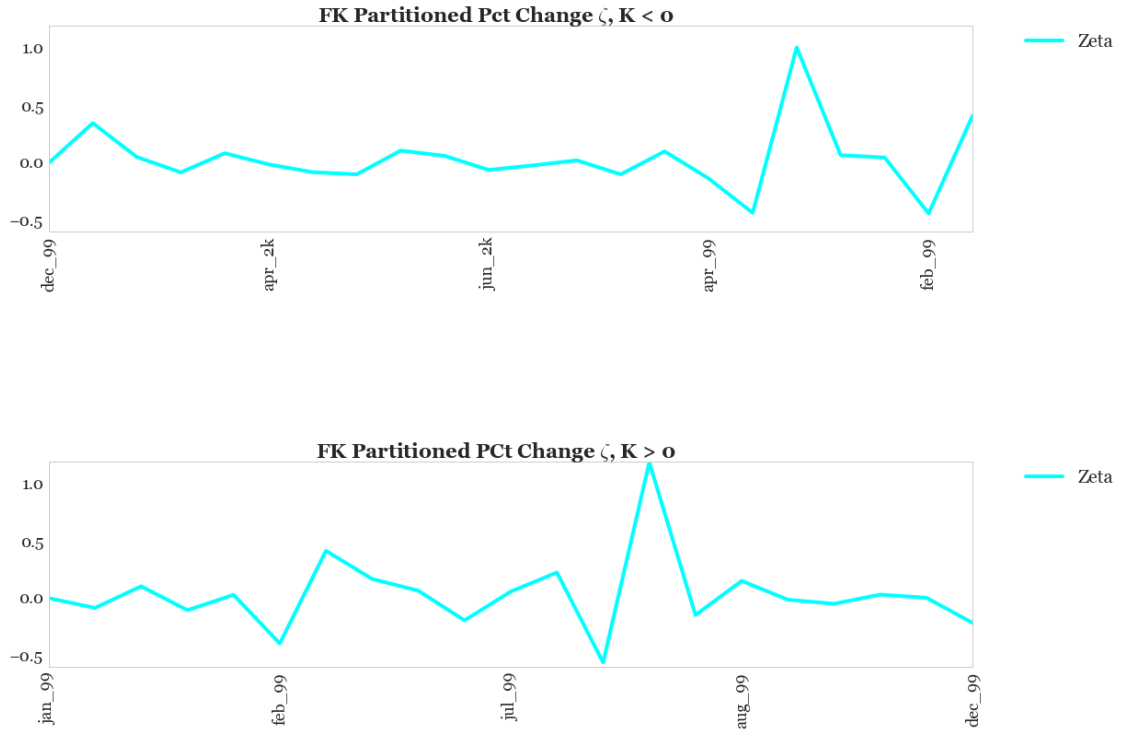

```
In [969]: (1-zeta).ix[idx_f_less0].plot(figsize=(18,4), cmap='cool',fontsize=18, ro
          plt.suptitle("FK Partitioned $1-\zeta$, K < 0", fontsize=22)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

          (1-zeta).ix[idx_f_more0].plot(figsize=(18,4), cmap='cool',fontsize=18, ro
          plt.suptitle("FK Partitioned $1-\zeta$, K > 0", fontsize=22)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
```

```
Out[969]: <matplotlib.legend.Legend at 0x1a918961128>
```





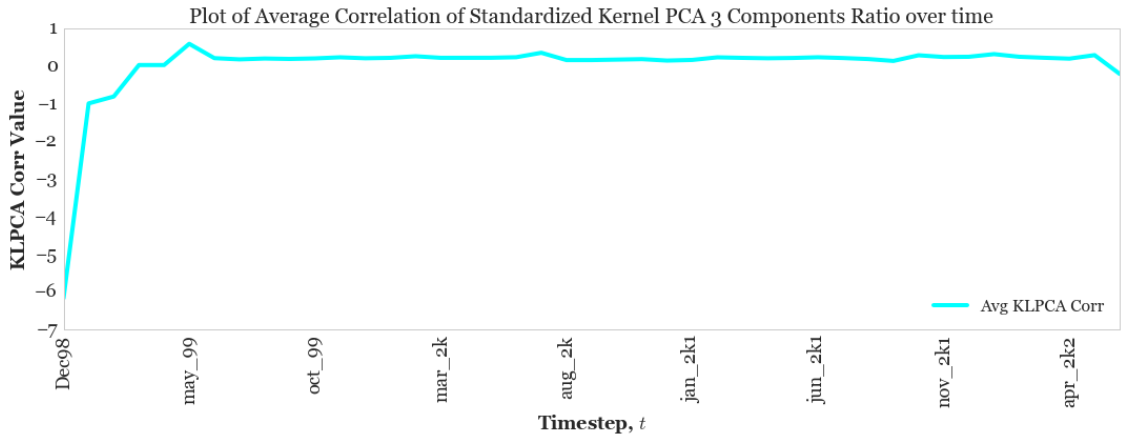
```
In [970]: zeta.ix[idx_f_less0].pct_change().fillna(0).plot(figsize=(18,4), cmap='co
          plt.suptitle("FK Partitioned Pct Change $\zeta$, K < 0", fontsize=22)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

          zeta.ix[idx_f_more0].pct_change().fillna(0).plot(figsize=(18,4), cmap='co
          plt.suptitle("FK Partitioned PCt Change $\zeta$, K > 0", fontsize=22)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

Out[970]: <matplotlib.legend.Legend at 0x1a9189d7198>
```



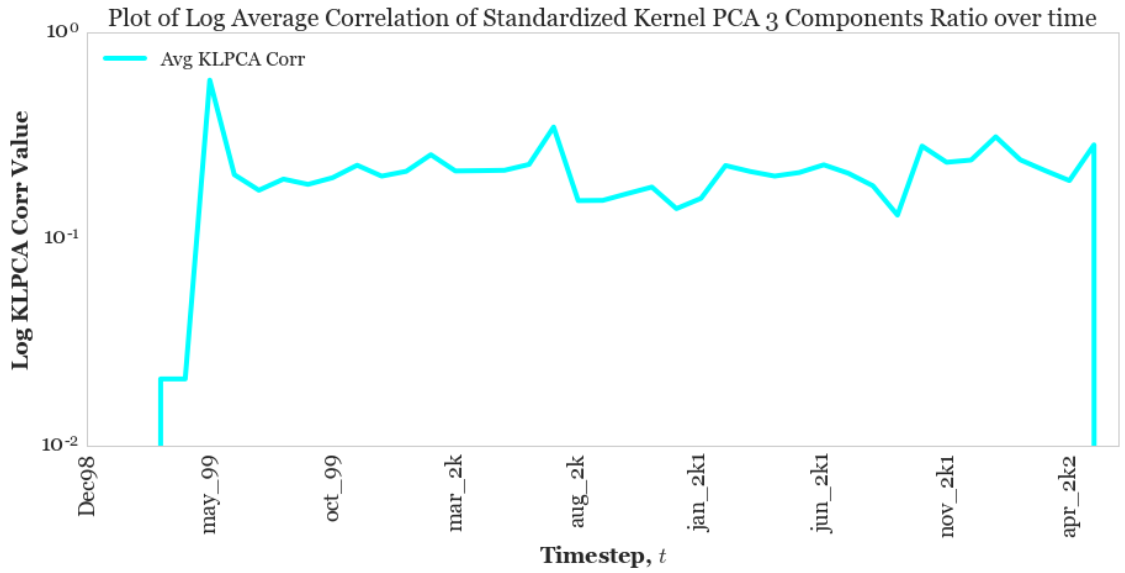
13 Kernel PCA 3 Component Ratio Correlation

```
In [638]: np.mean(np.correlate(std_klpca_ratio(G_nov98),std_klpca_ratio(G_dec98)))

Out[638]: -797.64345338943622

In [610]: def avg_klpca_corr(net1,net2):
              avg = np.mean(np.correlate(std_klpca_ratio(net1),std_klpca_ratio(net2
              return avg


In [611]: avg_klpca_corr(G_nov98,G_dec98)

Out[611]: -797.64345338943622
```

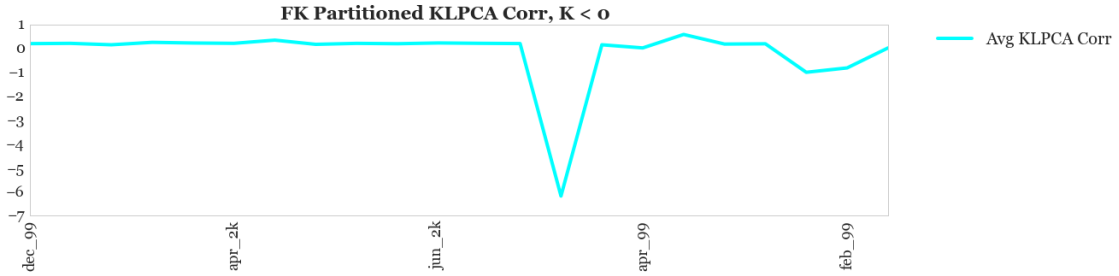
```
In [612]: avg_klpca3_corr = []
          for i in all_graphs:
              x = int(i)
              y = x +1
              avg_klpca3_corr.append(avg_klpca_corr(all_graphs[x],all_graphs[y]))

In [613]: klpca = pd.DataFrame(avg_klpca3_corr,columns=['Avg KLPCA Corr'])

In [614]: klpca=klpca.T
          klpca.columns = list(stat_all.T.columns)[1:]
          klpca=klpca.T
          klpca.head()

Out[614]:          Avg KLPCA Corr
          Dec98       -797.643453
          jan_99      -148.926087
          feb_99      -125.903234
          mar_99       -21.172278
          apr_99       -21.172278

In [642]: klpca_std = (klpca - klpca.mean())/klpca.std()
          klpca_std.head(), klpca_std.tail()

Out[642]: (         Avg KLPCA Corr
           Dec98         -6.186219
           jan_99        -1.000341
           feb_99        -0.816295
           mar_99         0.020930
           apr_99         0.020930,          Avg KLPCA Corr
           feb_2k2        0.240676
           mar_2k2        0.213727
           apr_2k2        0.190984
           may_2k2        0.284378
           jun_2k2       -0.215817)

In [880]: klpca_std.plot(fontsize=18, cmap='cool', rot =90)
          plt.title("Plot of Average Correlation of Standardized Kernel PCA 3 Compo
          plt.xlabel("Timestep, $t$", fontsize=18)
          plt.ylabel("KLPCA Corr Value", fontsize=18)
          plt.legend(fontsize=16, loc=4)
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)
          plt.tight_layout()
```

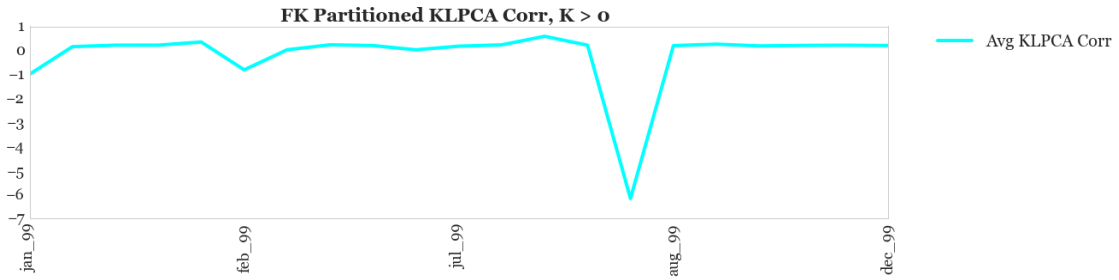



```
In [684]: klpca_std.plot(fontsize=18, cmap='cool', rot =90, logy=True, use_index=Tr
          plt.title("Plot of Log Average Correlation of Standardized Kernel PCA 3 C
          plt.xlabel("Timestep, $t$", fontsize=18)
          plt.ylabel("Log KLPCA Corr Value", fontsize=18)
          plt.legend(fontsize=16, loc=2)
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)

Out[684]: (array([  1.00000000e-03,   1.00000000e-02,   1.00000000e-01,
                    1.00000000e+00,   1.00000000e+01]),
           <a list of 5 Text yticklabel objects>)
```

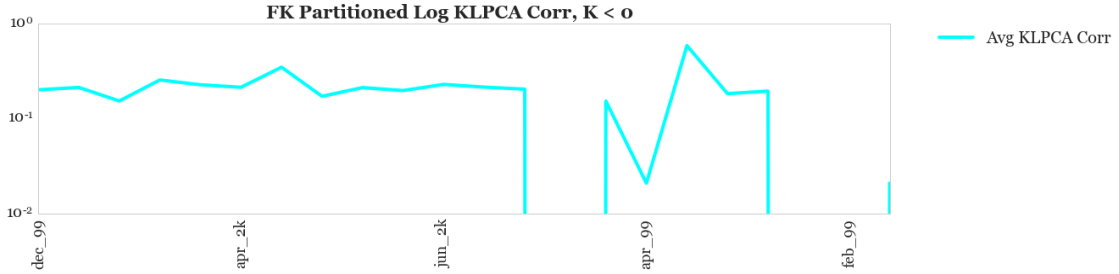


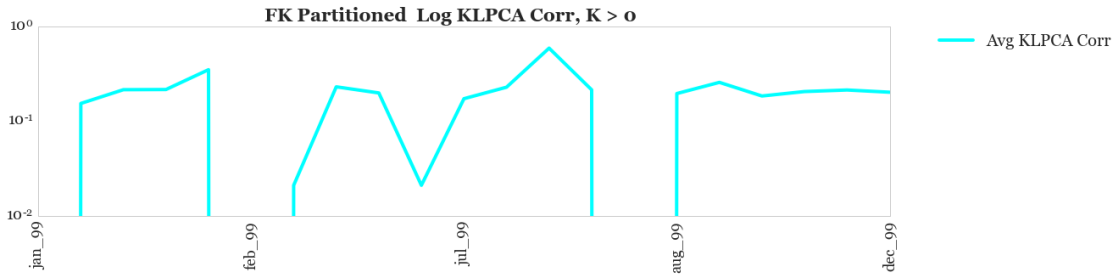
```
In [973]: klpca_std.ix[idx_f_less0].plot(figsize=(18,4), cmap='cool',fontsize=18, r
          plt.suptitle("FK Partitioned KLPCA Corr, K < 0", fontsize=22)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

          klpca_std.ix[idx_f_more0].plot(figsize=(18,4), cmap='cool',fontsize=18, r
          plt.suptitle("FK Partitioned KLPCA Corr, K > 0", fontsize=22)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

Out[973]: <matplotlib.legend.Legend at 0x1a918aca780>
```





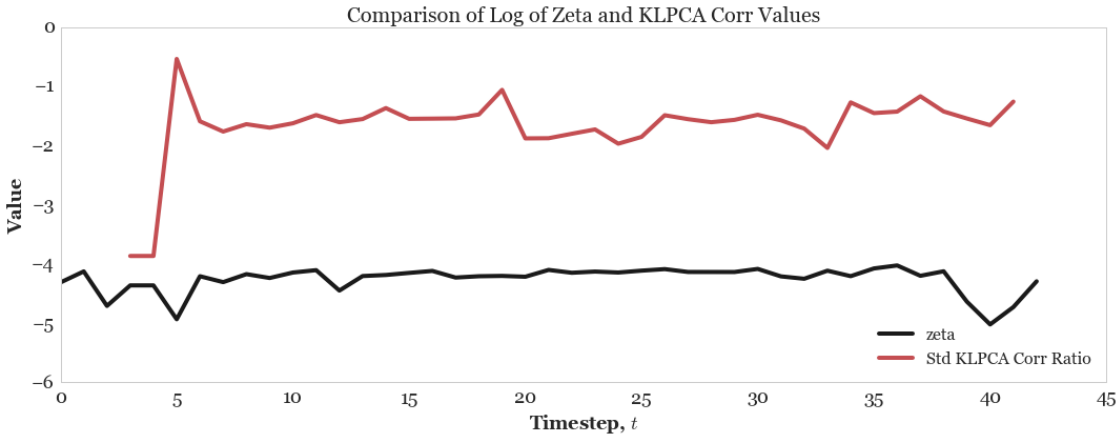

```
In [975]: klpca_std.ix[idx_f_less0].plot(figsize=(18,4), cmap='cool',fontsize=18, r
          plt.suptitle("FK Partitioned Log KLPCA Corr, K < 0", fontsize=22)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

          klpca_std.ix[idx_f_more0].plot(figsize=(18,4), cmap='cool',fontsize=18, r
          plt.suptitle("FK Partitioned  Log KLPCA Corr, K > 0", fontsize=22)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

Out[975]: <matplotlib.legend.Legend at 0x1a91caffc18>
```





13.1 Comparison of Zeta and KLPCA

```
In [886]: plt.plot(np.log(zeta.values),'k', label='zeta')
          plt.plot(np.log(klpca_std.values),'r', label='Std KLPCA Corr Ratio')
          plt.title("Comparison of Log of Zeta and KLPCA Corr Values", fontsize=20)
          plt.xlabel("Timestep, $t$", fontsize=18)
          plt.ylabel("Value", fontsize=18)
          plt.legend(fontsize=16, loc=4)
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)
          #plt.ylim(,klpca_std.values.max())
          plt.tight_layout()
```




```
In [709]: zeta.sort_index(ascending=True, axis=1).head(n=10)
```

```
Out[709]:             Zeta
          Dec98   0.013380
          jan_99  0.016119
```

```
          feb_99  0.008999
          mar_99  0.012727
          apr_99  0.012727
          may_99  0.007179
          jun_99  0.014862
          jul_99  0.013451
          aug_99  0.015394
          sep_99  0.014422
```

```
In [710]: klpca_std.sort_index(ascending=True, axis=1).head(n=10)
```

```
Out[710]:         Avg KLPCA Corr
          Dec98        -6.186219
          jan_99       -1.000341
          feb_99       -0.816295
          mar_99        0.020930
          apr_99        0.020930
          may_99        0.585006
          jun_99        0.203913
          jul_99        0.171475
          aug_99        0.194185
          sep_99        0.183072
```

```
In [711]: zeta.sort_index(ascending=True, axis=1).tail(n=10)
```

```
Out[711]:              Zeta
          sep_2k1  0.016317
          oct_2k1  0.014879
          nov_2k1  0.016949
          dec_2k1  0.017831
          jan_2k2  0.014968
          feb_2k2  0.016155
          mar_2k2  0.009669
          apr_2k2  0.006581
          may_2k2  0.008821
          jun_2k2  0.013633
```

```
In [712]: klpca_std.sort_index(ascending=True, axis=1).tail(n=10)
```

```
Out[712]:          Avg KLPCA Corr
          sep_2k1        0.129990
          oct_2k1        0.280595
          nov_2k1        0.234164
          dec_2k1        0.240161
          jan_2k2        0.311208
          feb_2k2        0.240676
          mar_2k2        0.213727
          apr_2k2        0.190984
          may_2k2        0.284378
          jun_2k2       -0.215817
```

In [ ]: