



CITY UNIVERSITY
LONDON

MSc Data Science Project Report 2016

*Analysing the evolution of
communication patterns in email
data through an extended
dynamic network analysis toolkit*

Arshad Ahmed

Supervised by: Dr Cagatay Turkay
Submission Date: 23 September 2016

Declaration

By submitting this work, I declare that this work is entirely my own except those parts duly identified and referenced in my submission. It complies with any specified word limits and the requirements and regulations detailed in the assessment instructions and any other relevant programme and module documentation. In submitting this work, I acknowledge that I have read and understood the regulations and code regarding academic misconduct, including that relating to plagiarism, as specified in the Programme Handbook. I also acknowledge that this work will be subject to a variety of checks for academic misconduct.

Signed: Arshad Ahmed

Abstract

This study explores the use of similarity measures in the analysis of dynamic networks. Dynamic networks arise in many modern applications such as email , social networks and telecommunications among others. Their defining feature is their evolution through time. Thus characterising this change requires the use of similarity measures that can be compared at different time intervals. For this study I use the Enron Email Network data from John Hopkins which is in the form of (timestamps, to , from) tuples to compare a large range of existing measures and propose some novel measures. The contributions of this study are the following. First, I consider a large range of existing network measures to determine the signal in the graph time series. Second, I use these measures to benchmark the novel measures which are of two categories: one based on the notion of the complex trace and the second on matrix decomposition. Also by turning the Normalised Graph Laplacian into a waveform audio signal I implement measures from Music Information Retrieval. Third, I propose novel visualisation techniques for multiple attribute analysis on graphs: the Frequency-Wavenumber Plot, the Radon Plot, Waveform Plot and the Log Panel. I show that these novel measures when calculated from the normalised Graph Laplacian are very stable and capture the network dynamics well. Also they are able to highlight additional areas of interest that are not obvious from the existing measures. These attributes can be used as global measures to characterise network change, visualised as attribute maps to consider local changes and finally to highlight interesting nodes. Also I propose a global NRMS measure to aggregate all these network attributes into one metric.

Keywords: Dynamic Network Analysis, Attribute Analysis, Attribute Visualisation

Contents

| | | |
|----------|---|-----------|
| 1 | Introduction | 9 |
| 1.1 | Background | 9 |
| 1.2 | Motivations and Aims | 10 |
| 1.3 | Beneficiaries | 11 |
| 1.4 | Objectives | 12 |
| 1.5 | Overview of Methodology | 12 |
| 1.6 | Report Structure | 13 |
| 2 | Critical Context | 14 |
| 2.1 | Paradigms of Network Analysis | 14 |
| 2.1.1 | Social Network Analysis (SNA) | 14 |
| 2.1.2 | Dynamic Network Analysis (DNA) | 14 |
| 2.1.3 | Super Networks Analysis (SuNA) | 14 |
| 2.2 | The Limitations of traditional SNA | 15 |
| 2.3 | Dynamic Network Analysis as an Extension to SNA | 15 |
| 2.3.1 | The Meta Matrix | 16 |
| 2.3.2 | Probabilistic Ties | 16 |
| 2.3.3 | Multi Agent Network Models | 16 |
| 2.4 | Network Measures | 16 |
| 2.5 | Overview of Similarity Methods | 17 |
| 2.5.1 | Distance Based Approach | 18 |
| 2.5.2 | Feature Based Approach | 18 |
| 2.5.3 | Probabilistic Approach | 18 |
| 2.6 | Graph Similarity | 18 |
| 2.7 | Spectral and Tensor Methods | 19 |
| 2.8 | Visual Methods | 20 |
| 3 | Methods | 21 |
| 3.1 | Analytical Tools | 21 |
| 3.2 | Analysis Approach | 21 |
| 3.2.1 | Graph Matrices | 23 |
| 3.3 | Benchmark Measures | 25 |
| 3.3.1 | Degree Centrality | 26 |
| 3.3.2 | Closeness Centrality | 26 |
| 3.3.3 | Betweenness Centrality | 26 |
| 3.3.4 | Eigenvector Centrality | 26 |
| 3.3.5 | Katz Centrality | 26 |
| 3.3.6 | Load Centrality | 27 |

| | | |
|----------|--|-----------|
| 3.3.7 | Density | 27 |
| 3.3.8 | Diameter | 27 |
| 3.4 | Complex Trace Attributes | 27 |
| 3.4.1 | The Hilbert Transform | 27 |
| 3.4.2 | The Complex Trace | 27 |
| 3.4.3 | Instantaneous Amplitude | 28 |
| 3.4.4 | Power | 28 |
| 3.4.5 | Derivative of Instantaneous Amplitude | 28 |
| 3.4.6 | 2nd Derivative of Instantaneous Amplitude | 28 |
| 3.4.7 | Instantaneous Phase | 29 |
| 3.4.8 | Instantaneous Frequency | 29 |
| 3.4.9 | Instantaneous Acceleration | 29 |
| 3.4.10 | Amplitude Weighted Instantaneous Frequency | 30 |
| 3.4.11 | Amplitude weighted Instantaneous Phase | 30 |
| 3.5 | Matrix and Matrix Decomposition Attributes | 30 |
| 3.5.1 | Norm of Forward Abel Transform | 30 |
| 3.5.2 | Mean Gaussian Curvature | 31 |
| 3.5.3 | Kernel PCA 3 Components Ratio | 31 |
| 3.5.4 | Norm Non-Negative Matrix Factorisation Ratio | 32 |
| 3.5.5 | Resistance Distance | 32 |
| 3.5.6 | Stationarity Ratio | 32 |
| 3.5.7 | Subgraph Stationarity | 33 |
| 3.5.8 | Power Spectral Density | 33 |
| 3.6 | Music Attributes | 33 |
| 3.6.1 | The Fourier Transform | 33 |
| 3.6.2 | Zero Crossing Rate | 34 |
| 3.6.3 | Spectral Centroid | 34 |
| 3.7 | Aggregation Schemes | 34 |
| 3.7.1 | Persistence and Emergence | 34 |
| 3.7.2 | NRMS of Network Attributes | 35 |
| 3.8 | Visualisations | 35 |
| 3.8.1 | F-K Plot | 35 |
| 3.8.2 | Radon Plot | 36 |
| 3.8.3 | Log Panel | 36 |
| 3.8.4 | Audio Waveform Plot | 36 |
| 4 | Results | 37 |
| 4.1 | Results Overview | 37 |
| 4.2 | Network Visualisation | 37 |
| 4.2.1 | Yearly Network Visualisations | 38 |
| 4.2.2 | Monthly Network Visualisations | 41 |
| 4.3 | Exploratory Analysis: Centrality Measures | 44 |
| 4.3.1 | Yearly Analysis | 44 |
| 4.3.2 | Monthly Analysis | 50 |
| 4.4 | Benchmark Measures | 56 |
| 4.5 | Attribute Analysis | 57 |
| 4.5.1 | Average Attributes | 63 |
| 4.5.2 | Correlation Analysis | 65 |

| | | |
|----------|--|------------|
| 4.5.3 | Manifold Visualisation of Attribute Volume | 67 |
| 4.6 | Feature Ranking with Regression Analysis | 69 |
| 4.7 | FK and Radon Plots | 70 |
| 4.8 | Node Level Trends | 74 |
| 5 | Discussion | 77 |
| 5.1 | Discussion of Original Aims and Objectives | 77 |
| 5.2 | Research Question | 79 |
| 6 | Evaluation, Reflection and Conclusions | 81 |
| 6.1 | Evaluation of Results | 81 |
| 6.2 | Generalisation of Analytical Work flow | 87 |
| 6.3 | Reflections and Conclusions | 88 |
| | Bibliography | 90 |
| | Appendix | 94 |
| | A Project Proposal | 95 |
| | B Project Gantt Chart | 105 |
| | C Project Test List | 107 |
| | D Jupyter Notebook: Python Code and Results | 108 |

List of Figures

| | | |
|------|--|----|
| 1.1 | Euler's graphical representation of the Konigsberg Bridge Problem. | 10 |
| 4.1 | Node Link Diagram of yearly networks | 38 |
| 4.2 | Reordered Matrix Diagram of yearly networks | 39 |
| 4.3 | Audio Waveform plot of yearly networks | 40 |
| 4.4 | Node Link Diagram of monthly networks | 41 |
| 4.5 | Reordered Matrix of monthly networks | 42 |
| 4.6 | Audio Wavefrom of monthly networks | 43 |
| 4.7 | Yearly Degree Histogram | 44 |
| 4.8 | Yearly Closeness Centrality Histogram | 45 |
| 4.9 | Yearly Betweenness Histogram | 46 |
| 4.10 | Yearly Eigenvector Centrality Histogram | 47 |
| 4.11 | Yearly Katz Centrality Histogram | 48 |
| 4.12 | Yearly Load Centrality Histogram | 49 |
| 4.13 | Monthly Degree Histogram | 50 |
| 4.14 | Monthly Closeness Centrality Histogram | 51 |
| 4.15 | Monthly Betweenness Histogram | 52 |
| 4.16 | Monthly Eigenvector Centrality Histogram | 53 |
| 4.17 | Monthly Katz Centrality Histogram | 54 |
| 4.18 | Monthly Load Centrality Histogram | 55 |
| 4.19 | Plot of Benchmark Measures over Years | 56 |
| 4.20 | Plot of Benchmark Measures over Months | 56 |
| 4.21 | Plot of Signal to Noise Ratio of Attributes calculated from 3 different Graph Matrices | 57 |
| 4.22 | Plot of Count of Number attributes with $SNR > 1$ from the different graph matrices | 57 |
| 4.23 | Plot of Entropy of attributes from the different graph matrices | 58 |
| 4.24 | Plot of Mean Entropy of different graph matrices | 58 |
| 4.25 | Plot of Complex Attributes over Years | 59 |
| 4.26 | Plot of Complex Attributes over Months | 59 |
| 4.27 | Plot of Matrix Attributes over Months | 60 |
| 4.28 | Plot of Matrix Attributes over Months | 60 |
| 4.29 | Plot of Music Attributes over Years | 61 |
| 4.30 | Plot of Music Attributes over Months | 61 |
| 4.31 | Boxplot of All Attributes | 62 |
| 4.32 | Plot of Average Attributes over Years | 63 |
| 4.33 | Plot of Average Attributes over Months | 64 |
| 4.34 | Reordered Correlation Heatmap showing correlation > 0.7 | 65 |

| | |
|---|----|
| 4.35 Correlation Network of Attributes. The thickness of the borders of the nodes indicates high degree while low thickness of borders around the nodes indicate low degree. | 66 |
| 4.36 Correlation Network Degree Histogram | 66 |
| 4.37 Non-Metric Multidimensional Scaling of Attribute Volume | 67 |
| 4.38 TSNE Plot of Attribute Volume with Euclidean Distance | 67 |
| 4.39 TSNE Plot of Attribute Volume with Correlation Distance | 68 |
| 4.40 TSNE Plot of Attribute Volume with Canberra Distance | 68 |
| 4.41 Feature Ranking by Gradient Boosting Regressor for predicting Average Degree | 69 |
| 4.42 Regression Deviance Plot | 69 |
| 4.43 Frequency Wavenumber (FK) Plot of Attribute Volume | 70 |
| 4.44 (top) Radon Plot of Attribute Volume, (bottom) Annotated Radon Plot | 70 |
| 4.45 Heatmap of Frequency of Attribute Volume. This is the F component derived from the FK Plot | 71 |
| 4.46 Heatmap of Wavenumber of Attribute Volume. This is the K component derived from the FK Plot | 71 |
| 4.47 Heatmap of Radon Transform of Attribute Volume. | 72 |
| 4.48 Log Panel of selected attributes. Benchmark attributes are shown in red. Seismic Attributes shown in blue. Music Attributes shown in yellow. Matrix Attributes shown in green and NRMS aggregation measure in black. | 73 |
| 4.49 Plot of Common Node Degree over years | 74 |
| 4.50 Plot of Common Node Betweenness Centrality over years | 74 |
| 4.51 Plot of Common Node Closeness Centrality over years | 75 |
| 4.52 Plot of Common Node Eigenvector Centrality over years | 75 |
| 4.53 Plot of Common Node Katz Centrality over years | 76 |
| 4.54 Plot of Common Node Load Centrality over years | 76 |

Chapter 1

Introduction

1.1 Background

Networks arise in many modern day applications such as email, social, transport and telecommunication applications. These networks represent convenient structures to analyse phenomenon that would be very difficult to analyse otherwise. However, with most modern networks a key feature is that these networks evolve over time. These dynamic networks require analytical methods that would allow easy comparison between them at the different time steps. Thus similarity measures that allow us to assess and track change over time in such networks are instrumental for us to be able to study such networks. Therefore similarity analysis on dynamic networks is of critical importance if we are to be able to understand evolution of such phenomenon over time. This study is motivated by the need to explore the effectiveness of traditional Network Analysis metrics such as centrality analysis in the context of dynamic networks.

The traditional metrics were developed mainly for the analysis of static graphs thus for modern applications their suitability needs to be assessed and their shortcomings determined. In this study I find that although these measures can be utilised in a dynamic network context they are useful for benchmarking of other new measures. As a result the field of Dynamic Network Analysis has evolved to meet this challenge with the aim to extend our toolkit to handle evolving networks.

In this study I propose novel measures of similarity which are motivated by the fields of Seismic Data Analysis and Music Information Retrieval. These novel measures proposed here are calculated from the Normalised Graph Laplacian and they behave similar to traditional centrality measures while being sensitive to smaller changes in the network that traditional measures do not pick up on as well. Also I show that since these networks evolve in time these new measures are very effective in helping to predict fundamental network properties such as Average Degree with good accuracy at a time step in the future. This would aid greatly in planning for example in a telecommunication the degree measure in the future could be indicative of future load. In addition novel visualisations to support analysis of these networks are also proposed such as the Waveform, Frequency-Wavenumber, Radon and Log Panel Plot.

The origins of graphs theory can be traced back to Leonhard Euler and his approach

to solving the Konigsberg Bridge Problem. This city was located on the Pregel River in Prussia. The river divided this city into 4 distinct areas which included an island all of which were connected by a total of 7 bridges. Euler's representation of this problem of the individual areas as nodes and the bridges as edges is considered one of the first applications of graph theory.[1]

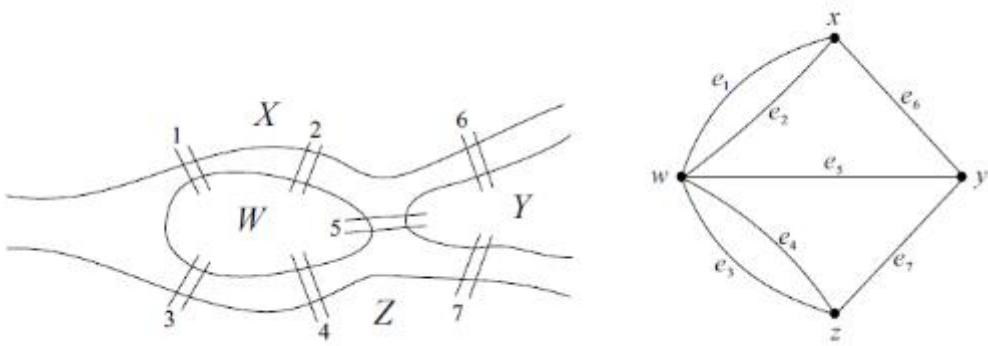


Figure 1.1: Euler's graphical representation of the Konigsberg Bridge Problem.

A graph, G can be described as a triple which consists of a set of edges E (G), a set of vertices or nodes V (G) and a relationship that connects the vertices to these edges. Finite graphs are those that have V and E as a finite set. Simple graphs are those that have no loops or multiple edges. A path is simple graph in which the vertices can be ordered where two vertices can be adjacent only if they are consecutively ordered. A cycle is defined as a simple graph where the vertices can be cyclically ordered such that two vertices are adjacent only if they are consecutive in cyclical ordering. A subgraph can be thought of cycles and paths within a larger graph, where the edge relations between the subgraph and the large graph are the same. [1]

1.2 Motivations and Aims

As mentioned that the analysis of dynamic networks is necessitated by the need to understand and model complex phenomenon. Undoubtedly there is great value in understanding the dynamics of such networks. The beneficiaries of this work are practitioners who are interested in or need to study dynamic networks. These could range from intelligence agencies monitoring changes in terrorist networks, social media companies monitoring and understanding the dynamics of their networks to telecommunications communications identifying interesting network dynamics that can drive their business forward.

The main aim of this study is to explore through thorough analysis of the Enron Email network to demonstrate the behaviour of traditional and novel metrics in a dynamic network context. I want to compare, contrast and evaluate the trends in this network while showing the value of novel measures to support such analysis. The analysis conducted is multifaceted and presents not only analytical measures but a number of visual and aggregation techniques that help to both understand network and node level dynamics.

Underlying all these analysis is the fact that these measures serve as a snapshot of

a network at a point in this time. Thus sampling these measures at the different time intervals it is possible to derive a time series of attributes from the graph time series. This time series approach allows utilisation of methods from signal processing because by treating the graphs at the different time steps as a time series and the derived attributes also as a time series I can apply methods from signal processing

From the field of Seismic Data Analysis I present a number of attributes which are based on the notion of the Complex Trace. This involves using the Hilbert Transform and using the Real and Imaginary components to derive attributes such as Amplitude, Phase and Frequency. From this a number of other attributes are derived. In addition matrix decomposition methods such as Kernel Principal Components Analysis (KPCA) and Non-negative Matrix Factorisation (NMF) are used to derive additional measures.

From the field of Music Information Retrieval I implement two measures the Zero Crossing Rate and the Spectral Centroid. This is possible by treating the networks as a audio signal by the use of the Fourier Transform on the Normalised Graph Laplacian.

Both sets of similarity measures that I will call Seismic and Music attributes for convenience are well established in their respective but their application to dynamic networks is novel. The motivation to utilise these measures is driven by the need to have measures that are scalable, have a relatable interpretation and are particularly suited for dynamic analysis. For example the seismic attributes are commonly analysed for the identification of hydrocarbon reservoirs and in 4D seismic two surveys are compared at different time steps to characterise change as a result of production. So these measures become particularly suited for analysis with a time component.

Therefore the key motivations for this project can be summarised as firstly to develop an understanding of the use of traditional metrics to define a signal in a graph time series. Secondly, to complement existing measures with novel ones which will enhance our analytical capabilities of the structure of dynamic graphs. Third, to develop visualisation techniques to enable multiple attribute analysis and finally aggregation measures which can serve as a proxy for network activity.

1.3 Beneficiaries

The main dataset used in this study is for an email network. But these methods proposed here are generalisable to any dynamic or static network. This is because the attributes are calculated from the matrix of the graph structure like the Normalised Graph Laplacian. Therefore any problem that can be modelled as a network can be expressed as a Normalised Laplacian Matrix and these attributes can be easily calculated from it. This opens up these methods to practitioners and analysts from a wide variety of fields as a result.

The main beneficiaries are practitioners or analysts who are involved or have an interest in the study of dynamic networks. This can range from social media companies trying to understand structural changes in their network and being able to identify easily the key players. It could be useful to intelligence agencies analysing threat

networks they have more tools to characterise the structural change in networks and identify times and nodes of interest. It could also benefit telecommunications companies from example to understand how fast their network is expanding or contracting and more importantly predict some fundamental network property such as Average Degree at a time step in the future. Understanding potential trends in advance has the capacity to drive changes in strategy, enhance planning and deliver increased value for the business.

1.4 Objectives

The objectives that will help us answer our research question are as follows:

- What are some of the graph similarity measures that have been proposed in the literature?
- How have these been used in a practical context such as email data?
- How can such measures be applied to dynamic networks?
- How are such measures evaluated?
- How do the novel measures proposed compare to traditional measures? How can they be interpreted in the context of graph analysis?
- How can these attributes be visualised and aggregated?
- Can we outline a robust workflow for the analysis of such networks utilising all these techniques?

1.5 Overview of Methodology

The methodology followed in this study is as follows:

1. Source an appropriate email network network dataset with timestamps
2. Break data into yearly and monthly sets with no temporal aggregation
3. For the graph time series at the monthly and yearly level analyse the networks through traditional measures such as Centrality, Algebraic Connectivity, Density and Average Clustering Coefficient. These form the benchmark measures against which other measures are compared.
4. For the novel measures proposed determine which of the 3 graph matrices: Modularity, Adjacency and Normalised Laplacian yield the most stable attributes. This is done by analysis of the Signal to Noise Ratio, Mean Absolute Deviation, numerical magnitude of the attributes and their ability to model the signal in the benchmark measures.
5. Pick the Matrix yielding the most stable attributes using this to explore correlation among the measures, new visualisation approaches and aggregation schemes.
6. Scale all attributes to [-1,1] interval for comparability.
7. Explain network dynamics captured by the measures through node level analysis

1.6 Report Structure

This report is structured into 6 main chapters. In Chapter 2, I explore the critical context for this study where the recent work in this field is surveyed and relevant material summarised. In Chapter 3, I present an outline in detail all the measures used in this analysis, their calculation and their typical interpretation in the fields from which they are taken. In Chapter 4, I present my findings as a consequence of all this analysis. In Chapter 5, I discuss the outcomes of this study in the context or its original and revised objectives. Also the research question is answered in this chapter. In Chapter 6, a critical evaluation of the results are presented as well as suggestions for the generalisation of the analytical procedure. The Reflections and Conclusions of this study are also presented in this chapter.

Chapter 2

Critical Context

2.1 Paradigms of Network Analysis

The literature suggests 3 main paradigms of network analysis these inform the framing of research questions and investigative approach. These can be broadly described as: [2]

- Social Network Analysis (SNA)
- Dynamic Network Analysis (DNA)
- Supernetwork Analysis (SuNA)

2.1.1 Social Network Analysis (SNA)

SNA is concerned with the study of relationships between entities and its focus of research is of two types: whole network analysis and self-centred network analysis. Whole network analysis is concerned with understanding the structure of relationships between different roles in a group and is used to investigate network structure changes with the time and the contact pattern of network entities. Self-centred network analysis is concerned with how the individual behaviour of network entities are influenced by the membership of the network. [2]

2.1.2 Dynamic Network Analysis (DNA)

DNA was proposed as an extension to the SNA. The strength of DNA is that it is able to handle large scale dynamic, multi-modal, multi-lateral network with various levels of uncertainty. The edges are probabilistic and the nodes behave like agents in a multi agent environment so this enables perturbations or changes in the network to propagate through the network and result in some global reconfiguration. The evolution of a network in The application of machine learning and multi agent modelling in the same environment is enabled by DNA's use of the meta matrix.[2]

2.1.3 Super Networks Analysis (SuNA)

Super networks can be thought of as networks of networks that exist above and beyond existing networks. These have the characteristic of being multi-layered, multi-dimensional, multi-attributed and multi-levelled with additional features such as congestion and coordination. These have been applied predominantly in supply chain management, finance, traffic and ecology among others. These networks are analysed using either variational inequality and/or hyper graph theory. [2]

2.2 The Limitations of traditional SNA

In Social Network Analysis (SNA), traditionally bounded networks are considered with maybe 2 or 3 connection or link types such as friendship or advice between a node types such as people sometimes another node type such as events are also considered together. [3]

If we consider more critically the interactions possible within our problem context of email networks we can have email networks within an organisation which are bounded and also with other organisations, clients and stakeholders and then the network does become unbounded. These networks can then be thought of as a higher order networks and as Carley[3] notes many tools developed for simpler networks do not scale well to increased network size and complexity and in some cases experience degradation through increased susceptibility to Type 1 and Type 2 errors.

The dynamics in these networks can arise from different processes depending on the context of the problem. Natural evolutionary processes would be learning, births, deaths and ageing Others could be as a result of intervention measures such removal or addition of nodes i.e. removing those who lead the system, communities forming or disintegrating. The data associated with such systems are also often incomplete and contain errors which make the process of analysis and evaluation of these systems.[4]

Analysis approaches that go beyond traditional SNA and link analysis are therefore necessary. Within the context of such dynamic networks analysis can be performed to identify of key individuals, locating hidden groups and estimate performance. The data analysis process on such networks then involve: [3], [4]

- Relationship identification among nodes
- Network structure characterisation
- Locating the elite within the network
- Identifying points of vulnerability
- Comparing networks

The approaches that enables effective analysis of such dynamic networks and help quantify their evolution over time is the motivation for this research.

2.3 Dynamic Network Analysis as an Extension to SNA

Dynamic network analysis (DNA) aims to extend the methods, tools and techniques used in traditional Social Network Analysis (SNA) to the analysis of networks which are able to handle big dynamic multi-mode, multi-link networks with varying levels of uncertainty. Dynamic networks also allow for probabilistic connection between nodes. [3], [4]

In Carley, Diesner, Reminga, et al.[4] DNA was explored within the context of terrorism networks. Here an additional layer of complexity is added by the fact that

an act of measurement changes its properties and this change propagates through the network and its state changes. Another key point is that the nodes in this network have the ability to learn. So the nodes themselves can be thought of being probabilistic compared to the more static nature of SNA nodes.

In a DNA representation system can be represented as relational data. This relational data structure can lend flexibility in defining multiple node types defined as multi-modal, have various types of connections among such nodes called multi-plex. The underlying attributes of both node, edges and the data change over time hence the dynamic part. [4]

In Carley, Diesner, Reminga, et al.[4] the key advances that allow for the analysis of such dynamic networks are identified as:

- The meta matrix
- Probabilistic edges between nodes
- Combining social networks with cognitive science and multi-agent systems

2.3.1 The Meta Matrix

The Meta matrix is a method used in operations research and organisational management that seeks to represent the entity and class relationships as a collection of networks. In the DNA context this translates as a multimode, multiplex approach to representing systems. Therefore, the Meta matrix can contain a social network, a membership network and knowledge network and allow us to explore and analyse the connections between them. [3]–[5]

2.3.2 Probabilistic Ties

The ties or connections in the Meta matrix are probabilistic with various factors affecting their probability. This allows for inclusion of the observers' uncertainty and the likelihood that the tie is present at the time of observation. These probabilities themselves and their temporal evolution maybe estimated by the Bayesian methods, cognitive inferencing and models of social and cognitive change. Carley

2.3.3 Multi Agent Network Models

As previously discussed the SNA treatment of nodes as static agents unable to learn is insufficient when dynamic networks are concerned. In DNA the nodes are able to take actions, learn from experience and alter their networks as a result. Some social and cognitive processes that influence the agent's interactions are relative similarity, relative expertise and co-workers. The dynamic behaviour of the network emerges from these interactions and experience a shared evolution. [3]

We briefly discuss some of the more common measures associated with networks which relate to their global and local properties. These will be important when we discuss similarity because one of the ways to assess similarity is to consider snapshots of a network attribute at different time intervals.

2.4 Network Measures

Centrality measures are a fundamental statistic in network analysis. Two paradigms of centrality definitions are suggested. One is the means based definition of centrality or the graph theoretic and the other is the ends-based definition which is a

dynamic model based view that focuses on the outcome for the nodes in a network where there is flows across the nodes . However, both approaches agree that this measure is a node level property. [6]–[8]

The network measures used in this analysis are:

- Degree Centrality
- Closeness Centrality
- Betweenness Centrality
- Katz Centrality
- Load Centrality
- Density
- Average Clustering Coefficient
- Algebraic Connectivity

These measures are described in detail in Chapter 3, but the reason they are chosen is that the centrality measures are well understood node level properties and Density, Average Clustering Coefficient and Algebraic Connectivity are good network level properties that indicate the growth or contraction of the network as well as its communities.

2.5 Overview of Similarity Methods

Similarity in a networks is classified as being of structural, content or keyword based. The Structural similarity or link based similarity considers the similarity of links between the nodes in the graph e.g. Cosine, Jaccard, Hub Promoted and Hub Depressed Index etc. Content similarity considers the attributes of the node in the graph. For example, on a social network this could be birth dates or hobbies of individuals. Keyword similarity aims to find similarity based on nodes representing word collections. Global Structural Similarity can be classified as being:

- Local vs. Global
- Parameter free vs. Parameter dependent
- Node Dependent vs. Path Dependent

The global structural measures aim to measure node similarity compared to the whole network. We will call them intra network similarity measures. [9]

Inter network similarity measures are described in [10]–[17]. These measures are classified by Ashby and Ennis[18] into three categories:

- Distance Based
- Feature Based
- Probabilistic

2.5.1 Distance Based Approach

The distance based approach is perhaps the earliest of the methods encountered which is based on edit distance Bunke[17]. Essentially this boils down to finding a sequence of operations such as deletion, insertion, or substitution minimising some cost function that will turn one graph into another. These involve detection and comparison of the graph isomorphism, subgraph isomorphism and maximum common subgraph detection utilising the edit distance. Although these methods are guaranteed to converge to an optimal solution their exponential complexity makes them unsuitable for large graphs.

2.5.2 Feature Based Approach

This involves calculation of a network attribute such as degree, closeness, betweenness, and/or eigenvector centrality for the graphs and then applying a similarity measure on them that will characterise their similarity or dissimilarity. This has the benefit of being scalable to very large networks as the aggregated statistics are much smaller than the network themselves.

2.5.3 Probabilistic Approach

The methods that fall under this approach in the literature are vast. Some approaches under the probabilistic framework for graph matching are discussed here [15], [19]–[21]. But simply stated these methods define a probability distribution over mappings or graph embedding’s [16], [21]. Graph embeddings are graphs whose nodes correspond to distinct points on a plane and the edges represents relationships connecting these points. The matching algorithm is strongly dependent upon the geometric information attached to the graphs[16], [21].

Graph matching allows for recovering point correspondences. In Zass and Shashua[15] the authors show that assuming that the assignment matrix that represents these correspondences are statistically independent the high order matching problem can be represented by a Kronecker product matrix. Also they show that that a high order tensor affinity tensor can be marginalised into a one dimensional vector of probabilities. This probability vector is then updated by projection to a vector assignment space and then minimizing a distance measure (Bregman measure) [22].

2.6 Graph Similarity

The problem of graph similarity or graph matching then becomes one of finding the equivalence of two graphs with potentially different number of nodes and edges and returning a measure within $[0, 1]$ that captures their similarity or dissimilarity. [9]–[13], [15], [18], [23]–[25]

The key idea of graph matching in the context of dynamic networks can be summarised as finding a subgraph or an attribute that we can compare between two time instances. For example, if we consider the Degree Centrality of a network at time step 0 and then again calculate this measure at time step 1 we can apply a similarity measure on this attribute to quantify the change within the network. This can be done by means of a distance metric such as cosine similarity and others are possible.

The evaluation of the change in metrics over time will be done through a statistical control process. This is a concept that comes from quality engineering and it essen-

tially involves calculating a statistic from a sequence of measurements of a random process and then comparing it so some control limit. This process translates to:

1. Calculating a cumulative sum control chart which is very good for detecting small changes in mean over time
2. Calculating a z-score for each time step $z_t = \frac{(x-\mu)}{\sigma}$
3. Construction of two charts to detect increase and decrease in the metric

2.7 Spectral and Tensor Methods

Spectral graph theory is the study of the eigenvectors and eigenvalues of graph matrices [20]. The spectrum of a finite graph is the spectrum of the adjacency matrix, which is the eigenvectors and eigenvalues derived from the eigendecomposition of this matrix. For an undirected graph without loops, the Laplace Matrix is the matrix indexed by a vertex set of v , with zero row sums if D is the degree matrix of a graph and A is the adjacency matrix then the Laplacian Matrix, L can be defined as $L = D - A$, where $Q = D + A$ is called the signless Laplacian Matrix of the graph. [20], [26], [27]

Spielman[20] note that since the eigenvalues of a graph do not depend on the vertex ordering of the graph then they could be used to distinguish between pairs of non-isomorphic graphs. If the eigenvalues for the graphs are different then two graphs can be considered different. But he notes that this approach has problems such as the eigenvectors being only determined up to sign i.e. v and $-v$ can both be eigenvectors, so spectral embedding comparison would result in having to check 2^K possible ways of flipping their signs. The eigenvectors can provide coordinates for each vertex in a graph which is independent of the vertex labels but for graphs for which non-trivial eigenvalues have high multiplicity the coordinate flips in addition to its rotation must also be considered. Also the coordinates denoted by the eigenvectors are not unique which means that all eigenspaces must be considered to guarantee uniqueness of the coordinates of the vertices. Hence, this approach is problematic in practice.

Kunegis, Fay, and Bauckhage[26], suggest that since evolution or changes in a graph over time will lead to changes in its spectrum therefore an Eigen decomposition of the adjacency matrix can be used to characterise the this change. They then use the networks spectrum for link prediction and also discuss a method to reducing this link prediction problem to a 1D curve fitting problem.

Duchenne, Bach, In-so, et al.[28] formulate the hypergraph matching problem as a maximization of a multilinear objective function over a tensor representing feature permutations. The tensor represents the affinity between tuples of features. A multidimensional power method is used to solve the problem and the solution is then projected onto to the closest assignment matrix. The power method utilises a tensor Eigen decomposition and is applied to point matching using some similarity measure.

2.8 Visual Methods

More recently, the authors in Behrisch, Bach, Riche, et al.[29] have proposed visualising dynamic networks and characterising change by visualising the adjacency matrix of these networks as a matrix cube. Representing the adjacency matrix as a stack of cubes rather than node link diagrams is found to be a much more useful paradigm for analysis of dynamic networks especially when these networks are dense.

Behrisch, Bach, Hund, et al.[30] have proposed Matrix Diagnostics which ranks matrix views according to the appearance of some visual patters such as lines and blocks. These are taken as a proxy for network features such as clustering. This approach has been designed to aid in the analysis, query and identification of matrices with similar patters when there are a large collection of matrices. This can also be used to judge the effectiveness of the matrix reordering methods. These methods are particularly suited to dynamic networks as it can identify most similar matrices in a graph time series by their appearance similarity.

Chapter 3

Methods

3.1 Analytical Tools

The dataset chosen for use in this study is the Enron email data set. However this data has many different versions with multitude of pre-processing applied to it and the provenance of which is difficult to ascertain. Therefore, for this study I use the version of the data from John Hopkins [31]. The data set is in (time, from, to) tuple format. Here the time is encoded as seconds elapsed from 1 Jan 1970. The from and to fields are node numberings. These could be potentially mapped to employee id's but a viable data set to perform this particular operation had not been found so it was decided not to perform this operation but continue the analysis with the nude numbers.

The analysis is performed entirely using open source tools namely the IPython using the following packages:

1. NetworkX - for graph analysis[32]
2. Numpy - for numerical computation [33]
3. Scipy - for statistical functions[34]
4. Pandas -for data structures and data analysis [35]
5. Matplotlib - for plotting [36]
6. Seaborn - for visualisation[37]
7. Scikit-learn - for matrix decompostion routines[38]
8. Librosa - for audio analysis[39]
9. PyAbel - for Abel Transforms [40]

3.2 Analysis Approach

Firstly, the network is decomposed into time steps at the year and monthly level. This was done by using the timestamps and converting them into dates. Once the dates had been derived the data was decomposed further into yearly time steps and the individual years were split further into monthly time steps. Not all months are

available for all years so for the period considered the yearly data is for the years 1998-2002 and the monthly data starts from November 1998 to April 2002. This gives 5 yearly time steps and 48 monthly time steps. At each step of the analysis the yearly and monthly time steps a range of network measures are used to characterise the change over time.

The first step is to conduct some exploratory analysis of the data. This is done through primarily checking that the data has been subset properly into the years and months and then by visualising the networks. The visualisation of the networks also confirmed that the networks at both the monthly and year level are not necessarily dense. This could result if there is some aggregation in the data for example the network for 1999 contained data for the 1998 and so on. Therefore as an additional check the network sizes are compared with and without aggregation. The networks sizes and visualisations resulting serve as an additional Quality Control (QC) measure to give us confidence in the remainder of this analysis.

Once the networks at the different time steps had been created the exploratory analysis was conducted using the measures listed above. These measures serve as the benchmark measures for the graph time series. The reason being that these measures are widely used and understood. They serve to establish the potential ground truth in this dataset and the signal that they represent can be used to asses the new measures proposed in this study.

The measures described in this section are then derived for both the monthly and yearly networks. All attributes are scaled to [-1,1] to enable comparison. Also this helps when I perform regression analysis for feature ranking.

These derived attributes are compared to the benchmark measures and deviations noted as points of interest. The correlation among all these measures are explored. This leads to the derivation of a correlation network of all attributes and correlation matrix which is reordered with cluster indices from hierarchical clustering and only plotted for measures where the correlation is > 0.7 . In addition I utilise Multi Dimensional Scaling of the final attribute volume to visualise the closeness of attributes in addition to the Correlation Network.

Also the issue of combining these various metrics into an overall global measure of network activity is explored. Here I use the Emergence and Persistence measures suggested by Wei and Carley[41]. Also I propose a novel measure of aggregation based on the Normalised RMS or NRMS that is commonly used in 4D seismic analysis to compare two surveys at different time points a so called repeatability measure. The NRMS measure can be thought of as a normalised RMS difference between two traces. In this case the attributes are aggregated by RMS at each time step.

Having aggregated all these measures into the Persistence and Emergence measure so assess the importance of these new attributes I performed regression analysis with a Gradient Boosting Regressor. This regression analysis is done in two stages. In the both cases I divide the data into 50/50 test /train set since the dataset is very sample

and I want to prevent over fitting. In the first case I use all the measures and use it to predict the Emergence measure of a network and perform a feature ranking on that basis. The second stage consists of predicting the Average Degree of the network using all the attributes including the aggregate Emergence measure. After this step I again perform feature ranking. Most these feature rankings for these network attributes show that the proposed attributes feature prominently. Especially with regards to the prediction of the average degree which is a fundamental property I show that the mean squared error on prediction is very low on this data using these attributes and that the derived attributes are particularly important in this.

In addition to all the above I isolate common nodes between the networks in the yearly data and then plot their benchmark measures. I find that there are 5 nodes common among the yearly data and that their profile mirrors the signal seen in the yearly data. Some nodes exhibit very strong change which suggests that a small number of nodes may be driving the change that we see in the graph time series. The rationale for this thinking is that since these nodes experience large fluctuations in their value and our analysis is based on taking the mean of measures. As the mean is not a robust estimator it is reasonable to expect that presence of such extreme values could skew the signal. But this not completely undesirable in this case because this helps us isolate the interesting periods in our time series and the nodes associated with them.

Finally I look at node level dynamics to explain some of the changes in the network. This is done by identifying common nodes between the yearly time scales and plotting their centrality trends over years.

3.2.1 Graph Matrices

The attributes are derived from the matrix of the graph structure. The graph structure can be represented by different types of matrices such as the adjacency matrix, the Laplacian matrix and the Modularity matrix among others. It is important to note that in this study whenever the Laplacian is referred to the Normalised Laplacian is being referred to for brevity. This is because as Anderson[42] note that the eigenvalues derived from the Normalised Laplacian relate well to other graph invariants for general graphs that is not the case with other matrices such as the adjacency matrix. Another advantage noted is that the definition is more consistent with the eigenvalues derived from spectral geometry. Spectral geometry is an extension of spectral graph theory which incorporates more of an geometric approach in deriving graph properties through methods such as random walks and mixing of Markov Chains among others.

Brouwer and Haemers[27] describes the **Adjacency matrix** of a graph as a [0,1] matrix indexed by the vertex or node set of a graph $A_{xy} = 1$ if there is an edge from node x to node y and 0 otherwise. In the case of Multigraphs or graphs with loops this 1 is replaced by the count of the edges between nodes x and y.

The Normalised Laplacian or Laplacian in our case is defined by Anderson in the following manner. If we consider a matrix, L of a Graph where d_v is the degree of node v and the matrix L is defined as follows:[42]

$$L = \begin{cases} d_v, & \text{if } u = v \\ -1, & \text{if } u \text{ and } v \text{ are adjacent} \\ 0, & \text{otherwise} \end{cases}$$

Then the Laplacian matrix of the Graph can be defined as:

$$\Gamma = \begin{cases} 1, & \text{if } u = v \text{ and } d_v \neq 0 \\ \frac{-1}{\sqrt{d_u d_v}}, & \text{if } u \text{ and } v \text{ are adjacent} \\ 0, & \text{otherwise} \end{cases}$$

This can be written as:

$$\Gamma = T^{-1/2} L T^{-1/2} \quad (3.3)$$

Here T is the diagonal matrix where containing the degree values, d_v .

The Modularity Matrix and Modularity function were developed as a means of better identifying community structure in graphs while minimising the influence of random factors. The modularity function, Q is the number of edges that fall in a community minus the expected number of edges in an equivalent network with edges placed at random. This value is high for well formed communities and low for poor communities. This is defined as :[14]

$$Q = \frac{1}{2m} \text{Trace}(X^T M X) \quad (3.4)$$

Here

$$\sum_v k = 2m$$

where v are the degrees of node k and X is the assignment matrix where $\mathbf{X} = (x_{ih})$, $x_{ih} = 1$ if node i belongs to community h and $x_{ih} = 0$ otherwise. M is the modularity matrix defined as:

$$\mathbf{M} = A - \frac{k k^T}{2m} \quad (3.5)$$

Here k is a vector of degree of nodes and A is the adjacency matrix.

Before continuing with the analysis utilising these attributes it was necessary to select which matrix the attributes will be calculated from. As will be shown in the Results Chapter that the attributes from the Laplacian are found to be most suitable. This was determined by firstly calculating all the attributes from the 3 different matrices: Laplacian, Adjacency and Modularity. Then for all the measures from each matrix the Signal to Noise Ratio was calculated as:

$$SNR = \frac{\mu}{\sigma} \quad (3.6)$$

Where the μ and σ represent the mean and standard deviation of each attribute from the respective matrices. From this plot it was noticed that the attributes from the Modularity Matrix had the highest SNR ratio meaning that the information content was greater than the background this is indicated by having a $SNR \geq 1$. From this plot the highest SNR ratio attributes were chosen and then these selected attributes were compared from the 3 different matrices to the benchmark metrics. This surprisingly showed that even though the Modularity Matrix gave attributes with high SNR that these attributes failed to model the trend the benchmark measures showed. The measures were far too constrained to capture the variability observed. Hence as an additional QC the Mean Absolute Deviation plots of the attributes from the 3 matrices were checked and here it was observed that the attributes from the Laplacian were both numerically more stable and had sufficient variability to capture the trends observed in the data and highlight additional areas of potential interest.

The next step after having picked a matrix for the derivation of the attributes were to calculate the averages of the attributes over time at the yearly and monthly scale. These were then compared against the benchmark measures.

It should be noted that a lot of these attributes proposed here are very commonly used in seismic attribute analysis to help interpret seismic volumes and delineate hydrocarbon reservoirs from surrounding geology. This is the inspiration for the interpretation of these measures here. The hydrocarbon reservoir in this case can be thought of as interesting points in our time series while the surrounding geology can be thought of as the background trend.

3.3 Benchmark Measures

The suite of benchmark measures are discussed here. These measures serve as snapshot of the network at each time step hence we can treat them as measures of similarity or dissimilarity of graphs within the graph time series.

Centrality measures are inherently node level properties and they can be broadly characterised as follows: [6], [8]

- Volume based measures – degree like centrality
- Length based measures – closeness like centrality
- Medial measures – betweenness like centrality

Volume and length based measures are what are called radial measures because they analyse walks that emanate from or terminate with a given node. Medial measures on the other hand are based on position so how many times does one encounter a node while trying to reach other nodes in the network.

We have mentioned some of the most popular centrality measures but there are numerous other variations mentioned in the literature and are beyond the scope of this work the interested reader is referred to the following starter references.[14]–[17]

3.3.1 Degree Centrality

Degree Centrality is a special case of the k-path centrality that counts the all the paths of a length, k that originate from a given node. the k path centrality behaves like the degree centrality when k=1 . This is simply the count of the number of edges incident upon a given node which is equivalent to summing the rows of the adjacency matrix. This is the case in an undirected network while in directed network we have to take the difference between the number of nodes incident upon a node and the number of nodes emanating from a node to get a measure of degree. Since the network under consideration in this study is undirected we will restrict our discussion to undirected interpretations of our measures. [6]

$$c_i^{Deg} = \sum_j a_{ij} \quad (3.7)$$

3.3.2 Closeness Centrality

Closeness Centrality is the graph theoretic distance or the geodetic distance from a given to all the other nodes in a network. This essentially the marginals of a geodetic distance matrix. [6], [8]

$$c_i^{Clos} = \sum_j d_{ij} \quad (3.8)$$

3.3.3 Betweenness Centrality

Betweenness centrality counts the number of times that a certain node, x needs to pass another node, y to get to another node, z through the shortest path between them. [6], [8]

$$c_i^{bet} = \sum_x \sum_y \frac{g_{xky}}{g_{xy}} \quad (3.9)$$

Here g_{xy} is the number of geodesic paths from nodes x to y and g_{xky} is the number of these geodesic paths that pass through the node k.

3.3.4 Eigenvector Centrality

The Eigenvector Centrality is defined as the principal eigenvector of the adjacency matrix of a network. It captures the intuition that nodes that have high eigenvector centrality scores are likely to be close to other nodes which themselves have high values for this measure. [6], [8]

3.3.5 Katz Centrality

Katz[43] introduced this centrality measure which derives the centrality value of a node based on the centrality of its neighbours. This is a generalisation of the Eigenvector centrality measure. The Katz Centrality is defined as:

$$c_i^{katz} = \alpha \sum A_{ij} c_j + \beta \quad \text{where; } \alpha < \frac{1}{\lambda_{max}} \quad (3.10)$$

Here \mathbf{A} is the adjacency Matrix of the graph with eigenvalues λ and β controls the initial centrality.

Katz Centrality measures is also a measure of relative influence of a node within the network because it takes into account the number of immediate neighbours and also

all the other nodes that connect to the node through these neighbours. When the β parameter is set to 0 the Katz centrality is identical to the Eigenvector Centrality. In this study this value is set to 1.

3.3.6 Load Centrality

Load Centrality is described by Goh, Kahng, and Kim, Newman[44], [45] as fraction of all shortest paths that pass through that node.

3.3.7 Density

This is defined as the total number of edges divided by the total number of possible edges. [23]

3.3.8 Diameter

The diameter of a network is the maximum geodesic distance between two nodes.[23]

3.4 Complex Trace Attributes

The Complex trace attributes make use of the Hilbert Transform. So before we introduce the measure an introduction to the Hilbert Transform is presented here.

3.4.1 The Hilbert Transform

The Hilbert Transform is an integral transform that extends a normal and real valued function into the complex plane. This allows the derivation of two useful attributes the Instantaneous Amplitude or Energy Envelope and the Instantaneous Frequency. These fundamental complex trace attributes form the basis for the derivation of the other attributes as they are derivatives of these quantities. A Hilbert transform does not change the domain of the function so a time domain function remains a time domain function while a frequency domain functions remains in the frequency domain. In the time domain this translates to a $\frac{\lambda}{4}$ shift for all frequencies and a -90° phase shift for all spectral components in the frequency domain. [46]

3.4.2 The Complex Trace

As mentioned that these attributes are inspired by their use in seismic attribute analysis so the Complex Trace or Analytical Signal is introduced here here. Essentially it is the signal after the Hilbert Transform has been applied to it so that it has a Real and Imaginary component. Their use in seismic attribute analysis is discussed in detail by Li and Zhao, Subrahmanyam and Rao[47], [48]. The descriptions included here are to build intuition around these attributes and to get a sense of what we can expect them to highlight.

The complex trace is defined as

$$C(t) = S(t) + iH(t) \quad (3.11)$$

Where:

$C(t)$ = Complex Trace

$S(t)$ = Real Data

$H(t)$ = Hilbert Transform of data

Based on the above the Complex Trace Attributes introduced are shown.

1. Instantaneous Amplitude, IA
2. Power

3. Instantaneous Phase, IP
4. Instantaneous Frequency, IF
5. Derivative of Instantaneous Amplitude , dIA
6. Second Derivative of IA , d2IA
7. Instantaneous Acceleration, IAcc
8. Amplitude Weighted Instantaneous Phase
9. Amplitude Weighted Instantaneous Frequency

3.4.3 Instantaneous Amplitude

Instantaneous Amplitude is widely used in traditional tectonic and stratigraphic interpretation. As one of the basic parameters of the amplitude attribute, it helps delineate the high- or low-amplitude anomaly (bright or dark spots). In this context this should highlight the bright and dark spots in the network when used as an attribute map and should show the highest and lowest points when used as a time series. As I show later this amplitude has a high correlation (0.7) with the traditional centrality measures so its behaviour is very similar to those. Therefore it is highly plausible that for this data it should suffice to look at this attribute instead of many different centrality measures.

$$\sqrt{S(t)^2 + H(t)^2} \quad (3.12)$$

3.4.4 Power

The Power is calculated as the square value of the amplitude. In this case I use the Instantaneous Amplitude. This allows us to better understand the signal envelope as this attribute is smoother than the Amplitude.

$$IA^2 \quad (3.13)$$

3.4.5 Derivative of Instantaneous Amplitude

The derivative of IA highlights the change in reflectivity and shows sharp interfaces and discontinuities. Effectively this should highlight the big changes in the IA and the smaller changes should make the attribute smooth. This is what we observe because the attribute highlights the peaks observed in the IA plot and the rest of the signal is fairly smooth for the time range.

$$\frac{d}{dt}(IA) \quad (3.14)$$

3.4.6 2nd Derivative of Instantaneous Amplitude

The second derivative of the IA highlights the interfaces very well - the places of change. This attribute is not too sensitive to the amplitude and can highlight even weak events. We see this to be the case where the first derivative highlights the individual peaks the second derivative smoothes the individual peaks and gives a smooth peak over the range of months where they occur and does a better job of

highlighting the peaks towards the end of the timeseries than the first derivative.

$$\frac{d^2}{dt^2}(IA) \quad (3.15)$$

3.4.7 Instantaneous Phase

Instantaneous Phase is expressed in degrees or radians at the selected sampling point. Instantaneous phase helps strengthen weak reflections in the inner parts of reservoirs and also strengthens the noise. Because the hydrocarbon accumulation often causes phase changes, this attribute can be used as a direct indication of hydrocarbons. The cosine of the instantaneous phase is derived from the instantaneous phase. It is commonly used to improve the variation display of the instantaneous phase because it has fixed boundary values (-1 to +1). In this context the change in the IP can be used as an indicator for the intervals which are the most interesting since they have a noticeable phase change. This is better illustrated by the cosine of IP especially the percentage change plot here we see the months with the greatest phase change really well and the peaks are well delineated in contrast to the just the IP plot.

$$\arctan\left(\frac{H(t)}{S(t)}\right) \quad (3.16)$$

3.4.8 Instantaneous Frequency

Instantaneous frequency is defined as the time derivative of the instantaneous phase and it is used for estimating seismic attenuation. Oil and gas reservoirs often cause the attenuation of high-frequency components, so this attribute is also conducive to measuring stratigraphic periodic intervals. In this context I expect this to show clearly the peaks of the signal. This is exactly what we see especially in the monthly trend where the two large peaks are clearly delineated with some smaller peaks highlighted the attribute is smooth over the rest of the series. So it has the property of finding the most significant peaks in our data.

$$\frac{d}{dt}(IP) \quad (3.17)$$

3.4.9 Instantaneous Acceleration

Instantaneous Acceleration is defined as the rate of change of the instantaneous frequency, which is often used to indicate the rate of attenuation and absorption. As gas (or oil, or water) can cause the attenuation of seismic waves, this attribute can represent a fluid interface in the high-resolution data. As a derivative attribute I expect it to highlight the peaks and troughs very effectively and be smooth in areas of less pronounced change. This is what is observed as the attribute highlights the two large peaks but with the opposite polarity of IF.

$$\frac{d}{dt}(IF) \quad (3.18)$$

3.4.10 Amplitude Weighted Instantaneous Frequency

Amplitude weighted Instantaneous Frequency provides a reliable smooth instantaneous frequency estimation in order to reduce the interference damage. I expect this to better highlight frequency anomalies in the data and suppress insignificant anomalies. We see that the weighted IF is sharper in places where the IF is fairly smooth. This highlights the major peaks as well as minor anomalies not immediately obvious from the IF alone.

$$IA.IF \quad (3.19)$$

3.4.11 Amplitude weighted Instantaneous Phase

The amplitude weighted IP should cause the phase to become sharper with the peaks and troughs more accentuated. This has the effect of magnifying the signal as well as the noise. However, in this case we see that the trend in IP weighted and regular IP plot are identical with the only difference being the magnitude of the peaks.

$$IA.IP \quad (3.20)$$

3.5 Matrix and Matrix Decomposition Attributes

The novel matrix decomposition based metrics proposed in this study are:

1. Norm of the Forward Abel Transform
2. Mean Gaussian Curvature
3. Kernel PCA 3 Component Ratio Change
4. Norm NMF Ratio Change

In addition some measures suggested in the literature are also implemented such as:

1. Resistance Distance [49]
2. Stationarity Ratio [50]
3. Subgraph Stationarity [51]
4. Power Spectral Density [50]

3.5.1 Norm of Forward Abel Transform

The Abel transform finds a slice of a cylindrically symmetric 3D object and provides a 2D projection of it. In essence it does a dimensionality reduction by finding a lower dimensional subspace for the higher dimensional projection. The Inverse Abel Transform recovers the original higher dimensional data space. In this attribute I use the Forward Abel Transform through direct numerical integration of the Abel equations.

The Abel Transform attribute is calculated as follows:

1. For the Normalised Graph Laplacian, Γ take the Fourier Transform, $F(\Gamma)$.

2. Calculate the magnitude of the Fourier Transform of the Graph Laplacian

$$\sqrt{Freq_{real}^2 + Freq_{imag}^2}$$
3. Take the Forward Abel transform of the magnitude of the Fourier Transform
 $A(F(\Gamma))$
4. Take the Norm of the resulting matrix at each time step

This results in a lower dimensional representation of the Frequency content of the Laplacian Matrix. This is useful for highlighting change over time as this attribute and highlights when the underlying networks have expanded or contracted with regards to their activity.

3.5.2 Mean Gaussian Curvature

There are many ways to define curvature I use the Gaussian Curvature which is derived from the Hessian Matrix. A surface might be curved upward in places, curve downward in places, or even be flat in places. Also, at some given point, the surface may be curved upward in some directions and downward in others. The curvature measure helps us detect such change. Gaussian curvature can be positive, negative, or zero. A useful property of the curvature attribute is that it is independent of orientation of the surface. If we place vectors on this surface the would indicates where the curve bends i.e., the vectors are either diverging, converging, or parallel. This can correspond to the negative, positive or zero value of curvature.

$$K = \frac{f_{xx}f_{yy} + f_{xy}f_{yx}}{(1 + f_x^2 + f_y^2)^2} \quad (3.21)$$

The mean curvature is the trace of the eigenvalues of the Hessian Matrix

$$K_{mean} = \text{trace}(\lambda_{hessian}) \quad (3.22)$$

3.5.3 Kernel PCA 3 Components Ratio

Here I calculate the Kernel PCA 3 Component ratio from the Normalised Graph Laplacian. This is the ratio of the main element difference between two networks. The reason a Kernel PCA is used is because of its ability to handle non-linearity through kernels. Here the RBF kernel is used to fit the Normalised Graph Laplacian and then the ratio is calculated as follows:

Step 1: Fit and transform the Normalised Graph Laplacian keeping only the first 3 components: PC1, PC2 and PC3

Step 2: Calculate the Kernel PCA Ratio

$$KPCA_r = \frac{PC1 - PC3}{PC1 - PC2} \quad (3.23)$$

Step 3: For all the networks calculate the change in the KPCA Ratio as:

$$\Delta K_r = \frac{K_{rt0}}{K_{rt1}} \quad (3.24)$$

recursively to derive a rolling measure of the change in KLPCA Ratio over time.

The KLPCA is one of the main element analysis methods in seismic attribute analysis where it is used to calculate the correlation in a multitrace window. A small value represents a degree of intermittent or no correlation of geological phenomena. It is also used to detect discontinuities, such as faults and unconformities. Here the purpose is to locate big discontinuities in our networks over time. This represents a scalable and easy way to locate the biggest changes even when we are dealing with a large number of dynamic networks. Most of the values encountered are relatively small. But from the monthly plot we see that there is a large discontinuity in June and July 1999 whose scale dominates the plot. Hence the log of this attribute is used and we see other smaller signals emerge as a result.

3.5.4 Norm Non-Negative Matrix Factorisation Ratio

This is another metric derived from the Non-negative Matrix Factorisation of the graph matrix. The NMF involves finding two non-negative matrices (W , H) whose product approximates the non-negative matrix X . I take the Frobenius Norm of the NonNegative components derived from the data and calculate change in this ratio over time.

I calculate the NMF Ratio change as:

$$NMF_r = \frac{\|WH_{t1}\|}{\|WH_{t0}\|} \quad (3.25)$$

3.5.5 Resistance Distance

The resistance distance between vertices i and j of a graph G is defined as the effective resistance between the two vertices (as when a battery is attached across them) when each graph edge is replaced by a unit resistor. This resistance distance is a metric on graphs.

I calculate the resistance distance as:

M = Graph Matrix

N = Length of M

P = Moore-Penrose Pseudo Inverse of M

D = The diagonal of P

$$R_d = (D \otimes (N, 1))^T + (D \otimes (N, 1))^T - P - P^T \quad (3.26)$$

Here \otimes denotes the Kronecker or outer product

3.5.6 Stationarity Ratio

The Stationarity Ratio is based on the SVD decomposition of the graph matrix and then calculated as a ratio of the norm of the diagonal elements to the norm of the derived matrix. This is calculated as follows:

L = graph matrix

U = eigenvalues of L

C = covariance of L

$$CF = L(U^T C U)$$

$$S_r = \frac{\|diag(CF)\|}{\|CF\|} \quad (3.27)$$

3.5.7 Subgraph Stationarity

This is computed from the adjacency matrix of graphs and is done in two steps. Essentially this is comparing the common subgraphs between two networks and deriving a correlation score. This is done in two steps.

Step 1: Calculate Correlation, C_t between the graphs at time, t and time, $t+1$.

$$C(t) = \frac{A(t) \cap A(t+1)}{A(t) \cup A(t+1)} \quad (3.28)$$

Step 2: Calculate the Subgraph Stationarity, ζ

$$\zeta = \frac{\sum_{t=0}^{tmax-1} C(t, t+1)}{tmax - t0 - 1} \quad (3.29)$$

From the Subgraph Stationarity ζ we can calculate the amount of members that change at each time step as:

$$1 - \zeta \quad (3.30)$$

3.5.8 Power Spectral Density

The Power Spectral Density (PSD) describes the distribution of power over frequency for a given time series or signal. The Power in this case can be thought of as not necessarily as physical power but as squared of the signal. This is similar to the Power attribute we introduced earlier but this allows us to understand its distribution. [52]

This is related to the auto correlation function and thus we can utilise this attribute like the amplitude to identify Amplitude or Power Anomalies.

3.6 Music Attributes

3.6.1 The Fourier Transform

I briefly describe the Fourier Transform as this is essential in derivation of the Music Attributes. The Fourier Transform is used here to extract the frequency components of the Normalised Graph Laplacian. Since for audio applications we need either a stereo or a mono channel we combine the frequency components into 1 audio channel by averaging across the rows of the frequency components. This results in a single audio channel which represents the underlying graph. From this it is possible to extract a number of Music Attributes such as the Zero Crossing Rate and Spectral Centroid. These two are chosen as they have intuitive interpretations which can be easily understood. Other attributes related to beats and tempo are more difficult to explain thus could be the subject of further work but not actively explored in this study.

The Fourier Transform is discussed in detail by Tao[53]. As with the Hilbert Transform the focus here is to build intuition and the reader is referred to materials cited for more thorough treatment of the subject.

The Fourier Transform allows us to decompose functions in a systematic way into a superposition of symmetric functions such as trigonometric functions. These are related to physical concepts such as frequency and energy. The Fourier Transform is a reversible linear transform and is fundamental in the study of groups and is related

to many linear algebra topics such as representing a vector as a linear combination of an orthonormal basis or as linear combinations of eigenvectors of a given vector. Changing the basis of the eigenvectors allows for the calculation of the Fourier Basis on Graph which will allow for more broader range of signal processing methods to be applied. [50]

3.6.2 Zero Crossing Rate

The Zero Crossing Rate (ZCR) is perhaps one of the most widely used measure in the field of speech and audio analysis. This is simply the number of zero crossings or the number of time the signal crosses the zero line within a defined region or a window over the signal. [54], [55]

The ZCR measure can be evaluated over a fixed, moving or variable sized window. This measure estimates for the waveform complexity in the time domain. It is very useful in providing a general trends with regards to the level of the overall frequency content. For this study we utilise this measure in the frequency domain this is also called the Spectral Zero Crossing Rate but the Zero Crossing Rate is used in the discussion because the underlying data is audio. The ZCR in the frequency domain provides information related to the transients location within a time window. Since the dataset is essentially a graph time series. The audio waveforms can also be thought of as a Audio Time Series. Hence we are essentially characterizing the whole audio signal at each time step by their average ZCR. This gives a measure of similarity for comparing the waveforms. The higher ZCR is indicative of greater change in the signal envelope. In this case since we are using the average of the ZCR we can interpret this as highlighting the fact the signal here is changing more rapidly compared to others which also leads to these regions having higher Power. This helps identify interesting time periods in the graph time series. [55]

3.6.3 Spectral Centroid

The Spectral Centroid is measure for the characterisation of spectra and is widely used in digital music to classify the brightness of a sound. It calculates the centre of mass of a signal using the weighted mean of the frequencies. The frequencies are derived via the Fourier Transform and the weights used are the magnitudes which are similar to the Amplitude. This could be thought of as being similar to the Amplitude weighted Frequency attribute introduced earlier. But instead of the Hilbert Transform the the audio signal derived from the Fourier Transform is used instead. [56]

3.7 Aggregation Schemes

3.7.1 Persistence and Emergence

The Persistence measure used in this study utilised the time averaging of attributes. This can be stated as

$$P_{it} = \text{Agg}(m_1, \dots, m_n) \quad (3.31)$$

The aggregation measure here is averaging but could be any number of aggregation functions such as linear or exponential aggregations are also possible. This measure is essentially all the measures at a time step averaged into one value and then normalised by the length of the time series or $\max(t)$. The Emergence measure is then just the normalised Persistence measure depending on whether the Persistence values are positive or negative.

$$E_{i,t} = \begin{cases} 0, & \text{if } P_{i,t} = P_{i,t-1} \quad \text{or } t = 1 \\ \frac{P_{i,t} - P_{i,t-1}}{N_{i,t}} & \text{otherwise} \end{cases}$$

$$N_{i,t} = \begin{cases} \max(P_{i,t}, P_{i,t-1}) & \text{for non-negative Persistence} \\ ||P_{i,t}|| + ||P_{i,t-1}|| & \text{otherwise} \end{cases}$$

This is comprehensive measure that allows all the above measures to be combined into a single measure for network characterisation. The Emergence measure is an indicator of network activity bursts can indicate periods of growth while troughs can indicate contraction. They help give an high level overview of the trend in the graph time series and captures network dynamics well when we use the attributes because they characterise the trends in the network the traditional measures do not highlight as well.

3.7.2 NRMS of Network Attributes

The NRMS measure can be though of as a normalised RMS similarity measure in the sense that when two traces are similar the NRMS value will be close to 200 and when there is a great dissimilarity it will be less than 200. This constant helps to exaggerate the trends in the data while suppressing noise due to the use of the RMS operator.

The NRMS measure is defined as follows:[57]

$$NRMS = \sqrt{\frac{200RMS(a - b)}{RMS(a) + RMS(b)}} \quad RMS = \sqrt{\frac{\sum(x)^2}{n}}$$

3.8 Visualisations

3.8.1 F-K Plot

The F-K Plot is a visualisation technique used in seismic data analysis that isolates the signal in a central cone and separates the noise to enable easy filtering in the FK domain. The FK plot consists of the Frequency,f and Wavenumber, k. The Frequency is derived by taking the Fourier Transform of the attribute volume. The wave number is then calculated as the reciprocal of the frequency. Plotting this also highlights that most of the signal is concentrated near the the central cone with k values around 0 and that the outliers have high/low values and are thus separated from the signal cone. These frequency indices from the F-K plot can be used to order the data and we can highlight which the months that standout on the F-K plot.

In addition the Frequency, F and Wavenumber, k can be visualised individually as attribute maps with mappings to the months and attribute names. This allows for the identification of anomalies.

3.8.2 Radon Plot

The Radon Transform is an integral transform that has various applications ranging from image processing, medical imaging, computer vision and seismic analysis. The transform can be defined as

$$R(p, \tau)[f(x, y)] = \cup(p, \tau) \quad (3.34)$$

Here the p is the slope and τ is the intercept. In the image analysis context the Radon Transform computes projections along a axis. Since the Radon transform used here comes from an image analysis package this is the form utilised in this study. The plots of the projection of the Radon components are plotted.

The Radon transform is used in seismic data analysis to separate events based on their velocities. This is possible with the $\tau - p$ implementation of the Radon Transform. However, in image analysis the projection method is used. Since this particular implementation computes a sinogram consisting of projections of the input at different angles the analogy to velocity here is not possible. Typically the Radon plot is used where the p trace represents the inverse of velocity and the τ represents time at zero offset or intercept. But the transformed space is still very useful as a form of visualisation for the high dimensional attribute volume.

This plot helps easily visualise clusters in the data and is an additional visualisation tool that can also be used for outlier detection and filtering. The original data can also be reordered from the Radon Matrix in order to see which are the highest and lowest values in the context of the Radon plot. The resulting Radon Matrix can also be visualised as a Heatmap hence the trends observed in the Radon plot can be linked to the graph time series.

3.8.3 Log Panel

The Log Panel is a multi-attribute visualisation technique where the y-axis is the time series and the x-axis represents the range of the values. This helps to put multiple attributes side by side and enable the tracking of peaks of troughs in addition to being very conducive for multiple attribute interpretation.

3.8.4 Audio Waveform Plot

The Audio Waveform Plot is a plot the amplitude envelope of a waveform. In our case the audio signal is one channel so we get a monophonic plot. The length indicates the length of the frequency trace which is a proxy for the length of the matrix. The frequency content is determined by the Laplacian Matrix which is a proxy for Degree. So high Degree correlates with high frequency which correlates to spikes in the waveform plot. This is a very compact representation of the graph matrix as we are able to gain both an overview of the characteristic of the network as well as a sense of the node level dynamics driving those changes.

Chapter 4

Results

4.1 Results Overview

The presentation of the results are conducted in a manner that allows for increasing level of granularity. This means that I start with a global overview of the networks at the yearly and monthly aggregation levels. Then the visualisation modalities for these networks are presented. The exploratory analysis and the benchmark measures allows us to get an understanding of the signal in the graph time series. The features are then introduced along with some correlation and regression analysis that helps us derive additional value. Finally the remaining visualisation methods and node level trends are presented.

4.2 Network Visualisation

In this section I present the three modalities of network visualisation used in this study namely the node-link diagram, reordered matrix and audio waveform plots.

The yearly networks are presented first followed by the monthly networks. It is important to note that these visualisations helps us understand some of the strong signal that we will observe in our measures. This is primarily due changes in network size. For the monthly networks this corresponds to unusually low network activity during some months which causes the measures to spike in most cases.

4.2.1 Yearly Network Visualisations

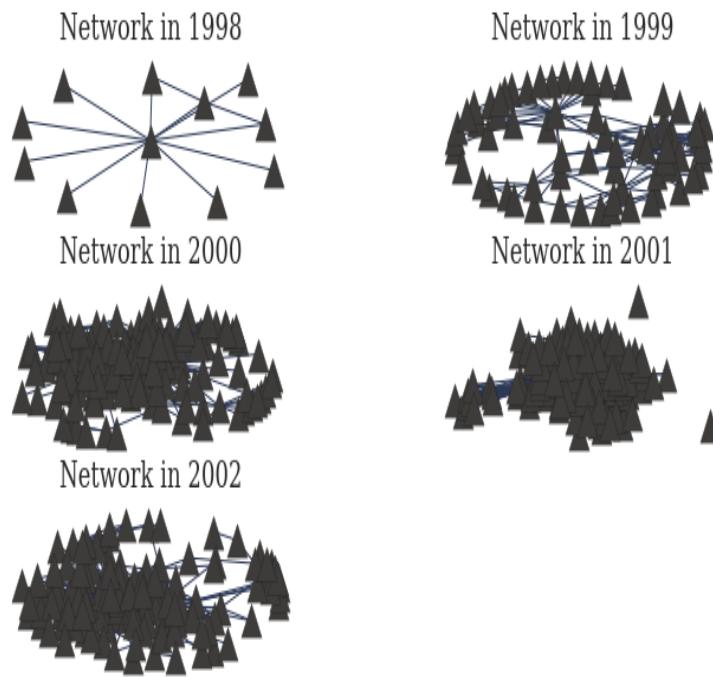


Figure 4.1: Node Link Diagram of yearly networks

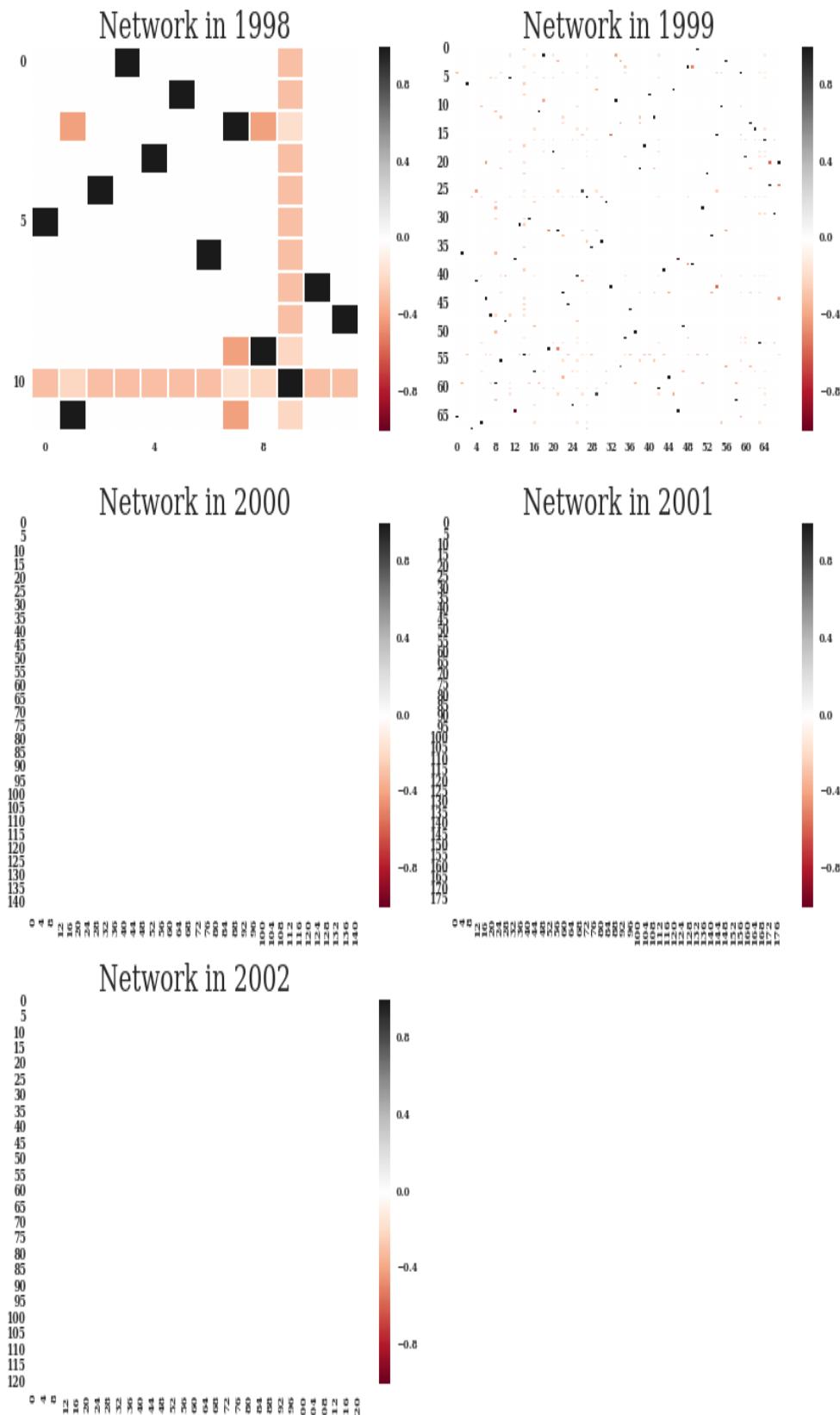


Figure 4.2: Reordered Matrix Diagram of yearly networks

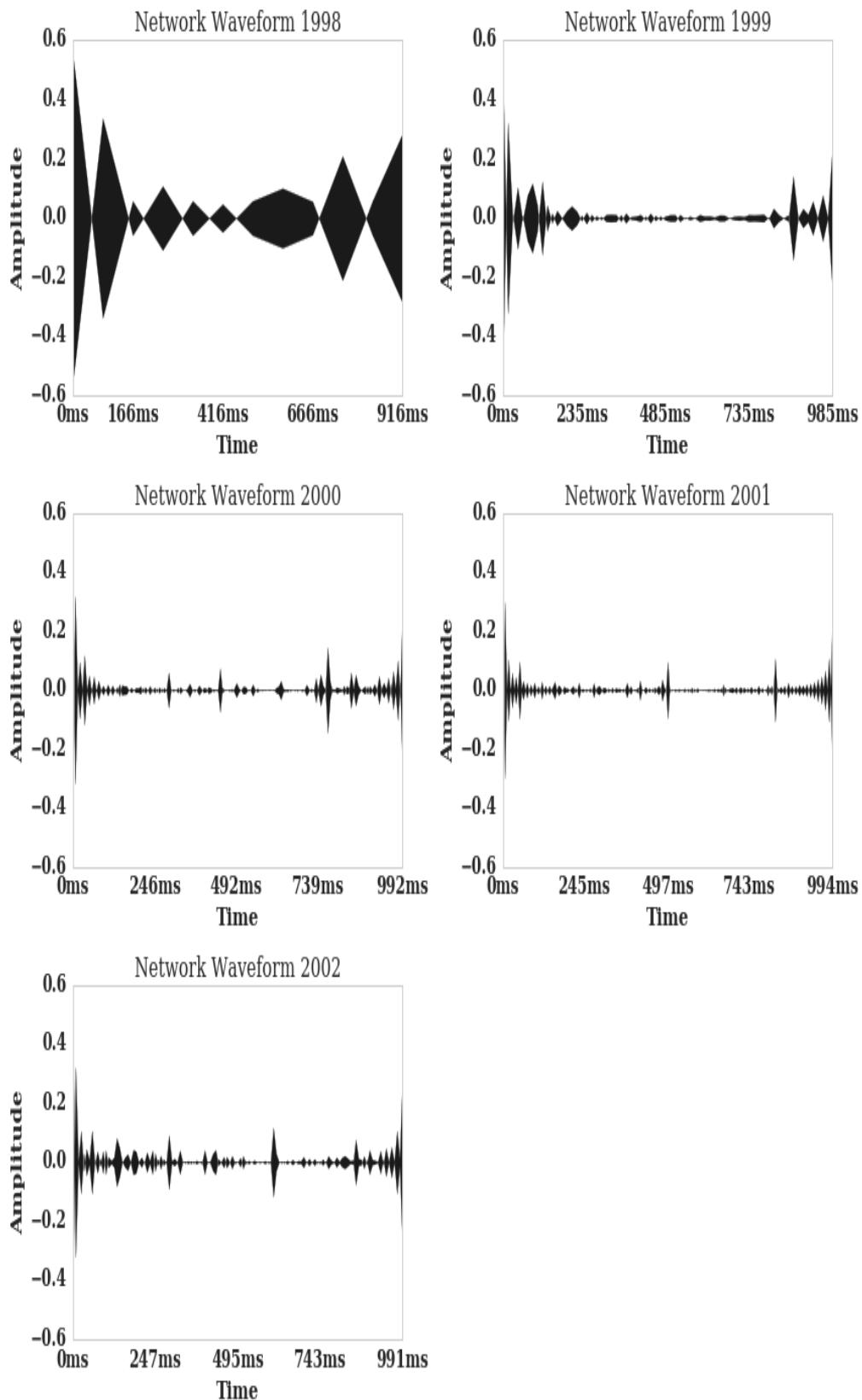


Figure 4.3: Audio Waveform plot of yearly networks

4.2.2 Monthly Network Visualisations

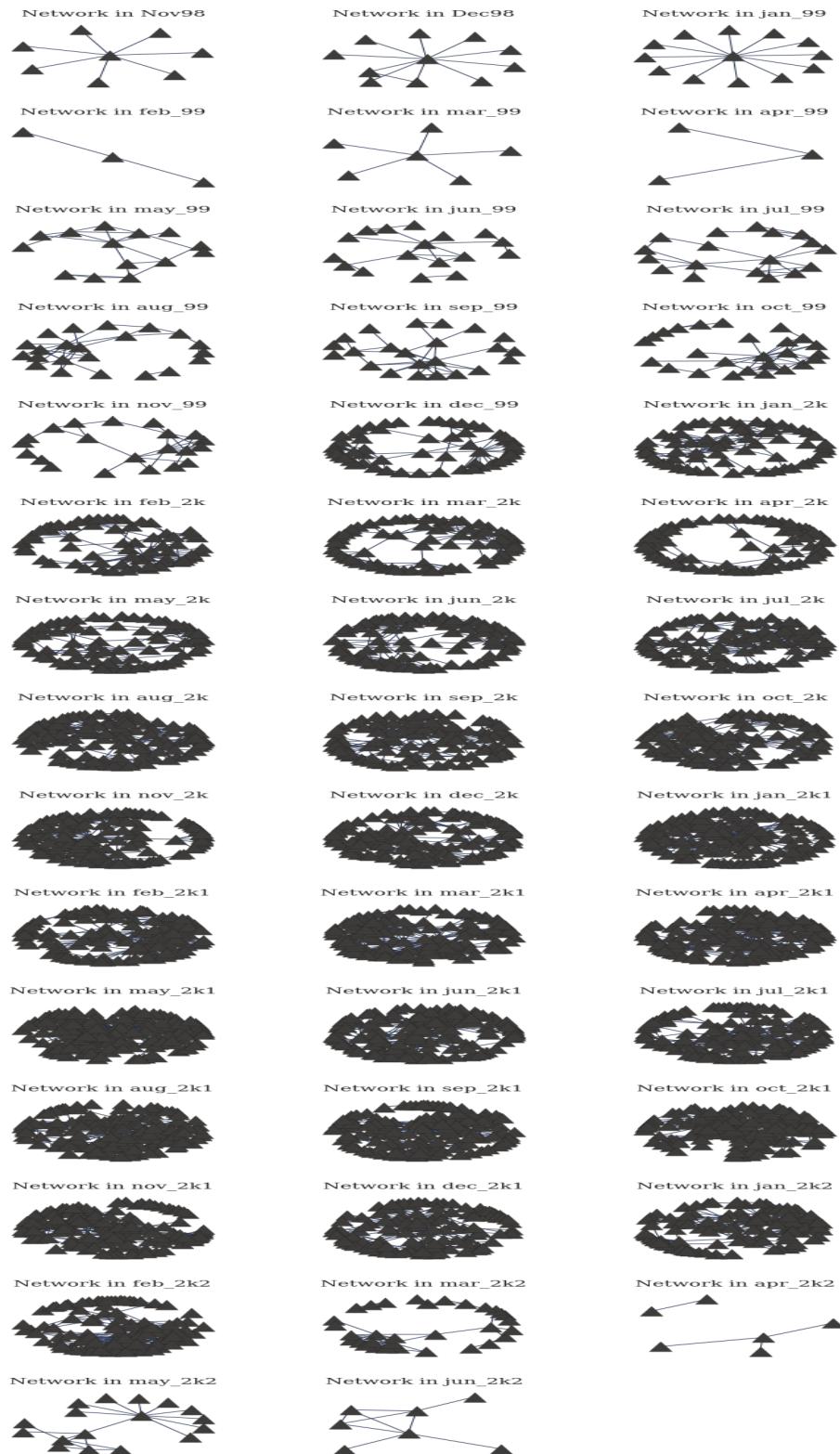


Figure 4.4: Node Link Diagram of monthly networks

Analysing the evolution of communication patterns in email data through an extended dynamic network analysis toolkit



Figure 4.5: Reordered Matrix of monthly networks

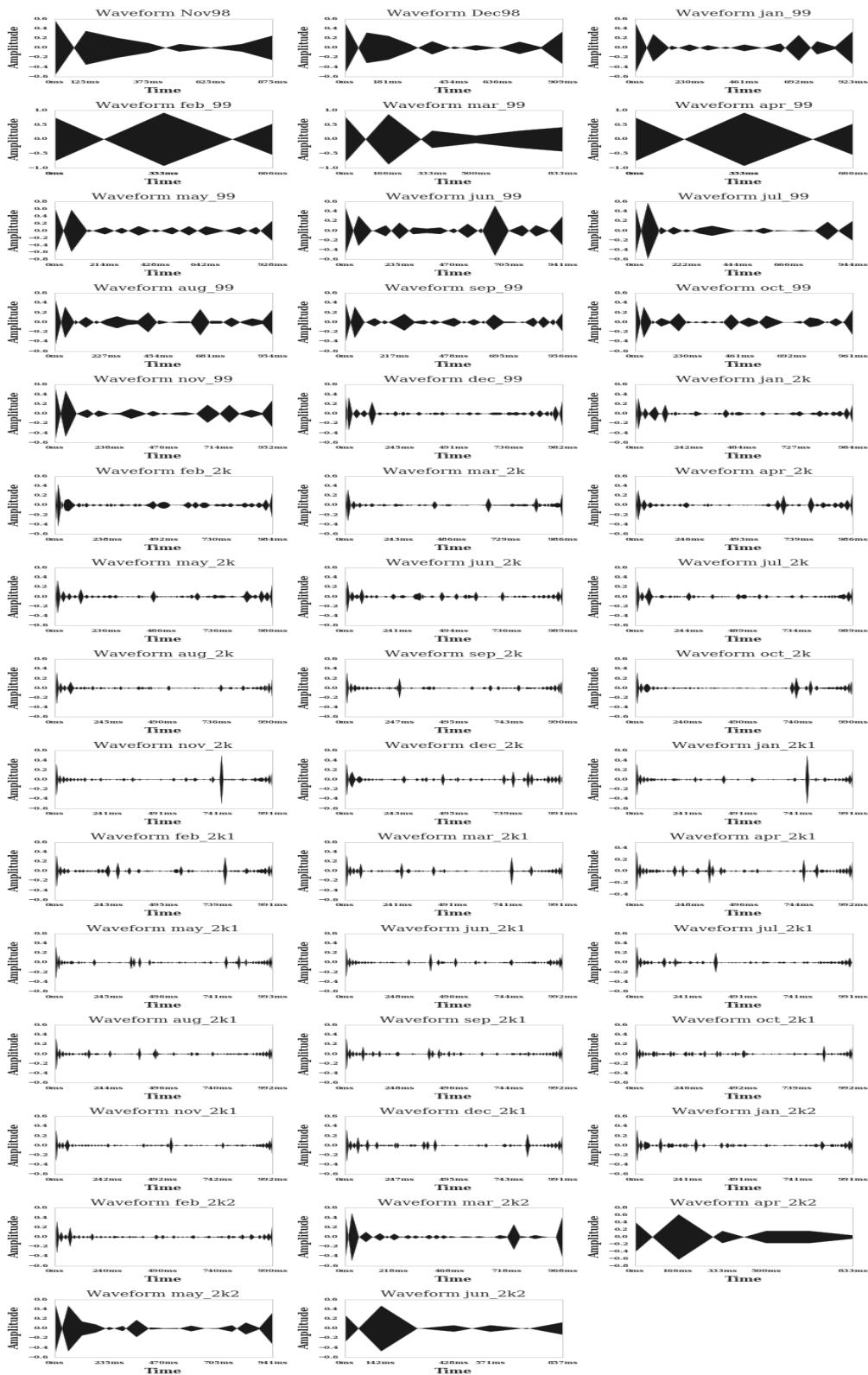


Figure 4.6: Audio Wavefrom of monthly networks

4.3 Exploratory Analysis: Centrality Measures

4.3.1 Yearly Analysis

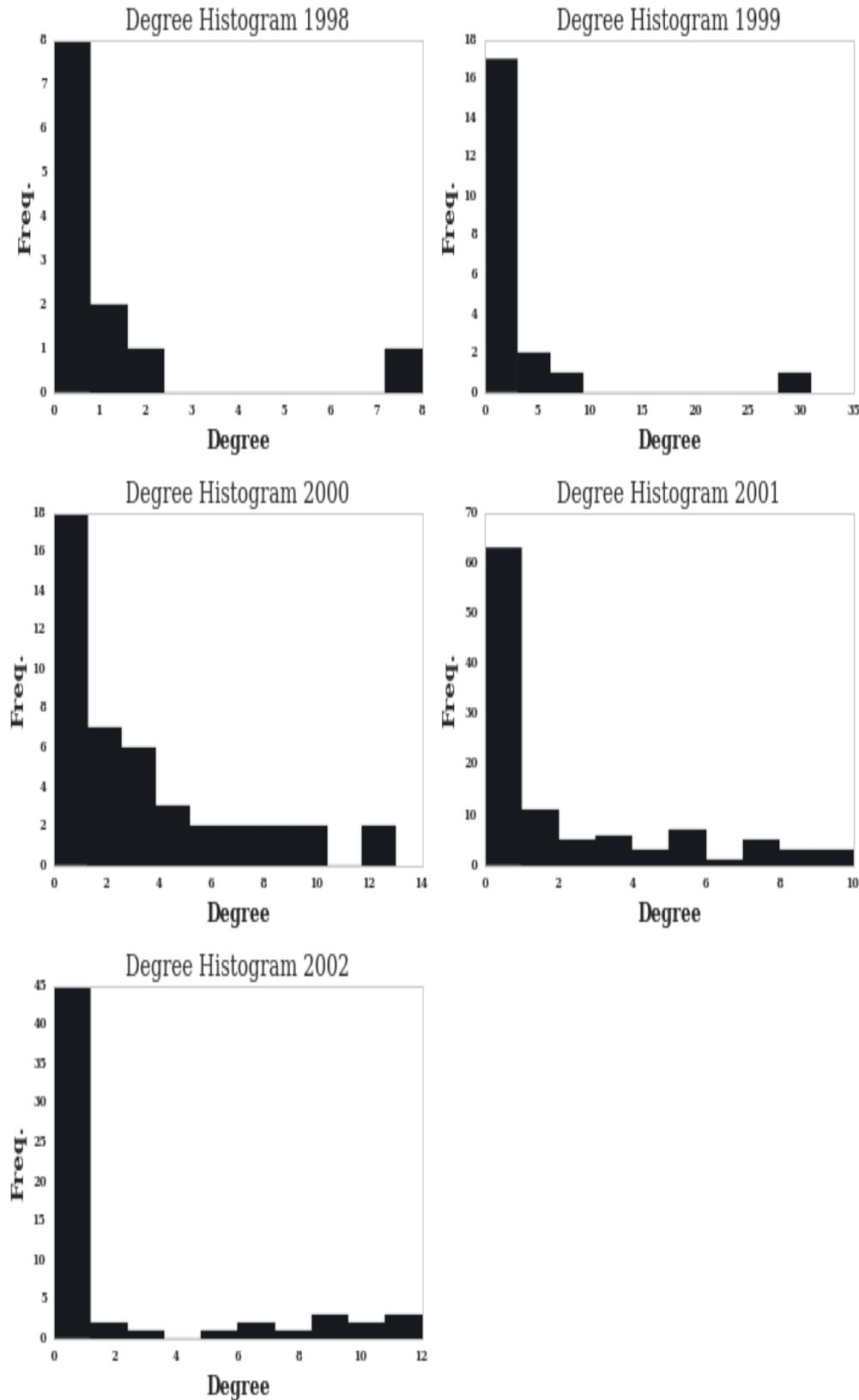


Figure 4.7: Yearly Degree Histogram

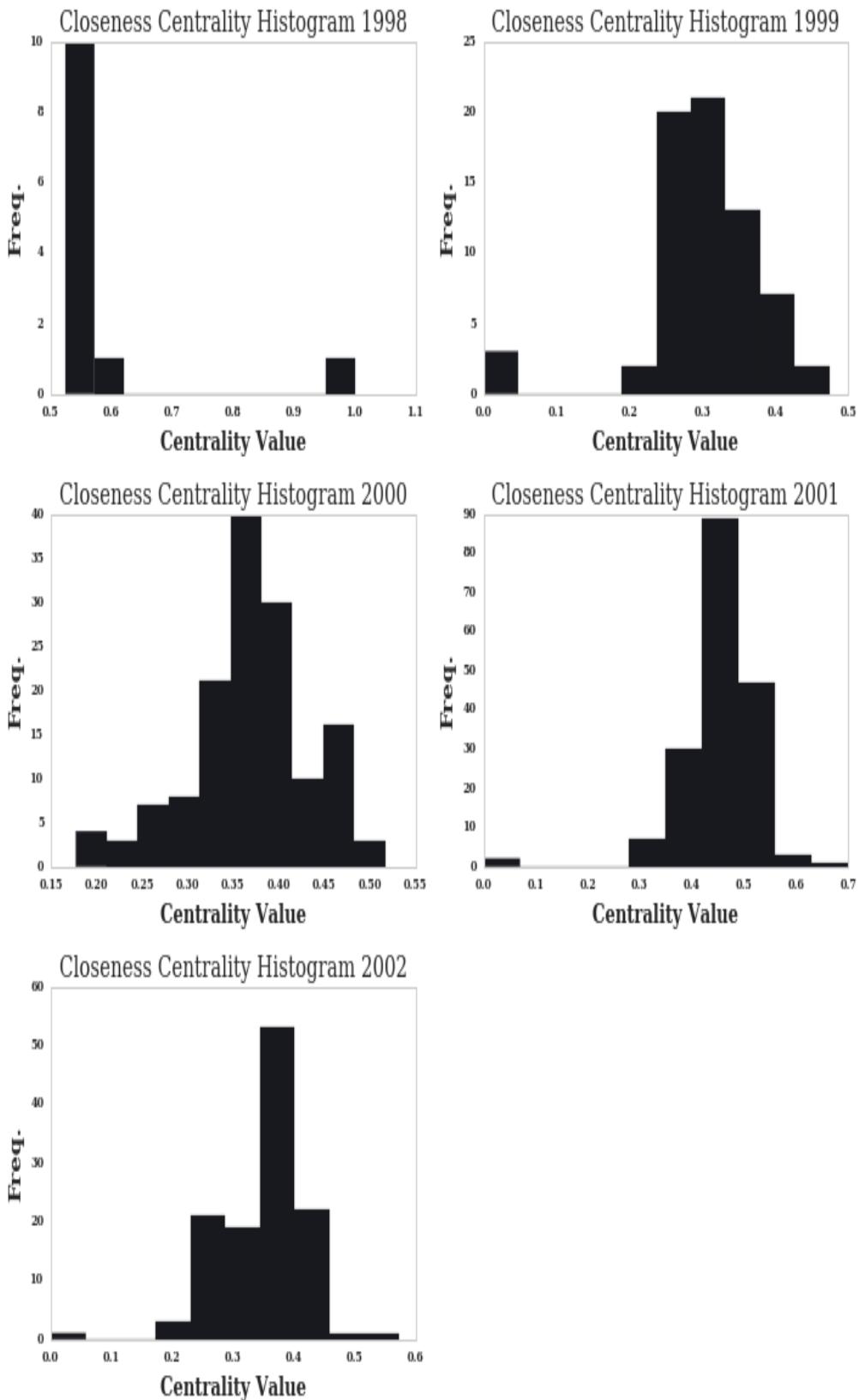


Figure 4.8: Yearly Closeness Centrality Histogram

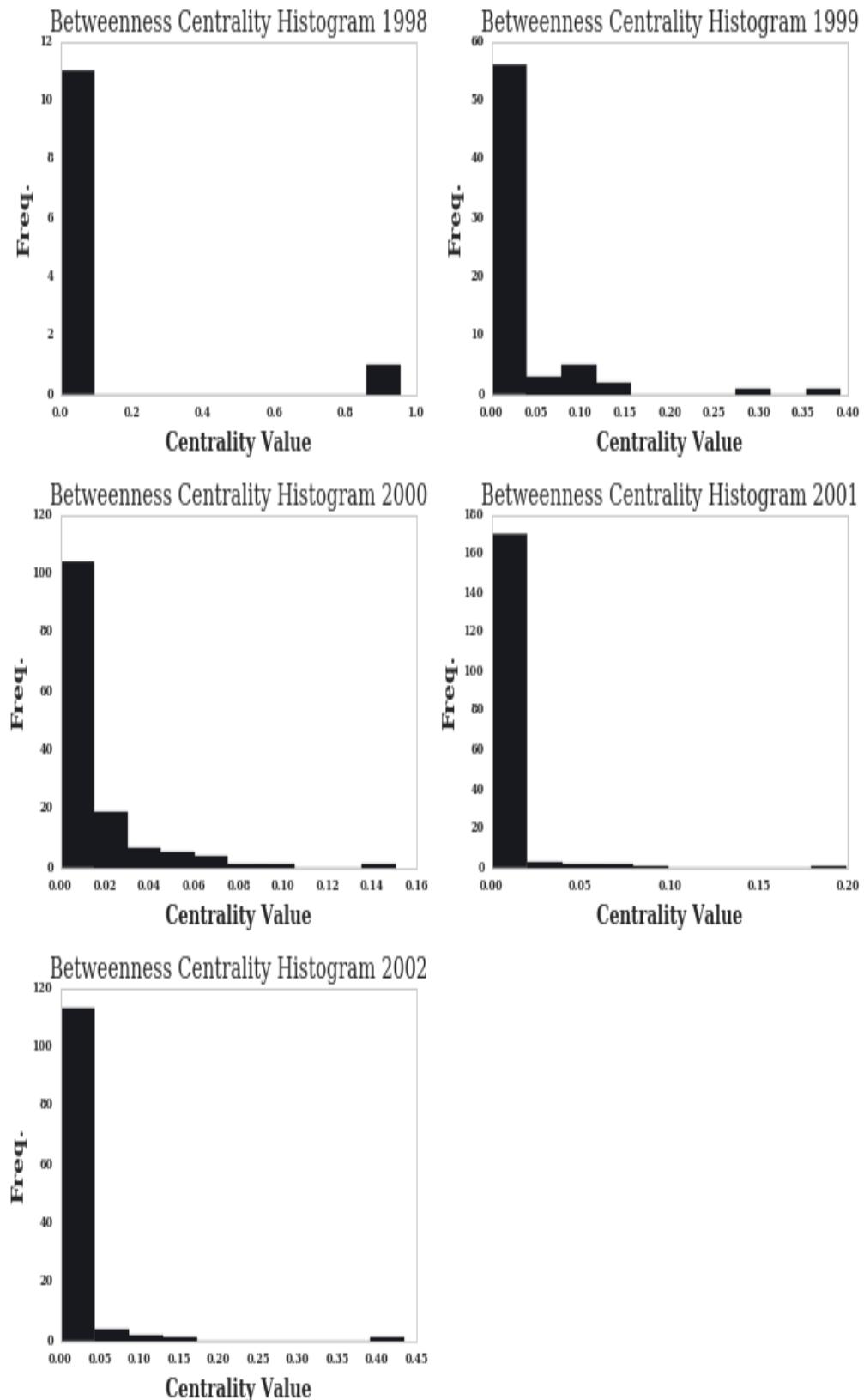


Figure 4.9: Yearly Betweenness Histogram

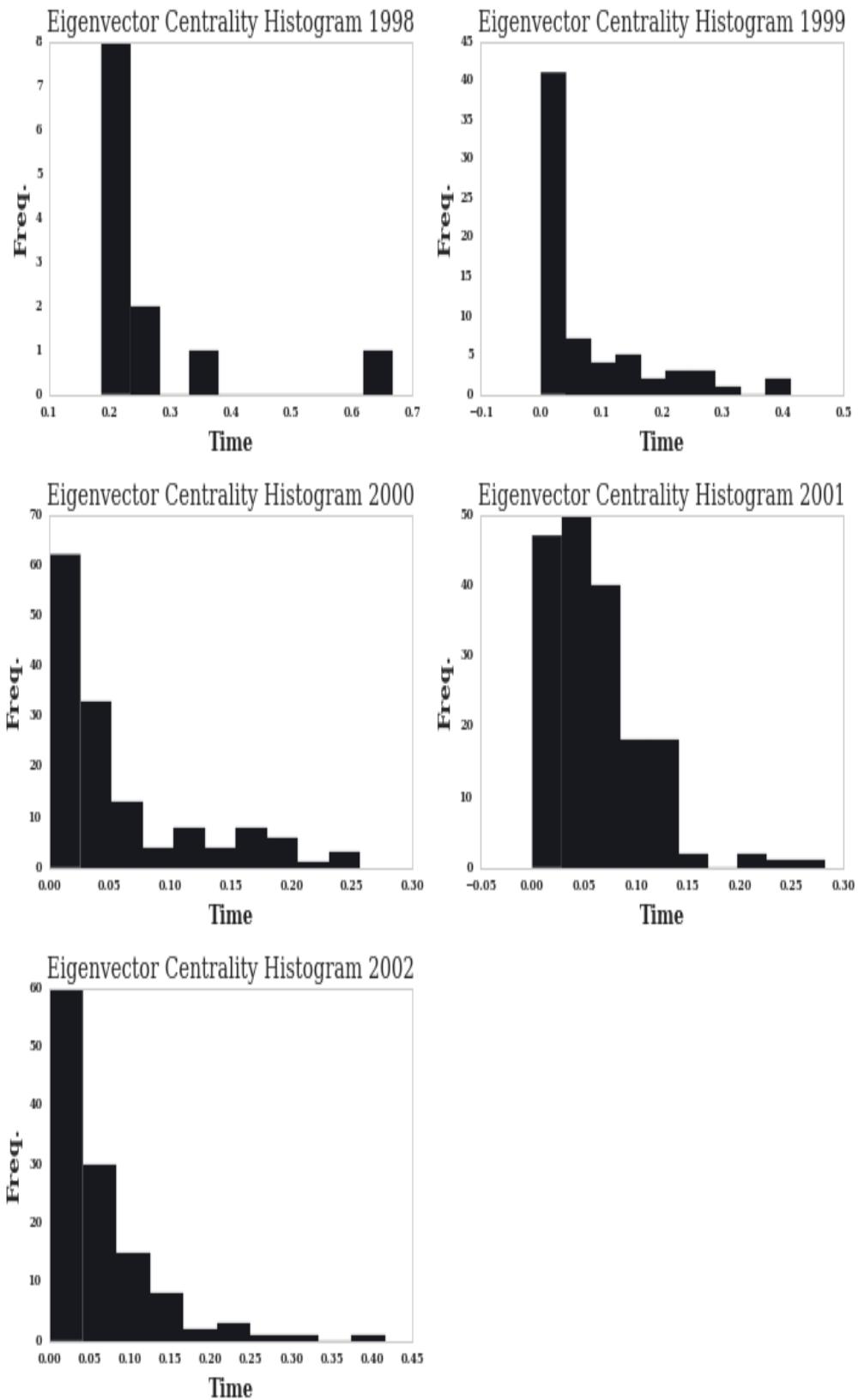


Figure 4.10: Yearly Eigenvector Centrality Histogram

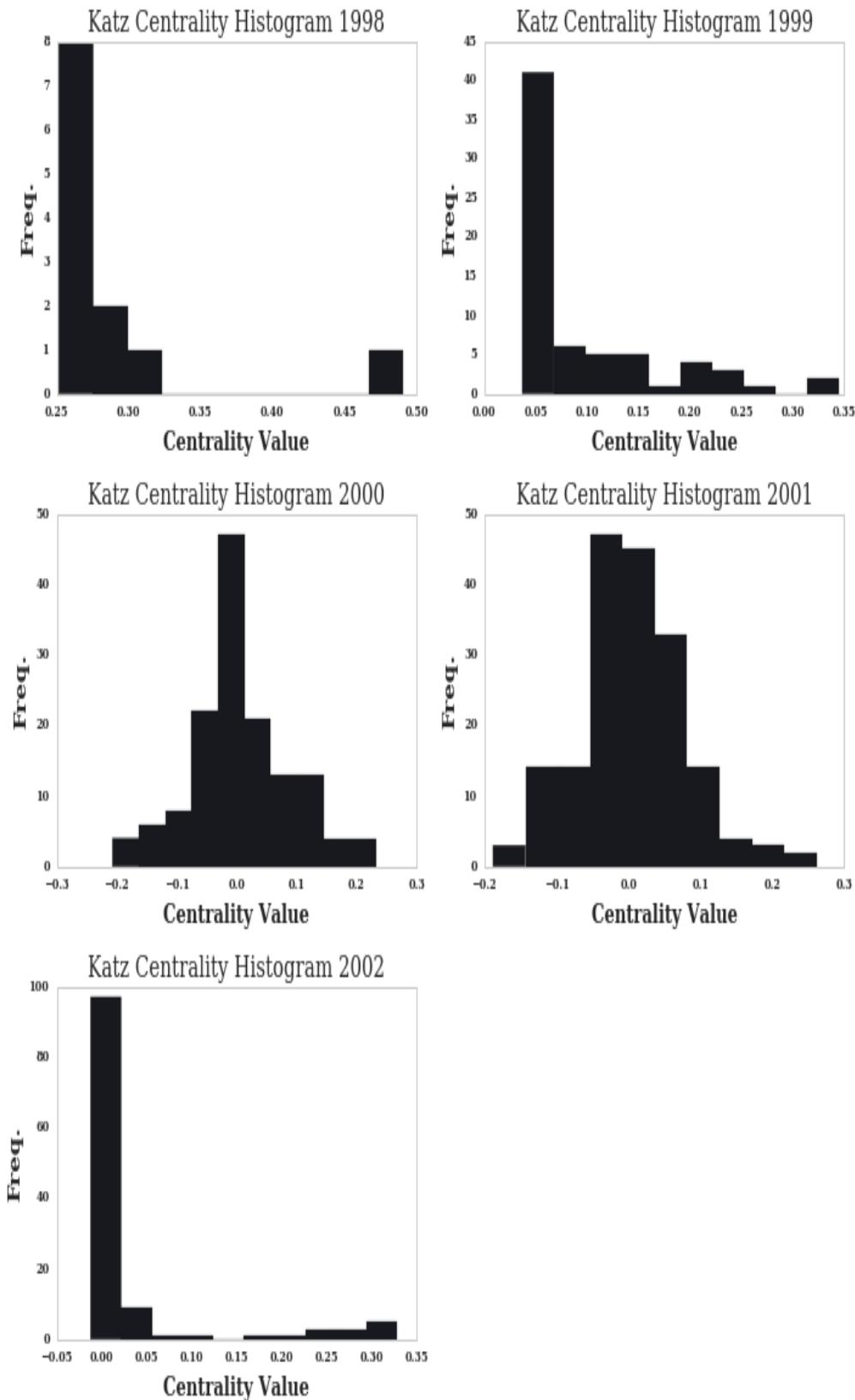


Figure 4.11: Yearly Katz Centrality Histogram

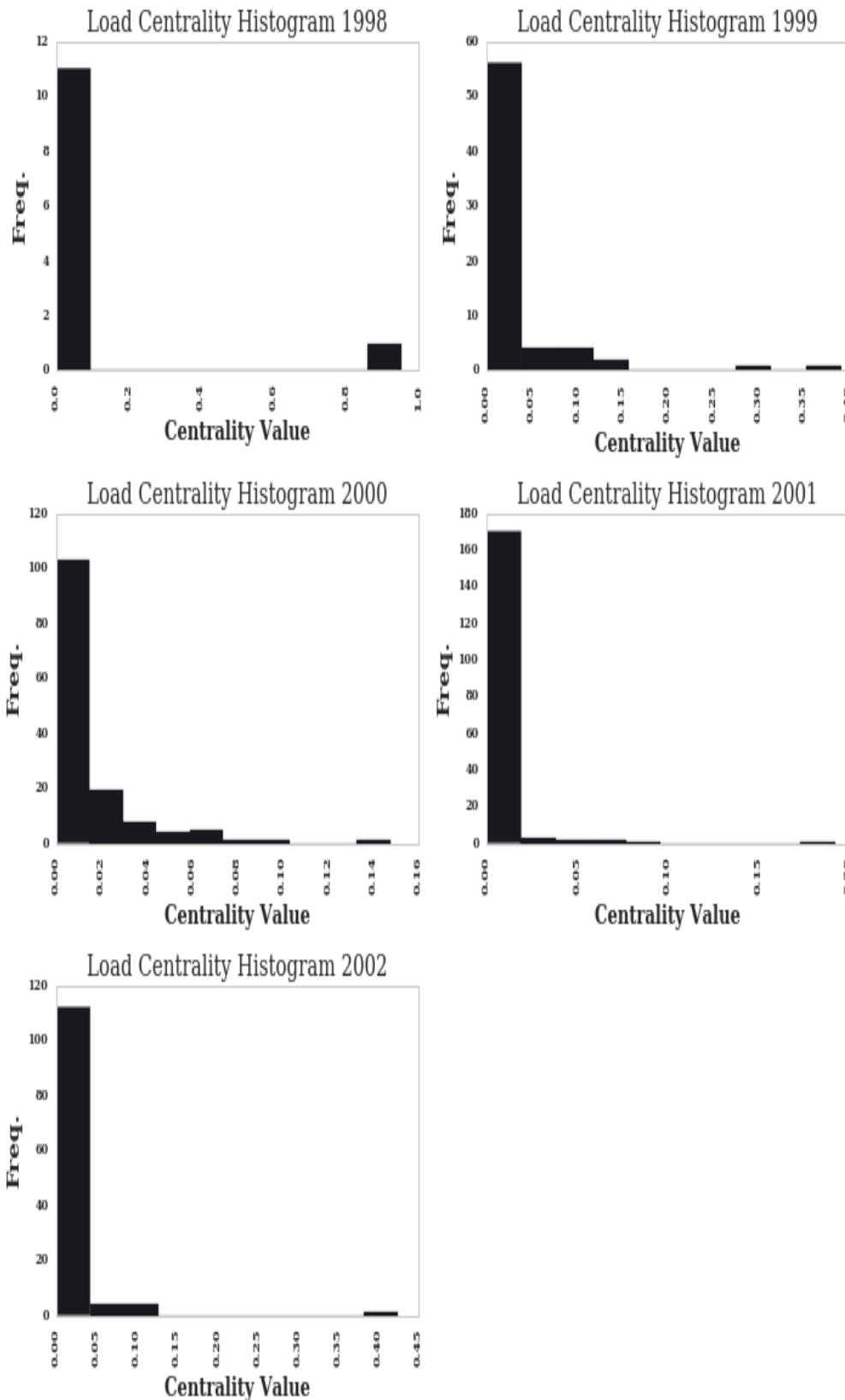


Figure 4.12: Yearly Load Centrality Histogram

4.3.2 Monthly Analysis

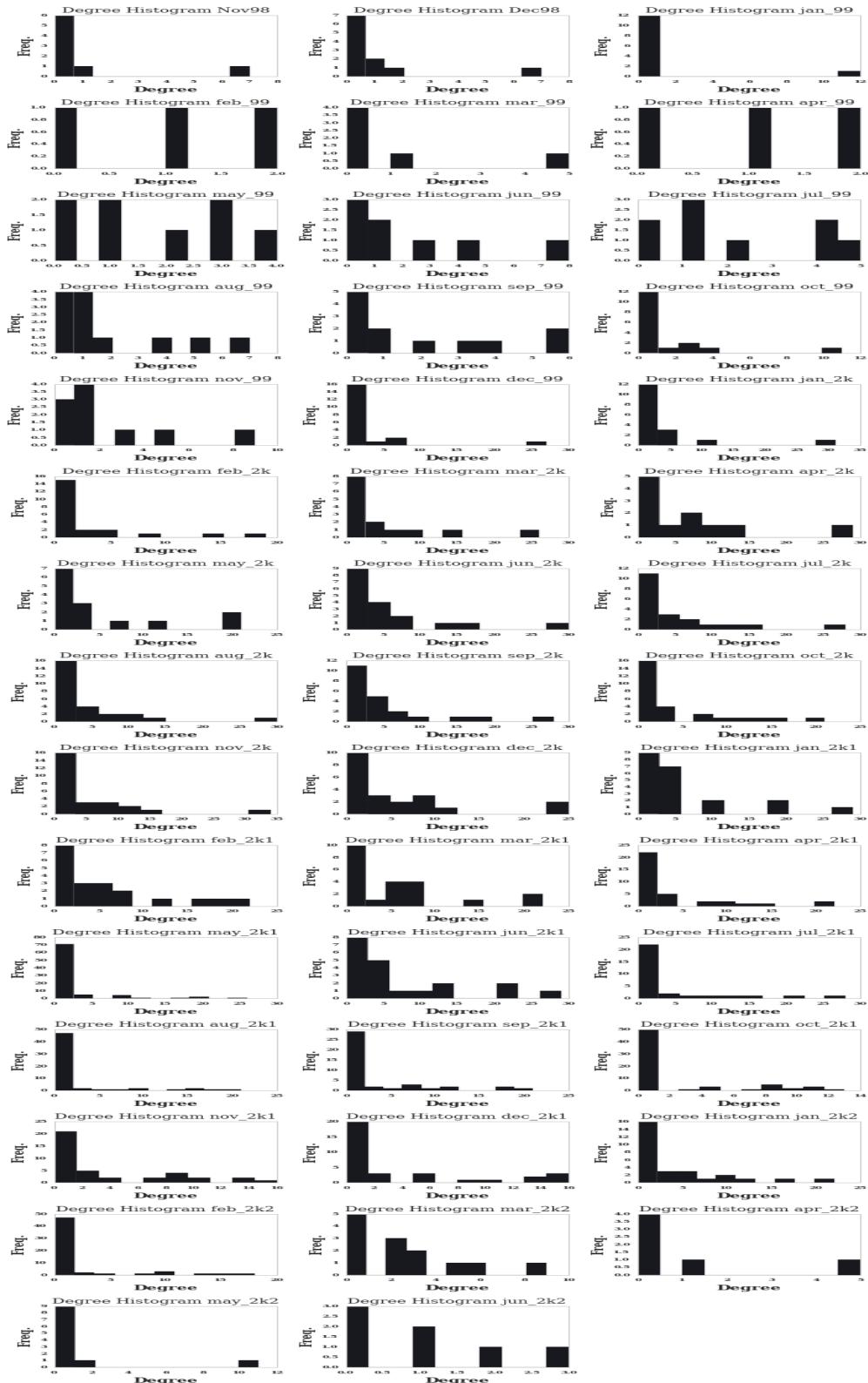


Figure 4.13: Monthly Degree Histogram

Analysing the evolution of communication patterns in email data through an extended dynamic network analysis toolkit

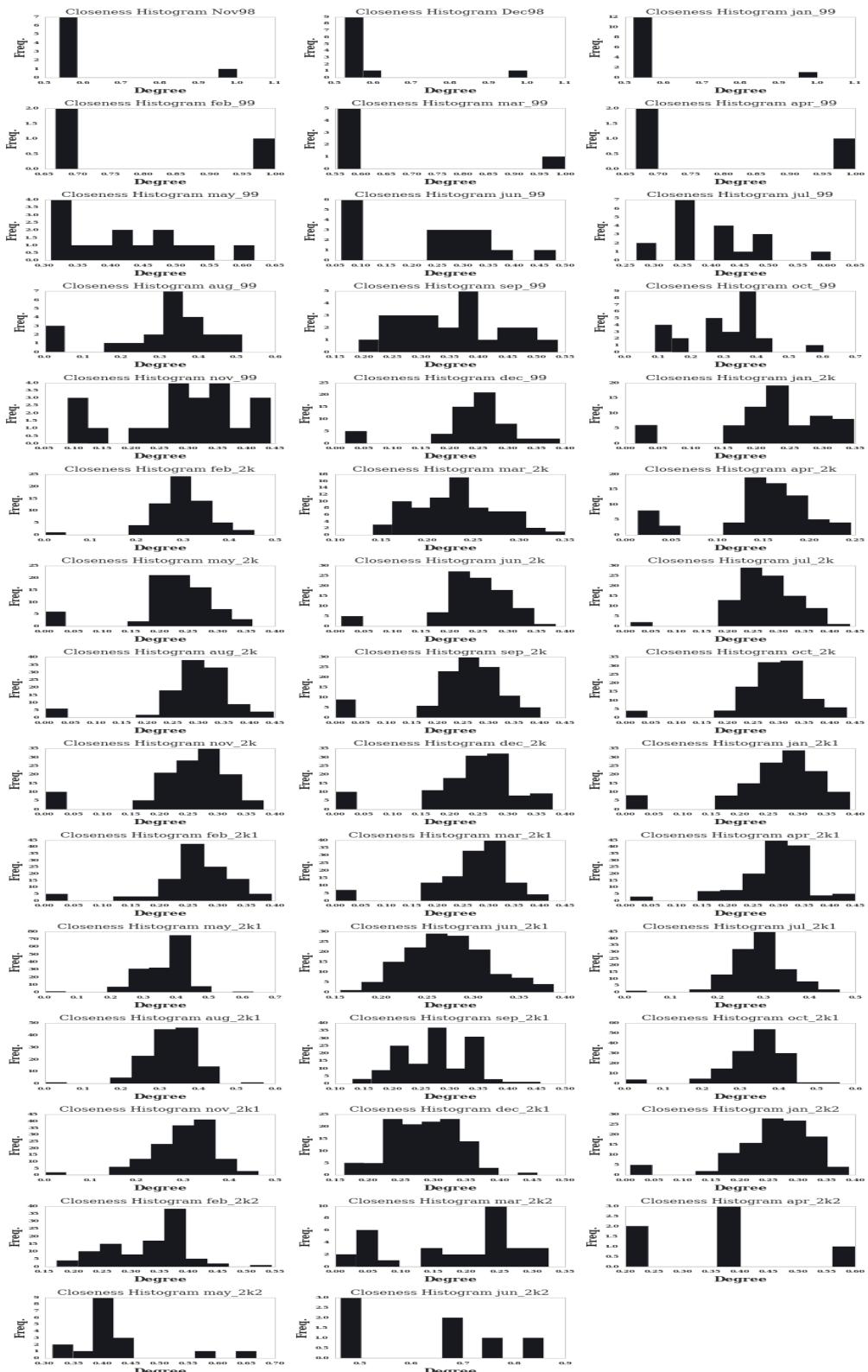


Figure 4.14: Monthly Closeness Centrality Histogram

Analysing the evolution of communication patterns in email data through an extended dynamic network analysis toolkit

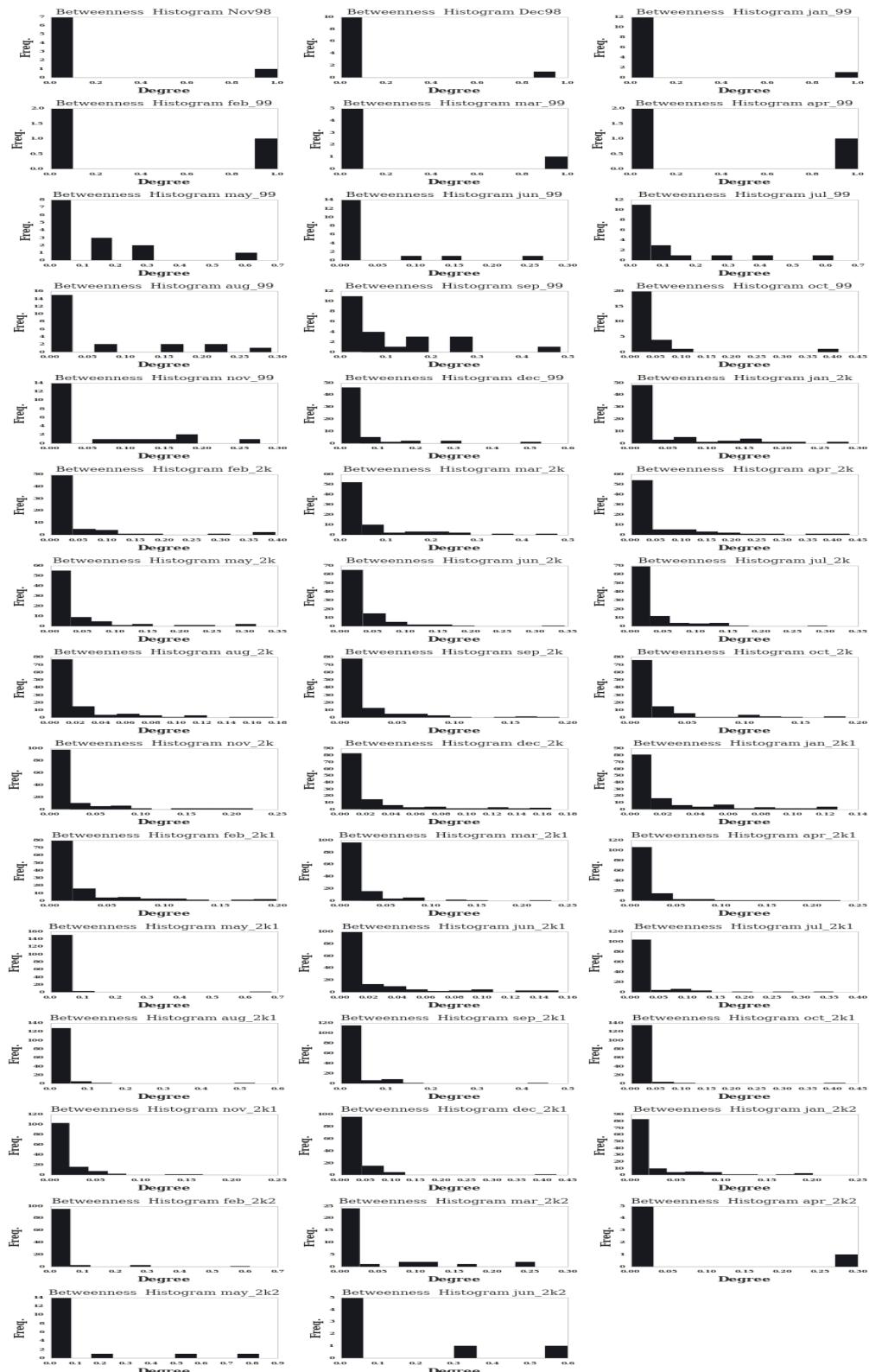


Figure 4.15: Monthly Betweenness Histogram

Analysing the evolution of communication patterns in email data through an extended dynamic network analysis toolkit

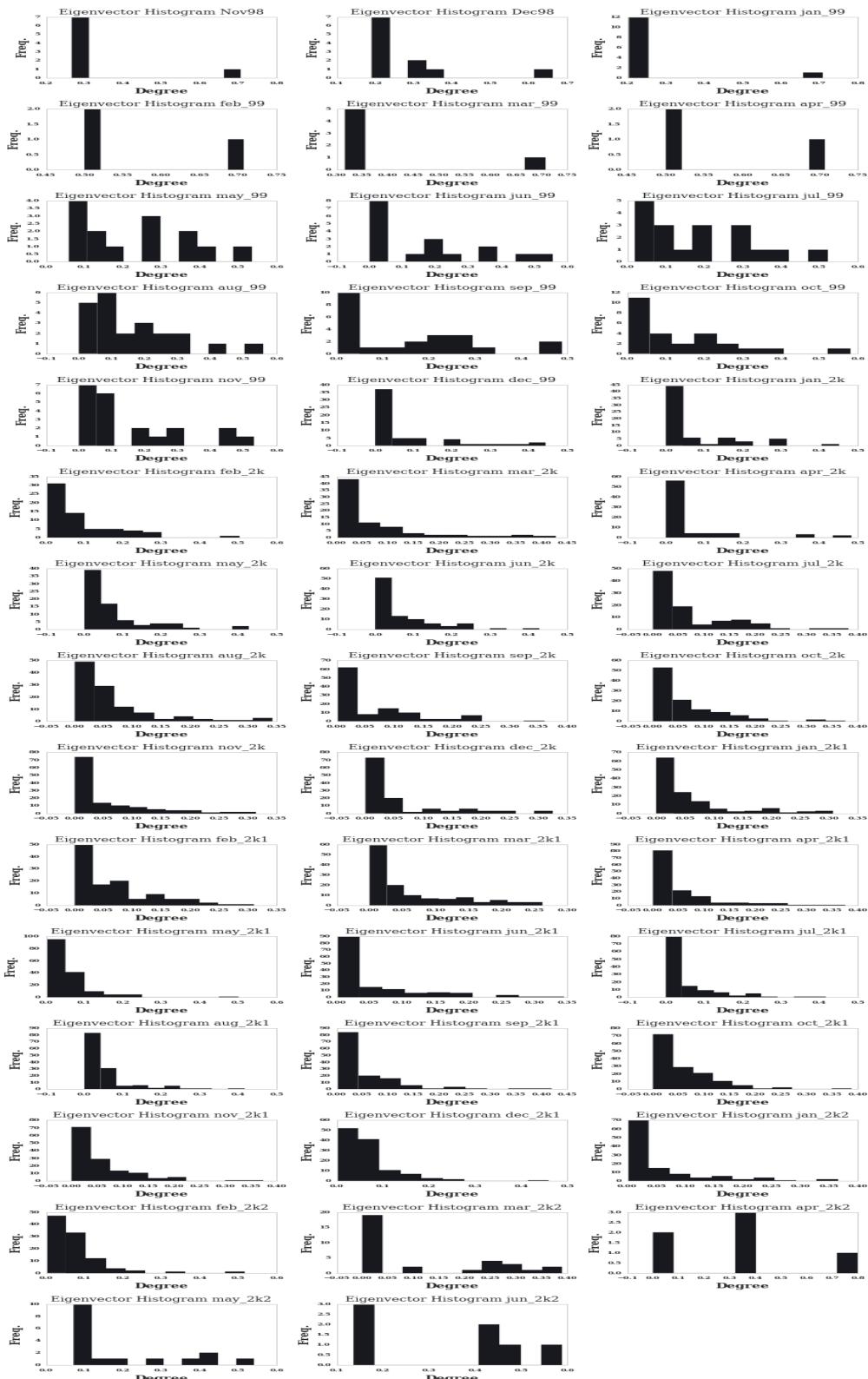


Figure 4.16: Monthly Eigenvector Centrality Histogram

Analysing the evolution of communication patterns in email data through an extended dynamic network analysis toolkit

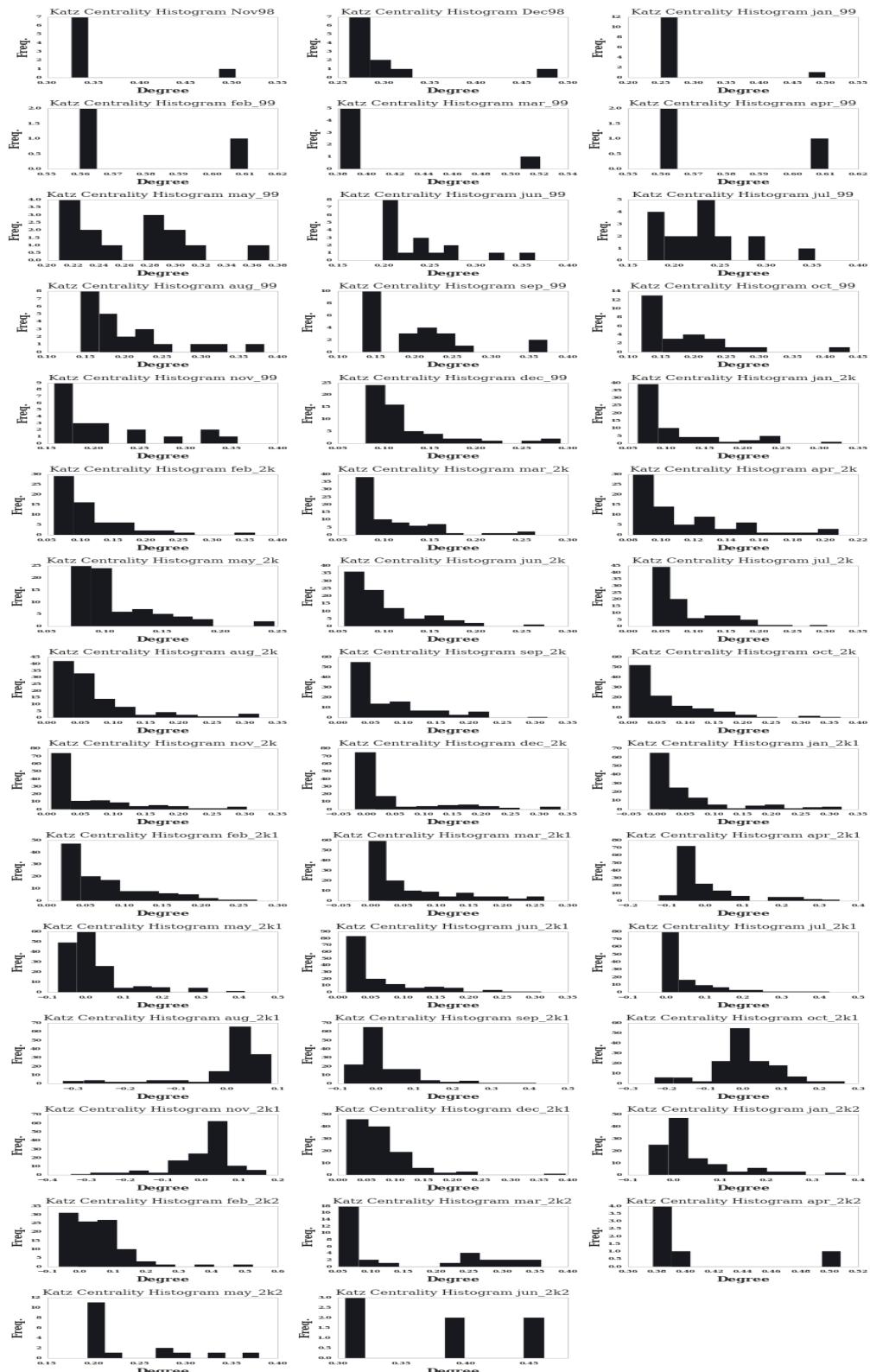


Figure 4.17: Monthly Katz Centrality Histogram

Analysing the evolution of communication patterns in email data through an extended dynamic network analysis toolkit

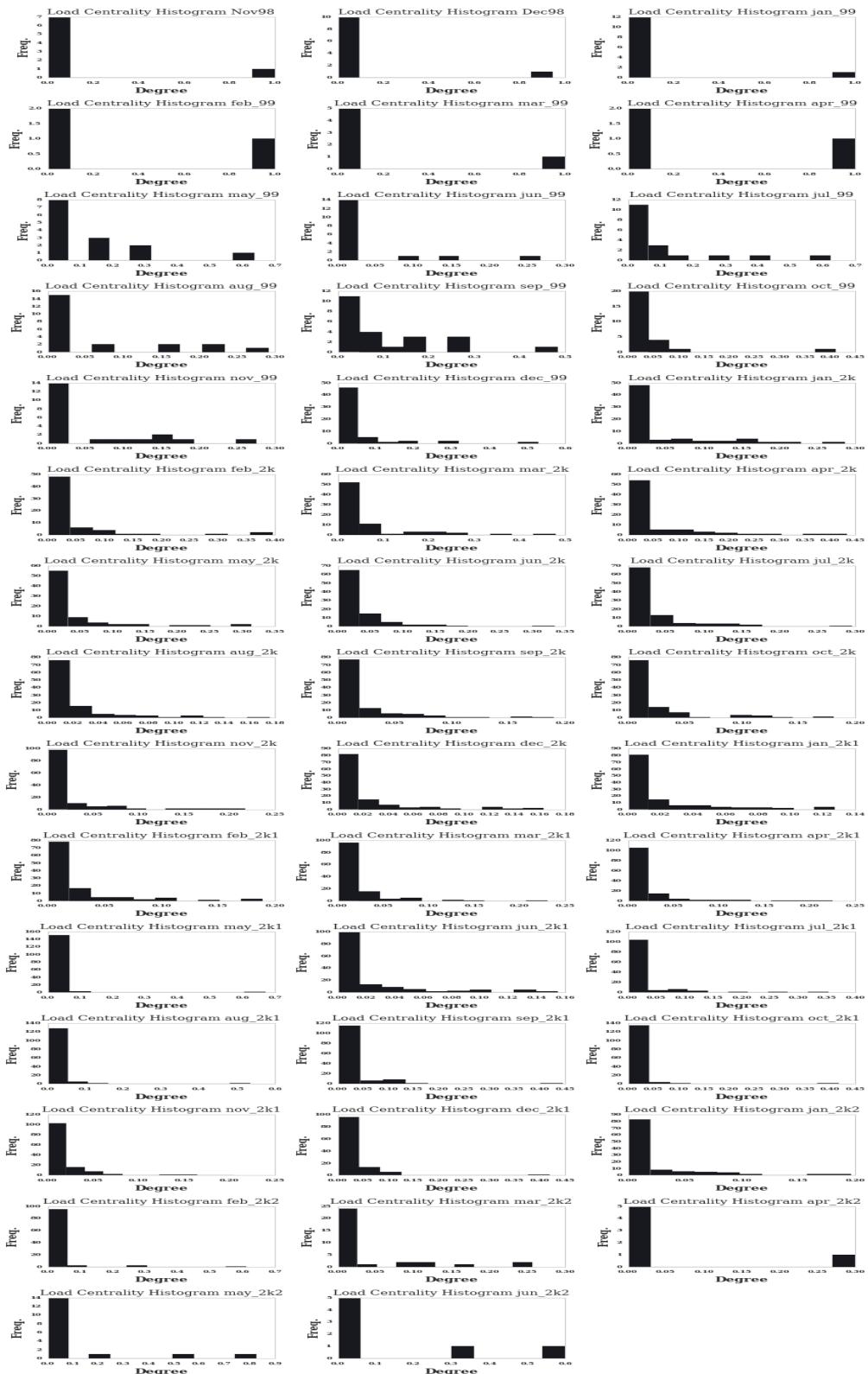


Figure 4.18: Monthly Load Centrality Histogram

4.4 Benchmark Measures

Here I plot the benchmark measures for the yearly and monthly networks to establish a signal in the graph time series. The measures proposed are judged firstly on their ability to model similar phenomenon and highlight additional interesting areas not obvious from these measures alone.

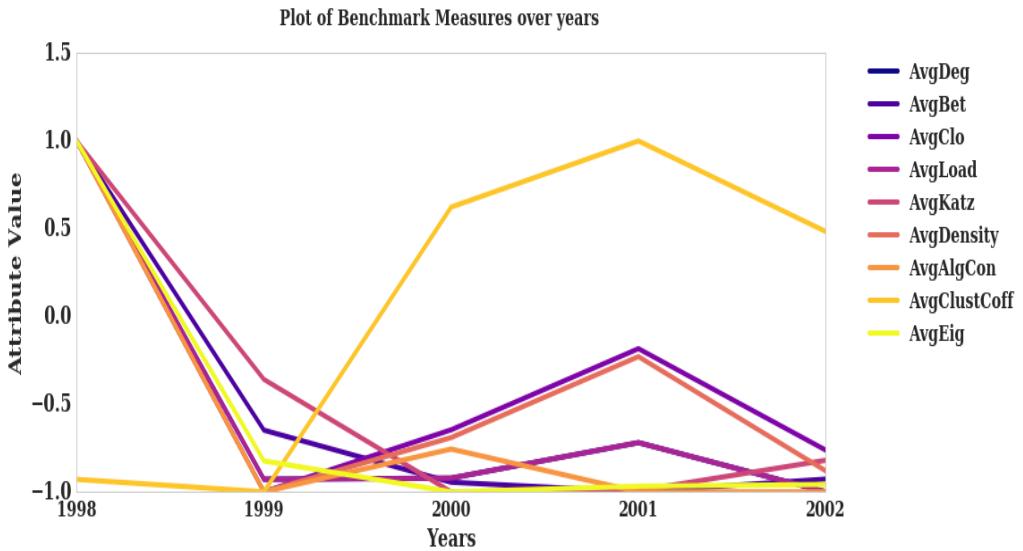


Figure 4.19: Plot of Benchmark Measures over Years

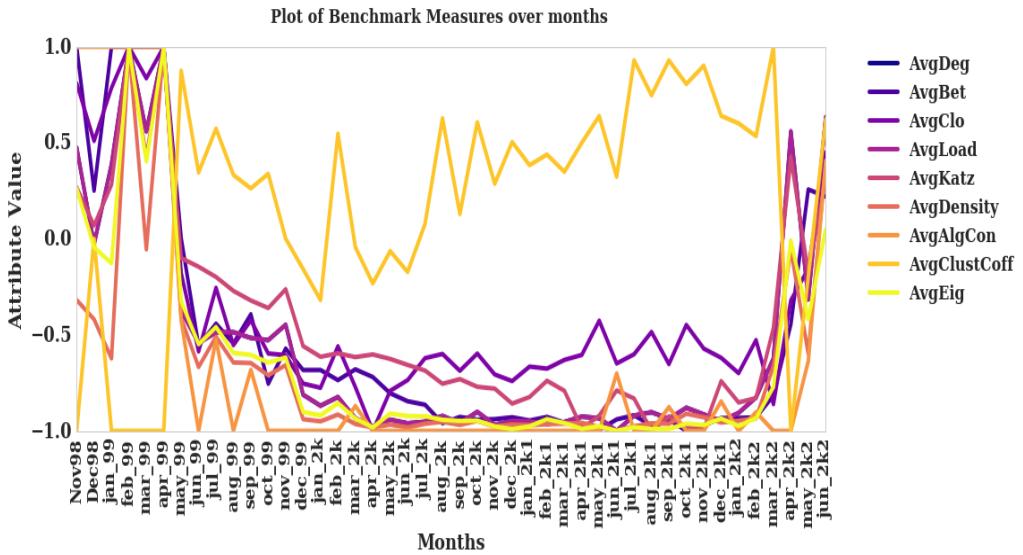


Figure 4.20: Plot of Benchmark Measures over Months

4.5 Attribute Analysis

Here the novel measures proposed in this study are presented. First, I explore how the attributes vary when derived from different graph matrices.

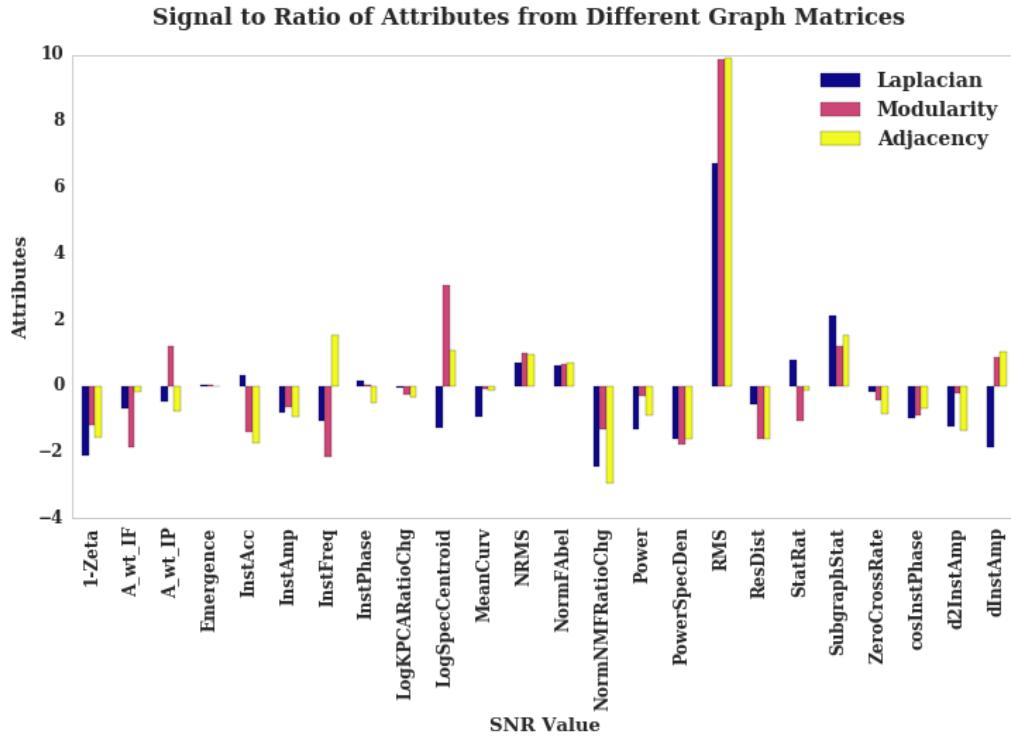


Figure 4.21: Plot of Signal to Noise Ratio of Attributes calculated from 3 different Graph Matrices

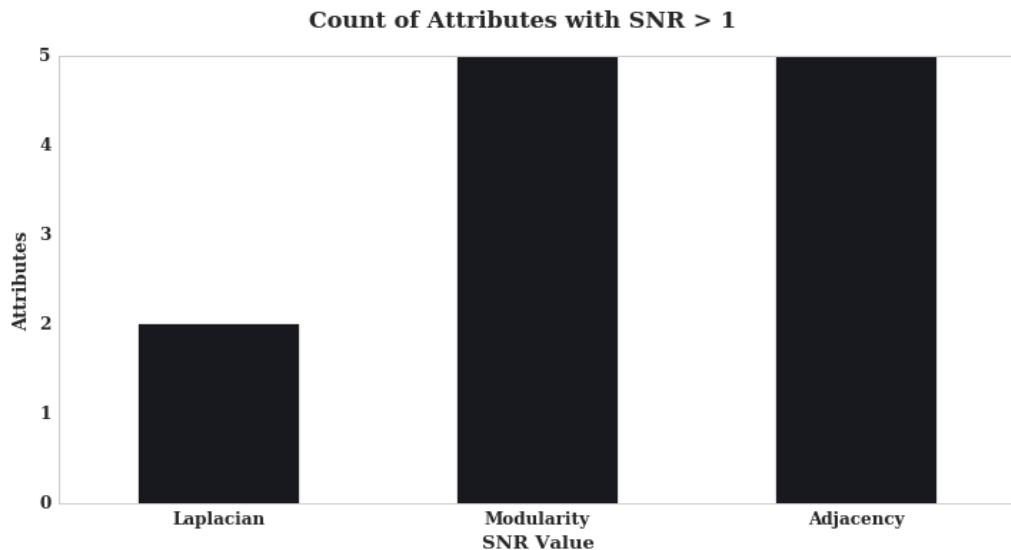


Figure 4.22: Plot of Count of Number attributes with $SNR > 1$ from the different graph matrices

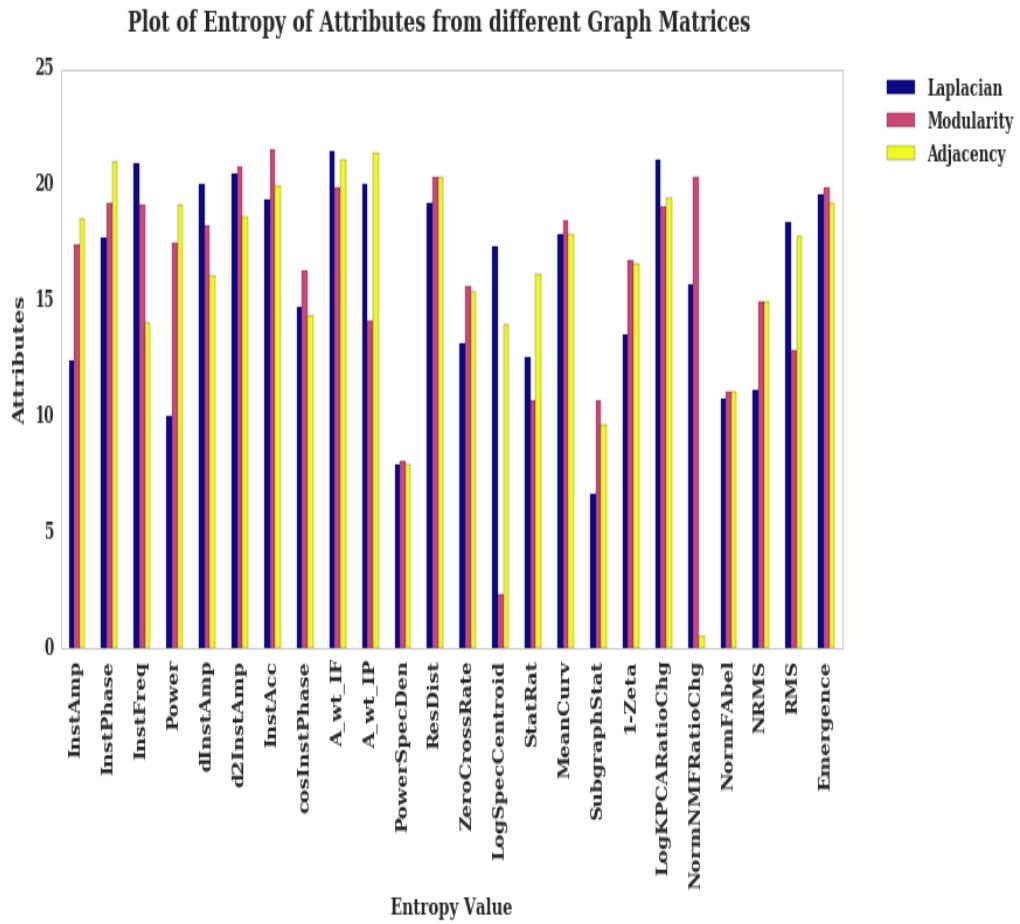


Figure 4.23: Plot of Entropy of attributes from the different graph matrices

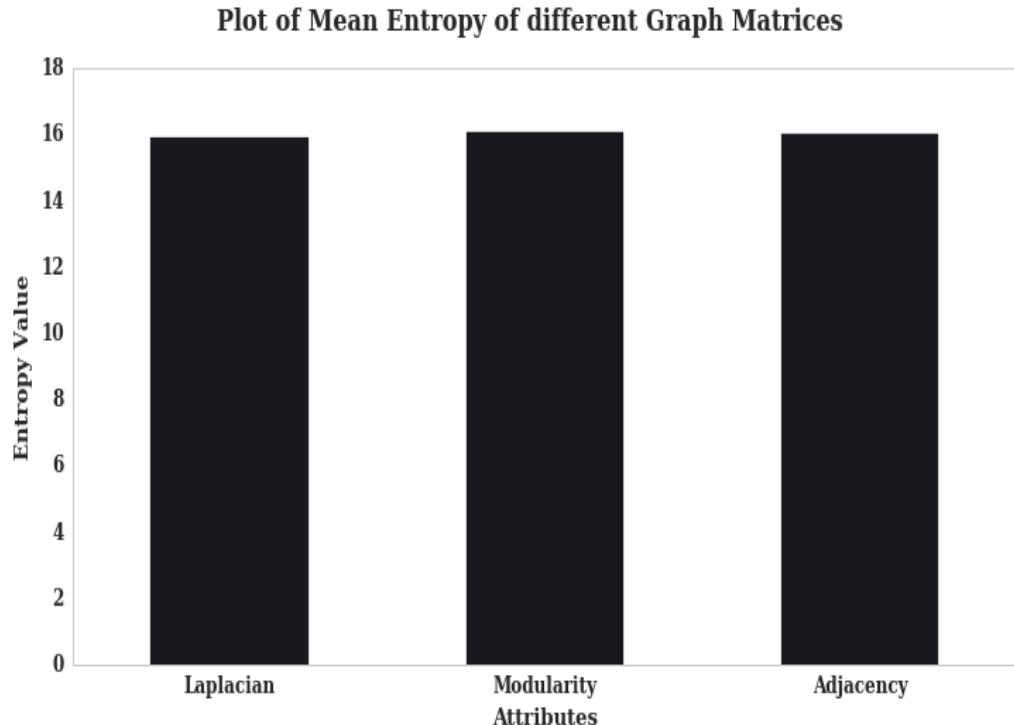


Figure 4.24: Plot of Mean Entropy of different graph matrices

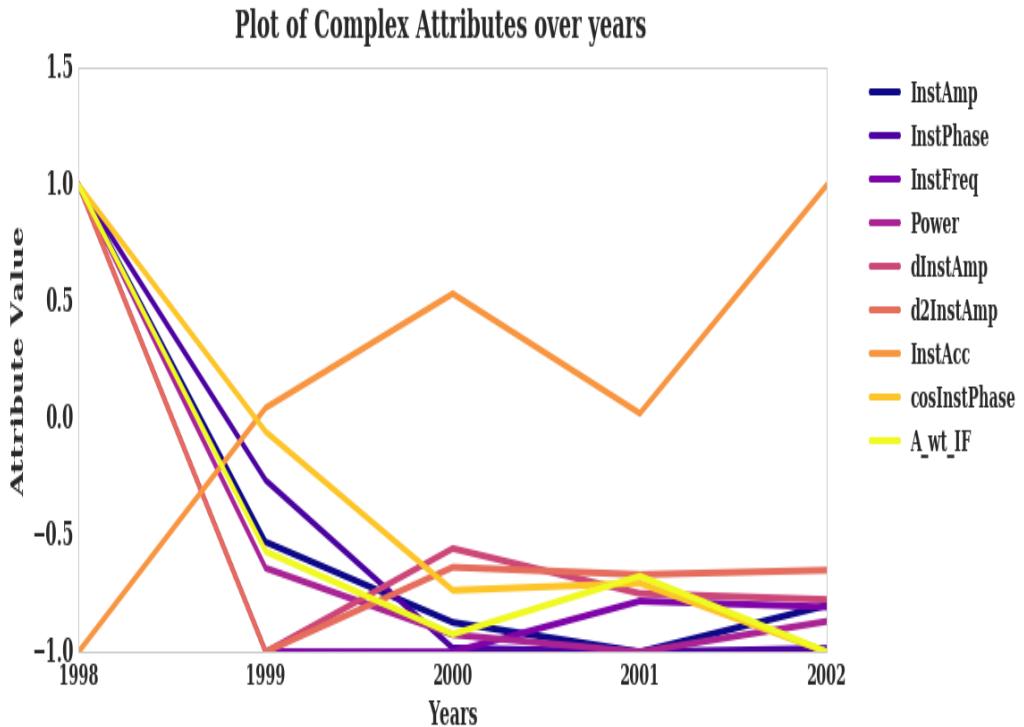


Figure 4.25: Plot of Complex Attributes over Years

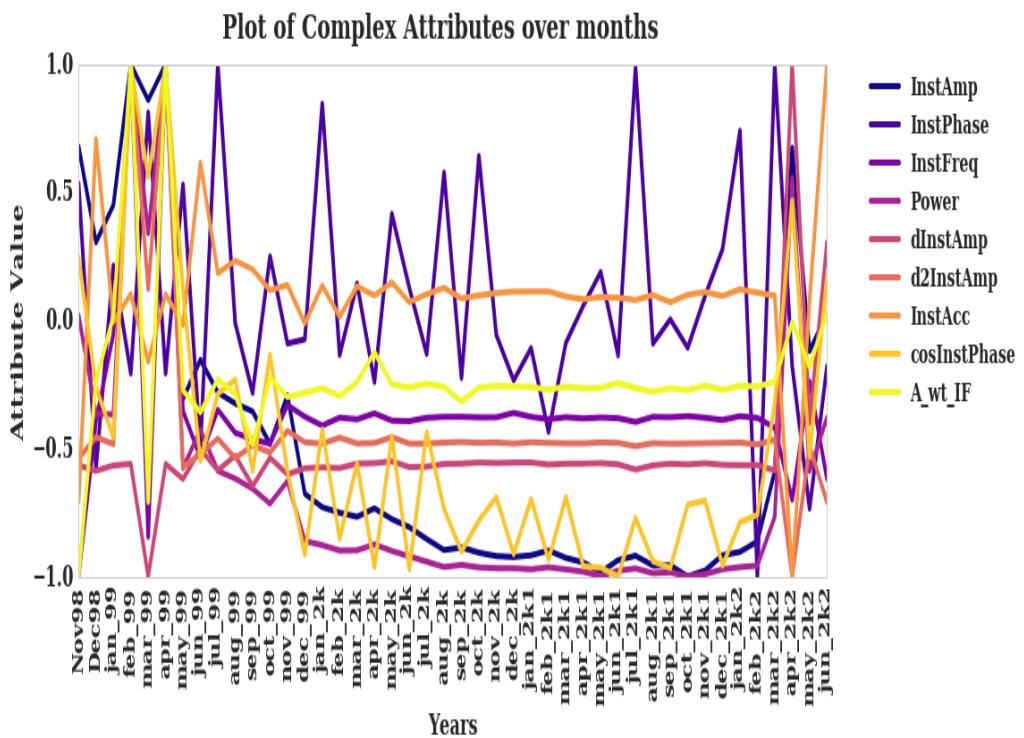


Figure 4.26: Plot of Complex Attributes over Months

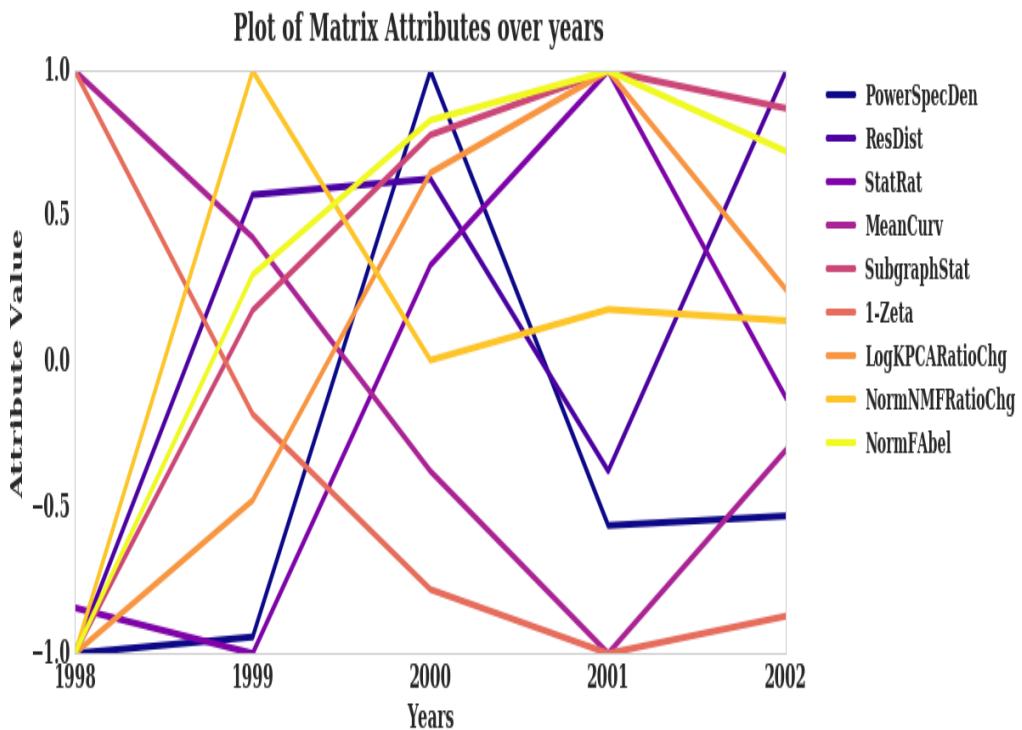
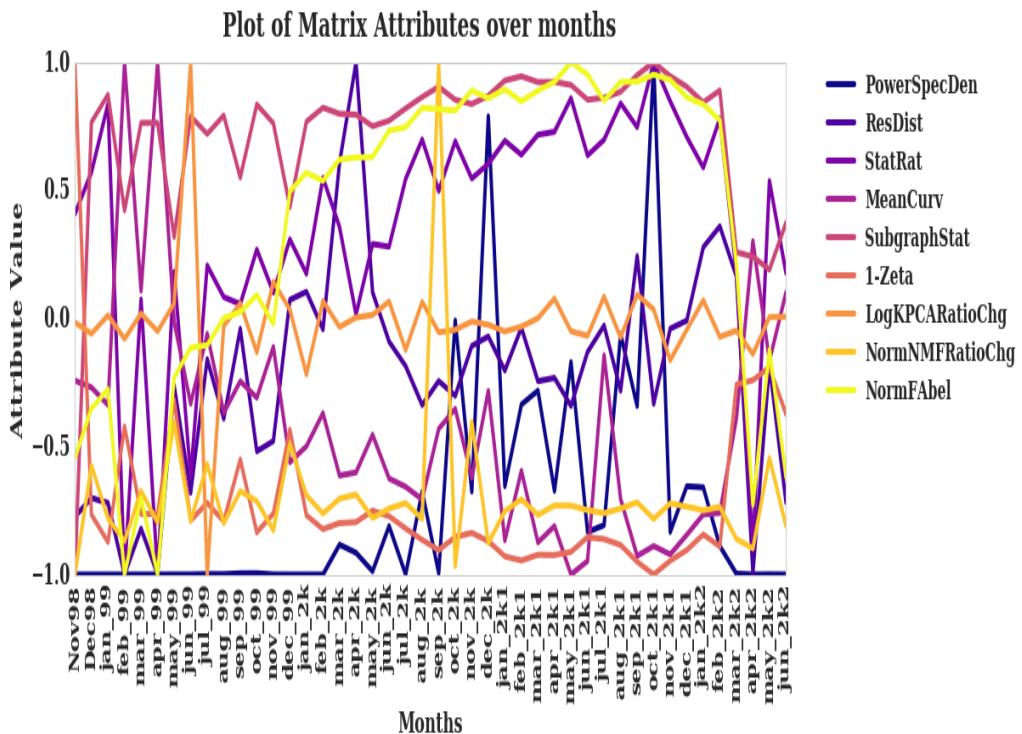


Figure 4.27: Plot of Matrix Attributes over Months



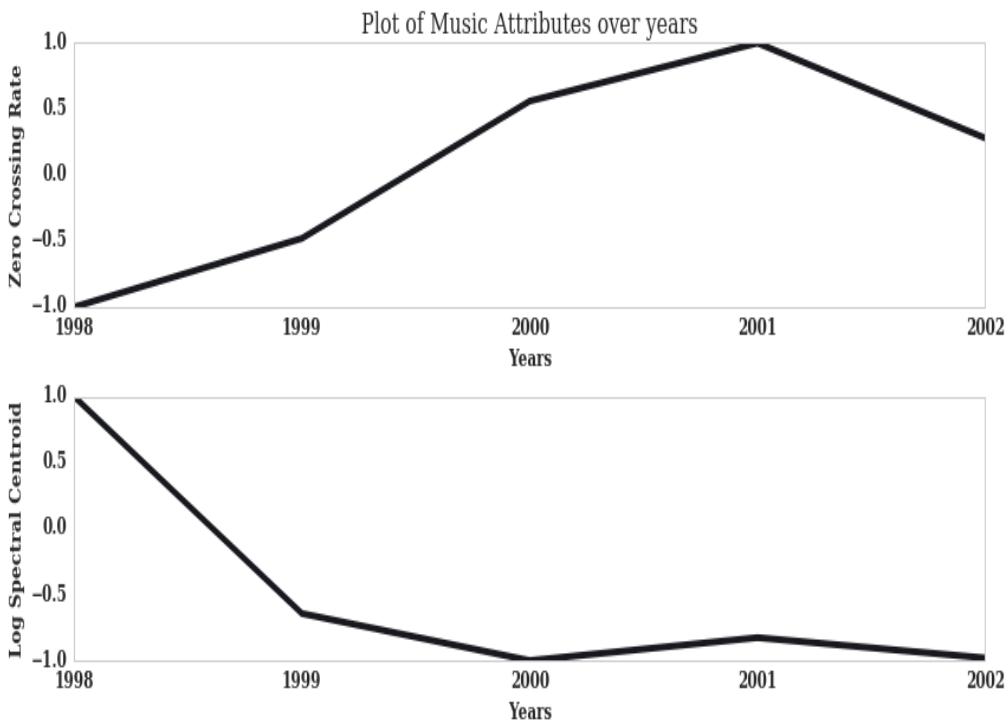


Figure 4.29: Plot of Music Attributes over Years

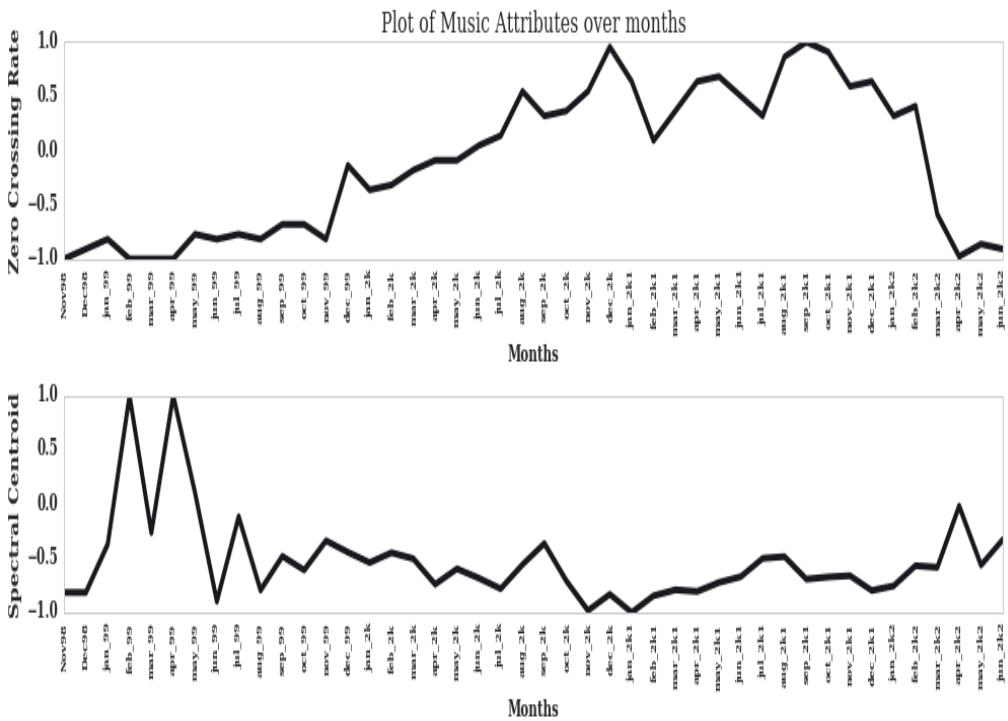


Figure 4.30: Plot of Music Attributes over Months

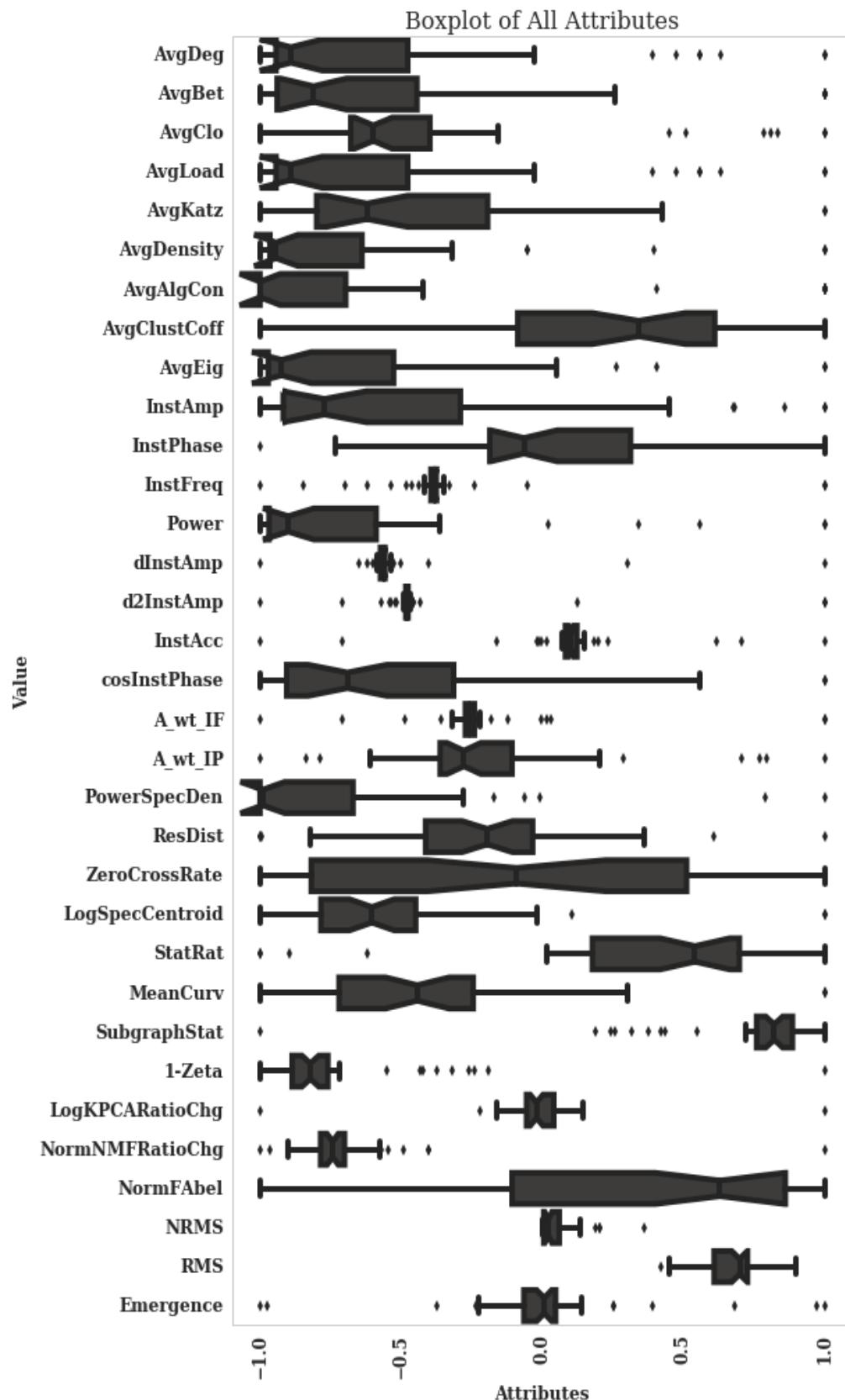


Figure 4.31: Boxplot of All Attributes

4.5.1 Average Attributes

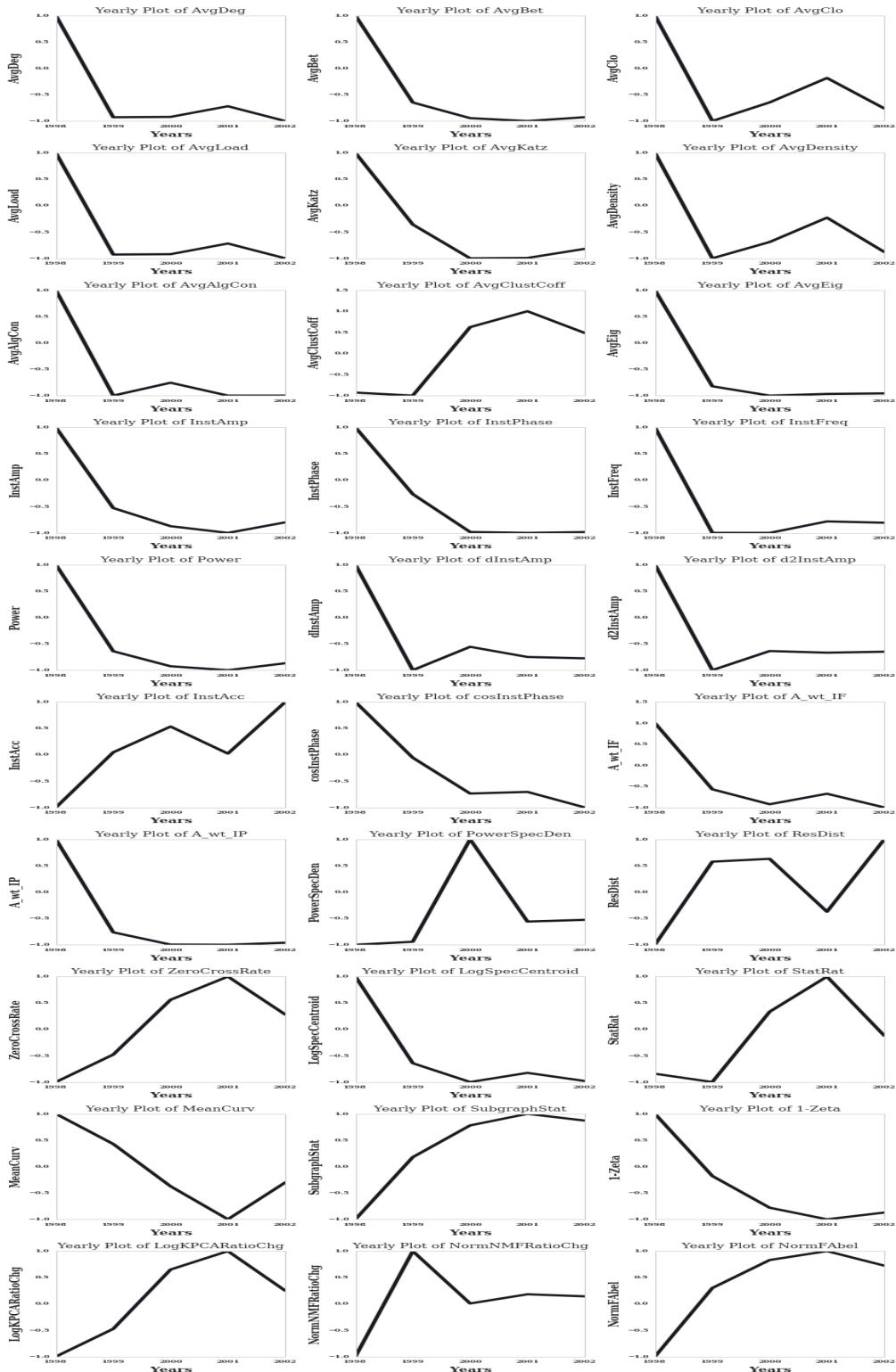


Figure 4.32: Plot of Average Attributes over Years

Analysing the evolution of communication patterns in email data through an extended dynamic network analysis toolkit

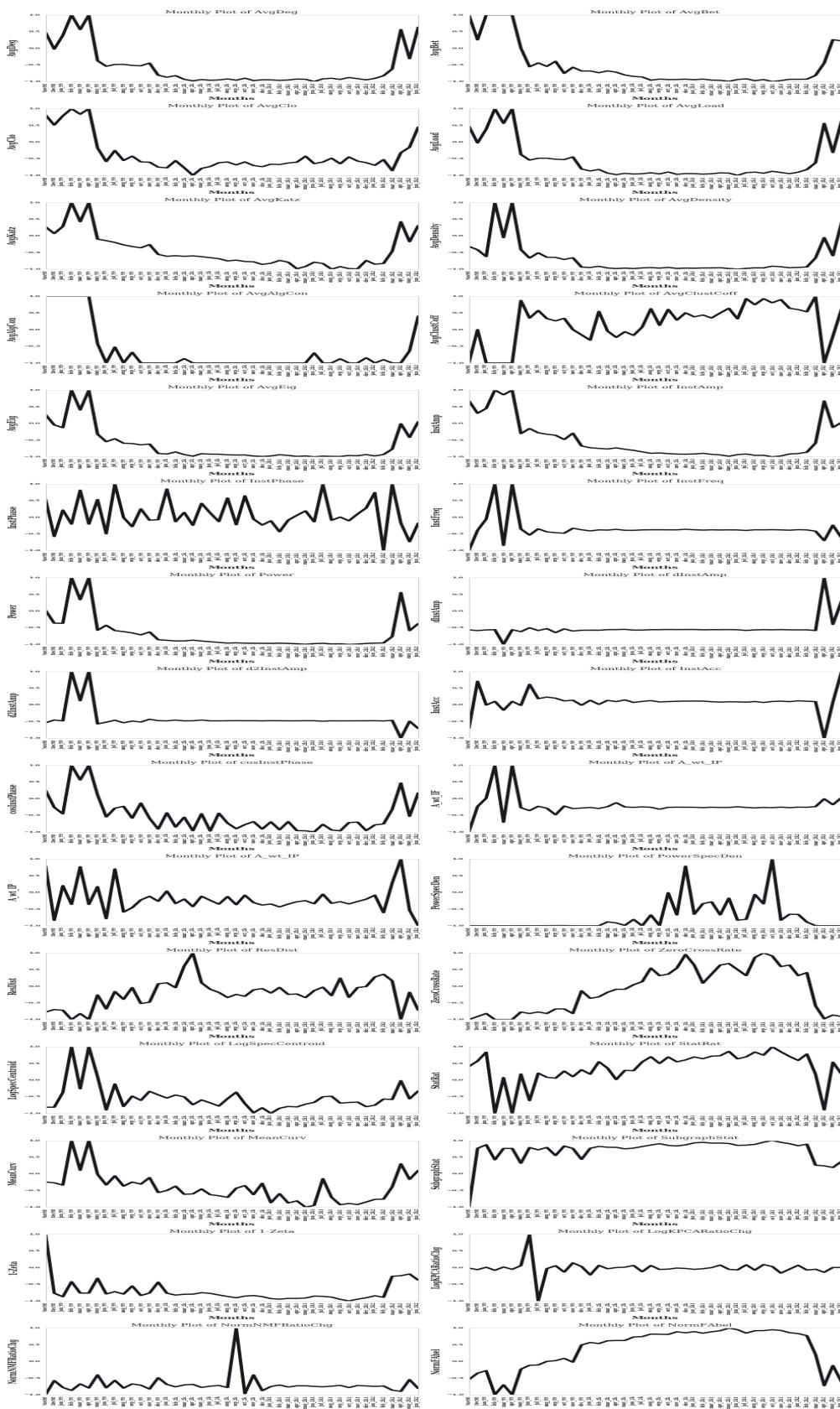


Figure 4.33: Plot of Average Attributes over Months

4.5.2 Correlation Analysis

The correlation analysis is performed using the monthly level attributes because the yearly data has a very small sample size.

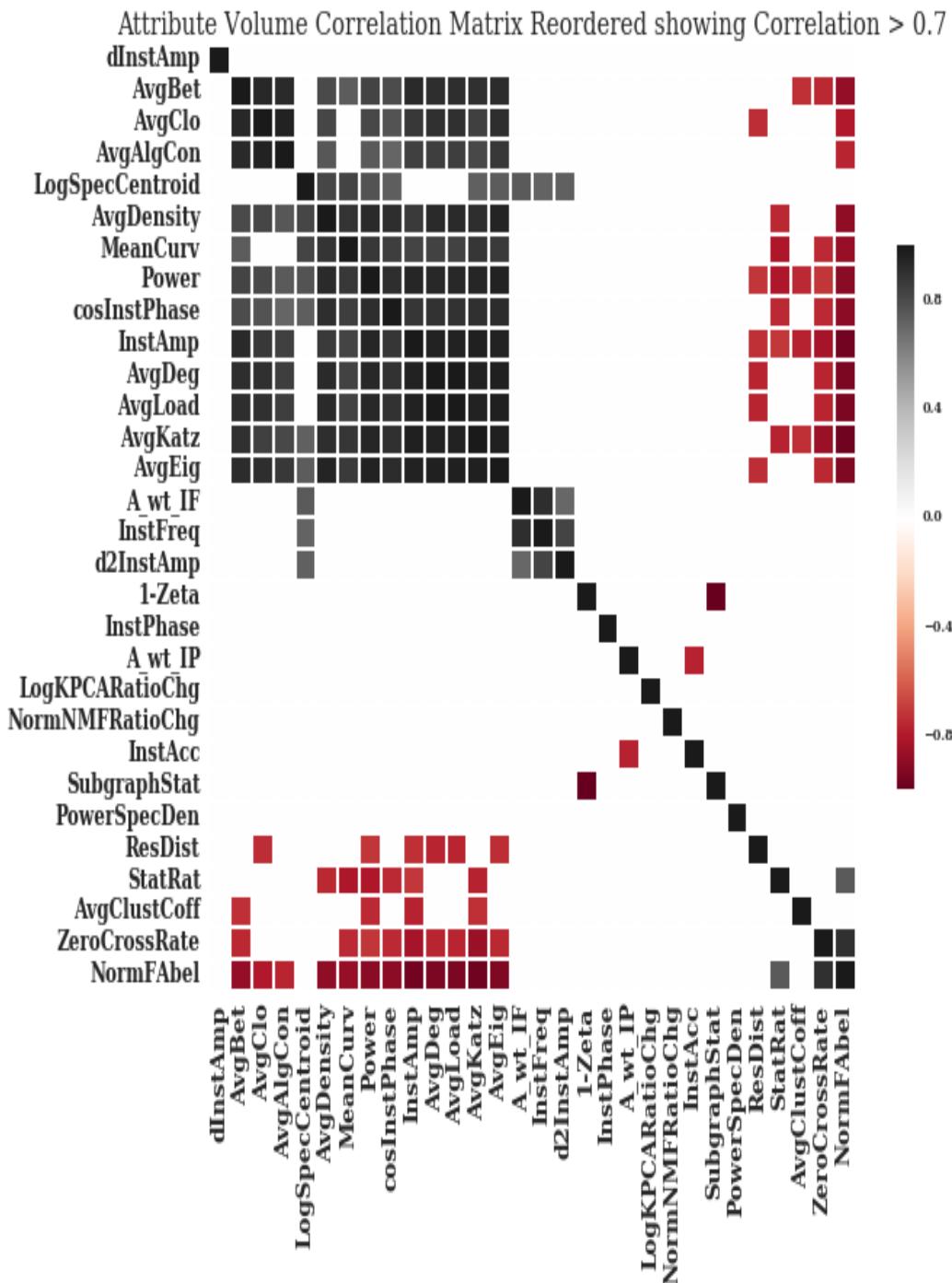


Figure 4.34: Reordered Correlation Heatmap showing correlation > 0.7

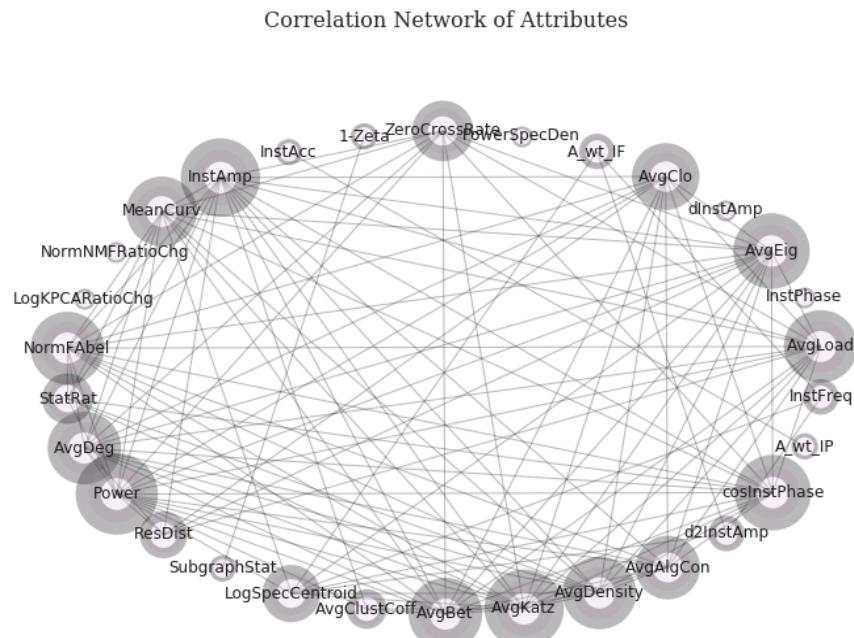


Figure 4.35: Correlation Network of Attributes. The thickness of the borders of the nodes indicates high degree while low thickness of borders around the nodes indicate low degree.

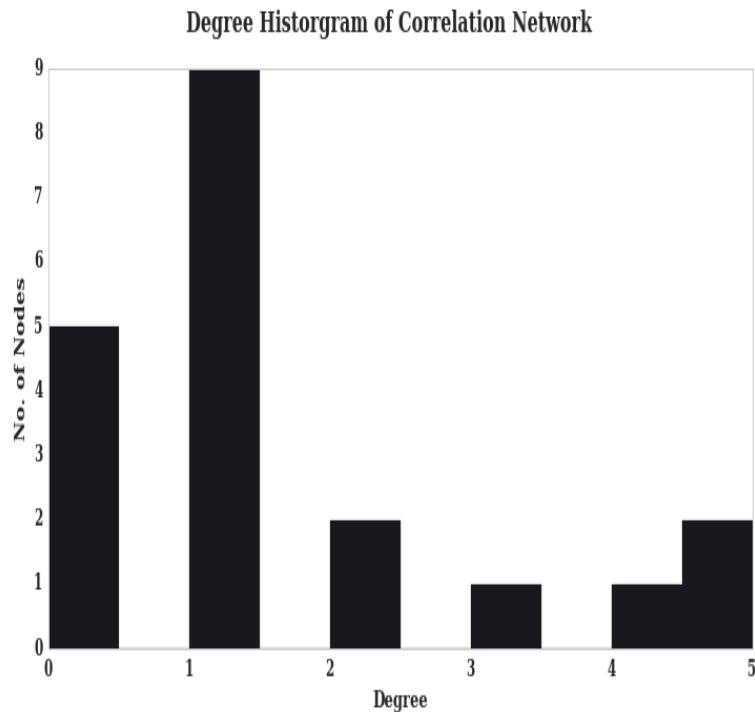


Figure 4.36: Correlation Network Degree Histogram

4.5.3 Manifold Visualisation of Attribute Volume

Non-Metric Multi Dimensional Scaling of Attribute Volume

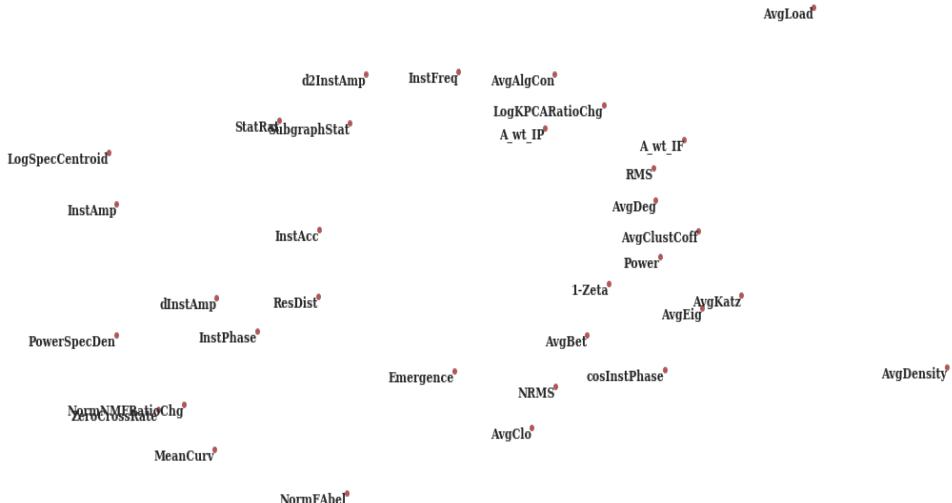


Figure 4.37: Non-Metric Multidimensional Scaling of Attribute Volume

TSNE Plot of Attribute Volume with Euclidean Distance

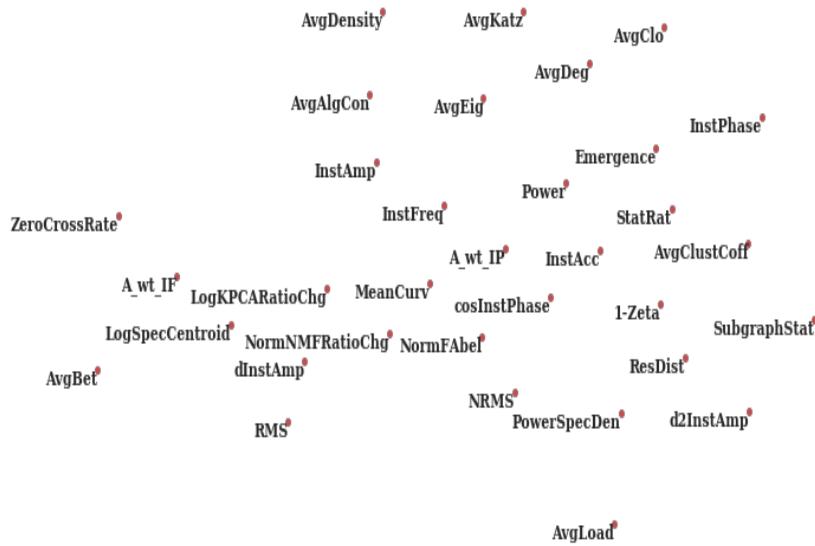


Figure 4.38: TSNE Plot of Attribute Volume with Euclidean Distance

TSNE Plot of Attribute Volume with Correlation Distance

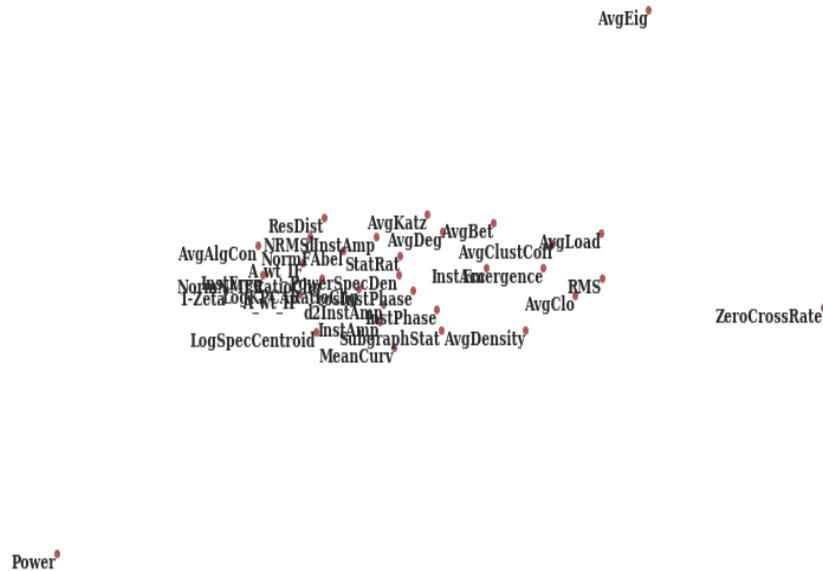


Figure 4.39: TSNE Plot of Attribute Volume with Correlation Distance

TSNE Plot of Attribute Volume with Canberra Distance



Figure 4.40: TSNE Plot of Attribute Volume with Canberra Distance

4.6 Feature Ranking with Regression Analysis

Here I used a Gradient Boosting Regressor with 10 boosting stages to predict the Average Degree of the network using the attribute volume. The test /train split is 50/50 to prevent over fitting. The Regressor produces a feature ranking of the variables and it is very useful to get a sense of the predictive potential of the measures.

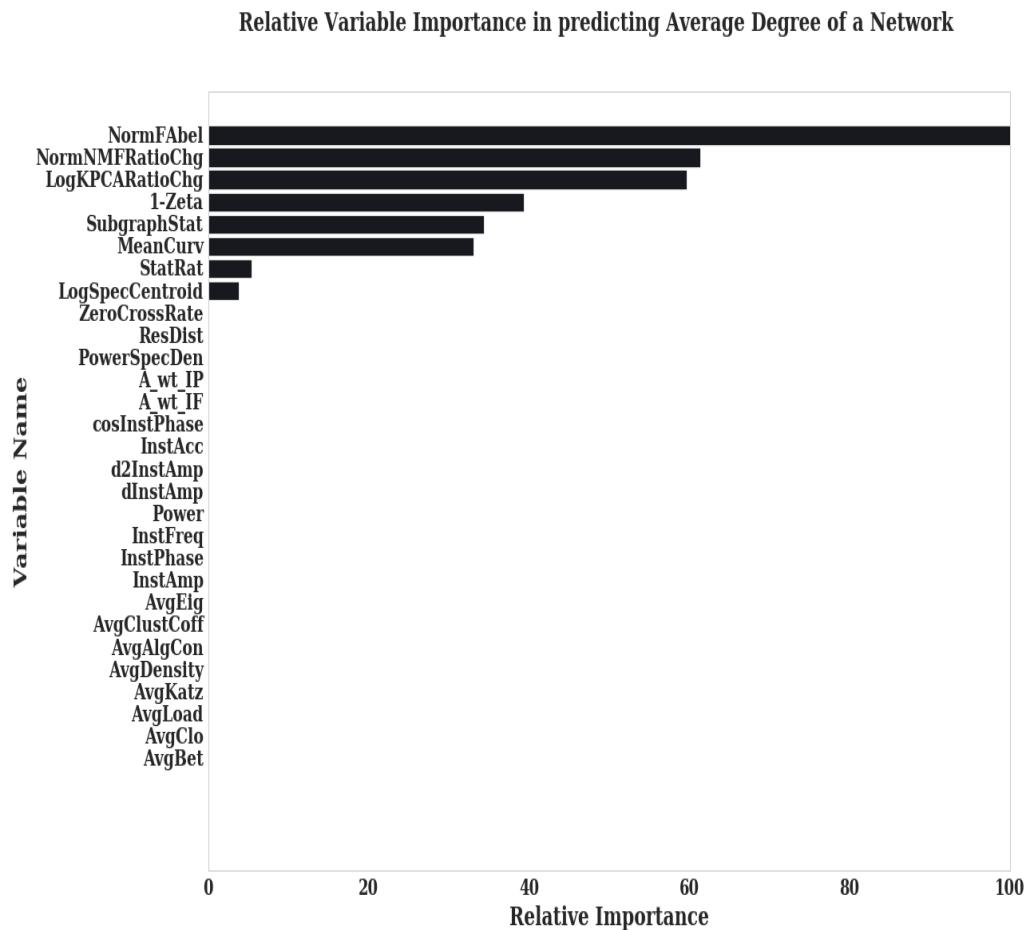


Figure 4.41: Feature Ranking by Gradient Boosting Regressor for predicting Average Degree

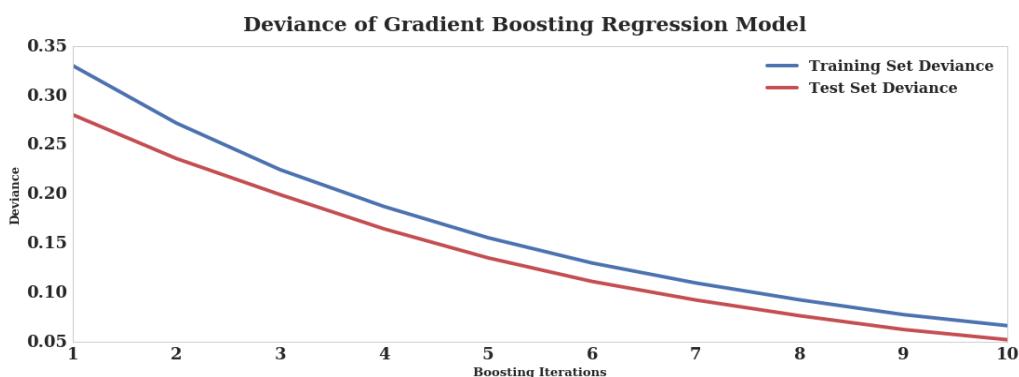


Figure 4.42: Regression Deviance Plot

4.7 FK and Radon Plots

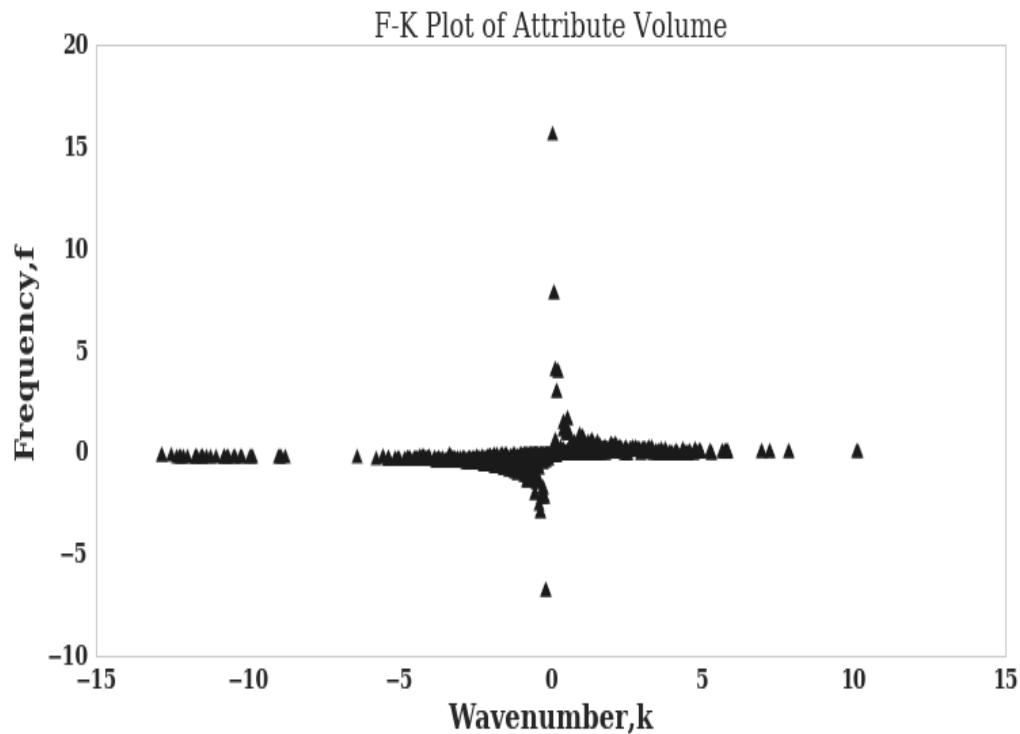


Figure 4.43: Frequency Wavenumber (FK) Plot of Attribute Volume

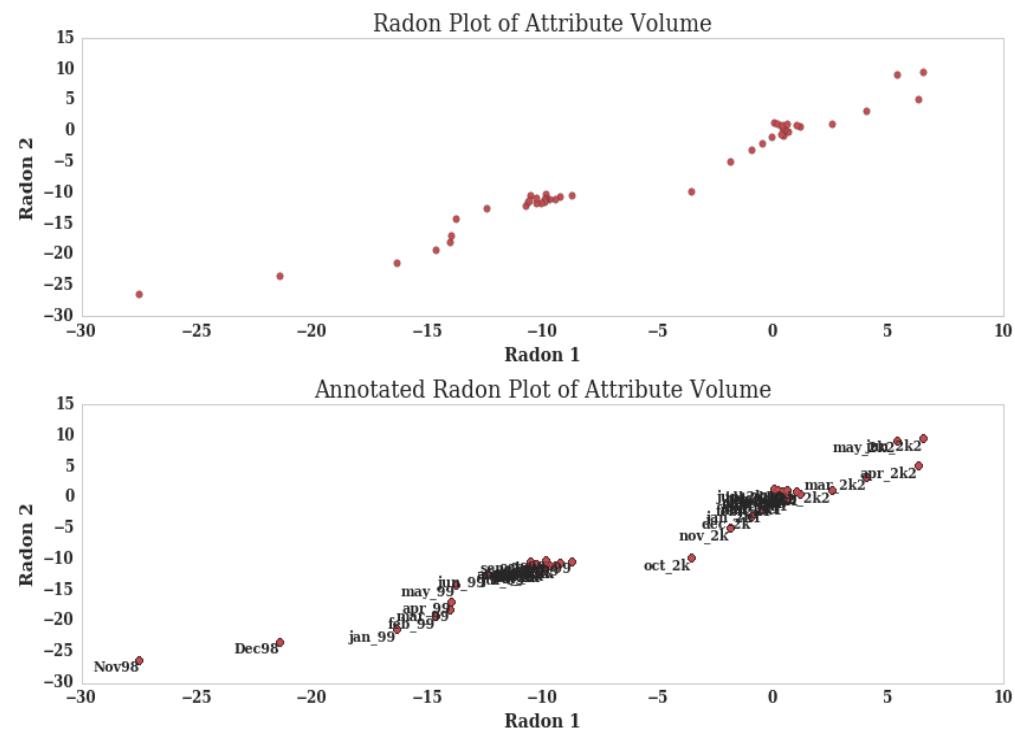


Figure 4.44: (top) Radon Plot of Attribute Volume, (bottom) Annotated Radon Plot

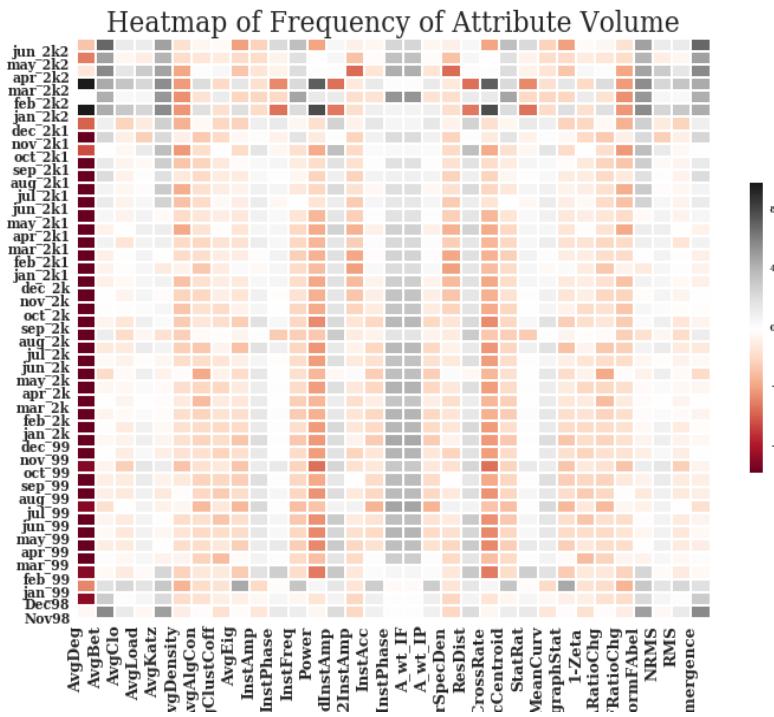


Figure 4.45: Heatmap of Frequency of Attribute Volume. This is the F component derived from the FK Plot

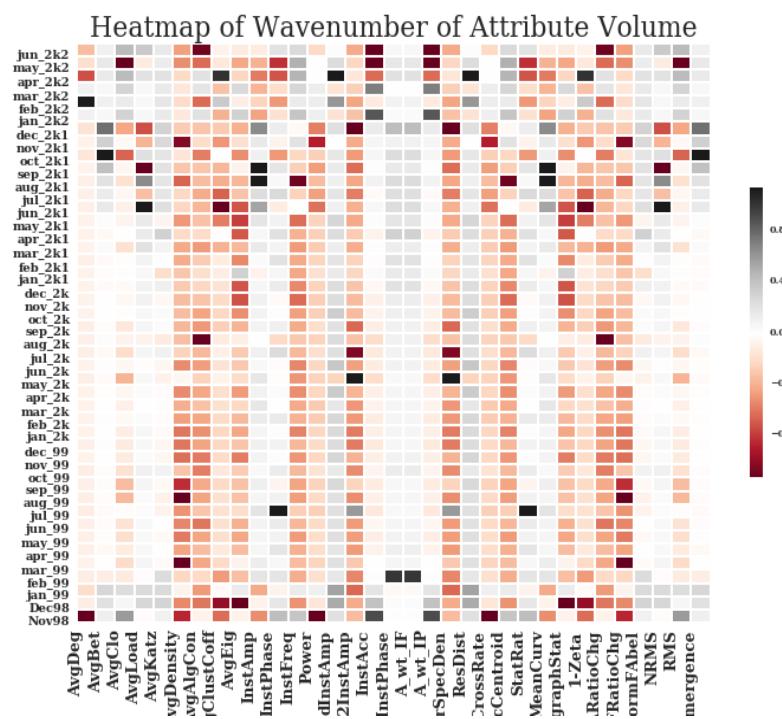


Figure 4.46: Heatmap of Wavenumber of Attribute Volume. This is the K component derived from the FK Plot

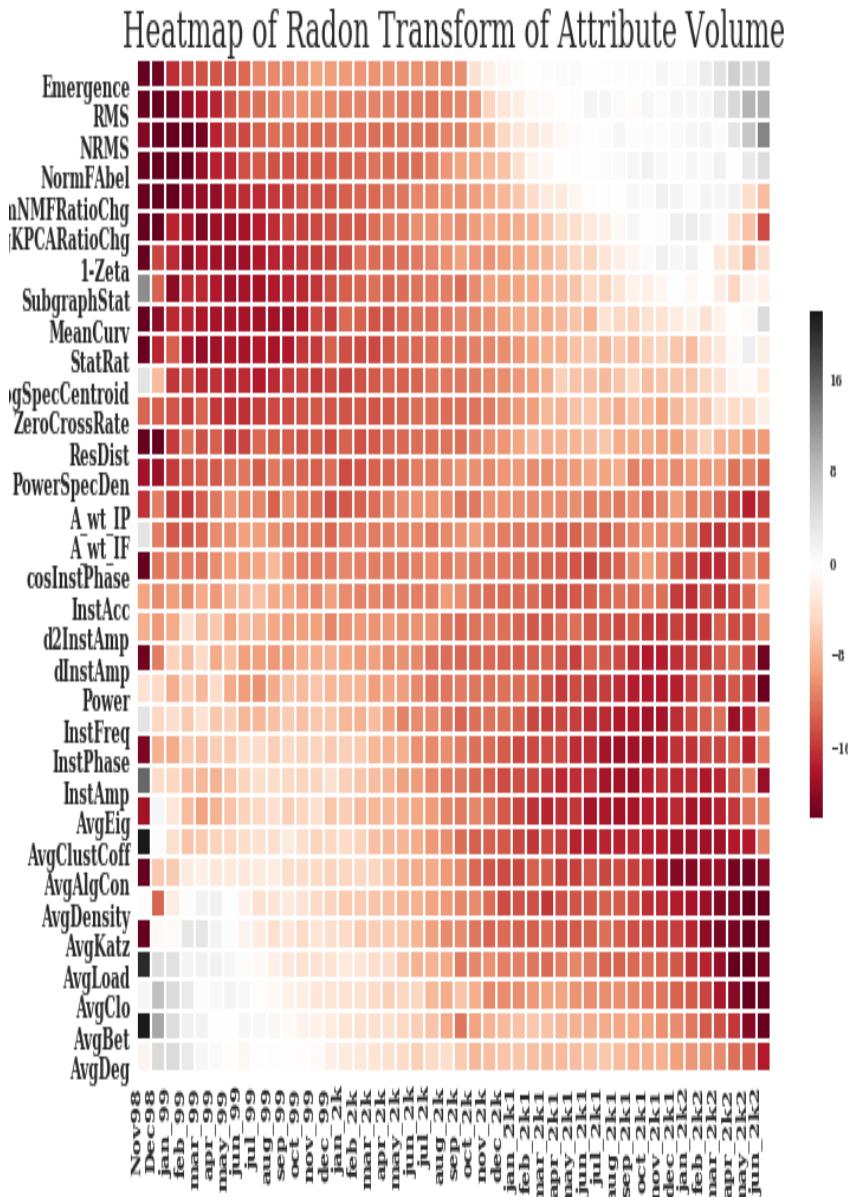


Figure 4.47: Heatmap of Radon Transform of Attribute Volume.

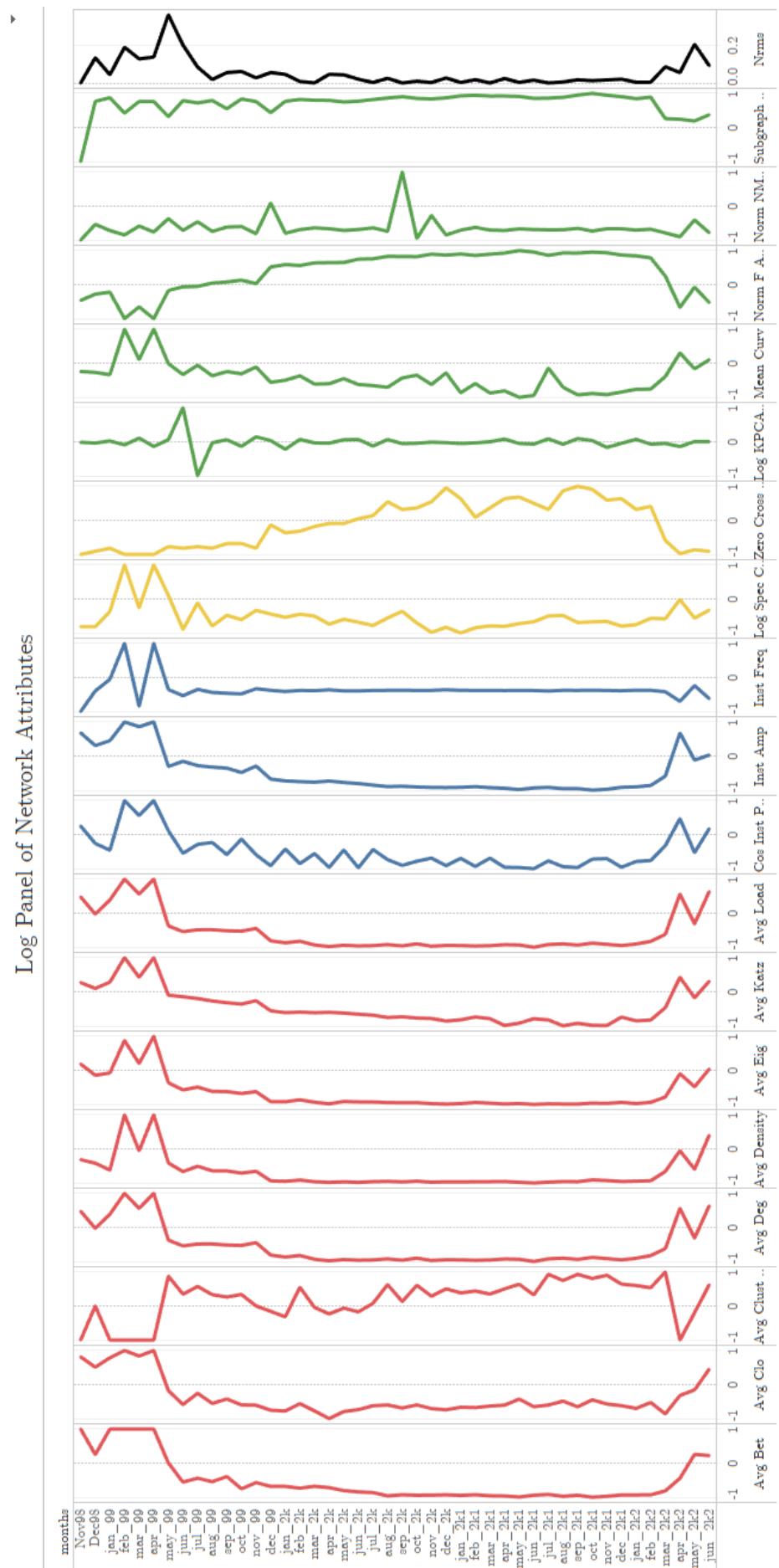


Figure 4.48: Log Panel of selected attributes. Benchmark attributes are shown in blue. Seismic Attributes shown in red. Music Attributes shown in yellow. Matrix Attributes shown in green and NRMS aggregation measure in black.

4.8 Node Level Trends

In this section I find common nodes across the 5 years and compare their centrality trends over time. The yearly trends for this part of the analysis is conducted as this is more plausible. The reason being that although these nodes are common across the years they do not have activity regularly across all months. Therefore it is deemed more informative to get an overview of their yearly centrality measure changes.

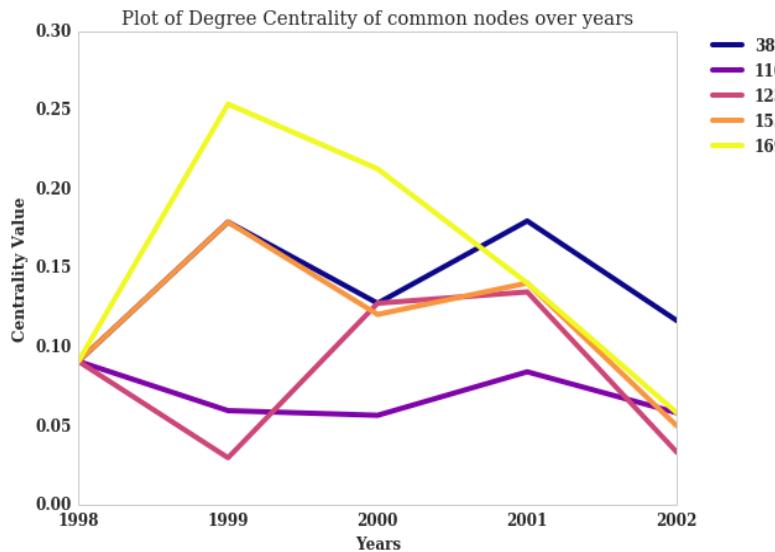


Figure 4.49: Plot of Common Node Degree over years

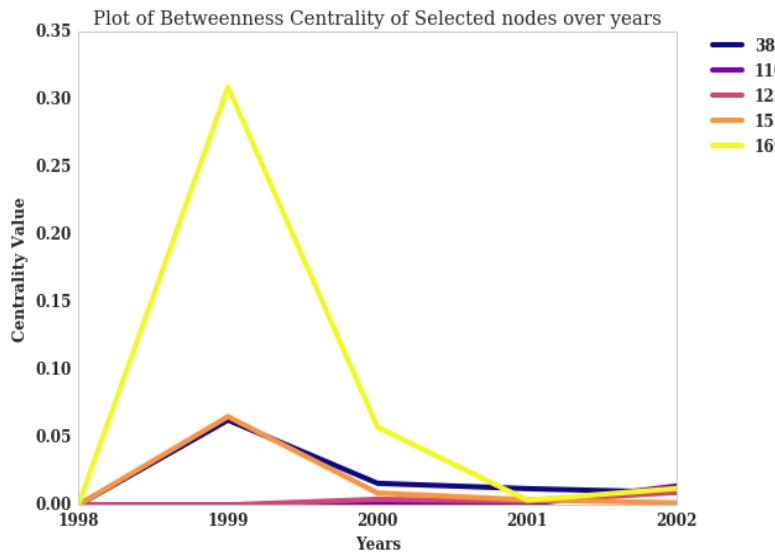


Figure 4.50: Plot of Common Node Betweenness Centrality over years

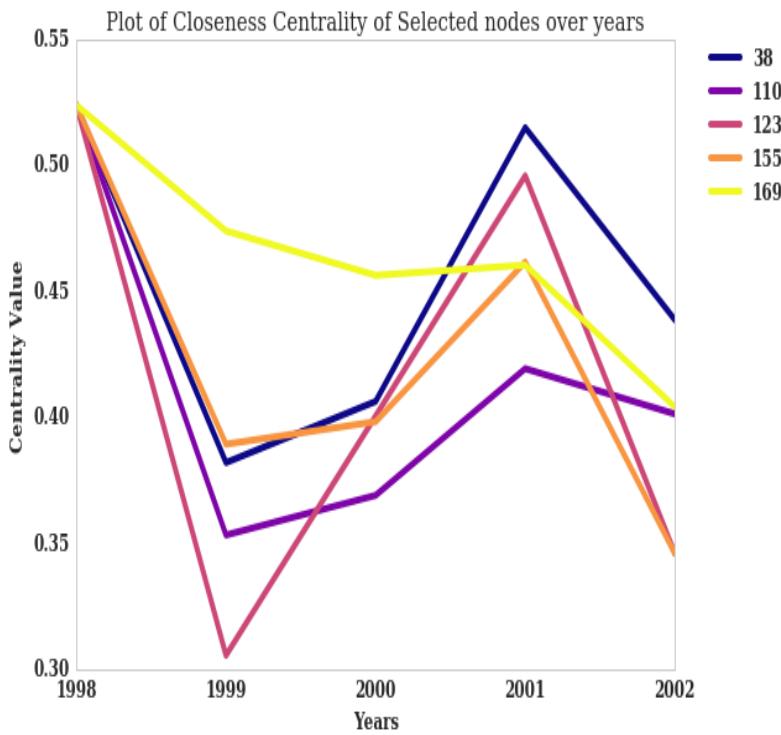


Figure 4.51: Plot of Common Node Closeness Centrality over years

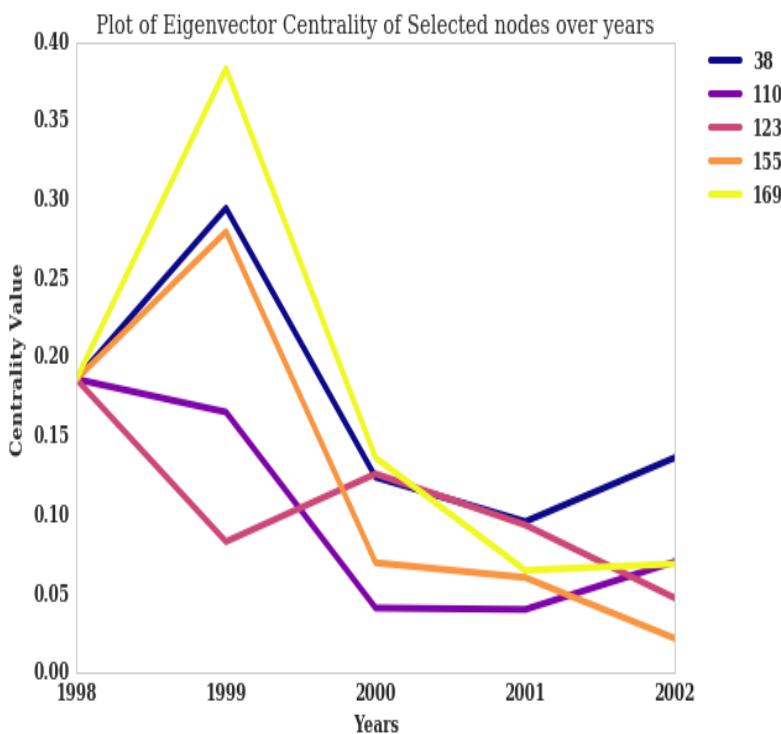


Figure 4.52: Plot of Common Node Eigenvector Centrality over years

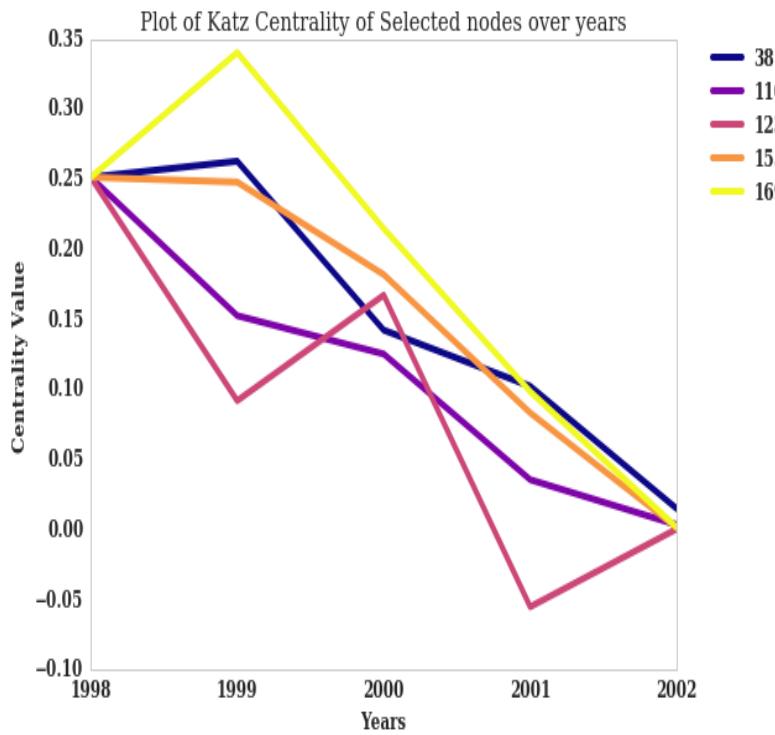


Figure 4.53: Plot of Common Node Katz Centrality over years

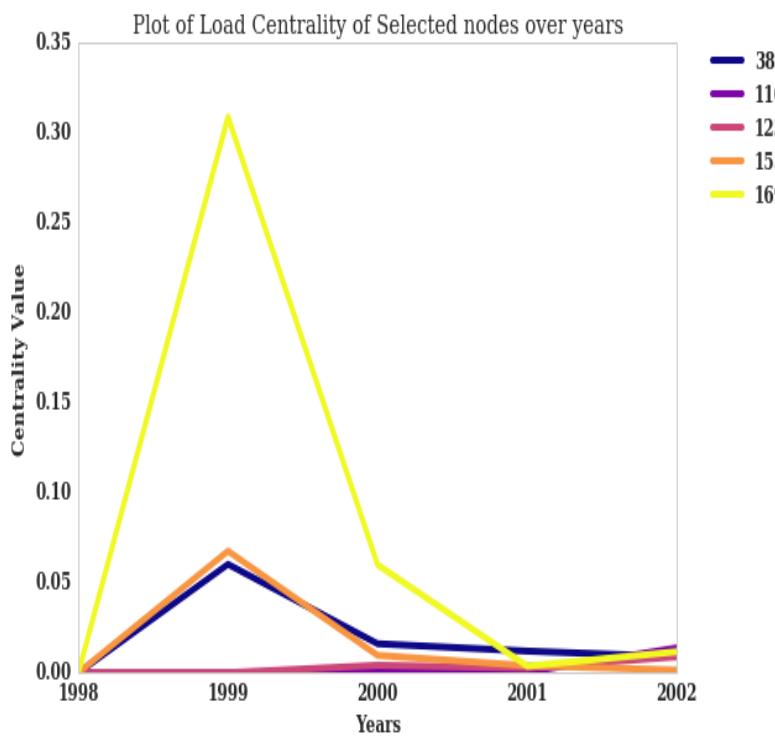


Figure 4.54: Plot of Common Node Load Centrality over years

Chapter 5

Discussion

5.1 Discussion of Original Aims and Objectives

The original objectives were as follows:

- What are some of the graph similarity measures that have been proposed in the literature?
- How have these been used in a practical context such as email data?
- How can such measures be applied to dynamic networks?
- How are such measures evaluated?

What are some of the graph similarity measures that have been proposed in the literature?

An extensive review has been presented in Chapter 2, regarding the issue of graph similarity and matching. Some of the methods suggested for similarity and graph matching are visual, spectral, tensor and distance based approaches. The different methods proposed have differing levels of complexity but far the most convenient approach was determined to be the feature based approach. This is because features are more compact snapshots of a network and we can have a large number of them to characterise the network and this gives us a way to treat the features or collections of features called the Attribute Volume in this study to be treated as a time series. This allows many analytical methods from time series analysis and signal processing to be applied to this volume. They allow us to easily analyse a large graph time series in a highly convenient manner.

How have these been used in a practical context such as email data? How can such measures be applied to dynamic networks?

The traditional network measures used in this study have typically been used in the case of static networks. However, in the case of dynamic networks the measures have been used as a time series with control charts used to determine signal. But in this study we use scaled attributes so we can detect spikes in the attribute time series and identify time steps of interest.

So for email networks Degree measures could represent the communication pattern in the network. Thus nodes with high centrality measures can be detected. Also the novel measures proposed can be used to highlight interesting time periods. So a time period identified as a amplitude or frequency anomaly for example could be investigated for the drivers of such change. This can lead to node level analysis through centrality measures as highlighted by the node level analysis presented in Chapter 4.

How are such measures evaluated?

The key thing to keep in mind when evaluating such measures is that there is not a right answer that we necessarily know of to begin with. There might be in some cases such as the Enron case where we know that in 2002 it went bankrupt when the scandal emerged. This is reflected in the graph time series through spikes in the attribute volume indicating a network that is undergoing some rapid change in this case the network was thinking out and dying. But in most cases we might be interested in finding the interesting times that we do now know off but would like to identify. With this in mind we can devise a more thorough evaluation procedure.

The evaluation approach used in this study is multi pronged in the sense that the first step was to try and establish some notion of ground truth this is done through the use of the benchmark measures. These measures are chosen as they are widely understood in the field and have well developed interpretations. From these measures we get a notion of the signal in the graph time series as to where the peaks and troughs are. For example the Average Clustering Coefficient is large when the networks are getting dense this correlates well to the Stationarity Ratio, Resistance Distance, Power Spectral Density and Zero Crossing Rate. These measures recover the characteristics of the Average Clustering Coefficient and highlights the characteristic two peaks of the other centrality measures. So comparison to existing benchmark measures is the first step. Secondly, I perform correlation analysis as well as an MDS and TSNE to get a notion of their correlations and closeness. Most of the attributes appear well correlated and some attributes are negatively correlated while a lot of the attributes have weak correlation. This indicates that from a correlation perspective only a few attributes are highly correlated so potentially have redundant information but encouragingly most of the attributes are negatively or weakly correlated to existing measures suggesting they are capturing dynamics that the traditional measures are not. The Correlation Network shows high degree for the correlated attributes while the Degree Histogram shows that 5 attributes are not connected to anything as there is not sufficient correlation such as the Instantaneous Phase and Derivative of the Amplitude attribute. The TSNE plots can make use of a range of distance functions so to get a sense how things are affected a few different distance measures such as the Euclidean, Canberra and Correlation Distance measures are tried. The Euclidean distance gives a somewhat comparable result to the MDS experiment whereas the Correlation distance gives one large cluster of attributes and the Canberra Distance gives a good separation among the attributes. This gives us an additional level of understanding that our novel attributes are in fact fairly unique and are capturing additional network dynamics that the traditional measures are not.

As a final level of evaluation and validation I propose regression testing to predict a fundamental network attribute based on the attribute volume. In this instance I chose the Average Degree this enables us to perform feature ranking and measure how well the novel attributes are at capturing network dynamics. If these metrics were not very good one would reasonably expect that the regression model would rank them in low importance but surprisingly we find that the novel measures are much more important to build a predictive model of the dynamic network. In addition it appears that measures that are not well correlated are more important than those that appear to be well correlated. From this we can conclude that these measures are capturing the basic signal hinted at by the benchmark measures, capturing the dynamics sufficiently well so they are highly correlated thus containing redundant information and they have predictive potential.

The original aims were as follows:

- To provide a fairly comprehensive overview of graph theory as is relevant to the understanding of the derivation of similarity measures
- Conducting an in depth literature review on the graph similarity measures proposed and that have been demonstrated to be useful in a practical context.
- Compare the utility and performance of these measures on appropriate data
- Explore the viability of developing a novel similarity measure based on Fourier and Hilbert Analysis of networks

Based on the discussion so far we can assert that all the aims and objectives for this project have been met. In addition they have been exceeded with metrics derived not just from Fourier and Hilbert spaces but Abel spaces and from Digital Music. Their interpretations have been discussed as well as their inclusion in a systematic and reproducible framework of analysis.

5.2 Research Question

How can similarity measures be used to analyse dynamic email networks? How can similarity measures in alternative spaces support the analysis of dynamic networks?

The research question has been addressed in many different forms over these chapters but to discuss it conclusively we can say that similarity measures can be used as a way of convenient analysis of dynamic networks. This is because the feature based approach as illustrated in this study enables us to treat our dynamics networks as a graph time series and thus allow us to apply signal processing and time series analysis approaches.

The feature based approach used in this study is used to derive a attribute volume that is then used to support multiple attribute interpretation and characterisation of the graph time series through novel visualisation techniques. Also it allows us to derive aggregate measures that serve as useful measures of network snapshot and support the overview first, zoom and details on demand type analysis. This is

discussed further in Chapter 6, where the generalisation of this approach is discussed.

Utilisation of similarity measures in the context of email networks or any dynamic network serve as convenient snapshots that allow us to build a time series to enable analysis of such networks at any level of granularity.

The derivation of attributes from alternative spaces such as Fourier, Hilbert and Abel spaces allow for novel attributes and visualisations to be applied. The Fourier Transform forms the basis of the Music Attributes that are derived because we turn the Normalised Laplacian into a frequency trace which can be then visualised as a waveform and then this waveform allows derivation of attributes such as the Zero Crossing Rate and Spectral Centroid. In addition the Fourier Transform forms the basis of the FK plot helps to visualise the large high dimensional attribute volume in a 2D space and identifies outliers. The Frequency and Wavenumber components of this plot can also be visualised as Heatmaps. Also the Abel Transform attribute takes the magnitude of the real and imaginary components of the frequency derived from the Fourier Transform and finds the lower dimensional representation of it. This attribute as the regression analysis shows is very useful for building a predictive model for this particular dynamic network. The Hilbert Transform forms the basis of the derivation of the Seismic Attributes as it enable the creation of a complex analytical trace from a real valued function. These attributes have proved useful in identifying anomalies in the time series.

Another class of attributes that were not in the original plan were the so called matrix decomposition attributes such as the PCA Ratio and NMF Ratio which have proved their ability to detect change points and have been shown to be useful for the Regression model.

Therefore the use of similarity measures can aid the analysis of dynamic networks by creating an attribute volume over time that can be used for network interpretation and characterisation. The use of alternative spaces allows for the derivation of new and creative attributes as well as visualisation approaches to facilitate analysis as well as predictive model building for dynamic networks.

Chapter 6

Evaluation, Reflection and Conclusions

6.1 Evaluation of Results

In this study I have presented a work flow for the systematic analysis of dynamic networks through the use of multitude of similarity measures and visualisation techniques. The methods surveyed and utilised range from traditional graph measures such as Centrality and Linear Algebra based measures to novel measures based on Hilbert, Fourier, Abel and Matrix Decomposition techniques. The intention was to present these novel methods from a theoretical stand point and validated them through their application on real data to conclusively demonstrate their merit. As an additional level of validation all these measures were ranked on their predictive capability for fundamental network measure such as Average Degree. From this analysis it is clear that the novel measures proposed in this study especially the measures that are not well correlated to existing measures such as Norm of the Abel Transform and Norm of the NMF Ratio Change are particularly well suited for this specific task.

The analysis was conducted at both the year and monthly level on three different graph matrix representations. The most suitable was determined to be the Normalised Graph Laplacian as discussed in Chapter 3. From this basis a large number of measures were derived for these networks which come from the field of Seismic Data Analysis and Music Information Retrieval (MIR). These derived measures are then compared against a set of benchmark measures to assess their suitability. These measures are interesting because they are able to capture the underlying signal that is suggested by the benchmark measures and highlight additional areas of interest.

For completeness the attributes are derived from 3 different graph matrices the Normalised Laplacian, Modularity and Adjacency Matrix as shown in 4.21 and 4.23. I use two information theoretic measures like the Signal to Noise Ratio (SNR) and Entropy to help assess which matrix is better. To do this all the attributes from the different matrices I calculate the SNR ratio and Entropy.

$$H = - \sum p(x) \log_2 p(x) \quad (6.1)$$

The SNR plot is useful as a first step to get an idea of the information content of the

attributes derived and which matrix might be better. From 4.22 we could conclude that any other matrix apart from the Laplacian would be a good choice because all the other matrices have a high number of attributes with a high SNR value meaning that their deviation is low compared to their mean. But upon inspection of the attributes with high and low SNR values this turns out to be misleading. This is because when the SNR is high the attributes behave identically from the three matrices and they represent well the trends highlighted by the benchmark measures. However, for the attributes that have a low SNR the attributes from the Modularity and Adjacency Matrix attributes do not model the trend in the benchmark measures and they fail to recover some of the basic characteristics of the signal observed such as the prominent peaks at the beginning and end of the time series due large variations in network size. But the Laplacian attributes do not suffer from this issue they model the trends observed in the benchmark measures well in both cases of high and low SNR value. Thus analysis by SNR alone was deemed insufficient. As an additional level of validation I opted to use the Entropy measure. As 4.24 shows the mean Entropy and the Entropy of the attributes 4.23 are very similar for the attributes from the different matrices. Thus the attributes must be judged on their ability to represent the basic characteristics of the signal from the benchmark measures and suggest potentially interesting areas. This is deemed to the Laplacian attributes in this study through inspection of the attributes against the benchmark measures so these attributes are used in all subsequent analysis.

The attributes from Seismic Data Analysis are predominantly derived from the Complex Trace which relies on the Hilbert Transform. The Music attributes rely on the frequency waveform which rely on the Fourier or Stockwell Transform but the Fourier transform is used in this study. In addition the Abel transform is used which finds a lower dimensional slice from a 3D cylindrical symmetric projection onto a lower 2D surface. The frequency component from the Normalised Laplacian is transformed into this subspace through the Abel transform and gives us one of the more interesting attributes.

The Norm of Abel Transform attribute we note from 4.33 shows that the attribute finds the initial two peaks that are associated with the Feb - Apr 1999 period. This period as we see from 4.4, 4.5 and 4.6 that these periods correspond to very small network sizes. In the node link diagram this is highlighted by a small number of connections. In the Matrix plot by a small number of edge connections and in the audio waveform plot by the blocky appearance of the plots. The audio waveform is particularly useful because as this is based on the frequency of the Normalised Laplacian a low number of peaks is indicative of low frequency meaning there are not many edge connections. This makes intuitive sense when we look at the remaining plots all which show that the waveform has many spikes along its length and does not share the distinctive blocky appearance. When considering a large number of networks as in this study we see that the node link diagrams quickly turn into a hairball once the network sizes grow. Another problem with this visualisation is that the placement of the nodes are recalculated each time and are not invariant so the physical location of the nodes are meaningless. The matrix visualisations on the other had are more useful to a certain extant and reordering them by cluster indices is a useful strategy. But unless the matrices can be visualised as a cube this

is also limited if static views are required. It would be very hard to present these visualisations in report form for a end user but their utility is more for the analyst. But the waveform plots can be intuitively understood in the sense that if we take the x-axis of the waveform to be indicative of the number pf nodes in the network. So the waveform length becomes symbolic of network size then the spikes correspond to the frequency of their connections. In a large network we would then expect to see lots of spikes and a longer length of the waveform. Although the Fourier Transform to derive requires a certain length in which case the trace is padded to a minimum length but even then the frequency of the spikes and their wavelength gives us an indication of the underlying network in a concise way in a manner that is invariant to position and order. This makes it a visualisation better suited for both static and interactive views. The node link and matrix diagram gives us an additional level of validation for the explanation of these plots. In the case of the Norm of Abel Transform attribute the trend of the attribute captures this intuition well. The reason being that the peaks in the Feb-Mar 1999 period at the beginning of the time series to the Apr - Jun 2002 period at the end of the time series correspond to the most dramatic changes in the network size. In the 1999 period the network is unusually small and at the end the network is in the process of thinning out which corresponds to the large spikes in the signal again. The remaining time periods in the time series correspond to increasing or larger size of the network in comparison to these periods. Hence the Centrality and the other traditional measures show these periods as having smaller values. This part of the series for these benchmark measures are dominantly smooth with the exception of the Average Clustering Coefficient. The reason being that an increasing size of the network usually results in more local clustering which this metric measures. This metric captures the intuition well that as the interactions in the network grow the metrics should show a similar trend. However, the centrality measures show this is in a counter intuitive way and so do most of the Seismic Attributes which are highly correlated to these centrality measures.

From the seismic attributes such as Amplitude and the attributes derived from it such as Power, the first and second derivative highlight these big changes in the network very well. These measures in Seismic Attribute analysis are used to detect amplitude anomalies and in this context these periods which cause the spikes observed correspond to such anomalies. The Amplitude anomalies and by extension the Power anomalies thus correspond to the time periods when the network is unusually small as compared to the other time periods. It is encouraging to see that the analogy from seismic holds well in the context of dynamic network analysis. The Phase attribute is used as a direct indicator of hydrocarbons as the presence of fluids in the rock causes phase changes for the waves passing through it in a seismic experiment. This highlights additional periods which undergo change. The raw phase attribute is not particularly smooth and difficult to interpret as it appears very noisy. Therefore it is common to look at the cosine of the Instantaneous Phase for display purposes as taking the cosine constrains the signal to a $[-1, 1]$ range and reverses the polarity of the signal to that which is more preferable i.e looking at peaks or troughs in the signal. The cosine of IP highlights additional areas in addition to these anomalies already noted well.

Also the Phase derived attributes such as the Frequency and Acceleration represent a smoother attribute than the raw Phase attribute. The Frequency attribute highlights the major anomalies particularly well and highlights some of the peaks highlighted by the Phase and cosine IP attributes and the Average Closeness Centrality measure. However these additional areas of potential interest are not very clear from these measures but the Acceleration highlights some of these additional periods such as Mar 2000 well. The Phase and Frequency attributes weighted by Amplitude are better in the sense that they are able to distinguish the peaks caused by the small network size and highlight additional spikes better such as Oct - Nov 1999 periods. The correlation between the Seismic Attributes and the Centrality and Assortativity measures is surprising because the centrality measures are predominantly calculated from the adjacency matrix while the Seismic Attributes are calculated from the Normalised Laplacian.

The matrix based attributes such as the Curvature, Resistance Distance, Stationarity, Subgraph Stationarity and Power Spectral Density highlight other features of the signal apart from these two peaks. The Power Spectral Density suggests that the periods between Aug - Oct 2001 are more interesting than the early 1999 and late 2002 periods. This attribute suppresses these parts of the time series and suggests this period as being more interesting. This is interesting because from what we know about the network the beginning and the end are due to network size being abnormally small but this part of the time series the networks are fairly dense thus it has the ability to highlight interesting areas among denser parts of the graph time series and suppress noisy parts. It is possibly that this period has the most dense part of the network which translates to higher Power Distribution over time compared to the rest of the time periods.

The Resistance Distance measures the resistance between nodes analogous to the flow of current in electrical networks thus it could reasonably be assumed that given more paths in a network that the connections will find the path of least resistance. It is this intuition that we expect this measure to capture. So we see peaks corresponding to the early and late periods but the bigger peaks correspond to later time periods when the networks are more dense. This suggests that during these periods even though the networks are getting dense that they are not fully connected. This is suggested by the Algebraic Connectivity as well but the Resistance Distance is clearer about these time periods in the sense that it highlights peaks in these areas such as Feb - Jun 2000.

The Music Attributes are interesting because they capture fairly different behaviour from the waveforms. The Zero Crossing Rate which is the number of times the signal crosses zero over time is greater when the network is dense and less so when the networks are sparse. The Spectral Centroid measure which calculates the centre of gravity of a spectrum is lower for denser parts than it is for the sparse parts which is expected because it is essentially weighted mean of the frequency components which are likely to get smoothed out when the network is denser in contrast to when it is more sparse.

The Mean Curvature attribute finds the July to Nov 2001 period as being partic-

ularly interesting which is corroborated by the other matrix attributes and not as clear from the benchmark measures. In Seismic Attribute attributes changes in curvature are associated with structures such as faults, fractures and discontinuities. It is encouraging that the discontinuities in this chase the sparse networks of the graph time series are highlighted in addition to interesting parts from the denser section of the graph time series.

The stationarity attributes such as the Stationarity and the Subgraph Stationarity highlight the sparse parts of the network as troughs and denser parts as peaks and point to the July to Nov 2001 as having undergone the most change. In the context of the stationarity measures this translates to large positive changes in the size of the network. The 1-zeta attribute which shows the proportion of members that change at a time step shows that the early and later parts of the time series as having undergone the greatest change in the proportion of members.

The KLP PCA Ratio is used to detect large changes in seismic data such as unconformities from this we see that the largest change in this attribute is the Mar - July 1999 range this just after the sparse parts of the network at the point where the network starts to become dense. This is very interesting as it is a change point in the network that is not clear from the other measures. Most of the other measures highlight either aspects of the sparse part or the dense part of the network but the KLP PCA ratio highlights this change point that would have been missed otherwise.

The Norm NMF Ratio highlights the periods identified by the other attributes but refines the range from Aug - Nov 2000. The other changes are far less prominent in this attribute.

The Abel Transform attribute captures the intuition very well that the sparse parts of the network shows up as smaller peaks and from the change point there is peak for the areas of the time series where the network is dense. The trough at the end of the time series corresponds again to the thinning of the network. This measure captures very well the dynamics of the whole time series highlighting the insights gained from all the remaining measures.

In terms of the trends for the years all the attributes agree that the biggest change in the series was between 1999-2001. This corresponds to the densification of the network and is picked up by all measures with different levels of clarity. The yearly aggregation is not very interesting as there are not many samples but this is the main signal from the yearly time series.

From the MDS and TSNE plots Fig 4.37, 4.40 we get a sense of the clustering of the attributes. As expected the Centrality measures are close to each other but the surprise is the Load and Density metrics which are separate from everything else on the MDS plot. Also on the TSNE plot with the Euclidean Distance ?? the Load stands out as being separate from the rest while the Norm attributes such as the NMF and Abel attributes are close together. The Centrality measures are also close together as are the seismic attributes to which they are well correlated. Another curious feature with this distance metric for the TSNE is that the aggregation measures are

not close to each other but are fairly well separated while with the other two distance metrics they are fairly close to each other. The Correlation distance seems to suggest that most of the attributes have low correlation distance hence the presence of the central large cluster with the Average Eigenvector Centrality, Zero Crossing Rate and Power attributes standing out. This is counter to what the correlation matrix has shown for starters that most of the attributes are weakly correlated, while some are negatively correlation and some are strongly correlated. So this distance metric is not recommended for this application. The Canberra distance is better because it gives good separation among the attributes. Also interestingly the Music Attributes are placed close to each other. So overall the MDS visualisation is probably better to use as its lack of options for the dissimilarity metrics makes the analytical options limited which is helpful in this case.

The Radon and FK Plots in addition to their attribute map give us an additional way to visualise the whole attribute volume over time. From the Radon plot it is clear that there is some structure in this data with 3 apparent clusters consisting of the early part when the network is very sparse the growth and the final decay of the network. Also the FK plot allows us to visualise the attribute volume not only together but as slices through the FK attribute volume. The Log Panel gives us a way to easily analyse multiple attributes in the style of well log analysis used for hydrocarbon exploration. The idea here is that we have panels of different attributes and in this the case the attributes which are highly correlated are clustered together in the panel display. This allows for easy peak and trough tracking across the time series for multiple attributes at the same time. This gives as an easy way to gain insight from the graph time series utilising not just traditional graph measures but the Seismic and Music Attributes introduced in this study.

Also I have identified 5 key nodes at the yearly aggregation level. The trends in the centrality of these nodes broadly mimic the trend observed at the yearly scale. These nodes appear to experience the largest change between 1999-2001. Node 167 seems to be particularly important as it seems to have high centrality values across all the measures over the time period compared to the rest of the nodes. For node 199 values for the centrality measures rise sharpest from 1999-2000 but falls thereafter while node 38 has a sharp rise in the measures from 2000-2001 compared to the rest of the nodes. Although it was not possible to locate employee id data for these nodes so they could be merged but one could posit that these nodes could probably represent senior managers whose influence changed but did not diminish completely over time.

From the correlation analysis shown in Fig 4.35,4.34 and 4.36 we see that the traditional metrics are well correlated to a lot of the new metrics proposed. Most particularly it appears that the centrality measures are particularly well correlated to some of the Seismic and Music attributes such as Instantaneous Amplitude, Cosine of Instantaneous Phase, Mean Curvature, Spectral Centroid and Power. The Spectral Centroid appears to be strongly correlated to the Eigenvector and Katz Centrality which are based on the eigenvalue analysis of the graph matrices as opposed to the other centrality measures. The remaining attributes such as the matrix attributes and the Zero Crossing Rate do not seem to be well correlated to the other metrics but rather negatively correlated to most of the metrics. Particularly inter-

esting are the Abel attribute which is negatively correlated and the NMF attribute which is not correlated to the other metrics but appear to have high predictive capacity. The predictive importance of these attributes is shown in Fig 4.41 and the dominant attributes are the Abel and NMF attributes which are not well correlated to the other measures. Thus a case can be made that these measures are capturing important network dynamics while the correlated measures are capturing similar information to the centrality measures. In addition it could be argued that the matrix attributes such as the Norm of the Abel Transform, Norm of the NMF Ratio Change, Stationarity type attributes and Curvature capture vital network dynamics that enable the prediction of fundamental network attribute at a time step in the future. Thus we have a framework for linking all our results.

The aggregation schemes are important as they enable us to develop a single metric that serves as a measure of network activity. Therefore having a suite of measures which capture different aspects of network dynamics well should reflect in the final aggregation. But there is always a potential drawback that such aggregation measure smoothes out smaller changes. Hence the Log Panel of multiple attributes can provide additional context. Of the 3 measures introduced the NRMS is preferable because it is a normalised RMS measure so it already has a degree of noise suppression due to the RMS operator. But since all the attributes are scaled to the $[-1, 1]$ range this is perhaps less important. But what it highlights well is that it picks up bursts in the network in addition to smaller changes, is numerically better constrained and overall much smoother than the RMS and Emergence measure.

Therefore it can be said that from these measures we can identify not only interesting time steps but also interesting nodes over time. The node level analysis is only presented for the centrality measures because since they are well correlated with a lot of the Seismic and Music attributes so they would not tell us much new information. Also since these nodes seem to mimic the trend in the time series it is easier to think about them in terms of traditional centrality measures because the matrix attributes are more network level attributes.

6.2 Generalisation of Analytical Work flow

Therefore generalising this analytical work flow for any dynamic network scenario is possible. This can be done by the approach followed in this study. First, for the network under consideration decide the granularity at which to conduct the analysis such as yearly, monthly, weekly or daily. Although depending on the type of network there might not be sufficient network density at too fine a granularity such as daily for meaningful analysis but this can be evaluated against the data available and the application. Once the granularity has been decided segment the network into the required granularity and for each time step derive the Normalised Graph Laplacian. The attribute volume can be generated easily from this and an aggregation scheme can be chosen such as the RMS, NRMS and/or Emergence. Then as more data is available the process can be repeated at different time step resulting in a snapshot of the network through these metrics. The correlation analysis will suggest which measures are likely to be redundant and thus a subset of the more interesting measure can be selected. This can be seen as a pruning step. Not all measures will be useful in every application and it is therefore important to discern how each measure

performs depending on the domain. Since these measures have only been applied in an email network context they might behave differently in another domain. This should be checked for different application domains.

Once the measures and the aggregation schemes are picked a potential dashboard could consist of an overview panel of the aggregation measure highlighting network activity. Then the Log Panel of selected metrics could provide additional insight of network dynamics. The Radon plots can be used to assess structure in the data and the FK plot can be used for outlier detection.

When interesting time steps are identified the Waveform plot of the network would give a good overview of the underlying network with further investigation through the Matrix and Node link views.

6.3 Reflections and Conclusions

In this study I have presented a large body of metrics for the analysis of dynamic networks which have been inspired by the fields of Seismic Analysis and Music Information Retrieval. The original aims of the project were to investigate current features and the potential of alternative mathematical spaces such as the Fourier and Hilbert space to support such analysis. I have shown that there is great merit in the use of such integral transforms to derive powerful attributes to capture network dynamics. As a result the original project aims have been comfortably met and exceeded.

There is not much that could be done differently regarding the project planning and execution. This is because good planning and management has allowed for the efficient management of the research and effective collation of the results well before the deadline for the project. Project management best practices such as sensible aims, agile development and timely communication with feedback being continuously assimilated. This has led to timely tweaks to the approach which has enabled a robust and relevant set of measures and results to be developed.

In conclusion it can be said that the feature based approach to the analysis of dynamic networks has great merit as the network statistics and attributes are much smaller than the potentially large and dense networks that they can be derived from. Also, a proof of concept has been presented for attributes derived from alternative mathematical spaces such as Fourier, Hilbert and Abel spaces. To support the analysis of these multiple attributes visualisation approaches have been proposed and demonstrated to be effective. Also a correlation and regression based feature ranking and evaluation approach has also been introduced. The MDS and TSNE plots have shown that most of the derived attributes are clustered together but not entirely close to the traditional metrics. Therefore the aims of the project have been met and novel measures introduced which have allowed for the original aims to be met and exceeded.

The study has not presented a generalised work flow for the systematic and reproducible analysis of dynamic networks but has proposed creative and novel attributes to support and visualise the results. As a result we have succeeded in providing a

reference for practitioners or analysts interested in such analysis along with an evaluation of the metrics that are useful.

Overall, this study has met and exceeded both its original objectives and revised objectives stated in Appendix A and Chapter 1 of this report.

Bibliography

- [1] Allen Dickson. “Introduction to Graph Theory The Konigsberg Bridge Problem”. In: *October* October (2006), pp. 1–15.
- [2] Anurat Chapanond, Mukkai S. Krishnamoorthy, and Bülent Yener. “Graph theoretic and spectral analysis of Enron email data”. In: *Computational and Mathematical Organization Theory* 11.3 (2005), pp. 265–281. ISSN: 1381298X. DOI: 10.1007/s10588-005-5381-4.
- [3] Kathleen M. Carley. “Dynamic Network Analysis”. In: September (2001).
- [4] Kathleen M. Carley, Jana Diesner, Jeffrey Reminga, et al. “Toward an interoperable dynamic network analysis toolkit”. In: *Decision Support Systems* 43.4 (2007), pp. 1324–1347. ISSN: 01679236. DOI: 10.1016/j.dss.2006.04.003.
- [5] Tom A B Snijders. “Introduction To Dynamic Social Network Analysis”. In: *BRiMS 2015* November (2015).
- [6] Stephen P. Borgatti, Kathleen M. Carley, and David Krackhardt. “On the robustness of centrality measures under conditions of imperfect data”. In: *Social Networks* 28.2 (May 2006), pp. 124–136. ISSN: 03788733. DOI: 10.1016/j.socnet.2005.05.001. URL: <http://www.sciencedirect.com/science/article/pii/S0378873305000353>.
- [7] Stephen P. Borgatti, Kathleen M. Carley, and David Krackhardt. “On the robustness of centrality measures under conditions of imperfect data”. In: *Social Networks* 28.2 (May 2006), pp. 124–136. ISSN: 03788733. DOI: 10.1016/j.socnet.2005.05.001. URL: <http://www.sciencedirect.com/science/article/pii/S0378873305000353>.
- [8] Stephen P. Borgatti. “Centrality and network flow”. In: *Social Networks* 27.1 (2005), pp. 55–71. ISSN: 03788733. DOI: 10.1016/j.socnet.2004.11.008.
- [9] Ahmad Rawashdeh and Anca L Ralescu. “Similarity Measure for Social Networks – A Brief Survey”. In: (2012).
- [10] Matthias Dehmer, Frank Emmert-Streib, and Jürgen Kilian. “A similarity measure for graphs with low computational complexity”. In: *Applied Mathematics and Computation* 182.1 (Nov. 2006), pp. 447–459. ISSN: 00963003. DOI: 10.1016/j.amc.2006.04.006. URL: <http://brainmaps.org/pdf/similarity1.pdf> <http://linkinghub.elsevier.com/retrieve/pii/S0096300306003523>.
- [11] D Koutra, a Parikh, a Ramdas, et al. “Algorithms for Graph Similarity and Subgraph Matching”. In: (2011). URL: <http://www.cs.cmu.edu/%7B~%7Daramdas/reports/DBreport.pdf>.

- [12] Sucheta Soundarajan, T Eliassi-Rad, and Brian Gallagher. “A guide to selecting a network similarity method”. In: *Sdm* 1 (2014). DOI: 10.1137/1.9781611973440.118. URL: <http://pubs.siam.org/doi/abs/10.1137/1.9781611973440.118>.
- [13] Laura A. Zager and George C. Verghese. “Graph similarity scoring and matching”. In: *Applied Mathematics Letters* 21.1 (2008), pp. 86–94. ISSN: 08939659. DOI: 10.1016/j.aml.2007.01.006.
- [14] Gaoxia Wang, Yi Shen, and Enjie Luan. “Measure of centrality based on modularity matrix”. In: *Progress in Natural Science* 18.8 (2008), pp. 1043–1047. ISSN: 10020071. DOI: 10.1016/j.pnsc.2008.03.015.
- [15] Ron Zass and Amnon Shashua. “Probabilistic graph and hypergraph matching”. In: *26th IEEE Conference on Computer Vision and Pattern Recognition, CVPR* (2008), pp. –7. ISSN: 1063-6919. DOI: 10.1109/CVPR.2008.4587500.
- [16] D. Conet, P. Foggia, C. Sansone, et al. “Thirty Years of Graph Matching in Pattern Recognition”. In: *International Journal of Pattern Recognition and Artificial Intelligence* 18.03 (2004), pp. 265–298. ISSN: 0218-0014. DOI: 10.1142/S0218001404003228.
- [17] H Bunke. “Recent developments in graph matching”. In: *Pattern Recognition, 2000. Proceedings. 15th ... 2* (2000), pp. 117–124. ISSN: 1051-4651. DOI: 10.1109/ICPR.2000.906030. URL: [http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=906030%5Cbackslash\\$http://ieeexplore.ieee.org/xpls/abs%7B%5C_%7Dall.jsp?arnumber=906030](http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=906030%5Cbackslash$http://ieeexplore.ieee.org/xpls/abs%7B%5C_%7Dall.jsp?arnumber=906030).
- [18] F. Ashby and Daniel Ennis. “Similarity measures”. In: *Scholarpedia* 2.12 (2007), p. 4116. ISSN: 1941-6016. DOI: 10.4249/scholarpedia.4116. URL: http://www.scholarpedia.org/article/Similarity%7B%5C_%7Dmeasures.
- [19] H Yaghi and H Krim. “Probabilistic graph matching by canonical decomposition”. In: *Image Processing, 2008. ICIP 2008. 15th IEEE International Conference on* (2008), pp. 2368–2371. DOI: 10.1109/ICIP.2008.4712268.
- [20] D.a. Spielman. “Spectral Graph Theory and its Applications”. In: *48th Annual IEEE Symposium on Foundations of Computer Science (FOCS’07)* 1 (2007), pp. 29–38. ISSN: 0272-5428. DOI: 10.1109/FOCS.2007.56.
- [21] Tib??rio S. Caetano, Julian J. McAuley, Li Cheng, et al. “Learning graph matching”. In: *IEEE Transactions on Pattern Analysis and Machine Intelligence* 31.6 (2009), pp. 1048–1058. ISSN: 01628828. DOI: 10.1109/TPAMI.2009.28. arXiv: 0806.2890v1.
- [22] A. Egozi, Y. Keller, and H. Guterman. “A Probabilistic Approach to Spectral Graph Matching”. In: *IEEE Transactions on Pattern Analysis and Machine Intelligence* 35.1 (2013), pp. 18–27. ISSN: 0162-8828. DOI: 10.1109/TPAMI.2012.51. URL: [http://ieeexplore.ieee.org/ielx5/34/6353858/06152128.pdf?tp=%7B%5C&%7Darnumber=6152128%7B%5C&%7Disnumber=6353858%5Cbackslash\\$http://ieeexplore.ieee.org/xpl/articleDetails.jsp?tp=%7B%5C&%7Darnumber=6152128%7B%5C&%7DsortType=desc%7B%5C_%7Dp%7B%5C_%7DPublication%7B%5C_%7DYear%7B%5C&%7DqueryText=Maximum+Likelihood+Estimation](http://ieeexplore.ieee.org/ielx5/34/6353858/06152128.pdf?tp=%7B%5C&%7Darnumber=6152128%7B%5C&%7Disnumber=6353858%5Cbackslash$http://ieeexplore.ieee.org/xpl/articleDetails.jsp?tp=%7B%5C&%7Darnumber=6152128%7B%5C&%7DsortType=desc%7B%5C_%7Dp%7B%5C_%7DPublication%7B%5C_%7DYear%7B%5C&%7DqueryText=Maximum+Likelihood+Estimation).

- [23] Robert a Hanneman and Mark Riddle. “Introduction to Social Network Methods”. In: *Riverside, CA: University of California, Riverside. On-line textbook* 46.7 (2005), pp. 5128–30. ISSN: 10744770. DOI: 10.1016/j.socnet.2006.08.002. URL: <http://www.faculty.ucr.edu/%7B~%7Dhanneman/nettext/>.
- [24] Colin McDiarmid and Fiona Skerman. “Modularity in random regular graphs and lattices”. In: *Electronic Notes in Discrete Mathematics* 43 (2013), pp. 431–437. ISSN: 15710653. DOI: 10.1016/j.endm.2013.07.063. URL: <http://linkinghub.elsevier.com/retrieve/pii/S1571065313002151>.
- [25] Maciej Komosinski and Marek Kubiak. “Quantitative measure of structural and geometric similarity of 3D morphologies”. In: *Complexity* 16.6 (2011), pp. 40–52. ISSN: 10762787. DOI: 10.1002/cplx.20367.
- [26] Jérôme Kunegis, Damien Fay, and Christian Bauckhage. “Spectral evolution in dynamic networks”. In: *Knowledge and Information Systems* 37.1 (Oct. 2013), pp. 1–36. ISSN: 0219-1377. DOI: 10.1007/s10115-012-0575-9. URL: <http://link.springer.com/10.1007/s10115-012-0575-9>.
- [27] Andries E. Brouwer and Willem H. Haemers. *Spectra of Graphs*. Vol. 1542. Universitext. New York, NY: Springer New York, 2012, pp. 33–36. ISBN: 978-1-4614-1938-9. DOI: 10.1007/978-1-4614-1939-6. arXiv: arXiv:1011.1669v3. URL: <http://link.springer.com/10.1007/978-1-4614-1939-6>.
- [28] Olivier Duchenne, Francis Bach, Kweon In-so, et al. “A Tensor-Based Algorithm for High-Order Graph Matching”. In: (2014).
- [29] Michael Behrisch, Benjamin Bach, Nathalie Henry Riche, et al. “Matrix Reordering Methods for Table and Network Visualization”. In: 35 (2016).
- [30] Michael Behrisch, Benjamin Bach, Michael Hund, et al. “Magnostics: Image-based Search of Interesting Matrix Views for Guided Network Exploration”. In: *IEEE Transactions on Visualization and Computer Graphics* (2016), pp. 1–1. ISSN: 1077-2626. DOI: 10.1109/TVCG.2016.2598467. URL: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=7534849>.
- [31] Y Park, C.E. Priebe, and D. J. Marchette. *Scan Statistics on Enron Hypergraphs*. 2008. URL: <http://www.cis.jhu.edu/%7B~%7Dparky/Enron/>.
- [32] Aric A. Hagberg, Daniel A. Schult, and Pieter J. Swart. “Exploring network structure, dynamics, and function using NetworkX”. In: *Proceedings of the 7th Python in Science Conference (SciPy2008)*. Pasadena, CA USA, Aug. 2008, pp. 11–15.
- [33] Stéfan van der Walt, S. Chris Colbert, and Gaël Varoquaux. “The NumPy Array: A Structure for Efficient Numerical Computation”. In: *Computing in Science Engineering* 13.2 (2011), pp. 22–30. DOI: <http://dx.doi.org/10.1109/MCSE.2011.37>. URL: <http://scitation.aip.org/content/aip/journal/cise/13/2/10.1109/MCSE.2011.37>.
- [34] Eric Jones, Travis Oliphant, Pearu Peterson, et al. *SciPy: Open source scientific tools for Python*. [Online; accessed 2016-08-04]. 2001–. URL: <http://www.scipy.org/>.
- [35] Wes McKinney. “Data Structures for Statistical Computing in Python”. In: *Proceedings of the 9th Python in Science Conference*. Ed. by Stéfan van der Walt and Jarrod Millman. 2010, pp. 51–56.

- [36] J D Hunter. “Matplotlib: A 2D graphics environment”. In: *Computing In Science & Engineering* 9.3 (2007), pp. 90–95.
- [37] Michael. Waskom. *Seaborn: statistical data visualization*. 2012. URL: <http://stanford.edu/%7B~%7Dmwaskom/software/seaborn/> (visited on 04/06/2016).
- [38] F Pedregosa and G Varoquaux. “Scikit-learn: Machine Learning in Python”. In: *Journal of Machine ...* 12 (2011), pp. 2825–2830. arXiv: 1201 . 0490. URL: <http://jmlr.csail.mit.edu/papers/volume12/pedregosa11a/pedregosa11a.pdf>.
- [39] Brian McFee, Matt McVicar, Colin Raffel, et al. *librosa: 0.4.1*. Oct. 2015. DOI: 10.5281/zenodo.32193. URL: <http://dx.doi.org/10.5281/zenodo.32193>.
- [40] Daniel D. Hickstein, Roman Yurchak, Dhrubajyoti Das, et al. *PyAbel (v0.7): A Python Package for Abel Transforms*. Mar. 2016. DOI: 10.5281/zenodo.47423. URL: <http://dx.doi.org/10.5281/zenodo.47423>.
- [41] Wei Wei and Kathleen M. Carley. “Measuring Temporal Patterns in Dynamic Social Networks”. In: *ACM Transactions on Knowledge Discovery from Data* 10.1 (2015), pp. 1–27. ISSN: 15564681. DOI: 10.1145/2749465. URL: <http://www.scopus.com/inward/record.url?eid=2-s2.0-84938355062%7B%5C%7DpartnerID=tZ0tx3y1>.
- [42] William N Anderson. “Eigenvalues of the Laplacian of a graph”. In: *Linear and Multilinear Algebra* 18.2 (1985), pp. 141–145. ISSN: 0308-1087. DOI: 10.1080/03081088508817681. URL: <http://www.informaworld.com/openurl?genre=article%7B%5C%7Ddoi=10.1080/03081088508817681%7B%5C%7Dmagic=crossref>.
- [43] Leo Katz. “A new status index derived from sociometric analysis”. In: *Psychometrika* 18.1 (1953), pp. 39–43. ISSN: 1860-0980. DOI: 10.1007/BF02289026. URL: <http://dx.doi.org/10.1007/BF02289026>.
- [44] K.-I. Goh, B. Kahng, and D. Kim. “Universal Behavior of Load Distribution in Scale-Free Networks”. In: *Physical Review Letters* 87.27 (Dec. 2001), p. 278701. ISSN: 0031-9007. DOI: 10.1103/PhysRevLett.87.278701. URL: <http://link.aps.org/doi/10.1103/PhysRevLett.87.278701>.
- [45] M E J Newman. “Scientific collaboration networks. II. Shortest paths, weighted networks, and centrality”. In: *Phys. Rev. E* 64.1 (June 2001), p. 16132. DOI: 10.1103/PhysRevE.64.016132. URL: <http://link.aps.org/doi/10.1103/PhysRevE.64.016132>.
- [46] Mathias Johansson. “The Hilbert transform”. PhD thesis. 1999, pp. 1–33. URL: <http://www.fuchs-braun.com/media/d9140c7b3d5004fbffff8007fffff0.pdf>.
- [47] Ming Li and Yimin Zhao. “Seismic Attribute Analysis”. In: *Geophysical Exploration Technology* (2014), pp. 103–131. DOI: 10.1016/B978-0-12-410436-5.00005-8. URL: <http://linkinghub.elsevier.com/retrieve/pii/B9780124104365000058>.
- [48] D Subrahmanyam and P H Rao. “Seismic Attributes-A Review”. In: () .
- [49] D. J. Klein and M. Randić. “Resistance distance”. In: *Journal of Mathematical Chemistry* 12.1 (Dec. 1993), pp. 81–95. ISSN: 02599791. DOI: 10.1007/BF01164627. URL: <http://link.springer.com/10.1007/BF01164627>.

- [50] Nathanael Perraudin, Johan Paratte, David Shuman, et al. “GSPBOX: A toolbox for signal processing on graphs”. In: (2016).
- [51] Amarnath Gupta. *Graph Analytics for Big Data - University of California, San Diego — Coursera*. URL: <https://www.coursera.org/learn/big-data-graph-analytics>.
- [52] Scott L. Miller and Donald Childers. “Power Spectral Density”. In: *Probability and Random Processes* (2012), pp. 429–471. DOI: 10.1016/B978-0-12-386981-4.50013-8. URL: <http://linkinghub.elsevier.com/retrieve/pii/B9780123869814500138>.
- [53] Terence Tao. “Fourier Transform”. In: *arXiv* (2012), pp. 1–5. ISSN: 00222852. DOI: 10.1006/jmsp.1998.7620.
- [54] Fabien Gouyon, François Pachet, and Olivier Delerue. “ON THE USE OF ZERO - CROSSING RATE FOR AN APPLICATION OF CLASSIFICATION OF PERCUSSIVE SOUNDS”. In: () .
- [55] Jesus David Terrazas Gonzalez and Witold Kinsner. “Zero-crossing analysis of Lévy walks for real-time feature extraction”. In: *2016 IEEE International Conference on Electro/Information Technology, 2016 EIT*. 2016. ISBN: 978-1-4673-9985-2. DOI: 10.1109/EIT.2016.7535276.
- [56] John M. Grey. “Perceptual effects of spectral modifications on musical timbres”. In: *The Journal of the Acoustical Society of America* 63.5 (1978), p. 1493. ISSN: 00014966. DOI: 10.1121/1.381843.
- [57] Ed Kragh and Phil Christie. “Seismic repeatability, normalized rms, and predictability”. In: *The Leading Edge* 21.7 (July 2002), pp. 640–647. ISSN: 1070-485X. DOI: 10.1190/1.1497316. URL: <http://library.seg.org/doi/10.1190/1.1497316>.

Appendix A

Project Proposal

TABLE OF CONTENTS

| | |
|--|---|
| 1. INTRODUCTION | 1 |
| 1.1. TITLE..... | 1 |
| 1.2. RESEARCH QUESTIONS..... | 1 |
| 1.3. PURPOSE | 1 |
| 1.4. OBJECTIVES | 1 |
| 1.5. MOTIVATIONS & AIMS | 2 |
| 2. CRITICAL CONTEXT | 2 |
| 2.1. LIMITATIONS OF TRADITIONAL SNA..... | 2 |
| 2.2. GRAPH TERMINOLOGY..... | 2 |
| 2.3. DYNAMIC NETWORK ANALYSIS AS AN EXTENSION TO SNA | 3 |
| 2.4. NETWORK MEASURES: LOCAL | 3 |
| 2.5. NETWORK MEASURES: GLOBAL..... | 4 |
| 3. APPROACHES..... | 4 |
| 3.1. SIMILARITY METHODS | 5 |
| 3.1. ORIGINAL IDEAS..... | 6 |
| 3.2. EVALUATION STRATEGIES..... | 8 |
| 3.1. DATA SOURCES..... | 8 |
| 4. WORK PLAN | 8 |
| 5. ETHICS | 8 |
| 6. RISK REGISTER..... | 8 |
| 7. REFERENCES | 9 |

1. INTRODUCTION

1.1. TITLE

Analysing the evolution of communication patterns in email data through an extended dynamic network analysis toolkit

1.2. RESEARCH QUESTIONS

How can similarity measures be used to analyse dynamic email networks? How can similarity measures in alternative spaces support the analysis of dynamic networks?

1.3. PURPOSE

The origins of graphs theory can be traced back to Leonhard Euler and his approach to solving the Konigsberg Bridge Problem. This city was located on the Pregel River in Prussia. The river divided this city into 4 distinct areas which included an island all of which were connected by a total of 7 bridges. Euler's representation of this problem of the individual areas as nodes and the bridges as edges is considered one of the first applications of graph theory. [1]

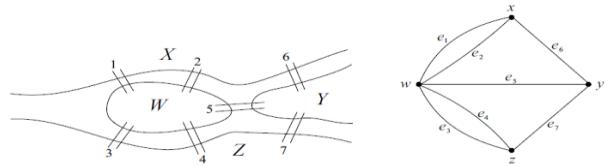


Figure 1: Euler's graphical representation of the Konigsberg Bridge Problem. [1]

Using Euler's insights into this problem in modern time's graphs have become a powerful tool with which to model and analyse communication and information networks. More specifically communities and their evolution in time [2]. The aim of this project is to explore the use of similarity measures to characterise such change in these networks. The viability of such analysis has been well demonstrated in the literature more specifically by [3], who modelled the email data released during the Enron investigation as a network and have made it available freely. Using this dataset [4] propose novel methods for community detection in real world information networks.

We intend to add to the existing body of knowledge regarding graph similarity measures and contribute a comparative and evaluative analysis of such measures on the emerging field of dynamic network analysis.

1.4. OBJECTIVES

The objectives that will help us answer our research question are as follows:

- ⊕ What are some of the graph similarity measures that have been proposed in the literature?
- ⊕ How have these been used in a practical context?
- ⊕ How can such measures be applied to dynamic networks?
- ⊕ How are such measures evaluated?

The aims of this project can be summarised as follows:

- ⊕ To provide a fairly comprehensive overview of graph theory as is relevant to the understanding of the derivation of similarity measures.
- ⊕ Conducting an in depth literature review on the graph similarity measures proposed and that have been demonstrated to be useful in a practical context.
- ⊕ Compare the utility and performance of these measures on appropriate data
- ⊕ Explore the viability of developing a novel similarity measure based on Fourier and Hilbert Analysis of networks

1.5. MOTIVATIONS & AIMS

The purpose of this research is twofold: firstly, we demonstrate the practical need for such analysis and secondly to develop a reference document for end user interested in applying such analysis to domains other than our stated domain of email networks.

The issue of graph similarity is critical in the analysis of dynamic networks. This is because when considering such networks an important concern is to understand the evolution of the network through time. Utilising similarity measures represents a very convenient method of achieving this as we can compare the same measure at different points in time to get a sense of network evolution. This is necessary because within our chosen context of email data, these methods can give insight into how communications patterns have evolved over time. Also we can gain insight into communities within such data and their birth, death and evolution. It can also serve as a means of anomaly detection. The set of analysis that such measures can enable are limited by one's creativity. The purpose is to show that there are very practical and real need to address the extension of similarity analysis to dynamic network analysis. We present many methods proposed in the literature and will also build on recent work on the application of Signal Processing methods on graphs to assess the potential of novel similarity methods.

The aim is to provide a comprehensive summary of graph similarity measures that can be applied to dynamic networks. We will explore their theoretical basis and methods of evaluation. In addition to the above it is hoped that the material developed as part of this project will provide a useful base reference to anyone interested in applying graph analytics to solve problems.

Also, we will explore the extent to which the measures that we encounter are implemented in common analysis packages such as IPython [5], NetworkX [6], Gephi [7] and Cytoscape [8] among others. The key point to make regarding tools selected here are that they are open source so they are freely available and the source code is viewable. This will allow if the need arises for extensions to be made to the code and shared freely.

2. CRITICAL CONTEXT

2.1. LIMITATIONS OF TRADITIONAL SNA

In Social Network Analysis (SNA), traditionally bounded networks are considered with maybe 2 or 3 connection or link types such as friendship or advice between a node types such as people sometimes another node type such as events are also considered together. [9]

If we consider more critically the interactions possible within our problem context of email networks we can have email networks within an organisation which are bounded and also with other organisations, clients and stakeholders and then the network does become unbounded. These networks can then be thought of as a higher order networks and as [9] notes many tools developed for simpler networks do not scale well to increased network size and complexity and in some cases experience degradation through increased susceptibility to Type 1 and Type 2 errors.

The dynamics in these networks can arise from different processes depending on the context of the problem. Natural evolutionary processes would be learning, births, deaths and ageing. Others could be as a result of intervention measures such removal or addition of nodes i.e. removing those who lead the system, communities forming or disintegrating. The data associated with such systems are also often incomplete and contain errors which make the process of analysis and evaluation of these systems. [10]

Analysis approaches that go beyond traditional SNA and link analysis are therefore necessary. Within the context of such dynamic networks analysis can be performed to identify of key individuals, locating hidden groups and estimate performance. The data analysis process on such networks then involve: [9], [10]

- ⊕ Relationship identification among nodes
- ⊕ Network structure characterisation
- ⊕ Locating the elite within the network
- ⊕ Identifying points of vulnerability
- ⊕ Comparing networks

The approaches that enables effective analysis of such dynamic networks and help quantify their evolution over time is the motivation for this research.

2.2. GRAPH TERMINOLOGY

A graph, G can be described as a triple which consists of a set of edges $E(G)$, a set of vertices or nodes V

(G) and a relationship that connects the vertices to these edges. Finite graphs are those that have V and E as a finite set. Simple graphs are those that have no loops or multiple edges. A path is simple graph in which the vertices can be ordered where two vertices can be adjacent only if they are consecutively ordered. A cycle is defined as a simple graph where the vertices can be cyclically ordered such that two vertices are adjacent only if they are consecutive in cyclical ordering. A subgraph can be thought of cycles and paths within a larger graph, where the edge relations between the subgraph and the large graph are the same. [1]

2.3. DYNAMIC NETWORK ANALYSIS AS AN EXTENSION TO SNA

Dynamic network analysis (DNA) aims to extend the methods, tools and techniques used in traditional Social Network Analysis (SNA) to the analysis of networks which are able to handle big dynamic multi-mode, multi-link networks with varying levels of uncertainty. Dynamic networks also allow for probabilistic connection between nodes. [9]

In [9] DNA was explored within the context of terrorism networks. Here an additional layer of complexity is added by the fact that an act of measurement changes its properties and this change propagates through the network and its state changes. Another key point is that the nodes in this network have the ability to learn. So the nodes themselves can be thought of being probabilistic compared to the more static nature of SNA nodes.

In a DNA representation systems can be represented as relational data. This relational data structure can lend flexibility in defining multiple node types defined as multi-modal, have various types of connections among such nodes called multi-plex. The underlying attributes of both node, edges and the data change over time hence the dynamic part. [10]

In [9] the key advances that allow for the analysis of such dynamic networks are identified as:

- ⊕ The meta matrix
- ⊕ Probabilistic edges between nodes
- ⊕ Combining social networks with cognitive science and multi-agent systems

The Meta Matrix

The Meta matrix is a method used in operations research and organisational management that seeks

to represent the entity and class relationships as a collection of networks. In the DNA context this translates as a multi-mode, multi-plex approach to representing systems. Therefore the Meta matrix can contain a social network, a membership network and knowledge network and allow us to explore and analyse the connections between them. [9]– [12]

Probabilistic Ties

The ties or connections in the Meta matrix are probabilistic with various factors affecting their probability. This allows for inclusion of the observers uncertainty and the likelihood that the tie is present at the time of observation. These probabilities themselves and their temporal evolution maybe estimated by the Bayesian methods, cognitive inferencing and models of social and cognitive change. [9]

Multi Agent Network Models

As previously discussed the SNA treatment of nodes as static agents unable to learn is insufficient when dynamic networks are concerned. In DNA the nodes are able to take actions, learn from experience and alter their networks as a result. Some social and cognitive processes that influence the agent's interactions are relative similarity, relative expertise and co-workers. The dynamic behaviour of the network emerges from these interactions and experience a shared evolution. [9]

We briefly discuss some of the more common measures associated with networks which relate to their global and local properties. These will be important when we discuss similarity because one of the ways to assess similarity is to consider snapshots of a network attribute at different time intervals.

2.4. NETWORK MEASURES: LOCAL

Centrality measures are a fundamental statistic in network analysis. In [13] two paradigms of centrality definitions are suggested. One is the means based definition of centrality or the graph theoretic and the other is the ends-based definition which is a dynamic model based view that focuses on the outcome for the nodes in a network where there is flows across the nodes [14]. However, both approaches agree that this measure is a node level property.

In both formulations of centrality measures noted above we characterise centrality measures as follows: [13]

- ⊕ Volume based measures – degree like centrality
- ⊕ Length based measures – closeness like centrality
- ⊕ Medial measures – betweenness like centrality

Volume and length based measures are what are called radial measures because they analyse walks that emanate from or terminate with a given node. Medial measures on the other hand are based on position so how many times does one encounter a node while trying to reach other nodes in the network.

Degree Centrality

Degree Centrality is a special case of the k-path centrality that counts the all the paths of a length, k that originate from a given node. Degree Centrality is then defined as the number of edges incident on a node. Which translates to summing all the rows of the adjacency matrix of a network. [13], [14]

Closeness Centrality

Closeness Centrality is the graph theoretic distance or the geodetic distance from a given to all the other nodes in a network. [13], [14]

Betweenness Centrality

Betweenness centrality counts the number of times that a certain node, x needs to pass another node, y to get to another node, z through the shortest path between them. [13], [14]

Eigenvector Centrality

The Eigenvector Centrality is defined as the principal eigenvector of the adjacency matrix of a network. It captures the intuition that nodes that have high eigenvector centrality scores are likely to be close to other nodes which themselves have high values for this measure. [13], [14]

We have mentioned some of the most popular centrality measures but there are numerous other variations mentioned in the literature and are beyond the scope of this work the interested reader is referred to the following starter references. [15]– [18]

2.5. NETWORK MEASURES: GLOBAL

Network Size

This can be defined by counting the number of nodes in a network. [19]

Density

This is defined as the total number of edges divided by the total number of possible edges. [19]

Diameter

The diameter of a network is the maximum geodesic distance between two nodes.

Modularity

The modularity function finds partitions within the graph where a large proportion of the edges fall entirely within that partition and biases against those partitions that have too few or unequal sized parts. [20]

3. APPROACHES

The purpose of detailing the some of the key properties and attributes are they can be used as to derive similarity measures on graphs. These measures will form the basis of our exploration of existing and new similarity measures for DNA.

The problem of graph similarity or graph matching then becomes one of finding the equivalence of two graphs with potentially different number of nodes and edges and returning a measure within [0, 1] that captures their similarity or dissimilarity. [18], [21]– [27]

The key idea of graph matching in the context of dynamic networks can be summarised as finding a subgraph or an attribute that we can compare between two time instances. For example, if we consider the Degree Centrality of a network at time step 0 and then again calculate this measure at time step 1 we can apply a similarity measure on this attribute to quantify the change within the network. This can be done by means of a distance metric such as cosine similarity and others are possible.

Therefore the analysis methodology can be generalised as follows:

- ⊕ For network X at time step, t extract some attribute
- ⊕ At time step, t +1 extract the same attribute
- ⊕ Perform similarity analysis on attribute
- ⊕ Track the change in the similarity metric over time through some control process.

The evaluation of the change in metrics over time will be done through a statistical control process as suggested in [28]. This is a concept that comes from quality engineering and it essentially involves calculating a statistic from a sequence of measurements of a random process and then comparing it to some control limit. This process translates to:

- ⊕ Calculating a cumulative sum control chart which is very good for detecting small changes in mean over time
- ⊕ Calculating a z-score for each time step (Eq. 1)
- ⊕ Construction of two charts to detect increase and decrease in the metric as shown.

$$Z_i = (x_i - \mu_0) / \sigma$$

$$C_t^+ = \max\{0, Z_t - k + C_{t-1}^+\}$$

$$C_t^- = \max\{0, -Z_t - k + C_{t-1}^-\}$$

Equation 1: Control chart to detect increase and decrease in metric over time

In the following section will present a brief treatment of the subject of similarity.

3.1. SIMILARITY METHODS

The problem of finding similarity between nodes which are similar in a network can be thought of as a problem of finding a set of nodes which are similar to a given node according to some attributes which are represented as connections [25].

Similarity in a networks is classified as being of structural, content or keyword based. The Structural similarity or link based similarity considers the similarity of links between the nodes in the graph e.g. Cosine, Jaccard, Hub Promoted and Hub Depressed Index etc. Content similarity considers the attributes of the node in the graph. For example on a social network this could be birth dates or hobbies of individuals. Keyword similarity aims to find similarity based on nodes representing word collections. Global Structural Similarity can be classified as being:

- ⊕ local vs. global
- ⊕ parameter-free vs. parameter-dependent
- ⊕ node-dependent vs. path-dependent

The global structural measures aim to measure node similarity compared to the whole network. We will call them intra network similarity measures. [25]

Inter network similarity measures are described in [22], [24], [26], [27], [29]–[32]. These measures are classified by [21] into three categories 1) Distance Based 2) Feature Based and 3) Probabilistic.

Distance based approach

The distance based approach is perhaps the earliest of the methods encountered which is based on edit distance [32]. Essentially this boils down to finding a sequence of operations such as deletion, insertion, or substitution minimising some cost function that will turn one graph into another. These involve detection and comparison of the graph isomorphism, subgraph isomorphism and maximum common subgraph detection utilising the edit distance. Although these methods are guaranteed to converge to an optimal solution their exponential complexity makes them unsuitable for large graphs.

Feature Based approach

We have already hinted at the feature based approach above in the Al-Qaeda example above. But more formally this involves calculation of a network attribute such as degree, closeness, betweenness, and/or eigenvector centrality for the graphs and then applying a similarity measure on them that will characterise their similarity or dissimilarity. This has the benefit of being scalable to very large networks as the aggregated statistics are much smaller than the network themselves.

A taxonomy of the methods that have been proposed to solve these problems are shown in Fig 3. We include this for illustrative purposes and their discussion will form part of the extended review performed for the final report.

Probabilistic Approach

The methods that fall under this approach in the literature are vast. Some approaches under the probabilistic framework for graph matching are discussed here [29], [33]–[35]. But simply stated these methods define a probability distribution over mappings or graph embedding's [31], [34]. Graph embeddings are graphs whose nodes correspond to distinct points on a plane and the edges represents relationships connecting these points. The matching algorithm is strongly dependent upon the geometric information attached to the graphs [31], [34].

Graph matching allows for recovering point correspondences. In [29] the authors show that assuming that the assignment matrix that represents these correspondences are statistically independent the high order matching problem can be represented by a Kronecker product matrix. Also they show that

that a high order tensor affinity tensor can be marginalised into a one dimensional vector of probabilities. This probability vector is then updated by projection to a vector assignment space and then minimizing a distance measure (Bregman measure) [33]. Spectral Methods involve looking at the spectra of a graph which are defined by the eigenvalues of the adjacency matrix. In [36] the high order matching is expressed a tensor Eigen decomposition and applied to point matching using some similarity measure.

Visual Representation

More recently, the authors in [37] have proposed visualising dynamic networks and characterising change by visualising the adjacency matrix of these networks as a matrix cube. Representing the adjacency matrix as a stack of cubes rather than node link diagrams is found to be a much more useful paradigm for analysis of dynamic networks especially when these networks are dense.

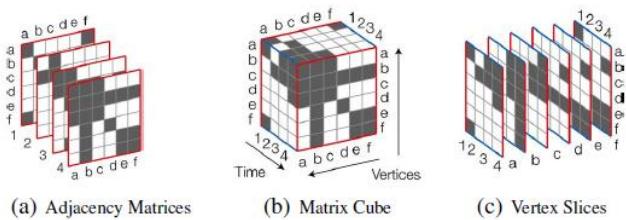


Figure 2: The matrix cube view of dynamic networks that allow for visual characterisation of network evolution over time. [37]

We will explore some representative methods from these classes of methods and assess their utility for DNA.

3.1. ORIGINAL IDEAS

Given the author's background as a Geophysicist in performing seismic data analysis, there was a strong interest in exploring potential ideas that could be imported from seismic analysis to the analysis of dynamic networks. The reasons for this is because seismic data is essentially a time series on which signal processing approaches are utilised. Since there is a time component to these networks the attributes of these networks could be viewed as a time series and some of these methods can then be tested for effectiveness.

Firstly, for visualisation of dynamic network attributes we suggest something similar to how well log analysis

is performed in reservoir characterisation of hydrocarbon reservoirs shown in Fig 3.

From Fig 6, we propose to develop a dynamic network attribute panel that allows analysis of selected network attributes over time such as Degree, Closeness, and Density etc. [38]

The Coherence attribute is multi trace similarity of the seismic waveform that is used for event detection and is very useful when the amplitude of the signal is low compared to the noise. For DNA we can explore the viability of developing a Coherence for network similarity measures or network attributes that also allows for event detection shown in Fig 7. More details regarding this method can be found here [39].

Some recent work has focused on the application of signal processing methods to graphs. In [40] a generalisation of the Fourier Transform and its associated methods such as convolutions to graphs. In [41], [42] transforms and tomograms for graph application are discussed. An interesting application is that of the Radon transform commonly used for noise removal in seismic data analysis, which the authors in [42] generalise to arbitrary pairs of non-commuting operators. These are positive bilinear transforms with a probabilistic interpretation and provide a full characterization of the signals while being robust to noise. These developments although very interesting do not have mainstream implementations in software packages and must be conducted in a more ad-hoc manner. This is a potential limitation of their evaluation in our study.

However, we present some preliminary analysis on a synthetic geographical threshold graph model which places n nodes uniformly at random in a rectangular domain with 500 nodes. We extract the Degree Centrality and then apply the Fourier and Hilbert Transform to it to extract its periodic features and complex trace information.

From Fig 3, 4 we note that these transforms serve as a useful visualisation tool of network attributes. The Fourier Transform picks out the maximum degree centrality while the Double Hilbert Transform shows some outliers.

A correlation between the Fourier or Hilbert transforms of a measure would give a similarity value shown in Fig 8. The autocorrelation of the measures with themselves would give an indication of the self-similarity of the network.

From the autocorrelation functions we see that these have the potential to highlight anomalies. We will explore these measures over time to assess their value in DNA.

In summary, our analysis approach will focus on network measures calculated through some of the methods we identified previously. We will consider the temporal component as an important aspect of network characterisation.

The key consideration of the analysis will be to use already implemented method and not be distracted by extensive development work as this is not the purpose of the research. Hence, we identify promising areas to focus on but if time constraints do not permit or the software packages are found to be lacking we will include them as future areas of research.

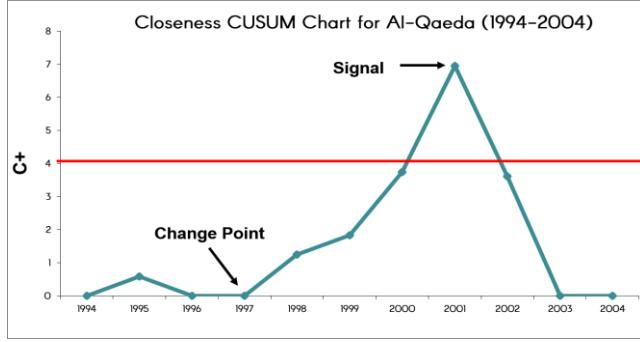
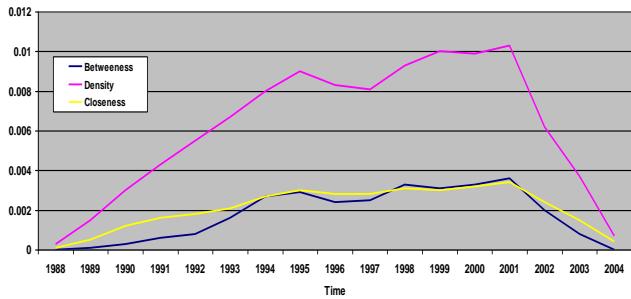


Figure 5: From [28] (top) shows the application of this method on the Al Qaeda network for the Betweenness, Density and Closeness measures. Bottom modified from [28] shows the change in Closeness Centrality for the network through the control chart. The control limit is shown in red

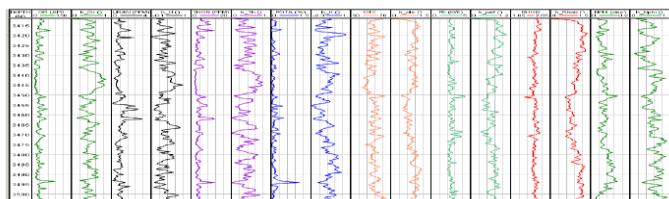


Figure 6: Well Log panel showing the different measurements typically recorded while drilling. These attributes are then used for characterisation.

Plot of Degree Centrality and with different transforms

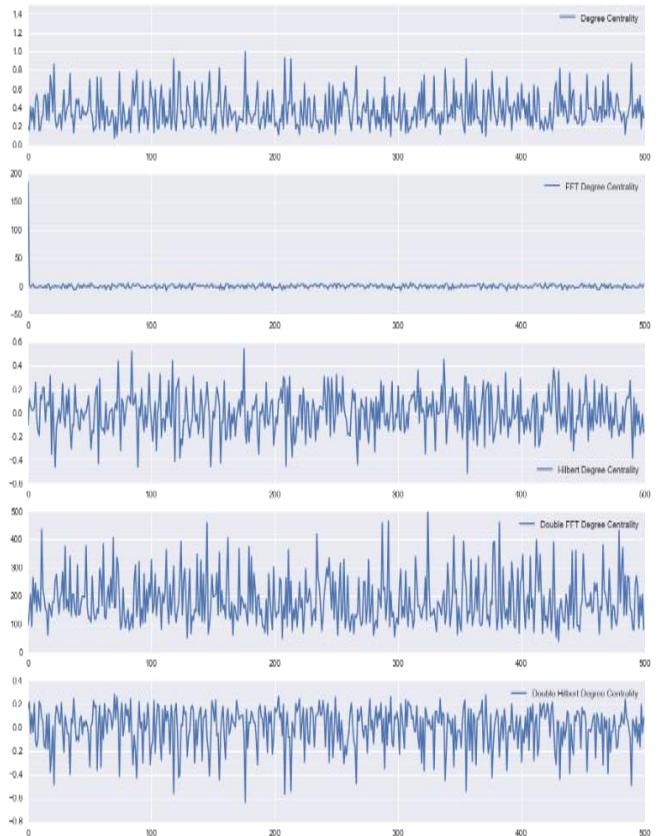


Figure 3: Plot of (1st) Degree Centrality (DC), (2nd) Fourier Transform of DC, (3rd) Hilbert Transform of DC, (4th) Double Fourier Transform of DC, (5th) Double Hilbert Transform of DC

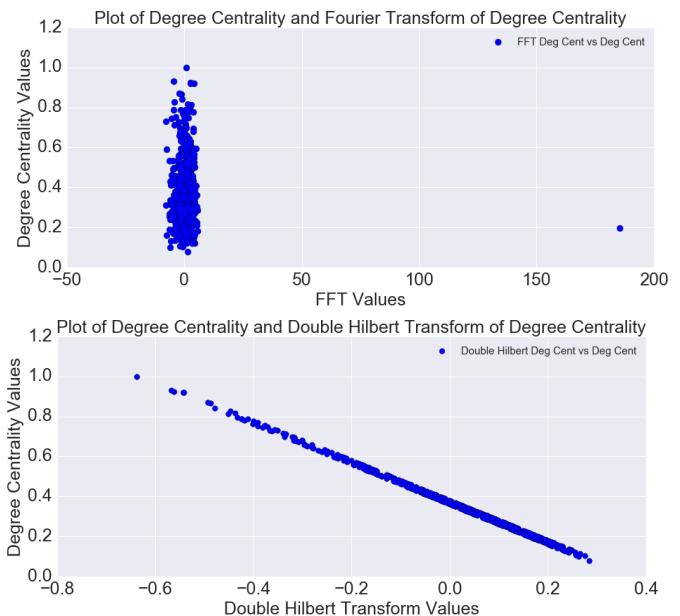


Figure 4: (top) Scatter plot of Fourier Transform of DC and DC (bottom) Double Hilbert Transform of DC and DC.

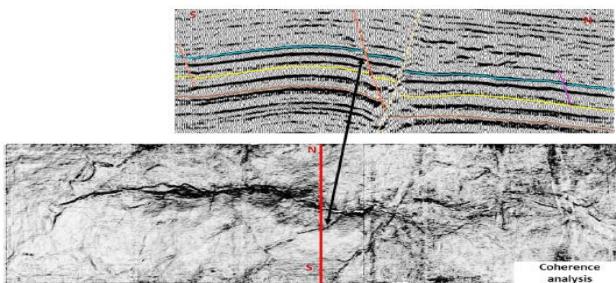


Figure 7: Example of a coherence attribute cube for a seismic volume. Coherence is essentially a multi trace similarity measure used for event detection. [39]

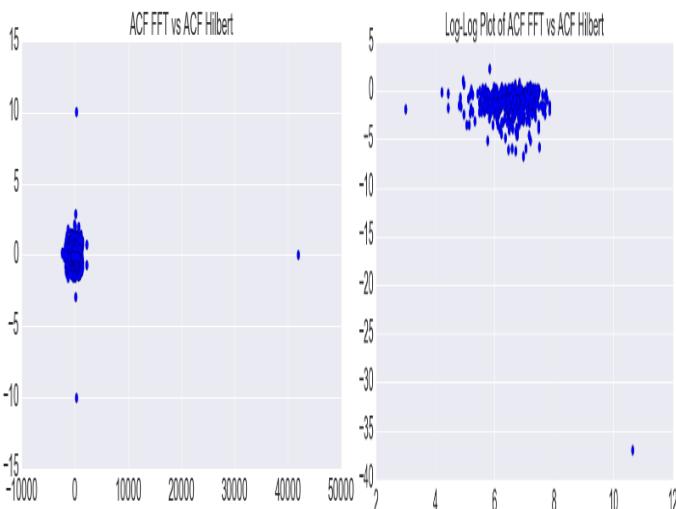


Figure 8: Plot of autocorrelation functions of the Fourier and Hilbert Transforms of Degree Centrality. (Left) Log-Log Plot of ACF's.

3.2. EVALUATION STRATEGIES

We have presented an array of methods and techniques proposed in the lecture and our own novel ideas based on the literature. Evaluation of such measures and their interpretation will form a key part of the research. The interpretation will be especially important when we consider our suggestions of measures of similarity in alternative spaces such as the Fourier and Hilbert spaces. These Fourier Transform extracts periodic features from a time series by extracting its frequency while the Hilbert transform extends the signal to the complex plane. What these mean in the context of our measures will be explored.

Therefore the evaluation strategy will be as follows:

- ⊕ For the dataset used search the literature to see what other authors have found especially with regards to interesting features, signals and/or structures

- ⊕ Then evaluate whether our measures are able to find these features and how difficult or easy they are in comparison with the original measures
- ⊕ If such signals are not found in the literature, we can look for them in the data through other analysis and use our measures to confirm alternatively we can insert a test signal in the data and see how well the measures are able to find them.

This is deemed the most practical way to evaluate the research given the short time horizon. Other methods such as involving groups or surveys to assess participant's interactions with these analytical tools would not be practical for this study.

3.1. DATA SOURCES

This research will utilise open and publicly available data so the results are reproducible. To this end we identify some sources of network data available to us:

- ⊕ Enron email data : <http://www.cs.cmu.edu/~enron/>
- ⊕ Stanford Large Network Collection: <http://snap.stanford.edu/data/#email>
- ⊕ UCI Network Data Repository : <https://networkdata.ics.uci.edu/resources.php>
- ⊕ Open Dynamic Network Data: <http://www.aviz.fr/~bbach/opendynamicnetworks>

4. WORK PLAN

The research will be conducted between the periods of June – September 2016. A detailed work plan is highlighted in the Gantt chart attached with this report.

5. ETHICS

Since this is predominantly a research and analysis task on open data there are ethical concerns. Completed ethics questionnaire is also appended with this report.

6. RISK REGISTER

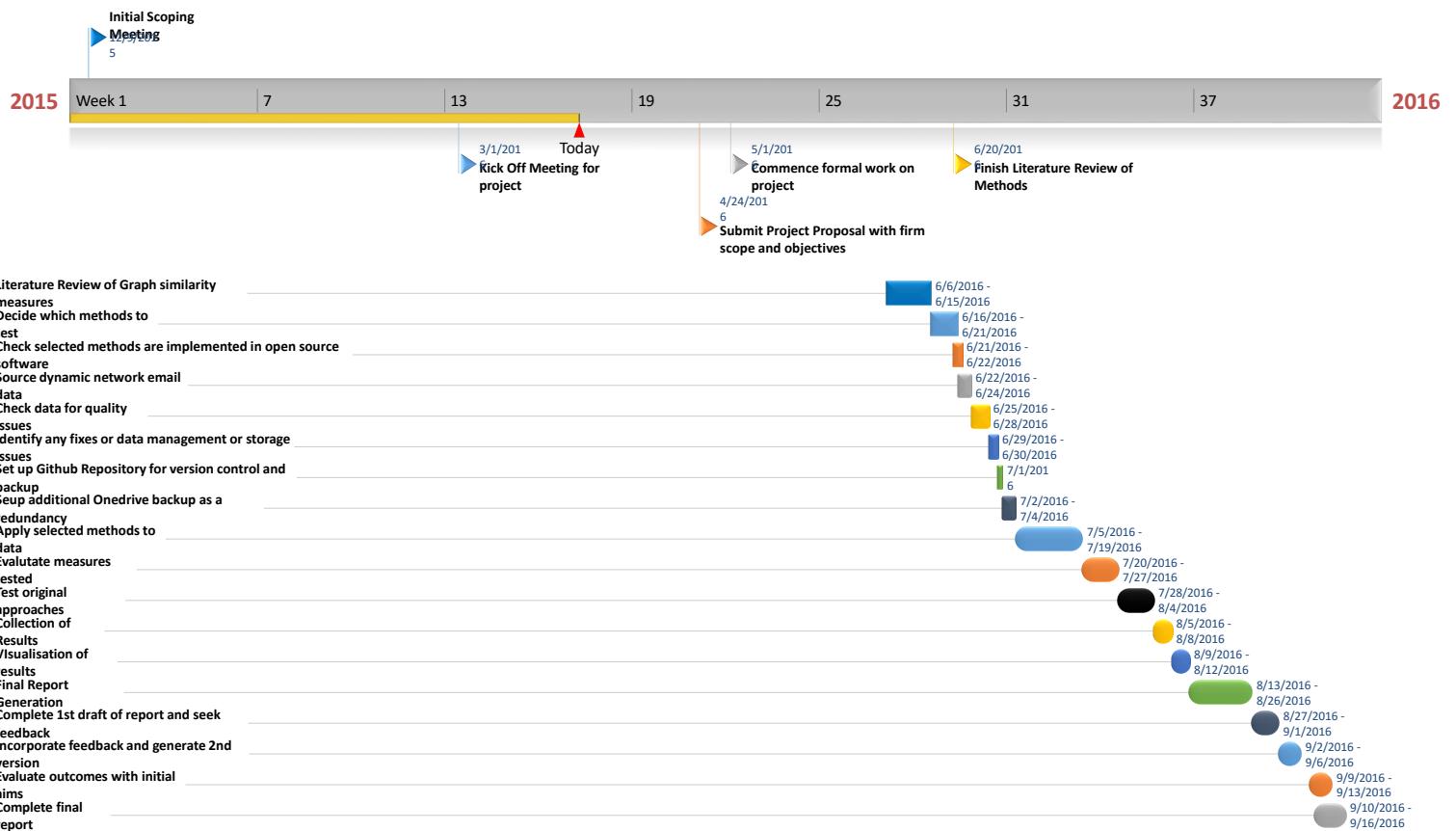
The standard types of technical and non-technical risks apply to this project as any other. A completed risk register is appended with this report.

7. REFERENCES

- [1] A. Dickson, "Introduction to Graph Theory The Konigsberg Bridge Problem," *October*, no. October, pp. 1–15, 2006.
- [2] A. Chapanond, M. S. Krishnamoorthy, and B. Yener, "Graph theoretic and spectral analysis of Enron email data," *Comput. Math. Organ. Theory*, vol. 11, no. 3, pp. 265–281, 2005.
- [3] B. Klimt and Y. Yang, "Introducing the Enron Corpus," *Mach. Learn.*, vol. stitutepl, p. wwceascaers2004168, 2004.
- [4] J. Leskovec, K. J. Lang, A. Dasgupta, and M. W. Mahoney, "Community Structure in Large Networks: Natural Cluster Sizes and the Absence of Large Well-Defined Clusters," *Internet Math.*, vol. 6, no. 1, pp. 29–123, 2011.
- [5] F. Pérez and B. E. Granger, "IPython: a System for Interactive Scientific Computing," *Comput. Sci. Eng.*, vol. 9, no. 3, pp. 21–29, May 2007.
- [6] H. Aric, D. Schult, and S. Pieter, "NetworkX." .
- [7] M. Bastian, S. Heymann, and M. Jacomy, "Gephi: An Open Source Software for Exploring and Manipulating Networks," *Third International AAAI Conference on Weblogs and Social Media*, 2009. [Online]. Available: <http://www.aaai.org/ocs/index.php/ICWSM/09/paper/view/154> \npapers2://publication/uuid/CCEBC82E-OD18-4FFC-9IEC-6E4A7F1A1972. [Accessed: 25-Mar-2016].
- [8] P. Shannon, A. Markiel, O. Ozier, N. S. Baliga, J. T. Wang, D. Ramage, N. Amin, B. Schwikowski, and T. Ideker, "Cytoscape: A software Environment for integrated models of biomolecular interaction networks," *Genome Res.*, vol. 13, no. 11, pp. 2498–2504, Nov. 2003.
- [9] K. M. Carley, "Dynamic Network Analysis," no. September, 2001.
- [10] K. M. Carley, J. Diesner, J. Reminga, and M. Tsvetovat, "Toward an interoperable dynamic network analysis toolkit," *Decis. Support Syst.*, vol. 43, no. 4, pp. 1324–1347, 2007.
- [11] K. M. Carley, "Ora: Organization risk analyzer," *Science* (80-.), no. January, p. 49, 2004.
- [12] T. A. B. Snijders, "Introduction To Dynamic Social Network Analysis," *BRIMS 2015*, no. November, 2015.
- [13] S. P. Borgatti and M. G. Everett, "A Graph-theoretic perspective on centrality," *Soc. Networks*, vol. 28, no. 4, pp. 466–484, 2006.
- [14] S. P. Borgatti, "Centrality and network flow," *Soc. Networks*, vol. 27, no. 1, pp. 55–71, 2005.
- [15] G. Costantini, S. Epskamp, D. Borsboom, M. Perugini, R. Mōttus, L. J. Waldorp, and A. O. J. Cramer, "State of the aRt personality research: A tutorial on network analysis of personality data in R," *J. Res. Pers.*, vol. 54, pp. 13–29, 2015.
- [16] K. R. Harrison, M. Ventresca, and B. M. Ombuki-Berman, "A meta-analysis of centrality measures for comparing and generating complex network models," *J. Comput. Sci.*, 2015.
- [17] R. Kaur and S. Singh, "A survey of data mining and social network analysis based anomaly detection techniques," *Egypt. Informatics J.*, 2015.
- [18] R. a. Hanneman and M. Riddle, "Measures of similarity and structural equivalence," *Introduction to social network methods*, 2011. [Online]. Available: http://www.faculty.ucr.edu/~hanneman/nettext/CI3_Structural_Equivalence.html. [Accessed: 26-Mar-2016].
- [19] R. a Hanneman and M. Riddle, "Introduction to Social Network Methods," *Riverside, CA Univ. California, Riverside. On-line Textb.*, vol. 46, no. 7, pp. 5128–30, 2005.
- [20] C. McDiarmid and F. Skerman, "Modularity in random regular graphs and lattices," *Electron. Notes Discret. Math.*, vol. 43, pp. 431–437, 2013.
- [21] F. Ashby and D. Ennis, "Similarity measures," *Scholarpedia*, vol. 2, no. 12, p. 4116, 2007.
- [22] M. Dehmer, F. Emmert-Streib, and J. Kilian, "A similarity measure for graphs with low computational complexity," *Appl. Math. Comput.*, vol. 182, no. 1, pp. 447–459, Nov. 2006.
- [23] M. Komosinski and M. Kubiak, "Quantitative measure of structural and geometric similarity of 3D morphologies," *Complexity*, vol. 16, no. 6, pp. 40–52, 2011.
- [24] D. Koutra, a Parikh, a Ramdas, and J. Xiang, "Algorithms for Graph Similarity and Subgraph Matching," 2011.
- [25] A. Rawashdeh and A. L. Ralescu, "Similarity Measure for Social Networks – A Brief Survey," 2012.
- [26] S. Soundarajan, T. Eliassi-Rad, and B. Gallagher, "A guide to selecting a network similarity method," *Sdm*, no. 1, 2014.
- [27] L. A. Zager and G. C. Vergheze, "Graph similarity scoring and matching," *Appl. Math. Lett.*, vol. 21, no. 1, pp. 86–94, 2008.
- [28] "Dynamic Social Network Analysis of Al-Qaeda." .
- [29] R. Zass and A. Shashua, "Probabilistic graph and hypergraph matching," *26th IEEE Conf. Comput. Vis. Pattern Recognition, CVPR*, pp. 0–7, 2008.
- [30] C. Wang, L. Wang, and L. Liu, "Improving graph matching via density maximization," *Proc. IEEE Int. Conf. Comput. Vis.*, pp. 3424–3431, 2013.
- [31] D. Conet, P. Foggia, C. Sansone, and M. Vento, "Thirty Years of Graph Matching in Pattern Recognition," *Int. J. Pattern Recognit. Artif. Intell.*, vol. 18, no. 03, pp. 265–298, 2004.
- [32] H. Bunke, "Recent developments in graph matching," *Pattern Recognition, 2000. Proceedings. 15th ...*, vol. 2, pp. 117–124, 2000.
- [33] A. Egozi, Y. Keller, and H. Guterman, "A Probabilistic Approach to Spectral Graph Matching," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 35, no. 1, pp. 18–27, 2013.
- [34] T. S. Caetano, J. J. McAuley, L. Cheng, Q. V. Le, and A. J. Smola, "Learning graph matching," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 31, no. 6, pp. 1048–1058, 2009.
- [35] H. Yaghi and H. Krim, "Probabilistic graph matching by canonical decomposition," *Image Process. 2008. ICIP 2008. 15th IEEE Int. Conf.*, pp. 2368–2371, 2008.
- [36] O. Duchenne, F. Bach, K. In-so, J. Ponce, O. Duchenne, F. Bach, K. In-so, J. Ponce, A. T. Algorithm, O. Duchenne, F. Bach, I. Kweon, and J. Ponce, "A Tensor-Based Algorithm for High-Order Graph Matching," 2014.
- [37] B. Bach, E. Pietriga, and J.-D. Fekete, "Visualizing dynamic networks with matrix cubes," *32nd Annu. ACM Conf. Hum. Factors Comput. Syst. CHI 2014*, pp. 877–886, 2014.
- [38] L. Aliouane, S. Ouafeul, and A. Boudella, "Well-Logs Data Processing Using the Fractal Analysis and Neural Network," 2012.
- [39] C. Outline, "Seismic Coherence Technique," pp. 63–74, 2014.
- [40] M. Begú, "Fourier Analysis on Graphs."
- [41] M. A. Man'ko, R. V. Mendes, and V. I. Man'ko, "Tomograms and other transforms: a unified view," *J. Phys. A. Math. Gen.*, vol. 34, no. 40, pp. 8321–8332, 2001.
- [42] A. Sandryhaila and J. M. F. Moura, "Discrete Signal Processing on Graphs," *IEEE Trans. Signal Process.*, vol. 61, no. 7, pp. 1644–1656, Apr. 2013.

Appendix B

Project Gantt Chart



Appendix C

Project Test List

This shows the different stages for the analysis process and an overview of the decisions made. This also helps with versioning and creates a provenance for the analytical process so that it could be replicated easily.

Table C.1: Test List for Analysis

Dynamic Network Analysis of Enron Email Network Data: Test List

| Test Number | Description |
|-------------|--|
| 00 | First look at data and initial checks |
| 01 | Break into networks but with cumulative yearly aggregation and look at centrality and other network statistics |
| 02 | Break data into monthly and yearly aggregations without cumulative aggregation and choose benchmark measures. |
| 02 | Derive Seismic attributes from Laplacian matrix to evaluate their plausibility |
| 03 | Deriving Seismic attributes from 3 graph matrices |
| 04 | Comparison of attributes from 3 graph matrices to benchmark measures |
| 05 | Consolidate analysis and remove analysis that |
| 06 | Derivation of Music attributes as a proof of concept and consolidated analysis for report. |
| 06.1 | Derivation of Seismic and Music attributes from 3 graph matrices |
| 06.2 | Comparison of all attributes from the 3 graph matrices using SNR and Entropy |

Appendix D

Jupyter Notebook: Python Code and Results

To enable full reproducibility and transparency of analysis and results all code and results presented in this study are attached in this section. The Jupyter Notebook format is convenient because it enables placing of code and its results together in a powerful way and can be exported to a wide variety of formats.

This shows clearly how every single attribute has been implemented with code, which libraries have been used and what the outputs were. This should enable practitioners to copy the code and use it in their own analysis as well as be able to reproduce any result presented here.

DNA_06

August 25, 2016

1 Table of Contents

- 1 Introduction
- 2 Network Visualisation
 - 2.1 Yearly Networks
 - 2.2 Monthly networks
- 3 Exploratory Analysis
 - 3.1 Yearly Networks
 - 3.2 Monthly Networks
- 4 Attribute Analysis
 - 4.1 Traditional Measures
 - 4.1.1 Centrality
 - 4.1.2 Assortativity & Linear Algebra
 - 4.2 Complex Trace Attributes
 - 4.3 Matrix
 - 4.4 Matrix Decomposition
 - 4.5 Music Attributes
 - 4.6 Aggregation Measures
- 5 Overview Plots
- 6 Correlation Analysis
 - 0.7-6.1“>6.1 Correlation > 0.7
- 7 Regression Analysis for Feature Ranking
- 8 Aggregation Measures
- 9 MDS and TSNE
- 10 FK and Radon Plot
- 11 Exotic Transforms
- 12 Which nodes are common to all years?

2 Introduction

Dynamic Network Analysis of Enron Email Network Data.

I use the Enron email network data from [John Hopkins](#) which has time, sender and receiver pair format data.

Here I consolidate earlier analysis to make presentation of the final results easier and more readable.

From the JHU data, I have done the following in Excel: - The first column represents seconds elapsed since 1 January 1970, so I convert this in to days - I then add these days to the date to

get time stamps for all nodes - From the timestamps, I extract the year field - The network can be partitioned by the required level of granularity

```
In [153]: import pandas as pd
import numpy as np
import networkx as nx
import seaborn as sns
import scipy as sc
import random
from scipy.signal import *
from numpy.linalg import *
from sklearn.decomposition import *
from sklearn.preprocessing import *
import librosa
import abel
#plotting parameters
%matplotlib inline
sns.set(style="whitegrid", color_codes=True, context='paper')

In [154]: from matplotlib import rcParams
rcParams['font.family'] = 'serif'
rcParams['font.sans-serif'] = ['CMU Serif']
rcParams['font.weight']= ['heavy']

In [155]: import matplotlib.pyplot as plt

In [156]: plt.rc('axes', grid=False, titlesize='large', labelsize='large', labelweight='black')
plt.rc('lines', linewidth=4)
plt.rc('figure', figsize = (12,6),titlesize='large',titleweight='black')
plt.rc('font', weight='heavy', size=11)
plt.rc('grid', linewidth=5)

In [157]: sns.palplot(sns.cubehelix_palette(10,hue=0.3, reverse=True, rot=-0.55, dark=False))
```



```
In [158]: sns.set_palette(sns.cubehelix_palette(10, hue=0.3, reverse=True, rot=-0.55))

In [159]: data = pd.read_excel("../Data/data_03.2.xlsx")

In [160]: years = sorted(set(data.year))
years = years[1:]
years
```

```
Out[160]: [1998, 1999, 2000, 2001, 2002]
```

```
In [161]: months = ['Nov98', 'Dec98', 'jan_99', 'feb_99', 'mar_99', 'apr_99', 'may_99', 'nov_99', 'dec_99', 'jan_2k', 'feb_2k', 'mar_2k', 'apr_2k', 'may_2k', 'nov_2k', 'dec_2k', 'jan_2k1', 'feb_2k1', 'mar_2k1', 'apr_2k1', 'oct_2k1', 'nov_2k1', 'dec_2k1', 'jan_2k2', 'feb_2k2', 'mar_2k2']
```

```
In [162]: df_98 = data[data.year==years[0]]
df_99 = data[data.year==years[1]]
df_2k = data[data.year==years[2]]
df_2k1 = data[data.year==years[3]]
df_2k2 = data[data.year==years[4]]
```

```
In [163]: def create_graph(df):
    tmp = df.values[:,1:3]
    G= nx.Graph()
    G = nx.from_edgelist(tmp)

    return G
```

```
In [164]: Gt0 = create_graph(df_98)
Gt1 = create_graph(df_99)
Gt2 = create_graph(df_2k)
Gt3 = create_graph(df_2k1)
Gt4 = create_graph(df_2k2)
```

```
In [165]: nov_98 = df_98[df_98.month==11]
dec_98= df_98[df_98.month==12]

G_nov98 = create_graph(nov_98)
G_dec98 = create_graph(dec_98)
```

```
In [166]: jan_99=df_99[df_99.month==1]
feb_99=df_99[df_99.month==2]
mar_99=df_99[df_99.month==3]
apr_99=df_99[df_99.month==4]
may_99=df_99[df_99.month==5]
jun_99=df_99[df_99.month==6]
jul_99=df_99[df_99.month==7]
aug_99=df_99[df_99.month==8]
sep_99=df_99[df_99.month==9]
oct_99=df_99[df_99.month==10]
nov_99=df_99[df_99.month==11]
dec_99=df_99[df_99.month==12]
```

```
G_jan_99=create_graph(jan_99)
G_feb_99=create_graph(feb_99)
G_mar_99=create_graph(mar_99)
```

```
G_apr_99=create_graph(apr_99)
G_may_99=create_graph(may_99)
G_jun_99=create_graph(jun_99)
G_jul_99=create_graph(jul_99)
G_aug_99=create_graph(aug_99)
G_sep_99=create_graph(sep_99)
G_oct_99=create_graph(oct_99)
G_nov_99=create_graph(nov_99)
G_dec_99=create_graph(dec_99)
```

```
In [167]: jan_2k=df_2k[df_2k.month==1]
feb_2k=df_2k[df_2k.month==2]
mar_2k=df_2k[df_2k.month==3]
apr_2k=df_2k[df_2k.month==4]
may_2k=df_2k[df_2k.month==5]
jun_2k=df_2k[df_2k.month==6]
jul_2k=df_2k[df_2k.month==7]
aug_2k=df_2k[df_2k.month==8]
sep_2k=df_2k[df_2k.month==9]
oct_2k=df_2k[df_2k.month==10]
nov_2k=df_2k[df_2k.month==11]
dec_2k=df_2k[df_2k.month==12]
```

```
G_jan_2k=create_graph(jan_2k)
G_feb_2k=create_graph(feb_2k)
G_mar_2k=create_graph(mar_2k)
G_apr_2k=create_graph(apr_2k)
G_may_2k=create_graph(may_2k)
G_jun_2k=create_graph(jun_2k)
G_jul_2k=create_graph(jul_2k)
G_aug_2k=create_graph(aug_2k)
G_sep_2k=create_graph(sep_2k)
G_oct_2k=create_graph(oct_2k)
G_nov_2k=create_graph(nov_2k)
G_dec_2k=create_graph(dec_2k)
```

```
In [168]: jan_2k1=df_2k1[df_2k1.month==1]
feb_2k1=df_2k1[df_2k1.month==2]
mar_2k1=df_2k1[df_2k1.month==3]
apr_2k1=df_2k1[df_2k1.month==4]
may_2k1=df_2k1[df_2k1.month==5]
jun_2k1=df_2k1[df_2k1.month==6]
jul_2k1=df_2k1[df_2k1.month==7]
aug_2k1=df_2k1[df_2k1.month==8]
sep_2k1=df_2k1[df_2k1.month==9]
oct_2k1=df_2k1[df_2k1.month==10]
nov_2k1=df_2k1[df_2k1.month==11]
```

```
dec_2k1=df_2k1[df_2k1.month==12]
```

```
G_jan_2k1=create_graph(jan_2k1)
G_feb_2k1=create_graph(feb_2k1)
G_mar_2k1=create_graph(mar_2k1)
G_apr_2k1=create_graph(apr_2k1)
G_may_2k1=create_graph(may_2k1)
G_jun_2k1=create_graph(jun_2k1)
G_jul_2k1=create_graph(jul_2k1)
G_aug_2k1=create_graph(aug_2k1)
G_sep_2k1=create_graph(sep_2k1)
G_oct_2k1=create_graph(oct_2k1)
G_nov_2k1=create_graph(nov_2k1)
G_dec_2k1=create_graph(dec_2k1)
```

```
In [169]: jan_2k2=df_2k2[df_2k2.month==1]
feb_2k2=df_2k2[df_2k2.month==2]
mar_2k2=df_2k2[df_2k2.month==3]
apr_2k2=df_2k2[df_2k2.month==4]
may_2k2=df_2k2[df_2k2.month==5]
jun_2k2=df_2k2[df_2k2.month==6]
jul_2k2=df_2k2[df_2k2.month==7]
aug_2k2=df_2k2[df_2k2.month==8]
sep_2k2=df_2k2[df_2k2.month==9]
oct_2k2=df_2k2[df_2k2.month==10]
nov_2k2=df_2k2[df_2k2.month==11]
dec_2k2=df_2k2[df_2k2.month==12]
```

```
G_jan_2k2=create_graph(jan_2k2)
G_feb_2k2=create_graph(feb_2k2)
G_mar_2k2=create_graph(mar_2k2)
G_apr_2k2=create_graph(apr_2k2)
G_may_2k2=create_graph(may_2k2)
G_jun_2k2=create_graph(jun_2k2)
G_jul_2k2=create_graph(jul_2k2)
G_aug_2k2=create_graph(aug_2k2)
G_sep_2k2=create_graph(sep_2k2)
G_oct_2k2=create_graph(oct_2k2)
G_nov_2k2=create_graph(nov_2k2)
G_dec_2k2=create_graph(dec_2k2)
```

```
In [170]: all_year_G =tuple([Gt0,Gt1,Gt2,Gt3,Gt4])
```

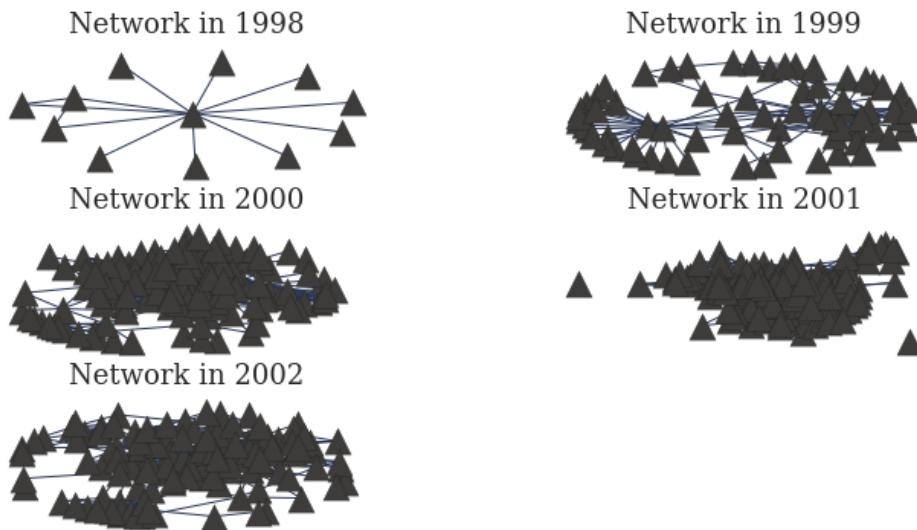
```
In [171]: all_month_G = tuple([G_nov98,G_dec98,G_jan_99,G_feb_99,G_mar_99,G_apr_99,
G_nov_99,G_dec_99,G_jan_2k,G_feb_2k,G_mar_2k,G_apr_2k,G_may
G_oct_2k,G_nov_2k,G_dec_2k,G_jan_2k1,G_feb_2k1,G_mar_2k1,G_
G_aug_2k1,G_sep_2k1,G_oct_2k1,G_nov_2k1,G_dec_2k1,G_jan_2k2
```

3 Network Visualisation

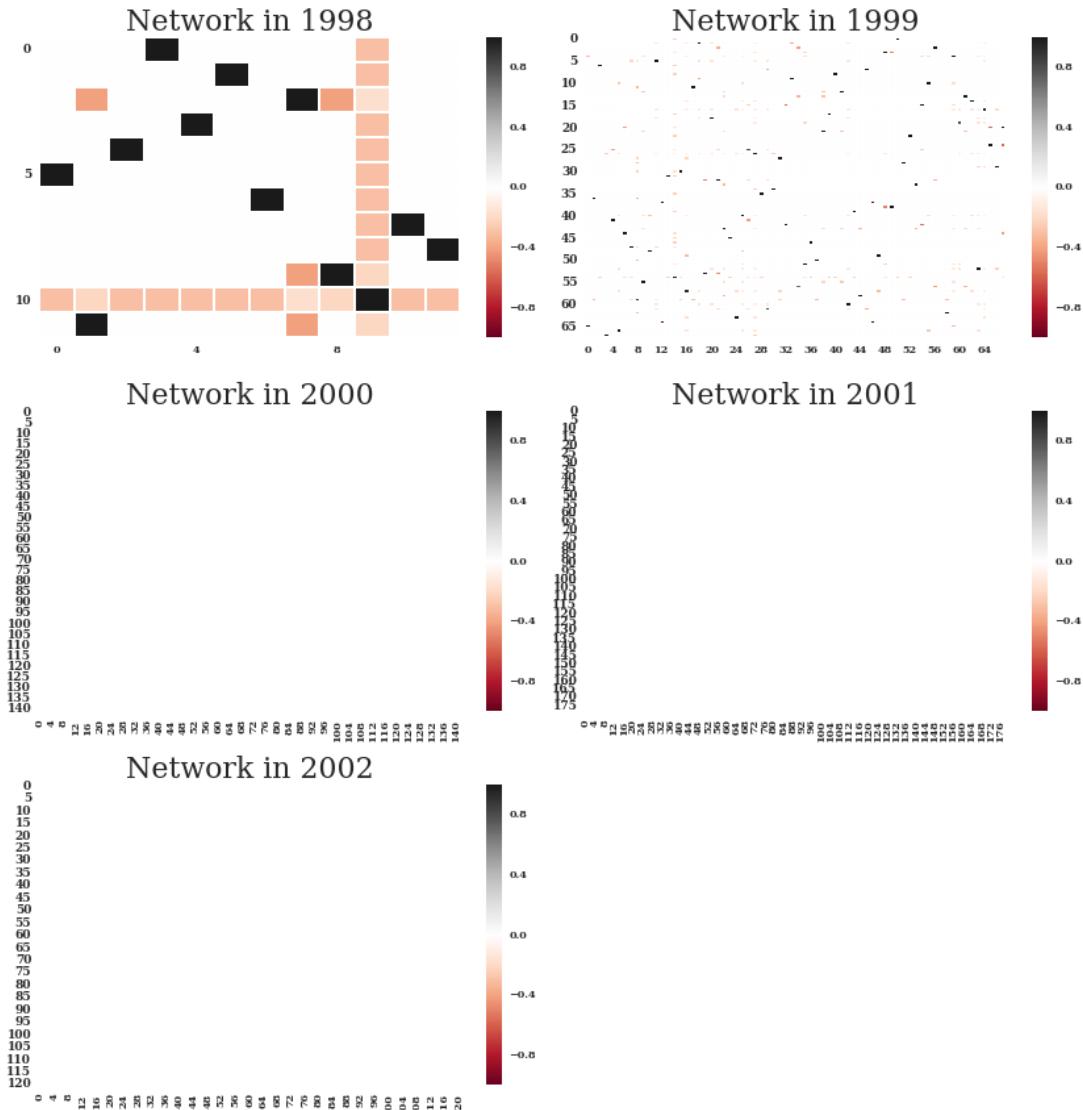
- Node Link Diagram
- Audio Waveform
- Matrix Visualisation

3.1 Yearly Networks

```
In [172]: plt.figure(figsize=(12,12))
#2B3856
#C6AEC7
for i in range(len(all_year_G)):
    plt.subplot(6,2,i+1)
    nx.draw_spring(all_year_G[i], node_color='#3D3C3A', node_shape='^', edge_color='blue')
    plt.title("Network in " + str(years[i]), fontsize=18)
plt.savefig('images/yearly_net.png')
```



```
In [173]: plt.figure(figsize=(12,12))
for i in range(len(all_year_G)):
    m = nx.normalized_laplacian_matrix(all_year_G[i]).todense()
    g = sns.clustermap(m)
    plt.close()
    ind = g.dendrogram_row.reordered_ind
    plt.subplot(3,2,i+1)
    sns.heatmap(m[ind][ind], cmap='RdGy', linewidths=1, xticklabels=4, yticklabels=4)
    plt.title("Network in " + str(years[i]), fontsize=22)
    plt.yticks(fontsize=9, rotation=360)
    plt.tight_layout()
plt.savefig('images/yearly_net_mat.png')
```



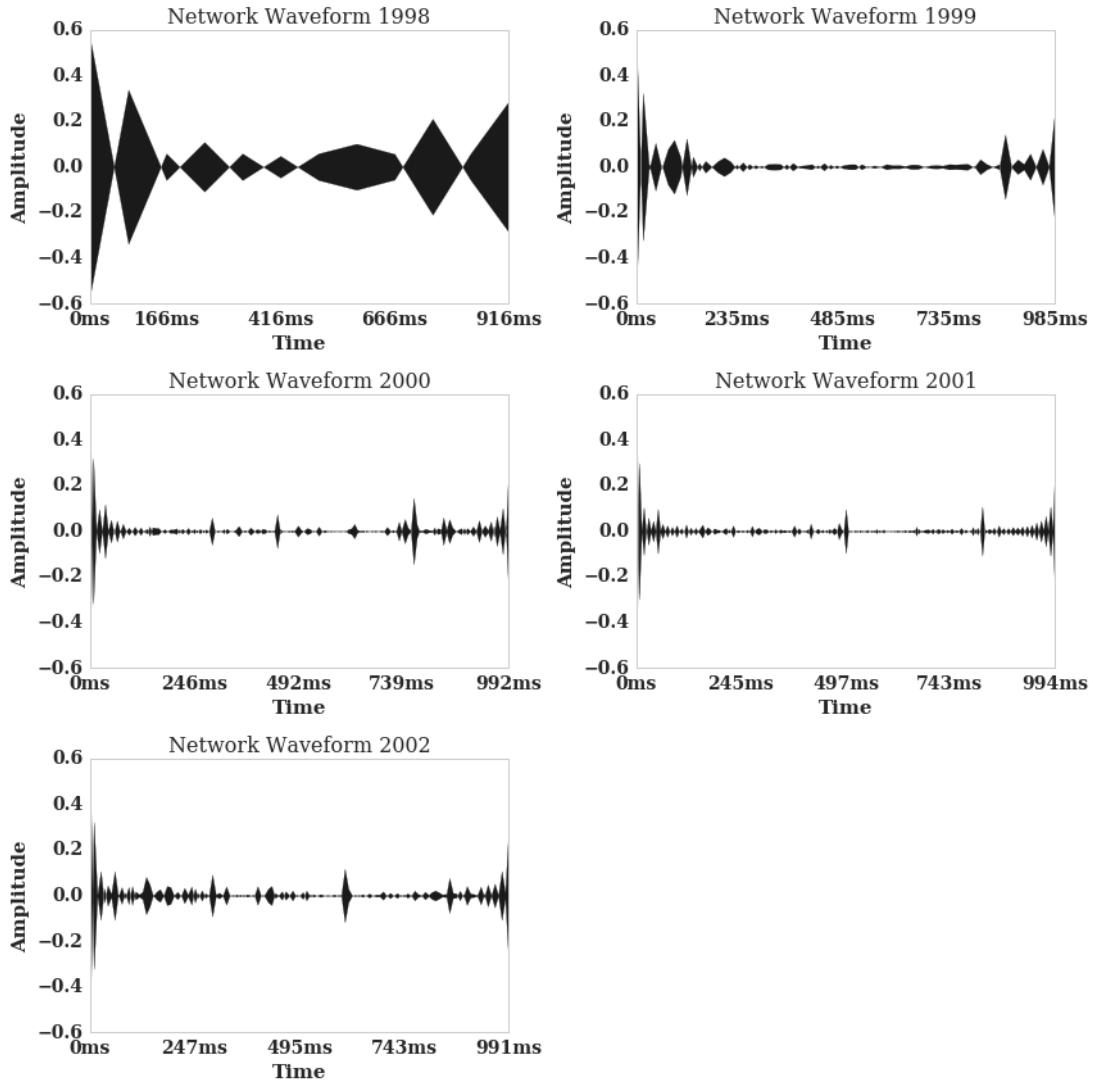
```
In [174]: plt.figure(figsize=(12,12))
for i in range(len(all_year_G)):
    f = sc.fftpack.rfft(nx.normalized_laplacian_matrix(all_year_G[i]).toarray())
    fs = f.shape[0]

    plt.subplot(3,2,i+1)
    librosa.display.waveplot(f, fs,color='k')
    plt.title("Network Waveform " + str(years[i]), fontsize=16)
    plt.xticks(fontsize=14)
    plt.yticks(fontsize=14)
    plt.xlabel("Time", fontsize=15)
    plt.ylabel("Amplitude", fontsize=15)
```

```

plt.grid(False)
plt.tight_layout()
plt.savefig('images/yearly_net_audio.png')

```

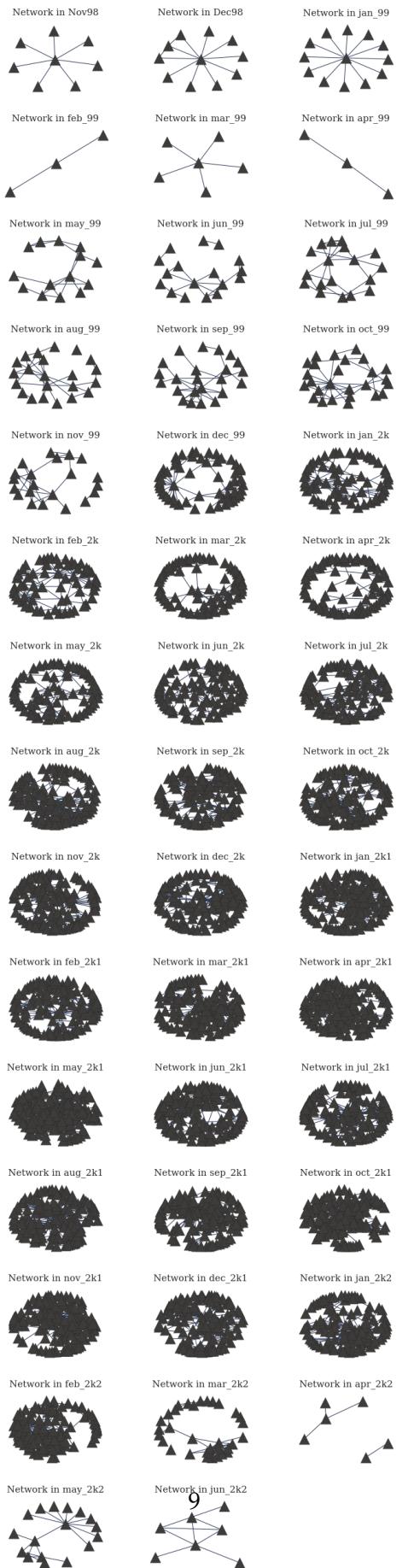


3.2 Monthly networks

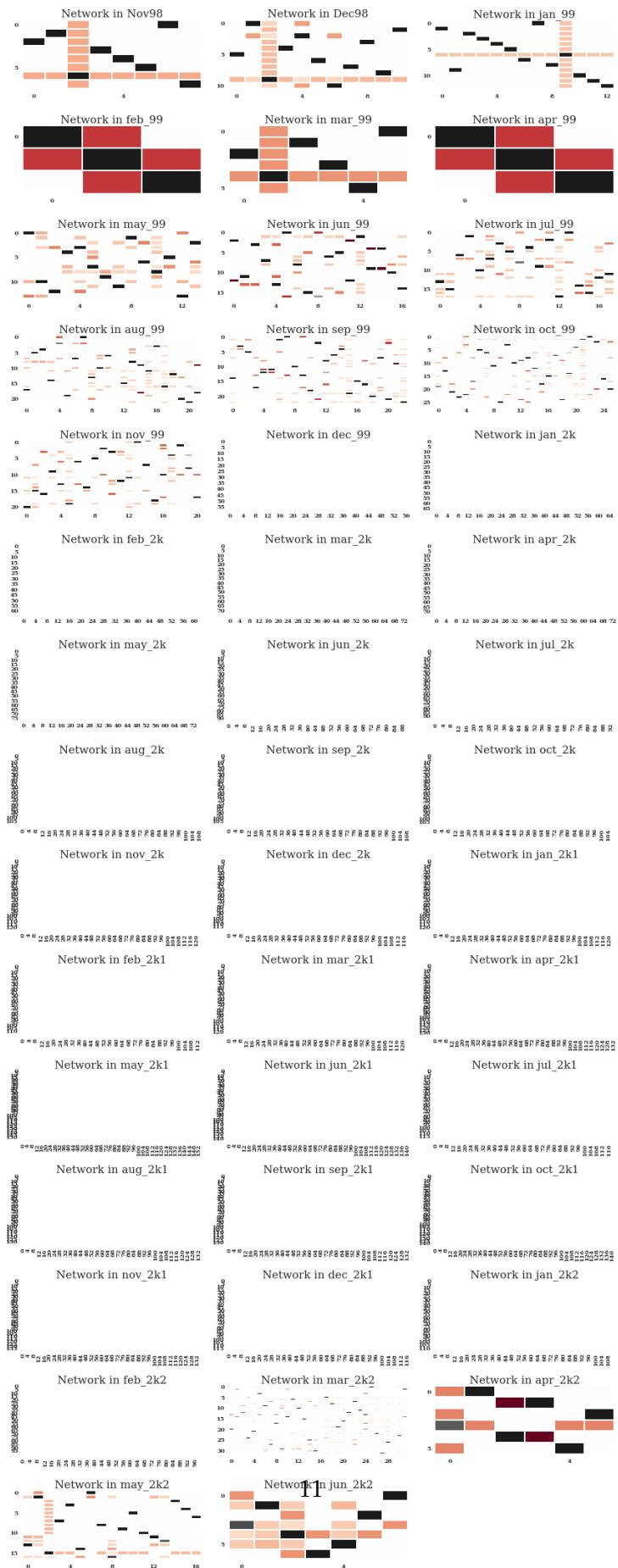
```

In [175]: plt.figure(figsize=(10,38))
for i in range(len(all_month_G)):
    plt.subplot(16, 3, i+1)
    nx.draw_spring(all_month_G[i], node_color='#3D3C3A', node_shape='^', e
    plt.title("Network in " + str(months[i]), fontsize=15)
    plt.tight_layout()
plt.savefig('images/mth_net_nodelink.png')

```



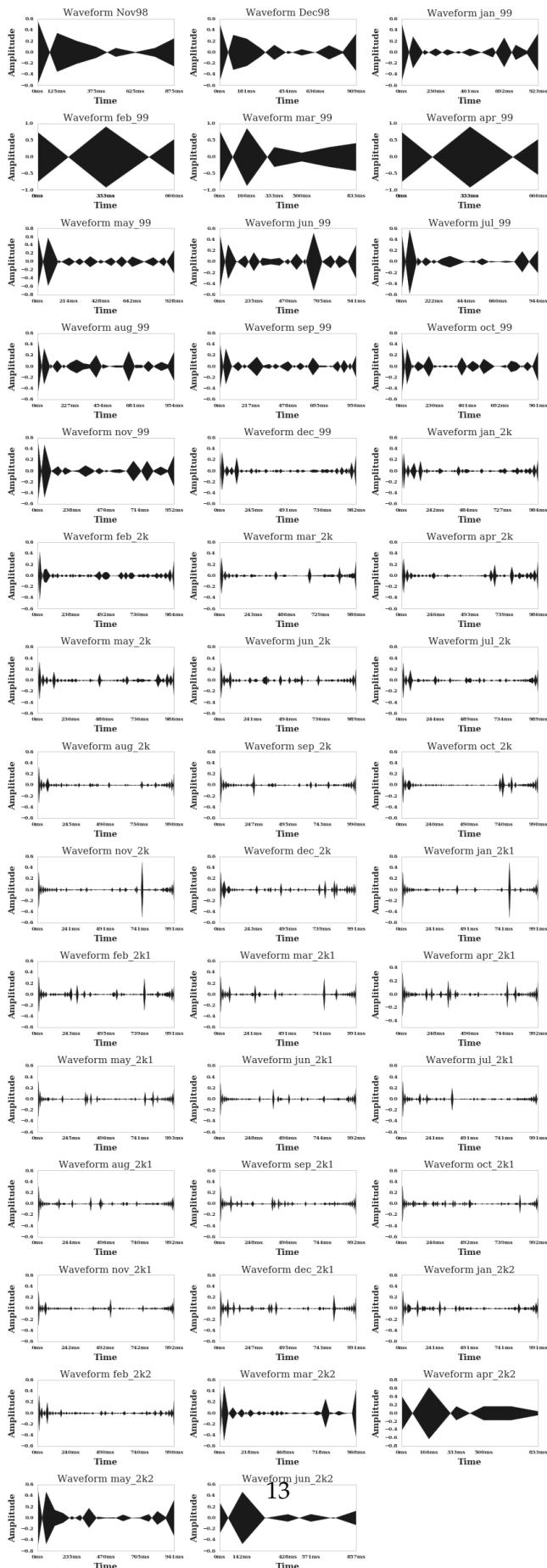
```
In [176]: plt.figure(figsize=(12,32))
for i in range(len(all_month_G)):
    m = nx.normalized_laplacian_matrix(all_month_G[i]).todense()
    g = sns.clustermap(m)
    plt.close()
    ind = g.dendrogram_row.reordered_ind
    plt.subplot(16, 3, i+1)
    sns.heatmap(m[ind][ind], cmap='RdGy', linewidths=1, xticklabels=4, yti
    plt.title("Network in " + str(months[i]), fontsize=15)
    plt.yticks(rotation=360)
    plt.tight_layout()
plt.savefig('images/mth_net_mat.png')
```



```
In [177]: plt.figure(figsize=(12,36))

for i in range(len(all_month_G)):
    f = sc.fftpack.rfft(nx.normalized_laplacian_matrix(all_month_G[i])).toarray()
    fs = f.shape[0]

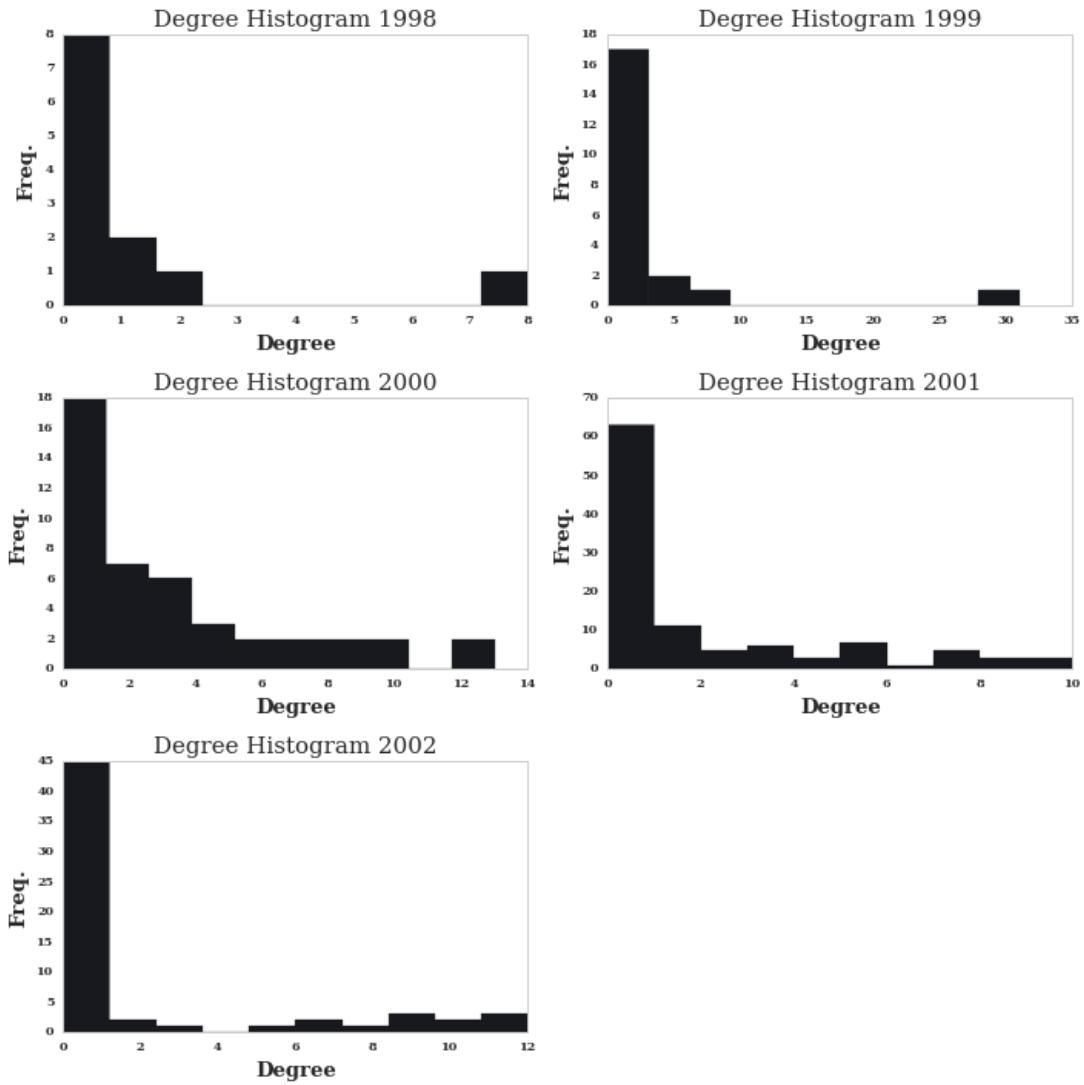
    plt.subplot(16, 3, i+1)
    librosa.display.waveplot(f, fs,color='k')
    plt.title("Waveform " + str(months[i]), fontsize=15)
    #plt.xticks(rotation=90)
    plt.xlabel("Time")
    plt.ylabel("Amplitude")
    plt.grid(False)
    plt.tight_layout()
plt.savefig('images/mth_net_audio.png')
```



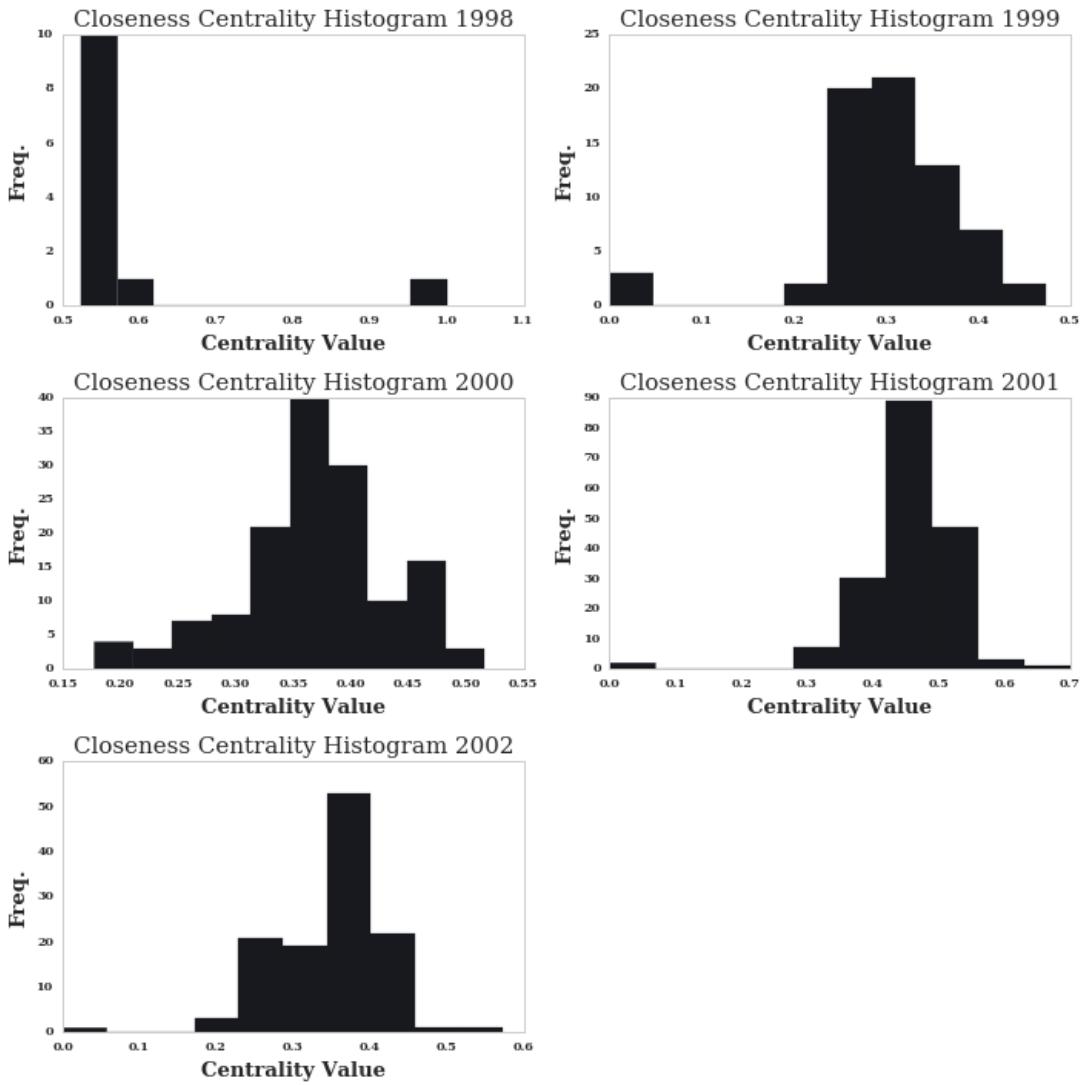
4 Exploratory Analysis

4.1 Yearly Networks

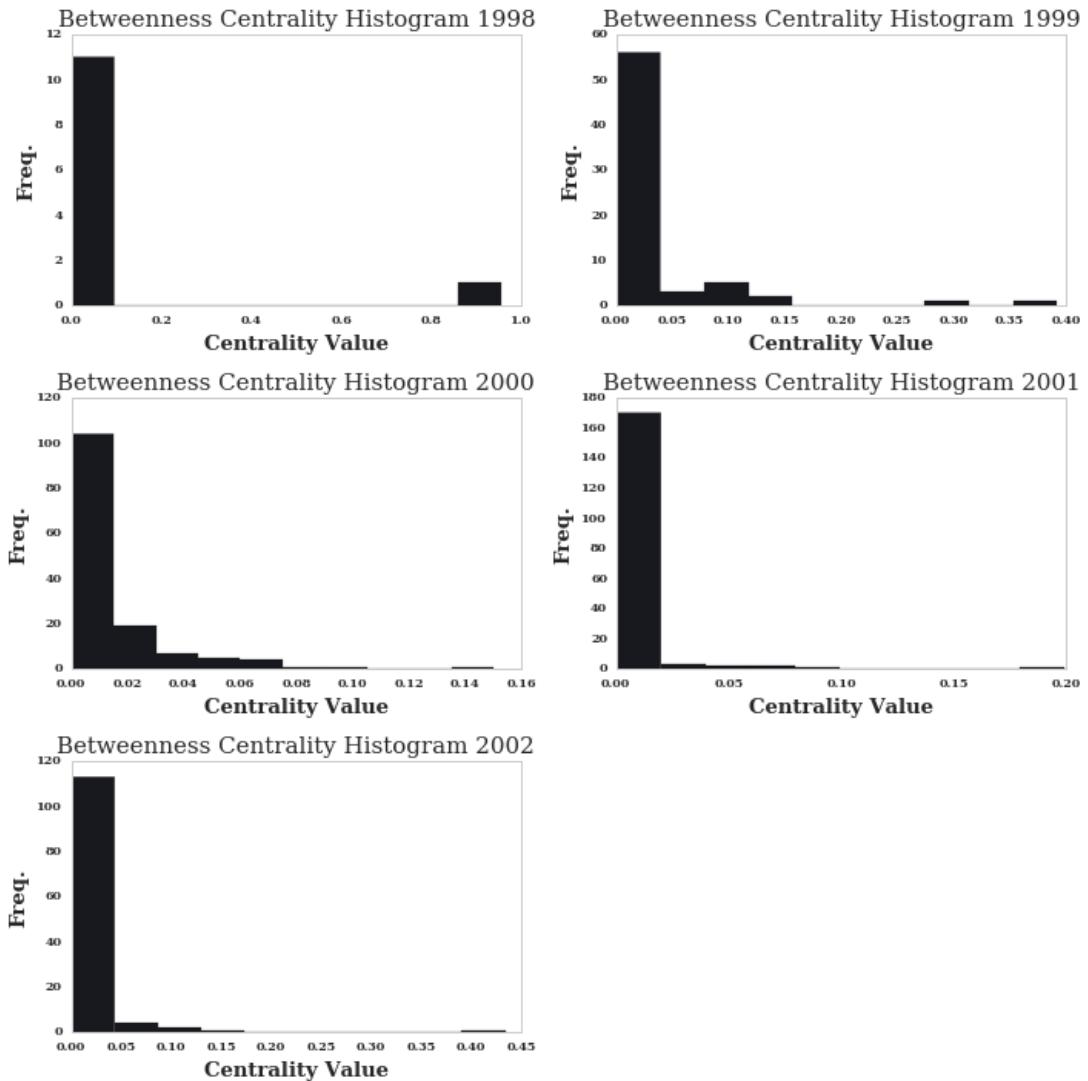
```
In [178]: plt.figure(figsize=(10,10))
    for i in range(len(all_year_G)):
        deg = nx.degree_histogram(all_year_G[i])
        plt.subplot(3, 2, i+1)
        plt.hist(deg)
        plt.title("Degree Histogram " + str(years[i]), fontsize=15)
        plt.xlabel("Degree")
        plt.ylabel("Freq.")
        plt.grid(False)
        plt.tight_layout()
    plt.savefig('images/year_deghist.png')
```



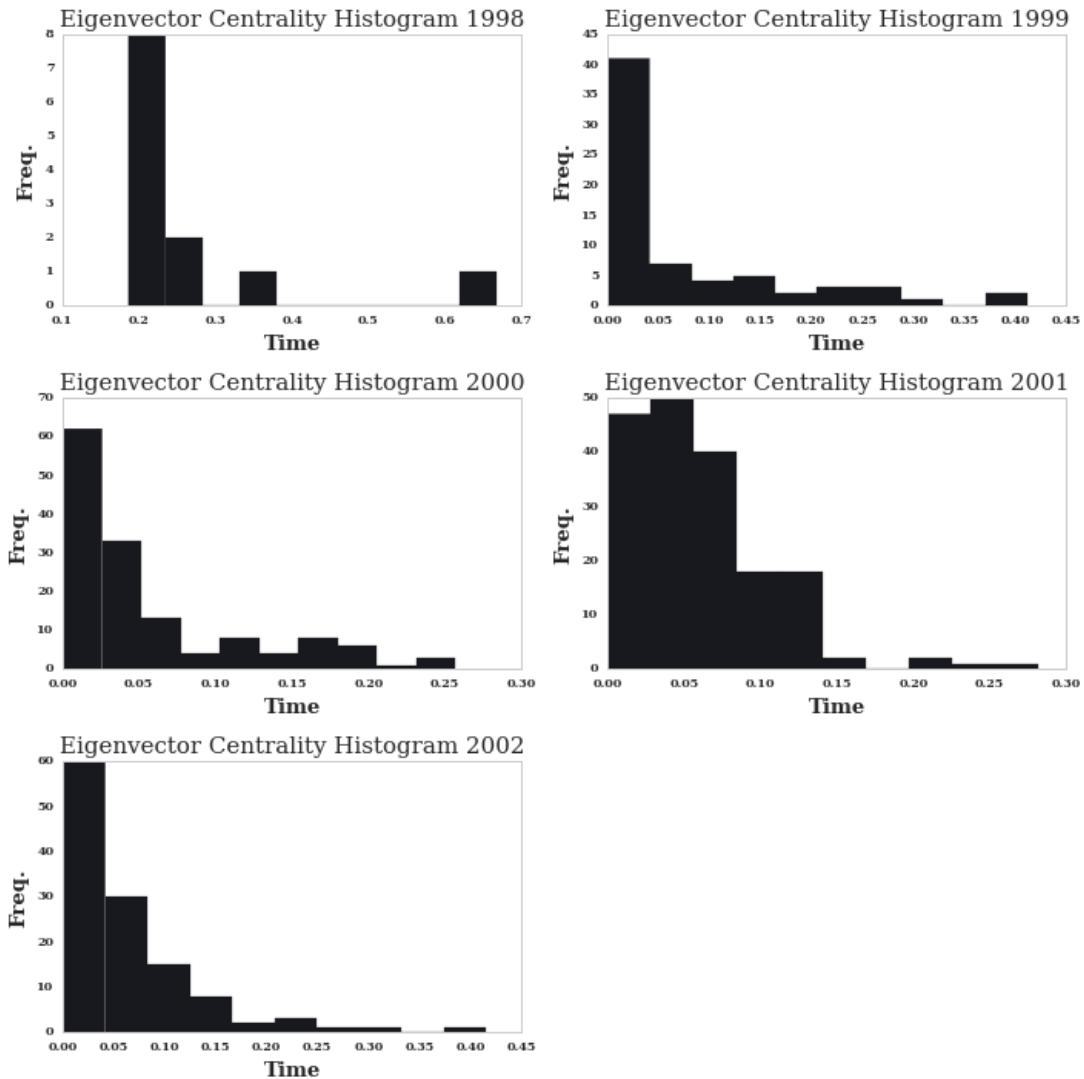
```
In [179]: plt.figure(figsize=(10,10))
for i in range(len(all_year_G)):
    deg = nx.closeness_centrality(all_year_G[i]).values()
    deg = sorted(deg)
    plt.subplot(3, 2, i+1)
    plt.hist(deg)
    plt.title("Closeness Centrality Histogram " + str(years[i]), fontsize=10)
    plt.xlabel("Centrality Value")
    plt.ylabel("Freq.")
    plt.grid(False)
    plt.tight_layout()
plt.savefig('images/year_clohist.png')
```



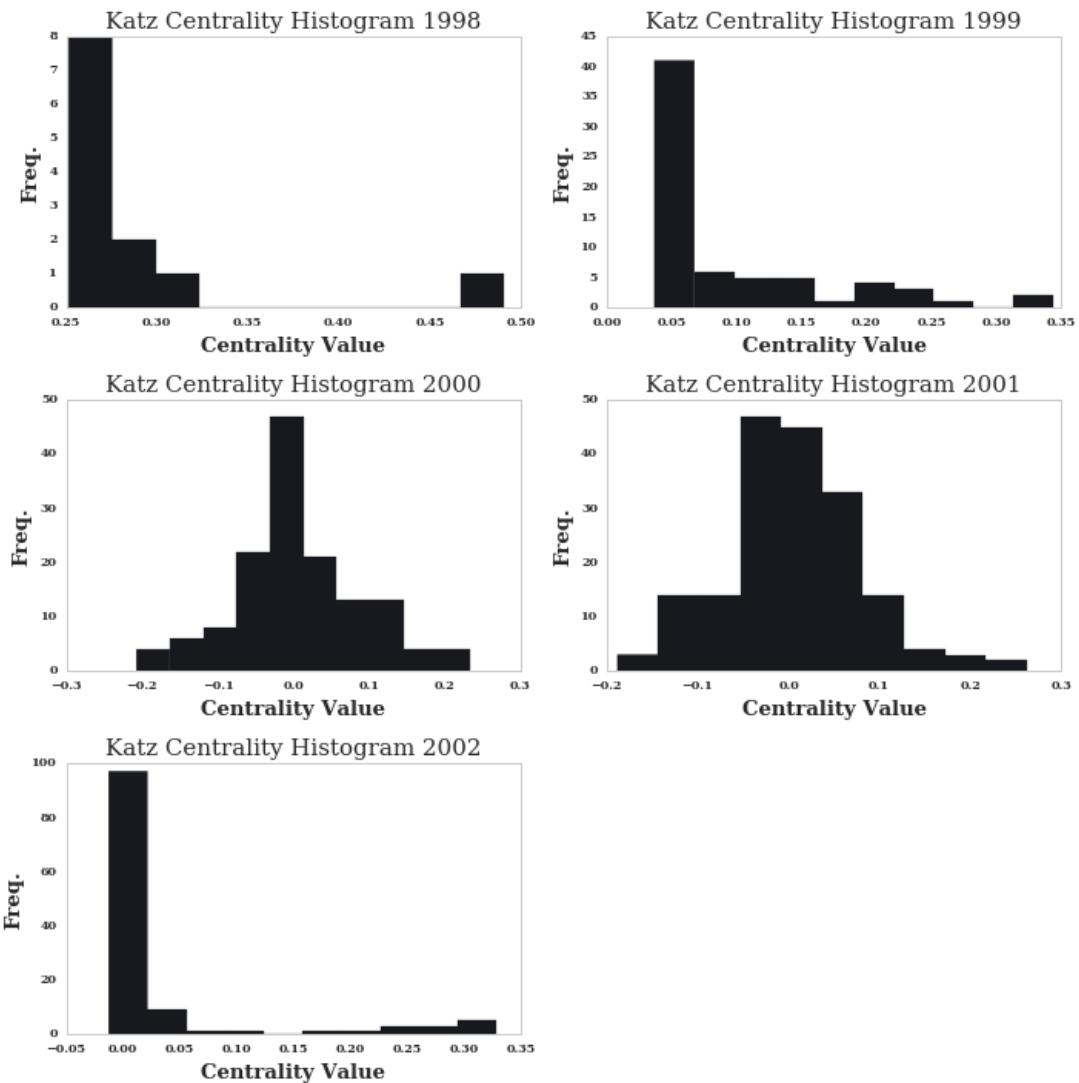
```
In [180]: plt.figure(figsize=(10,10))
for i in range(len(all_year_G)):
    deg = nx.betweenness_centrality(all_year_G[i]).values()
    deg = sorted(deg)
    plt.subplot(3, 2, i+1)
    plt.hist(deg)
    plt.title("Betweenness Centrality Histogram " + str(years[i]), fontstyle="italic")
    plt.xlabel("Centrality Value")
    plt.ylabel("Freq.")
    plt.grid(False)
    plt.tight_layout()
plt.savefig('images/year_bethist.png')
```



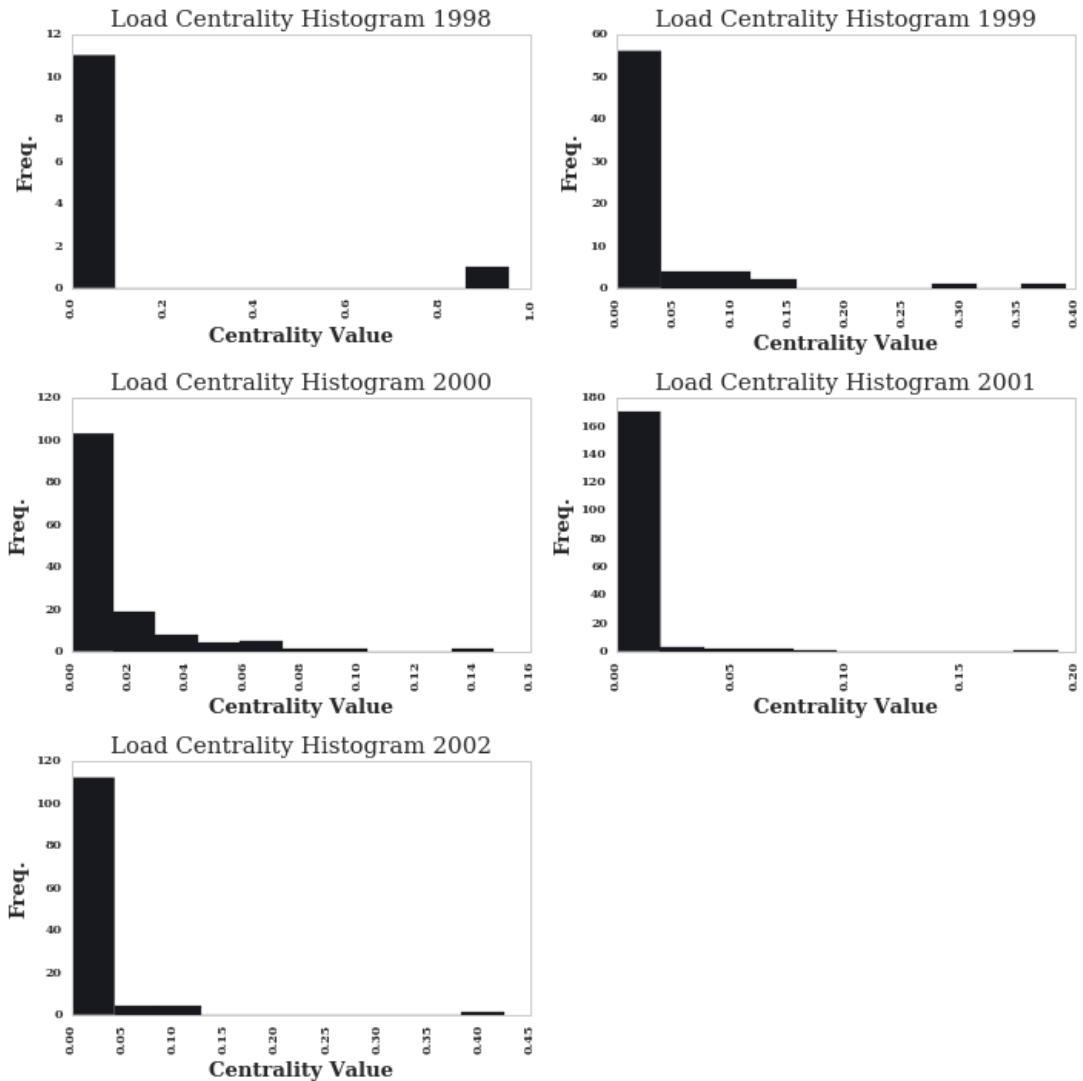
```
In [181]: plt.figure(figsize=(10,10))
for i in range(len(all_year_G)):
    deg = nx.eigenvector_centrality_numpy(all_year_G[i]).values()
    deg = sorted(deg)
    plt.subplot(3, 2, i+1)
    plt.hist(deg)
    plt.title("Eigenvector Centrality Histogram " + str(years[i]), fontsize=10)
    plt.xlabel("Time")
    plt.ylabel("Freq.")
    plt.grid(False)
    plt.tight_layout()
plt.savefig('images/year_eighist.png')
```



```
In [182]: plt.figure(figsize=(10,10))
for i in range(len(all_year_G)):
    deg = nx.katz_centrality_numpy(all_year_G[i]).values()
    deg = sorted(deg)
    plt.subplot(3, 2, i+1)
    plt.hist(deg)
    plt.title("Katz Centrality Histogram " + str(years[i]), fontsize=15)
    plt.xlabel("Centrality Value")
    plt.ylabel("Freq.")
    plt.grid(False)
    plt.tight_layout()
plt.savefig('images/year_katzhist.png')
```

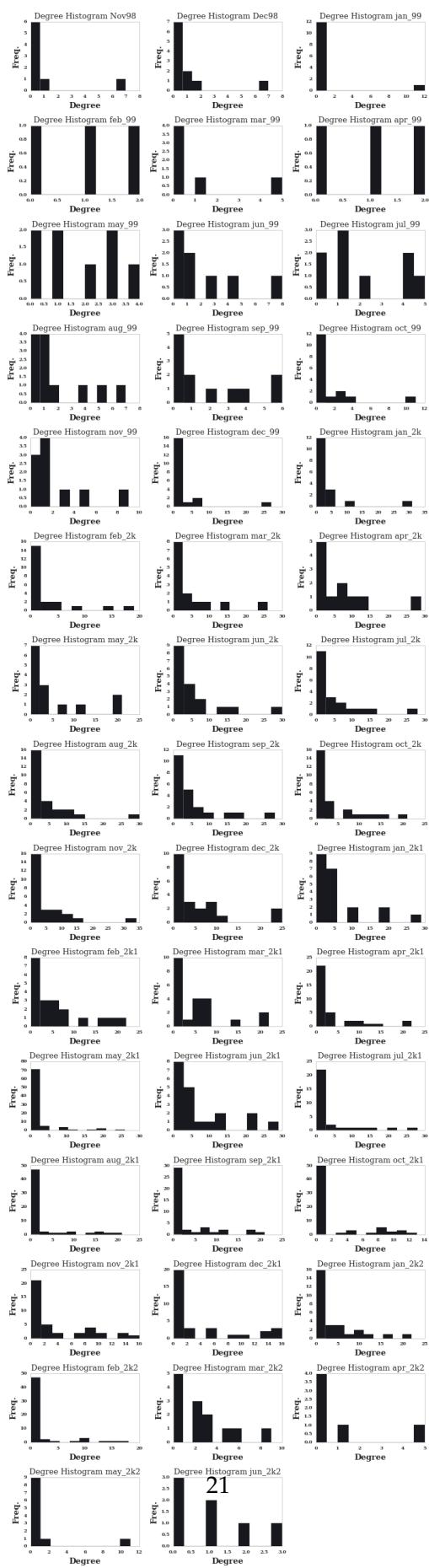


```
In [183]: plt.figure(figsize=(10,10))
for i in range(len(all_year_G)):
    deg = nx.load_centrality(all_year_G[i]).values()
    deg = sorted(deg)
    plt.subplot(3, 2, i+1)
    plt.hist(deg)
    plt.title("Load Centrality Histogram " + str(years[i]), fontsize=15)
    plt.xticks(rotation=90)
    plt.xlabel("Centrality Value")
    plt.ylabel("Freq.")
    plt.grid(False)
    plt.tight_layout()
plt.savefig('images/year_loadhist.png')
```

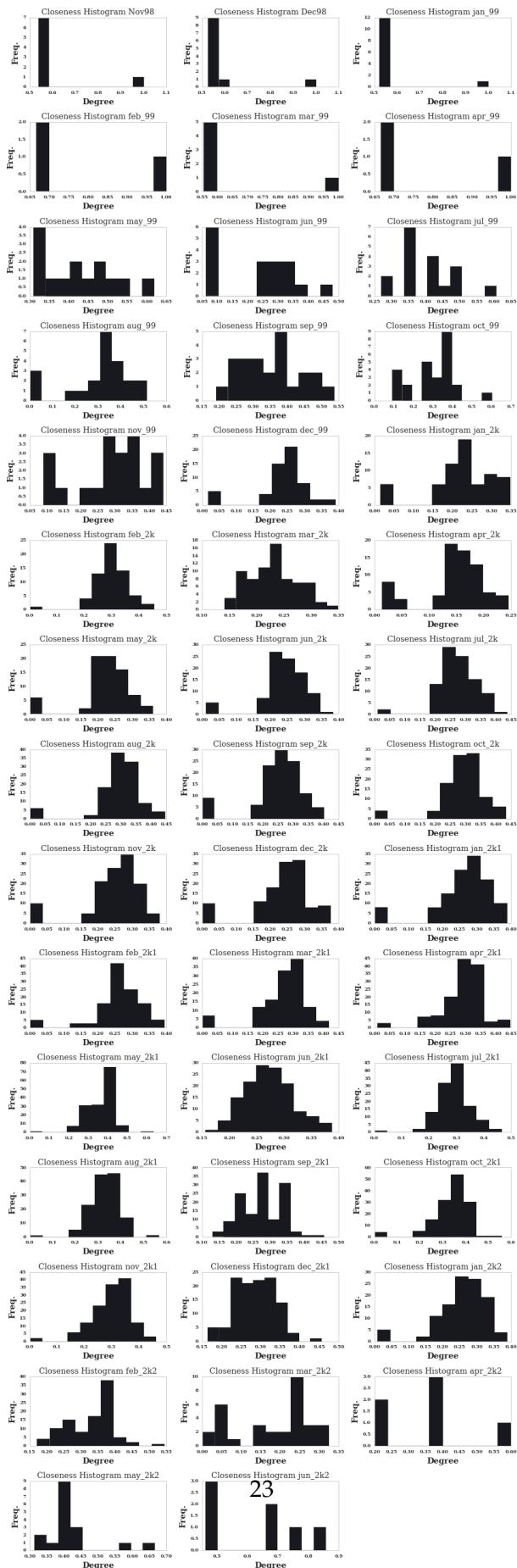


4.2 Monthly Networks

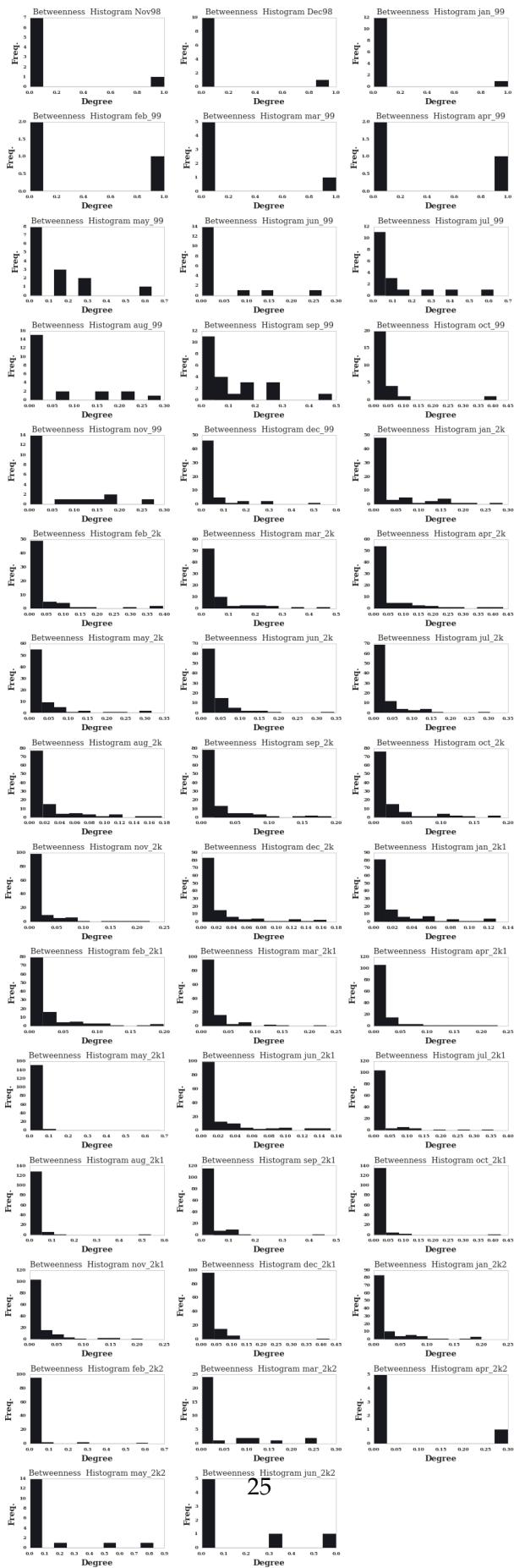
```
In [184]: plt.figure(figsize=(10, 38))
for i in range(len(all_month_G)):
    deg = nx.degree_histogram(all_month_G[i])
    plt.subplot(16, 3, i+1)
    plt.hist(deg)
    plt.title("Degree Histogram " + str(months[i]), fontsize=13)
    plt.xlabel("Degree")
    plt.ylabel("Freq.")
    plt.grid(False)
    plt.tight_layout()
plt.savefig('images/mth_deghist.png')
```



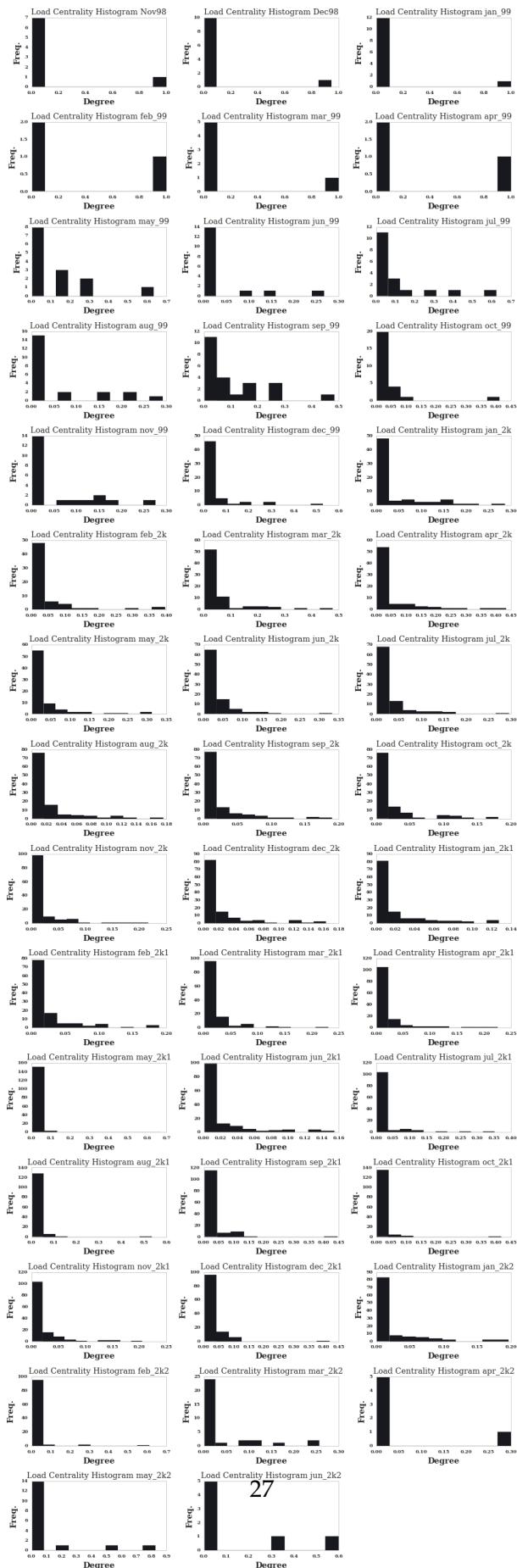
```
In [185]: plt.figure(figsize=(12,38))
for i in range(len(all_month_G)):
    deg = nx.closeness_centrality(all_month_G[i]).values()
    deg = sorted(deg)
    plt.subplot(16, 3, i+1)
    plt.hist(deg)
    plt.title("Closeness Histogram " + str(months[i]), fontsize=13)
    plt.xlabel("Degree")
    plt.ylabel("Freq.")
    plt.grid(False)
    plt.tight_layout()
plt.savefig('images/mth_clohist.png')
```



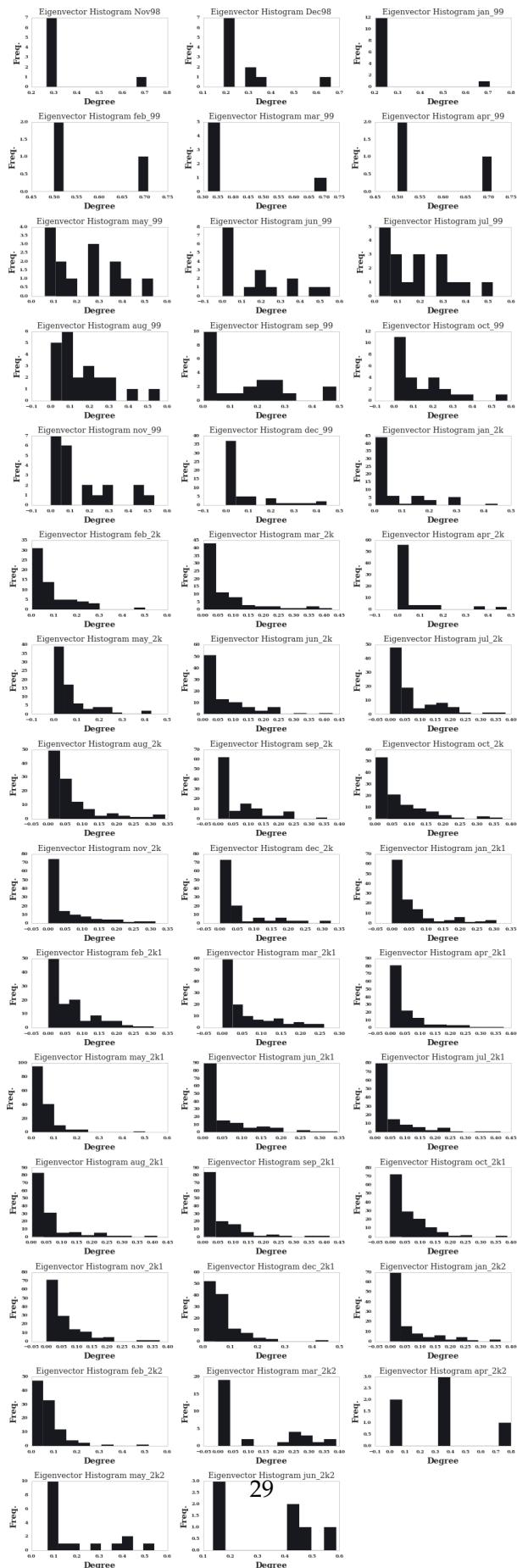
```
In [186]: plt.figure(figsize=(12,38))
for i in range(len(all_month_G)):
    deg = nx.betweenness_centrality(all_month_G[i]).values()
    deg = sorted(deg)
    plt.subplot(16, 3, i+1)
    plt.hist(deg)
    plt.title("Betweenness Histogram " + str(months[i]), fontsize=13)
    plt.xlabel("Degree")
    plt.ylabel("Freq.")
    plt.grid(False)
    plt.tight_layout()
plt.savefig('images/mth_bethist.png')
```



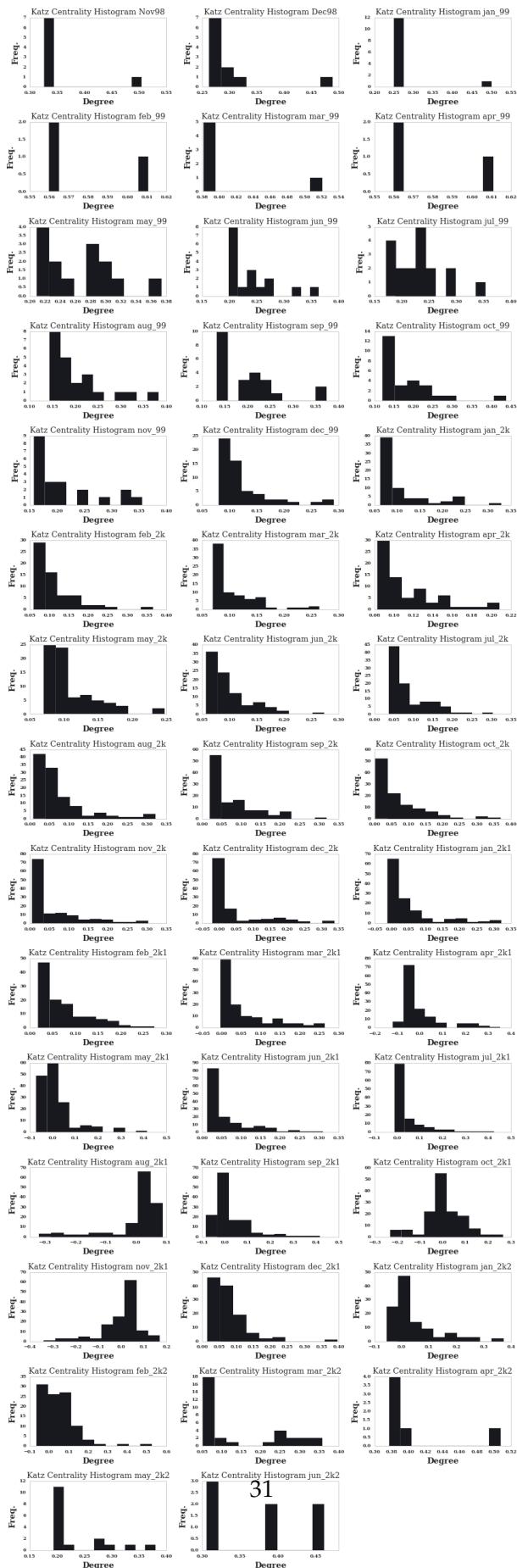
```
In [187]: plt.figure(figsize=(12,38))
for i in range(len(all_month_G)):
    deg = nx.load_centrality(all_month_G[i]).values()
    deg = sorted(deg)
    plt.subplot(16, 3, i+1)
    plt.hist(deg)
    plt.title("Load Centrality Histogram " + str(months[i]), fontsize=13)
    plt.xlabel("Degree")
    plt.ylabel("Freq.")
    plt.grid(False)
    plt.tight_layout()
plt.savefig('images/mth_loadhist.png')
```



```
In [188]: plt.figure(figsize=(12,38))
for i in range(len(all_month_G)):
    deg = nx.eigenvector_centrality_numpy(all_month_G[i]).values()
    deg = sorted(deg)
    plt.subplot(16,3, i+1)
    plt.hist(deg)
    plt.title("Eigenvector Histogram " + str(months[i]), fontsize=13)
    plt.xlabel("Degree")
    plt.ylabel("Freq.")
    plt.grid(False)
    plt.tight_layout()
plt.savefig('images/mth_eighist.png')
```



```
In [189]: plt.figure(figsize=(12,38))
for i in range(len(all_month_G)):
    deg = nx.katz_centrality_numpy(all_month_G[i]).values()
    deg = sorted(deg)
    plt.subplot(16,3, i+1)
    plt.hist(deg)
    plt.title("Katz Centrality Histogram " + str(months[i]), fontsize=13)
    plt.xlabel("Degree")
    plt.ylabel("Freq.")
    plt.grid(False)
    plt.tight_layout()
plt.savefig('images/mth_katzhist.png')
```



31

5 Attribute Analysis

5.1 Traditional Measures

5.1.1 Centrality

- Degree
- Betweenness
- Closeness
- Katz
- Load

5.1.2 Assortativity & Linear Algebra

- Density
- Average Clustering Coefficient
- Algebraic Connectivity

5.2 Complex Trace Attributes

- Instantaneous Amplitude
- Instantaneous Frequency
- Instantaneous Phase
- Derivative of Amplitude
- Second Derivative of Amplitude
- Power
- Cosine of Instantaneous Phase
- Amplitude weighted Frequency
- Amplitude weighted Phase
- Power Spectral Density

5.3 Matrix

- Resistance Distance
- Stationarity Ratio
- Subgraph Stationarity
- Curvature
- Norm of Abel Transform

5.4 Matrix Decomposition

- KLPCA Ratio Change
- Norm NMF Ratio Change

5.5 Music Attributes

- Zero Crossing Rate
- Spectral Centroid

5.6 Aggregation Measures

- Persistence
- Emergence
- NRMS of Attributes

```
In [190]: def calc_seisatt(net):  
    M = nx.normalized_laplacian_matrix(net).todense()  
    Ht = hilbert(M)  
    rHt = np.real(Ht)  
    iHt = np.imag(Ht)  
  
    #Basic attributes IA, IP, IF  
    IA = np.real(np.nan_to_num(np.sqrt(np.dot(rHt,rHt)+np.dot(iHt,iHt))))  
    IP = np.real(np.nan_to_num(np.arctan(iHt/rHt)))  
    IF,_ = np.real(np.nan_to_num(np.asarray(np.gradient(IP))))  
    P = np.square(IA)  
  
    #Derivatives  
    dIA,_ = np.nan_to_num(np.asarray(np.gradient(IA)))  
    d2IA,_ = np.nan_to_num(np.asarray(np.gradient(dIA)))  
    IAcc,_ = np.nan_to_num(np.asarray(np.gradient(IF)))  
  
    #Derived Attributes  
    cosIP = np.cos(IP)  
    IA_weit_IF = IA * IF  
    IA_weit_IP = IA*IP  
  
    return [IA,IP,IF,P, dIA,d2IA, IAcc,cosIP,IA_weit_IF,IA_weit_IP]  
  
In [191]: def stationarity_ratio(G):  
    #stationarity ratio with laplian  
    L = nx.normalized_laplacian_matrix(G).todense()  
    U = eigvals(L)  
    C = np.cov(L)  
    CF = np.dot(L,np.dot(np.dot(U.T,C),U))  
    r = np.linalg.norm(np.diag(CF))/np.linalg.norm(CF)  
  
    return r  
  
In [192]: #cite:`klein1993resistance`  
def resistance_distance(net):  
    M = nx.normalized_laplacian_matrix(net).todense()  
    pseudo = pinv(M)
```

```

N = M.shape[0]
d = np.diag(pseudo)
rd = np.kron(d,np.ones((N,1))).T+np.kron(d,np.ones((N,1))).T - pseudo

return rd.mean()

In [193]: def curvature(net):
    from skimage.feature import hessian_matrix, hessian_matrix_det, hessian
    M = nx.normalized_laplacian_matrix(net).todense()
    M = np.float64(M)
    fx, fy = np.gradient(M)
    Hxx, Hxy, Hyy = hessian_matrix(M)
    K = np.divide((np.dot(Hxx,Hxy)-np.dot(Hxy,Hxy)), \
                  (1+np.dot(fx,fx)+np.dot(fy,fy)))
    He1,_ = hessian_matrix_eigvals(Hxx,Hxy,Hyy)
    mean_curv = np.trace(He1)

    return mean_curv

In [194]: def kpca_ratio(net):
    from sklearn.decomposition import KernelPCA
    kpca = KernelPCA(n_components=3,kernel='rbf')
    m = nx.normalized_laplacian_matrix(net).todense()
    X_kpca = kpca.fit_transform(m)
    pc1 = X_kpca[:,0]
    pc2 = X_kpca[:,1]
    pc3 = X_kpca[:,2]
    kpca_rat = norm(pc1-pc3/(pc1-pc2))
    return kpca_rat

In [195]: def kpca_att(net):
    kpca_chg = []
    for i in range(len(net)-1):
        x = int(i)
        y = 1+x
        kpcar1= kpca_ratio(net[x])
        kpcar2= kpca_ratio(net[y])
        kpca_chg.append(np.divide(kpcar2,kpcar1))
    kpca_chgpad = np.zeros(len(kpca_chg)+1);
    kpca_chgpad[1:] = kpca_chg

    return kpca_chgpad

In [196]: def nmf_att(net):
    nmf_chg = []

    nmf = NMF(init='nndsvda',solver='cd', random_state=0, l1_ratio=1)
    for i in range(len(net)-1):
        x = int(i)

```

```

y = 1+x
m1= abs(nx.normalized_laplacian_matrix(net[x]).todense())
m2= abs(nx.normalized_laplacian_matrix(net[y]).todense())
nmf1 = norm(nmf.fit_transform(m1))
nmf2 = norm(nmf.fit_transform(m2))
nmf_chg.append(np.divide(nmf2,nmf1))
nmf_chgpad = np.zeros(len(nmf_chg)+1);
nmf_chgpad[1:] = nmf_chg

return nmf_chgpad

In [197]: def pad_shape(x,ref,offset=0):
    result = np.zeros_like(ref)
    result[0:x.shape[0]+0,0:x.shape[1]+0] = x

    return result

def subgraph_stat(net1,net2):
    net1_int_net2 = net1.copy()
    net1_int_net2.remove_nodes_from(n for n in net1 if n not in net2)
    net1_u_net2 = nx.disjoint_union(net1, net2)
    int_adjmat = nx.normalized_laplacian_matrix(net1_int_net2).todense()
    uni_adjmat = nx.normalized_laplacian_matrix(net1_u_net2).todense()
    int_adjmat_pad = pad_shape(int_adjmat,uni_adjmat)

    Ct = np.divide(norm(int_adjmat_pad),norm(uni_adjmat))

    return Ct

def zeta(n):

    Ct_m = []
    for i in range(len(n)-1):
        x = int(i)
        y = x +1
        Ct_m.append(subgraph_stat(n[x],n[y]))
    Ct_m_pad = np.zeros(len(Ct_m)+1);
    Ct_m_pad[1:] = Ct_m
    return Ct_m_pad

In [198]: def music_att(n):
    #music attributes
    f = sc.fftpack.rfft(nx.normalized_laplacian_matrix(n).todense()).mean(0)
    zcr = librosa.feature.zero_crossing_rate(f)[0,0]
    spc = librosa.feature.spectral_centroid(f)[0,0]

    return [zcr,spc]

In [199]: def norm_fabel(x):

```

```

import abel
fabel_att = []
for n in x:
    tmp = nx.normalized_laplacian_matrix(n).todense()
    tmp2 = sc.fftpack.fft2(tmp)
    fabel2 = abel.Transform(tmp2, direction='forward', method='direct')
    mag = np.sqrt(np.square(np.real(fabel2)) + np.square(np.imag(fabel2)))
    fabel_att.append(norm(mag))

return np.log10(fabel_att)

In [200]: def cal_avg_atts(x):

    #define attributes placeholders here
    deg_y = []
    bet_y = []
    clo_y = []
    load_y = []
    eig_y = []
    katz_y = []
    den_y = []
    algc_y = []
    cluscof_y = []
    IA_y = []
    IP_y = []
    IF_y = []
    P_y = []
    dIA_y = []
    d2IA_y = []
    IAcc_y = []
    cosIP_y = []
    IA_weit_IF_y = []
    IA_weit_IP_y = []
    psd_y = []
    rd_y = []
    zcr_y = []
    spc_y = []
    statr_y = []
    meank_y = []

    #matrix decompostion attributes
    zeta_y = zeta(x)
    nmf_ratio_y = nmf_att(x)
    kpca_chg_y= kpca_att(x)
    prop_members_chg_1_zeta = 1-zeta_y
    fabel = norm_fabel(x)

    for n in x:

```

```

deg = np.mean(sorted(set(nx.degree_centrality(n).values())))
bet = np.mean(sorted(set(nx.betweenness_centrality(n).values())))
clo = np.mean(sorted(set(nx.closeness_centrality(n).values())))
katz = np.mean(sorted(set(nx.katz_centrality_numpy(n).values())))
eig = np.mean(sorted(set(nx.eigenvector_centrality_numpy(n).values())))
load = np.mean(sorted(set(nx.degree_centrality(n).values())))
den = nx.density(n)
algc = nx.algebraic_connectivity(n)
clustcof = nx.average_clustering(n)

#all network metrics
deg_y.append(deg), bet_y.append(bet), clo_y.append(clo), load_y.append(load)
den_y.append(den), algc_y.append(algc), cluscof_y.append(clustcof)

#complex trace attributes
IA,IP,IF,P, dIA,d2IA, IAcc,cosIP,IA_weit_IF,IA_weit_IP = calc_sei()
IA_y.append(IA.mean())
IP_y.append(IP.mean())
IF_y.append(IF.mean())
P_y.append(P.mean())
dIA_y.append(dIA.mean())
d2IA_y.append(d2IA.mean())
IAAcc_y.append(IAcc.mean())
cosIP_y.append(cosIP.mean())
IA_weit_IF_y.append(IA_weit_IF.mean())
IA_weit_IP_y.append(IA_weit_IP.mean())

psd, _ = plt.psd(nx.laplacian_matrix(n).todense());
plt.close()
psd_y.append(psd.mean())

zcr, spc = music_att(n)
zcr_y.append(zcr)
spc_y.append(np.log10(spc))

#matrix attributes
rdm = resistance_distance(n)
rd_y.append(rdm)

statrat = stationarity_ratio(n)
statr_y.append(statrat)

meank = curvature(n)
meank_y.append(meank)

colnames = ['AvgDeg', 'AvgBet', 'AvgClo', 'AvgLoad', 'AvgKatz', 'AvgDensit

```

```

'AvgEig' , 'InstAmp' , 'InstPhase' , 'InstFreq' , 'Power' , 'dInstAmp'
'A_wt_IF' , 'A_wt_IP' , 'PowerSpecDen' , 'ResDist' , 'ZeroCrossRate'

attvol_y = pd.DataFrame([deg_y,bet_y, clo_y ,load_y ,katz_y ,den_y, a
                         P_y ,dIA_y ,d2IA_y,IAcc_y ,cosIP_y ,IA_weit_IF_y ,IA_weit
                         statr_y , meank_y]).T

attvol_y.columns = colnames
attvol_y['SubgraphStat']=zeta_y
attvol_y['1-Zeta'] = prop_members_chg_1_zeta
attvol_y['LogKPCARatioChg'] = np.log10(kpca_chg_y)
attvol_y.LogKPCARatioChg[0]=0
attvol_y['NormNMFRatioChg']= nmf_ratio_y
attvol_y['NormFAbel'] = fabel
attvol_y_sc = attvol_y.apply(lambda x: minmax_scale(x, feature_range))

return attvol_y_sc

In [201]: attvol_y = cal_avg_atts(all_year_G)

/home/arshad/anaconda3/lib/python3.5/site-packages/abel/transform.py:341: ComplexWarning:
self.IM = self.IM.astype('float64')

In [202]: attvol_m = cal_avg_atts(all_month_G)

/home/arshad/anaconda3/lib/python3.5/site-packages/abel/transform.py:341: ComplexWarning:
self.IM = self.IM.astype('float64')

```

6 Overview Plots

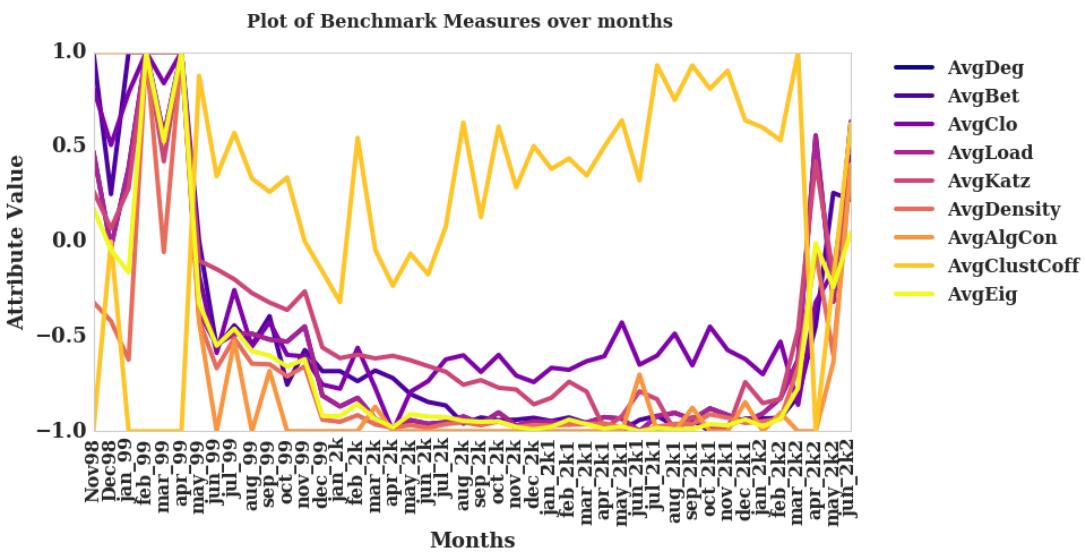
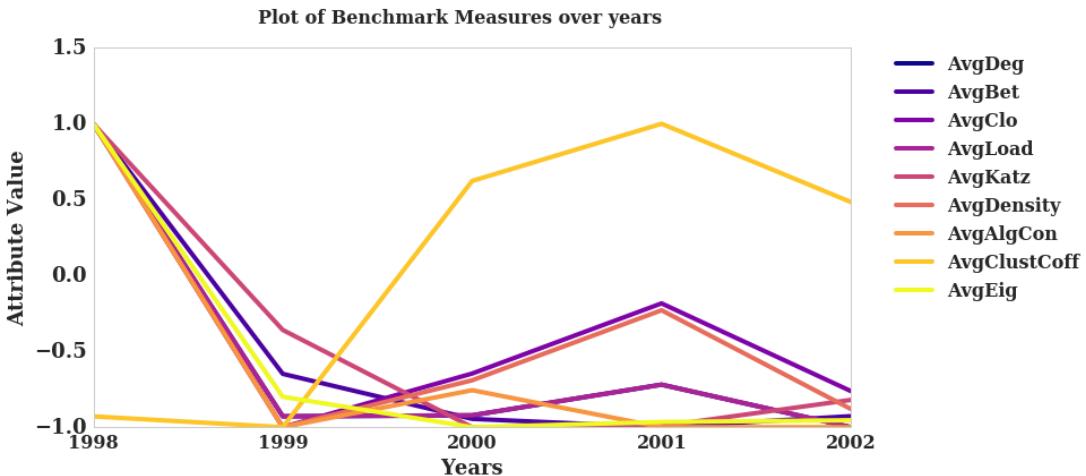
```

In [203]: attvol_y.iloc[:, :9].plot.line(use_index=True, cmap='plasma')
          plt.xlabel("Years", fontsize=18)
          plt.ylabel("Attribute Value", fontsize=18)
          plt.legend(fontsize=16, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0)
          plt.xticks(np.arange(len(years)), years, fontsize=16)
          plt.yticks(fontsize=18)
          plt.suptitle("Plot of Benchmark Measures over years", fontsize=16)

          attvol_m.iloc[:, :9].plot.line(use_index=True, cmap='plasma', rot=90)
          plt.xlabel("Months", fontsize=18)
          plt.ylabel("Attribute Value", fontsize=18)
          plt.legend(fontsize=16, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0)
          plt.xticks(np.arange(len(months)), months, fontsize=16)
          plt.yticks(fontsize=18)
          plt.suptitle("Plot of Benchmark Measures over months", fontsize=16)

Out [203]: <matplotlib.text.Text at 0x7fc0a8c80b8>

```



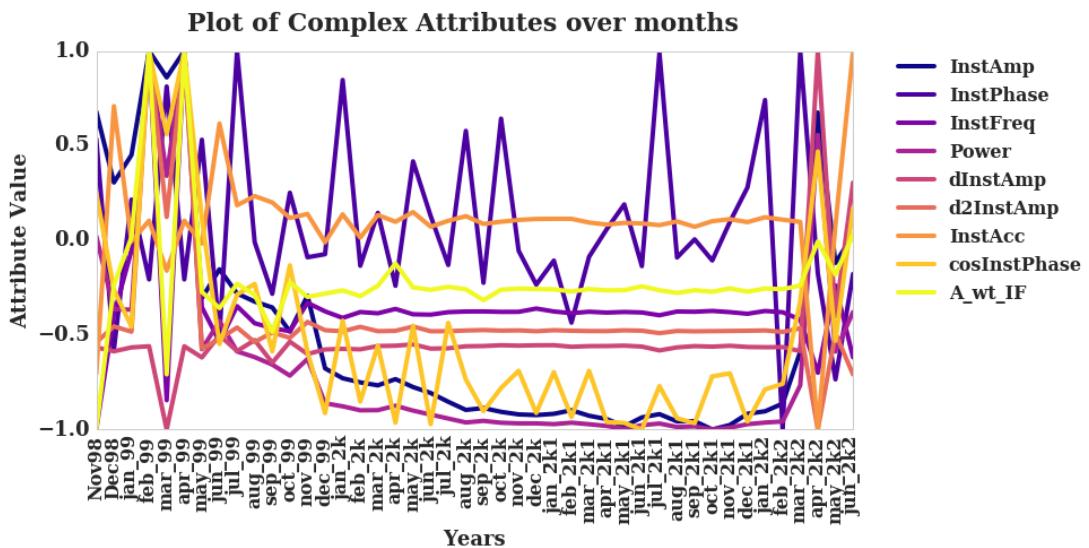
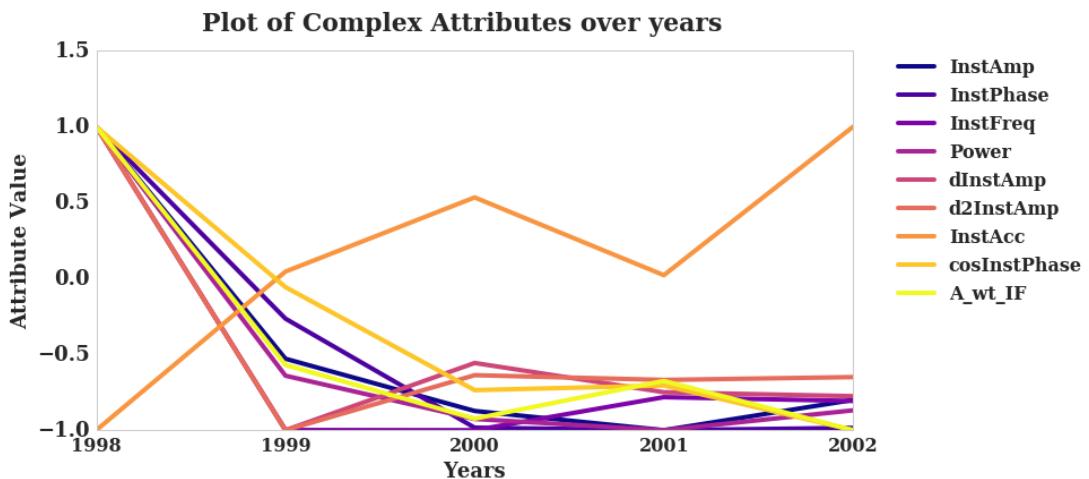
```
In [204]: attvol_y.iloc[:, 9:18].plot.line(use_index=True, cmap='plasma')
plt.xlabel("Years", fontsize=18)
plt.ylabel("Attribute Value", fontsize=18)
plt.legend(fontsize=16, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0)
plt.xticks(np.arange(len(years)), years, fontsize=16)
plt.yticks(fontsize=18)
plt.suptitle("Plot of Complex Attributes over years", fontsize=22)
plt.autoscale()

attvol_m.iloc[:, 9:18].plot.line(use_index=True, cmap='plasma', rot=90)
plt.xlabel("Years", fontsize=18)
```

```

plt.ylabel("Attribute Value", fontsize=18)
plt.legend(fontsize=16, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0)
plt.xticks(np.arange(len(months)), months, fontsize=16)
plt.yticks(fontsize=18)
plt.suptitle("Plot of Complex Attributes over months", fontsize=22)
plt.autoscale()

```



In [205]: plt.figure(figsize=(12, 6))

```

plt.subplot(2,1,1)
plt.title("Plot of Music Attributes over years", fontsize=16)

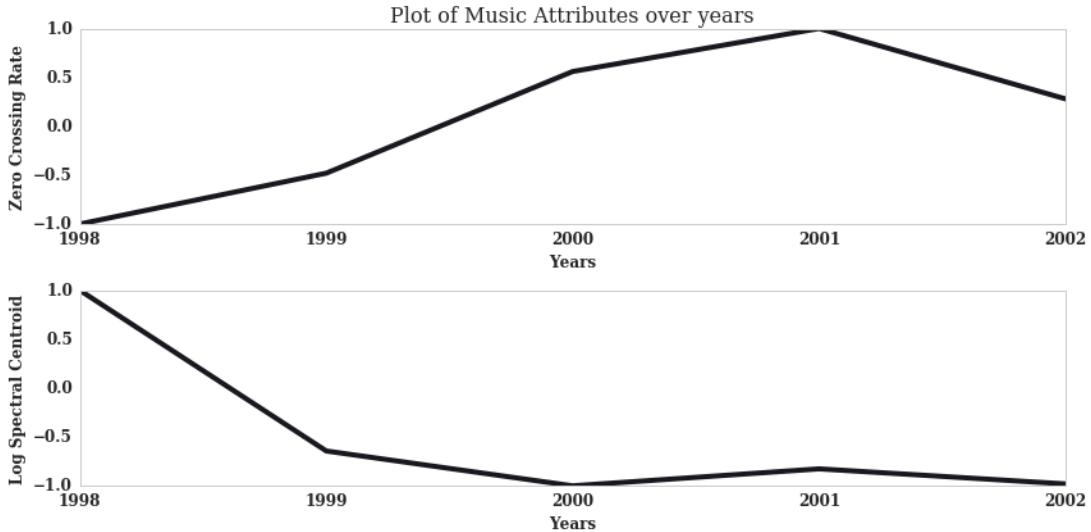
```

```

plt.plot(attvol_y.ZeroCrossRate)
plt.xlabel("Years", fontsize=12)
plt.ylabel("Zero Crossing Rate", fontsize=12)
plt.xticks(np.arange(len(years)), years, fontsize=12)
plt.yticks(fontsize=12)
plt.autoscale()

plt.subplot(2,1,2)
plt.plot(attvol_y.LogSpecCentroid)
plt.xlabel("Years", fontsize=12)
plt.ylabel("Log Spectral Centroid", fontsize=12)
plt.xticks(np.arange(len(years)), years, fontsize=12)
plt.yticks(fontsize=12)
plt.tight_layout()
plt.savefig('images/musicatt_yrs.png')

```



In [206]: plt.figure(figsize=(14, 6))

```

plt.subplot(2,1,1)
plt.title("Plot of Music Attributes over months", fontsize=16)
plt.plot(attvol_m.ZeroCrossRate.values)
plt.xlabel("Months", fontsize=12)
plt.ylabel("Zero Crossing Rate")
plt.xticks(np.arange(len(months)), months, rotation=90)
plt.yticks(fontsize=12)
plt.autoscale()

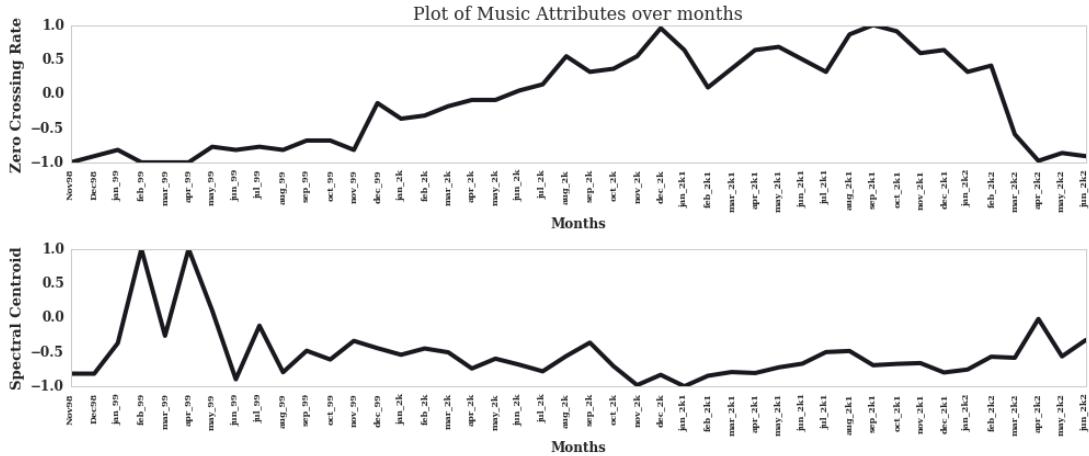
plt.subplot(2,1,2)
plt.plot(attvol_m.LogSpecCentroid.values)

```

```

plt.xlabel("Months", fontsize=12)
plt.ylabel("Spectral Centroid")
plt.xticks(np.arange(len(months)), months, rotation=90)
plt.yticks(fontsize=12)
plt.autoscale()
plt.tight_layout()
plt.savefig('images/musicatt_mth.png')

```

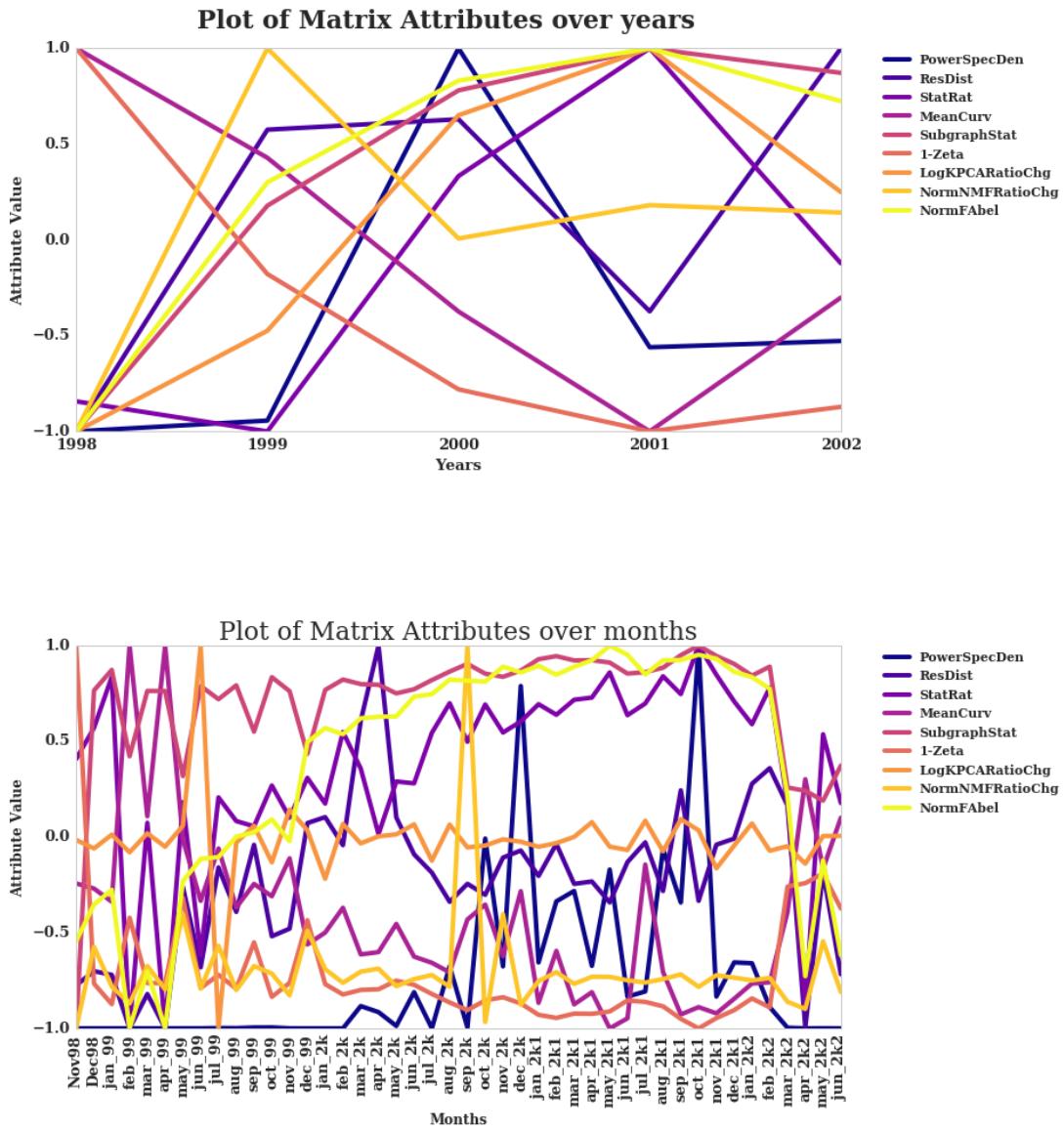


```

In [207]: attvol_y.drop(['ZeroCrossRate', 'LogSpecCentroid'], axis=1).iloc[:,19:].plot()
plt.xlabel("Years")
plt.ylabel("Attribute Value")
plt.legend(fontsize=11, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0)
plt.xticks(np.arange(len(years)), years, fontsize=13)
plt.yticks(fontsize=13)
plt.suptitle("Plot of Matrix Attributes over years", fontsize=22)
plt.autoscale()

attvol_m.drop(['ZeroCrossRate', 'LogSpecCentroid'], axis=1).iloc[:,19:].plot()
plt.xlabel("Months", fontsize=12)
plt.ylabel("Attribute Value", fontsize=12)
plt.legend(fontsize=11, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0)
plt.xticks(np.arange(len(months)), months, fontsize=13)
plt.yticks(fontsize=13)
plt.title("Plot of Matrix Attributes over months", fontsize=22)
plt.autoscale()

```



```
In [208]: all_cols = list(attvol_m.columns)
```

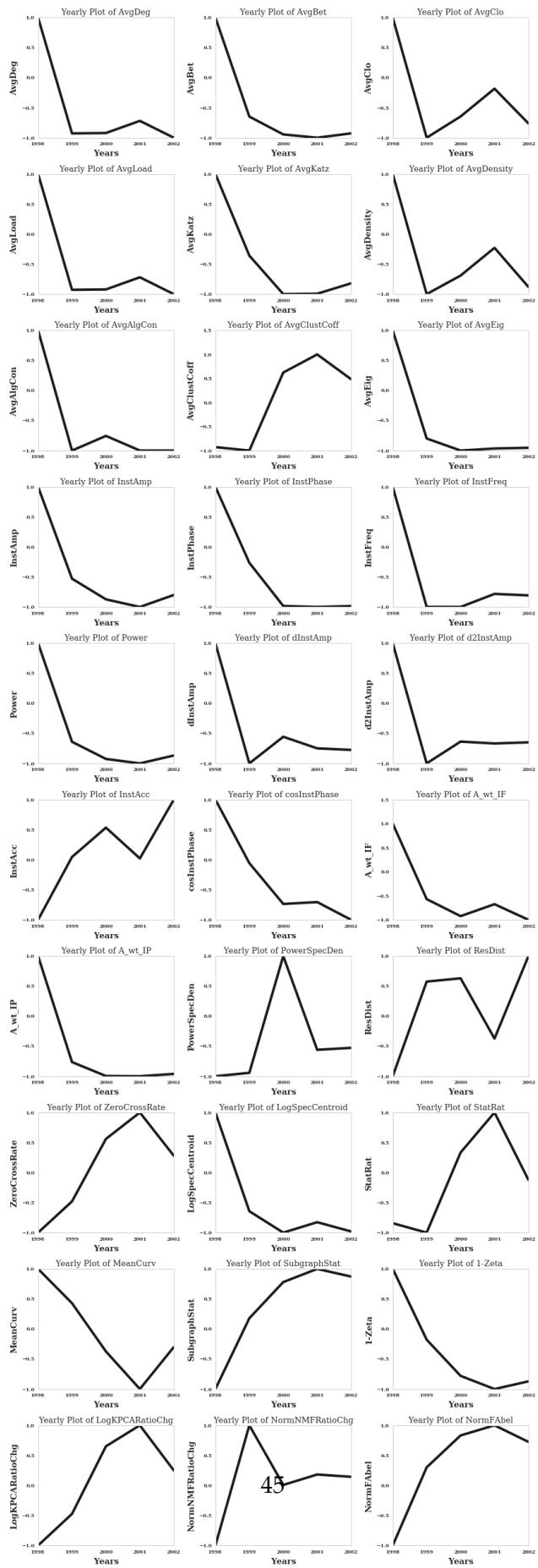
```
In [209]: attvol_y.shape
```

```
Out [209]: (5, 30)
```

```
In [210]: plt.figure(figsize=(12,38))
```

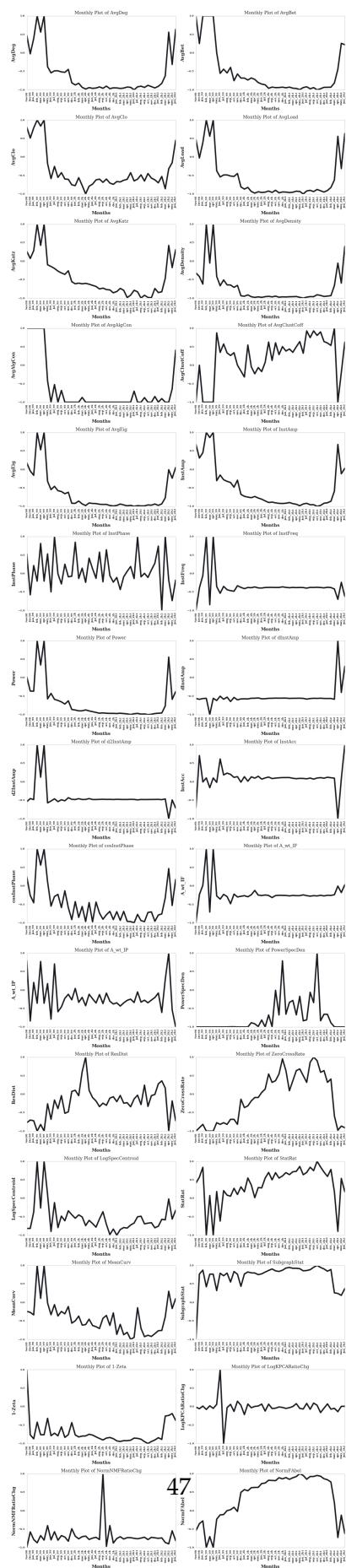
```
for i in range(len(all_cols)):
    plt.subplot(11,3,i+1)
    attvol_y.iloc[:,i].plot.line(use_index=True)
```

```
plt.xlabel("Years")
plt.ylabel(all_cols[i])
plt.xticks(np.arange(len(years)), years)
plt.title("Yearly Plot of " +all_cols[i], fontsize=13)
plt.tight_layout()
plt.savefig('images/avg_allatt_yrs.png')
```



```
In [211]: plt.figure(figsize=(14, 64))

for i in range(len(all_cols)):
    plt.subplot(15, 2, i+1)
    attvol_m.iloc[:,i].plot.line(rot=90)
    plt.xlabel("Months")
    plt.ylabel(all_cols[i])
    plt.xticks(np.arange(len(months)), months)
    plt.title("Monthly Plot of " +all_cols[i], fontsize=13)
    plt.tight_layout()
plt.savefig('images/avg_allatt_mth.png')
```



47

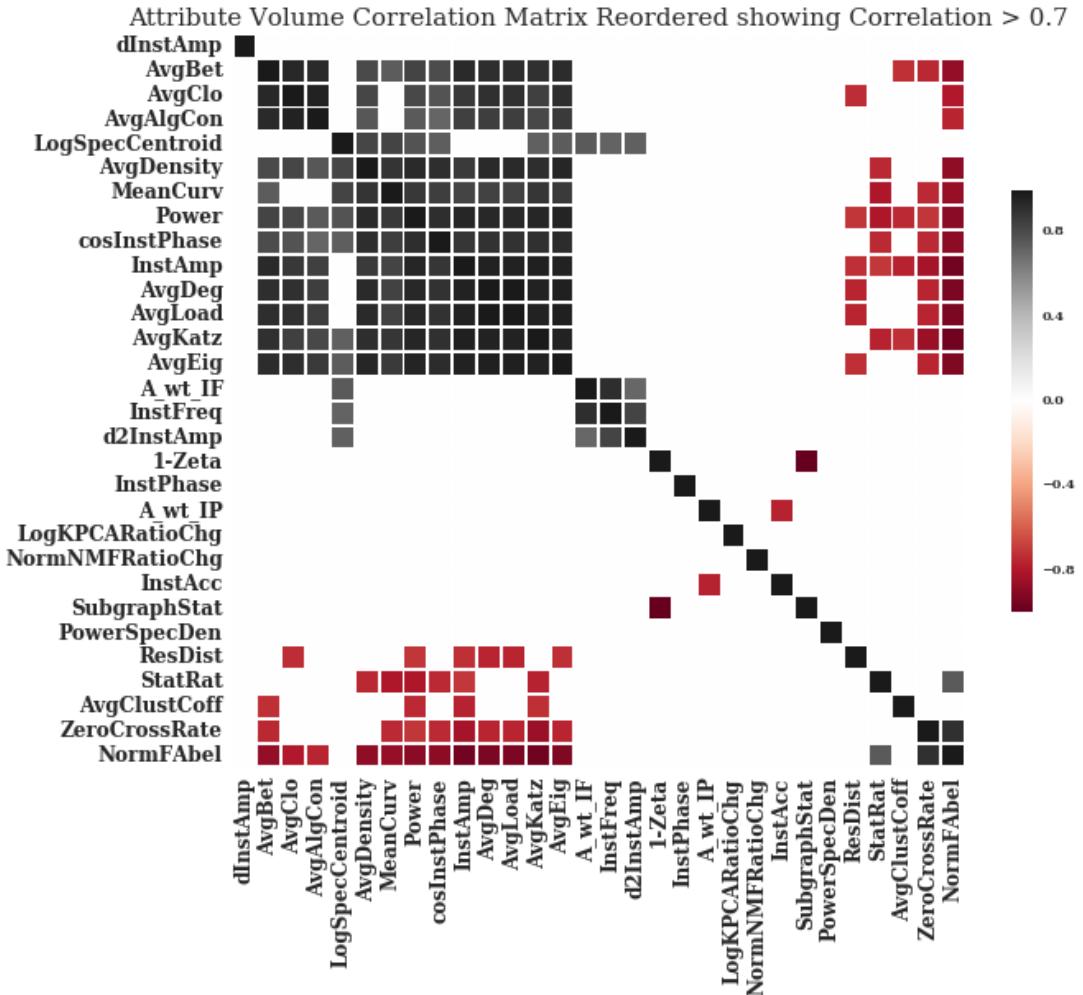
7 Correlation Analysis

```
In [212]: corr_m = attvol_m.corr()
g = sns.clustermap(corr_m, metric='chebyshev')
plt.close()
```

7.1 Correlation > 0.7

```
In [213]: threshold = 0.7
corr_m.values[np.where(np.abs(corr_m.values) < threshold)] = 0
```

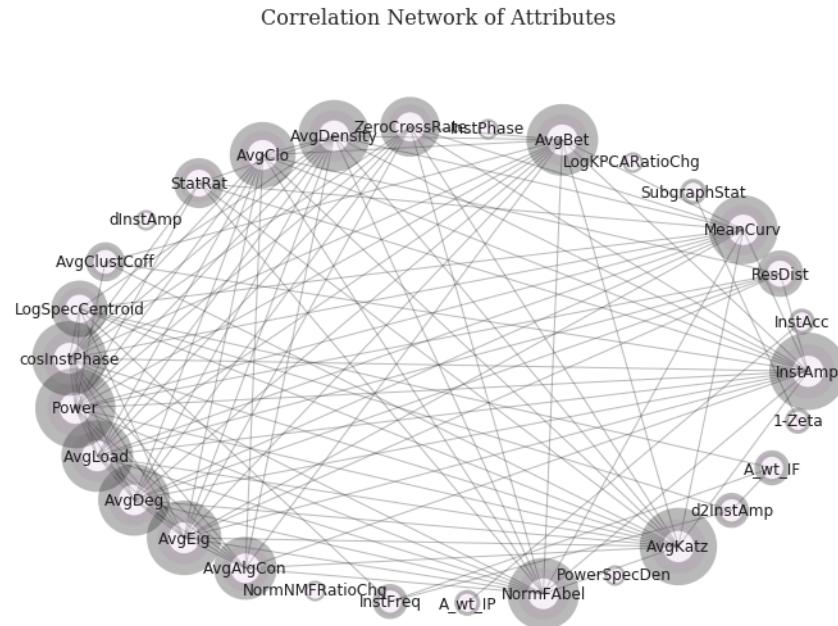
```
In [214]: plt.figure(figsize=(10,9))
sns.heatmap(corr_m.iloc[g.dendrogram_col.reordered_ind, g.dendrogram_row.
                      cmap='RdGy', robust=True, fmt='d', linewidths=1, square=True,
                      cbar_kws={'orientation':'vertical', 'shrink':0.5})
plt.title("Attribute Volume Correlation Matrix Reordered showing Correlat
plt.xticks(rotation=90, fontsize=12)
plt.yticks(rotation=360, fontsize=12)
plt.tight_layout()
plt.savefig('images/reordered_corrmat.png')
```



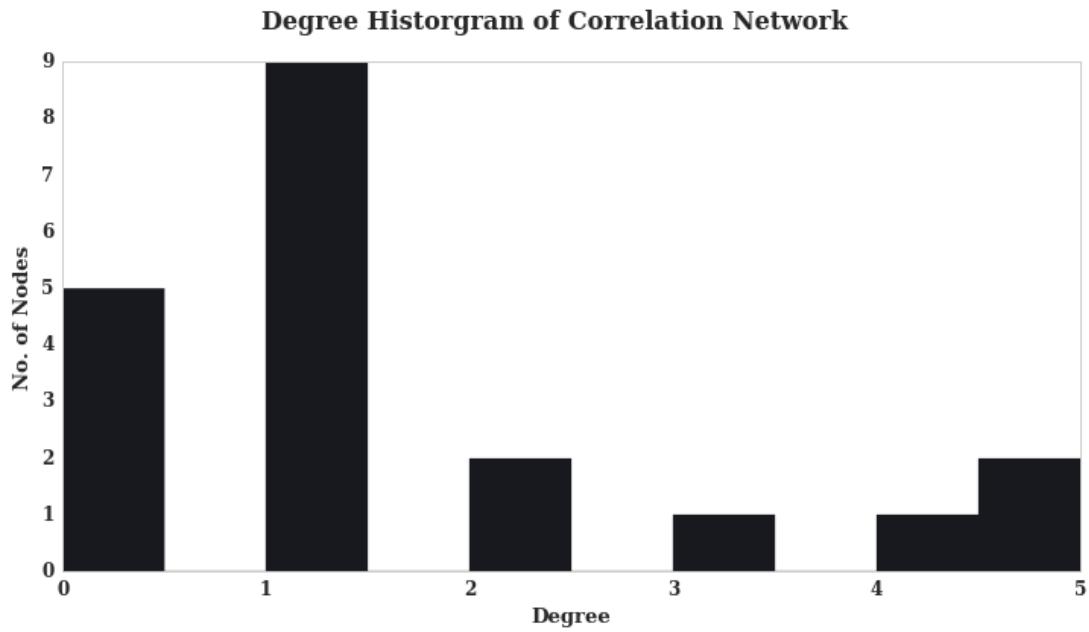
```
In [215]: names = corr_m.index.values
G_corr = nx.Graph(corr_m.values)
pos=nx.fruchterman_reingold_layout(G_corr, iterations=1000, k=200)
```

```
In [216]: #ref: https://github.com/traims/correlation-based-networks/blob/master/correlation_based_attributes.py
components = nx.connected_components(G_corr)
plt.figure(figsize=(12,8))
plt.title("Correlation Network of Attributes", fontsize=16)
plt.axis('off')
for i in components:
    component = G_corr.subgraph(i)
    nx.draw_networkx(component, with_labels = True, node_size = [component.nodes().size() * 1000], labels = dict([(x, names[x]) for x in component.nodes()]), pos=pos, edge_color='k', node_color='#E9CFEC', linewidths=[10], fontweight='bold', fontcolor='white', fontsize=11)
```

```
plt.tight_layout()  
plt.savefig('images/corrnet.png')
```



```
In [217]: plt.hist(nx.degree_histogram(G_corr))  
plt.suptitle("Degree Histogram of Correlation Network", fontsize=16)  
plt.xticks(fontsize=12)  
plt.yticks(fontsize=12)  
plt.xlabel("Degree")  
plt.ylabel("No. of Nodes")  
plt.savefig('images/corrnet_deghist.png')
```



8 Regression Analysis for Feature Ranking

```
In [218]: from sklearn.metrics import mean_squared_error
         from sklearn import ensemble
```

```
In [219]: def RMS(x, axis=None):
    rms = np.sqrt(np.mean(np.square(x), axis=axis))
    return rms
```

```
In [220]: X= attvol_m.drop(['AvgDeg'],axis=1)
          y = attvol_m.AvgDeg
```

```
In [221]: X[:5]
```

```
Out[221]:      AvgBet      AvgClo      AvgLoad      AvgKatz      AvgDensity      AvgAlgCon      \
0   1.000000   0.810523   0.475524   0.267944   -0.318532      1.0
1   0.250903   0.509903  -0.027972   0.069435   -0.419219      1.0
2   1.000000   0.785809   0.388112   0.282044   -0.622808      1.0
3   1.000000   1.000000   1.000000   1.000000   1.000000      1.0
4   1.000000   0.835787   0.559441   0.423789   -0.054825      1.0

      AvgClustCoff      AvgEig      InstAmp      InstPhase      ...
0     -1.000000   0.177677   0.679834   0.535455      ...
1     -0.000165  -0.044277   0.305093  -0.578269      ...
2     -1.000000  -0.161680   0.450396   0.214700      ...
3     -1.000000   1.000000   1.000000  -0.207114      ...      ResDist      \
0     -1.000000   0.177677   0.679834   0.535455      ...
1     -0.000165  -0.044277   0.305093  -0.578269      ...
2     -1.000000  -0.161680   0.450396   0.214700      ...
3     -1.000000   1.000000   1.000000  -0.207114      ...      -0.774637
4     -1.000000   0.835787   0.559441   0.423789      ...      -0.701077
5     -1.000000   0.509903  -0.027972   0.069435      ...      -0.720470
6     -1.000000   0.785809   0.388112   0.282044      ...      -1.000000
```

```

4      -1.000000  0.528792  0.860240   0.812100    ...   -0.820587

      ZeroCrossRate  LogSpecCentroid  StatRat  MeanCurv  SubgraphStat  1-
0      -1.000000     -0.815713  0.407864 -0.242635     -1.000000  1.00
1      -0.909091     -0.816057  0.570795 -0.269438      0.766117 -0.76
2      -0.818182     -0.371199  0.836955 -0.337965      0.873376 -0.87
3      -1.000000      1.000000 -1.000000  1.000000      0.421859 -0.42
4      -1.000000     -0.264125  0.074177  0.108395      0.763599 -0.76

      LogKPCARatioChg  NormNMFRatioChg  NormFAbel
0      -0.015620     -1.000000     -0.543745
1      -0.059551     -0.573875     -0.352246
2      0.012093     -0.780922     -0.276008
3      -0.079774     -0.869723     -1.000000
4      0.019315     -0.673657     -0.702049

[5 rows x 29 columns]

In [222]: X = X.astype(np.float32)
          offset = int(X.shape[0] * 0.5)
          X_train, y_train = X[:offset], y[:offset]
          X_test, y_test = X[offset:], y[offset:]

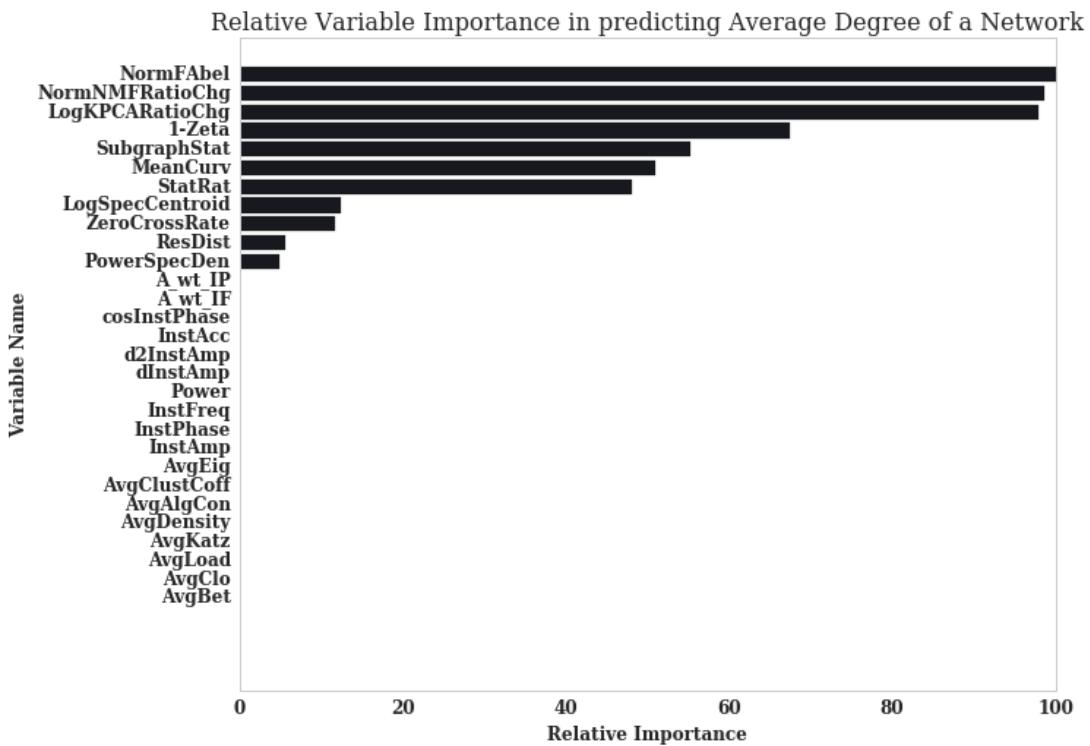
In [223]: params = {'n_estimators': 10, 'max_depth': 10, 'min_samples_split': 10,
               'learning_rate': 0.1, 'loss': 'ls'}
          clf = ensemble.GradientBoostingRegressor(**params)
          clf.fit(X_train, y_train)

Out[223]: GradientBoostingRegressor(alpha=0.9, init=None, learning_rate=0.1, loss='ls',
                                     max_depth=10, max_features=None, max_leaf_nodes=None,
                                     min_samples_leaf=1, min_samples_split=10,
                                     min_weight_fraction_leaf=0.0, n_estimators=10, presort='auto',
                                     random_state=None, subsample=1.0, verbose=0, warm_start=False)

In [224]: feature_importance = clf.feature_importances_

          # make importances relative to max importance
          plt.figure(figsize=(10,8))
          feature_importance = 100.0 * (feature_importance / feature_importance.max())
          sorted_idx = np.argsort(feature_importance)
          feature_names = list(X)
          pos = np.arange(sorted_idx.shape[0])
          plt.barh(pos, feature_importance[sorted_idx], align='center')
          plt.yticks(pos, feature_names, fontsize=12)
          plt.ylabel("Variable Name", fontsize=12)
          plt.xlabel('Relative Importance', fontsize=12)
          plt.title('Relative Variable Importance in predicting Average Degree of a Node')
          plt.xticks(fontsize=12)
          plt.savefig('images/feature_ranking.png')

```

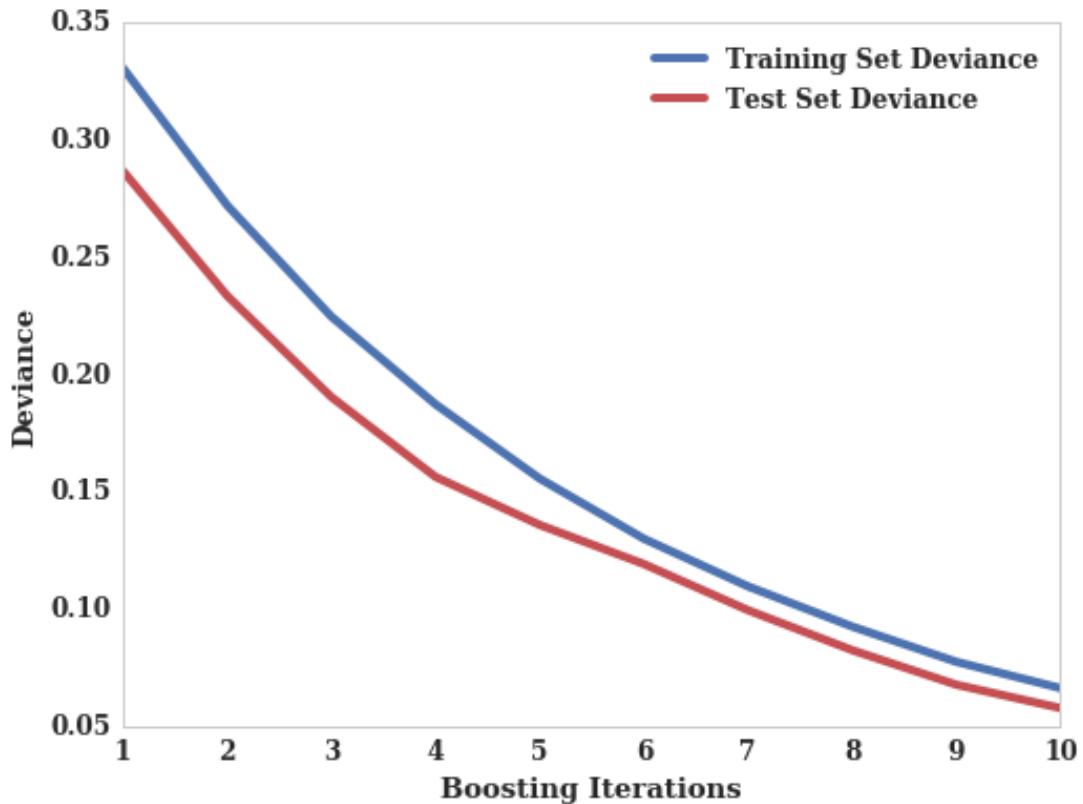


```
In [225]: test_score = np.zeros((params['n_estimators']),), dtype=np.float64)

for i, y_pred in enumerate(clf.staged_predict(X_test)):
    test_score[i] = clf.loss_(y_test, y_pred)

plt.figure(figsize=(8, 6))
plt.suptitle('Deviance of Gradient Boosting Regression Model', fontsize=16)
plt.plot(np.arange(params['n_estimators']) + 1, clf.train_score_, 'b-',
         label='Training Set Deviance')
plt.plot(np.arange(params['n_estimators']) + 1, test_score, 'r-',
         label='Test Set Deviance')
plt.legend(loc=1, fontsize=12)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)
plt.xlabel('Boosting Iterations')
plt.ylabel('Deviance')
plt.savefig('images/reg_deviance.png')
```

Deviance of Gradient Boosting Regression Model

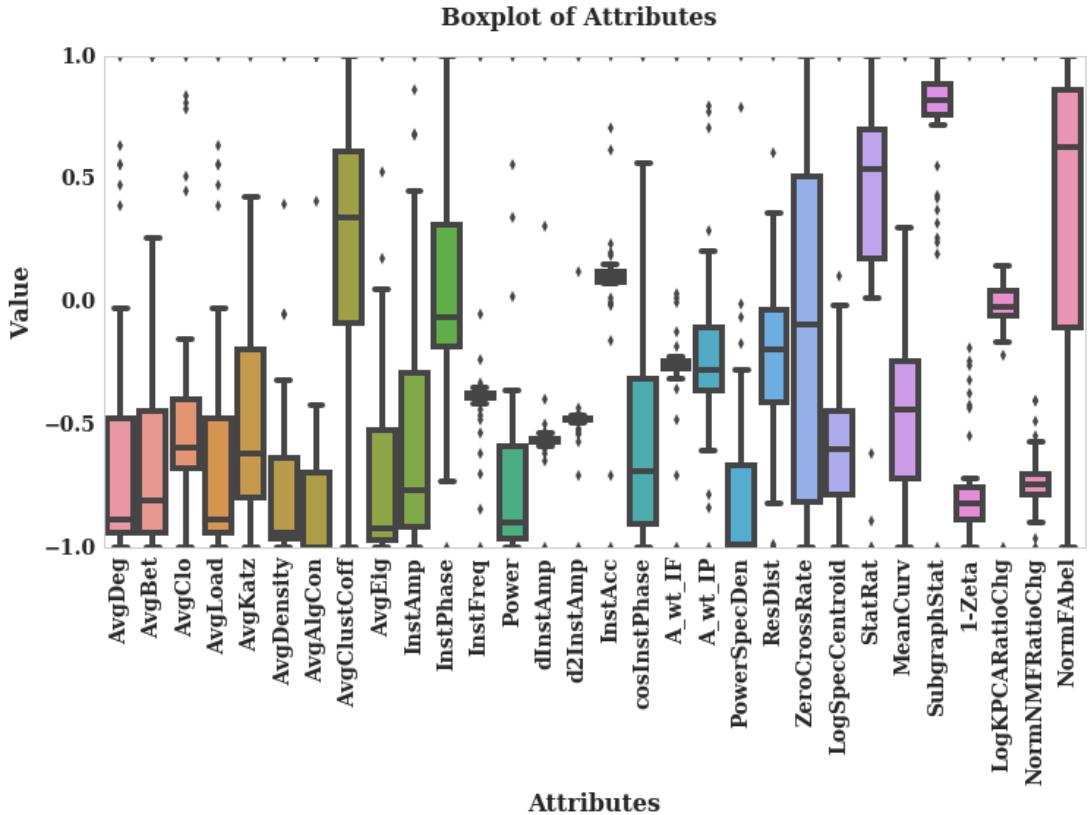


```
In [226]: mse = mean_squared_error(y_test, clf.predict(X_test))
          print("Gradient Boosting MSE: ", mse)
```

Gradient Boosting MSE: 0.0580291578899

```
In [227]: #attvol_m.plot.box(rot=90, fontsize=16, title='Boxplot of Attributes')
          plt.figure(figsize=(12, 6))
          plt.suptitle("Boxplot of Attributes", fontsize=16)
          sns.boxplot(attvol_m)
          plt.xlabel('Attributes', fontsize=16)
          plt.ylabel('Value', fontsize=16)
          plt.xticks(fontsize=14, rotation=90)
          plt.yticks(fontsize=14)
          plt.savefig('images/attvol_boxplot.png')
```

```
/home/arshad/anaconda3/lib/python3.5/site-packages/seaborn/categorical.py:2171: UserWarning
  warnings.warn(msg, UserWarning)
```



9 Aggregation Measures

```
In [228]: persistence_m = attvol_m.mean(axis=1)/(attvol_m.shape[0]-1)
persistence_y = attvol_y.mean(axis=1)/(attvol_y.shape[0]-1)
```

```
In [229]: def emergence(per):
    tmp = np.asarray(per)
    emerg = []
    for i in range(len(tmp)-1):
        x= int(i)
        y = x +1
        #print(tmp[y], tmp[x])
        if tmp[y]==tmp[x]:
            emerg.append(0)
        elif tmp[y] < (0) or tmp[x] < 0:
            res = (tmp[y]-tmp[x]) / (abs(tmp[y])+abs(tmp[x]))
            emerg.append(res)
        else:
            res = (tmp[y]-tmp[x]) /max([tmp[y],tmp[x]])
            emerg.append(res)
```

```

tmp2 = np.zeros(len(emerg)+1)
tmp2[1:] = emerg

return tmp2

def NRMS(n):
    nrms = []
    for i in range(len(n)-1):
        x= int(i)
        y = x +1
        a = n[x]
        b = n[y]
        nrms_ = np.divide( (RMS(a-b)) , (RMS(a)+RMS(b)) )
        nrms.append(nrms_)
    tmp2 = np.zeros(len(nrms)+1)
    tmp2[1:] = nrms

    return tmp2

In [230]: rms_m = attvol_m.apply(lambda x: RMS(x), axis=1)
nrms_m = NRMS(rms_m)
emerg_m = emergence(persistence_m)

In [231]: emerg_y = emergence(persistence_y)

In [232]: plt.figure(figsize=(12,8))

plt.subplot(3,1,1)
plt.plot(emerg_m,'r', label='Emergence')
plt.xticks(np.arange(len(months)), months, fontsize=12, rotation=90)
plt.yticks(fontsize=12)
plt.xlabel('Months', fontsize=16)
plt.legend(fontsize=12, loc=1)
plt.tight_layout()
plt.title("Comparison of Aggregation Schemes", fontsize=22)
plt.autoscale()

plt.subplot(3,1,2)
plt.plot(rms_m,'g',label='RMS of Attributes',)
plt.yticks(fontsize=12)
plt.xticks(np.arange(len(months)), months, fontsize=12, rotation=90)
plt.xlabel('Months', fontsize=16)
plt.legend(fontsize=12, loc=1)
plt.tight_layout()
plt.autoscale()

plt.subplot(3,1,3)

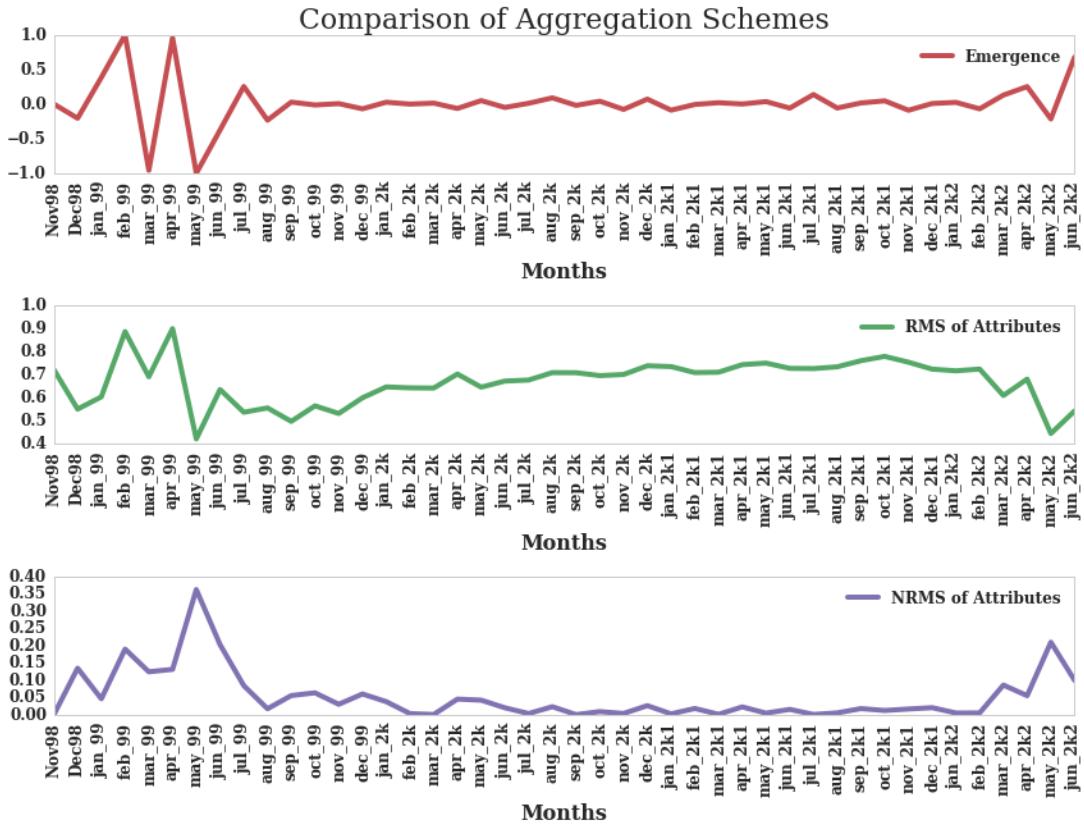
```

```

plt.plot(nrms_m, 'm', label='NRMS of Attributes')

plt.xticks(np.arange(len(months)), months, fontsize=12, rotation=90)
plt.yticks(fontsize=12)
plt.xlabel('Months', fontsize=16)
plt.legend(fontsize=12, loc=1)
plt.autoscale()
plt.savefig('images/agg_comp.png')

```



```
In [233]: final_attvol_m = attvol_m.copy()
```

```

final_attvol_m['NRMS'] = nrms_m
final_attvol_m['RMS'] = rms_m
final_attvol_m['Emergence'] = emerg_m

```

10 MDS and TSNE

```
In [234]: from sklearn.manifold import *
```

```
In [239]: mds = MDS(n_components=2, metric=False, random_state=0)
Y = mds.fit_transform(final_attvol_m)
```

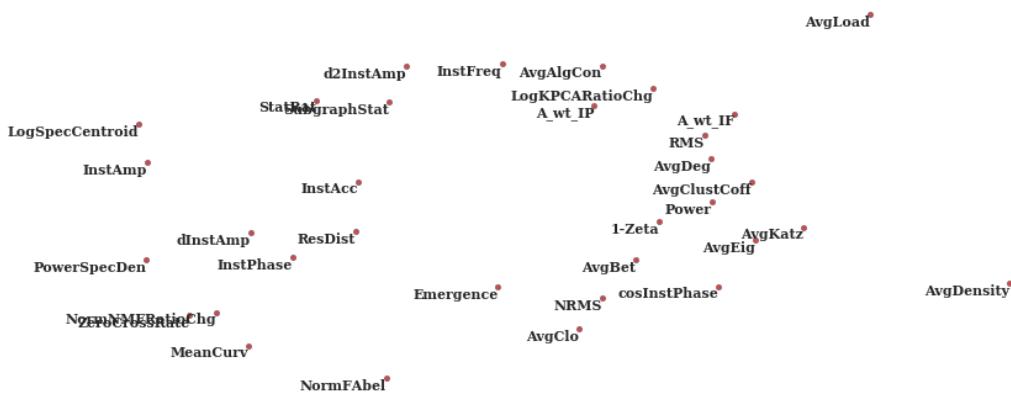
```

plt.figure(figsize=(12, 6))
plt.axis('off')
plt.suptitle("Non-Metric Multi Dimensional Scaling of Attribute Volume",
for i, txt in enumerate(final_attvol_m.columns):
    plt.scatter(Y[:, 1][i], Y[:, 0][i], c='r')
    plt.annotate(txt, (Y[:, 1][i], Y[:, 0][i])), horizontalalignment='right'

plt.autoscale()
plt.tight_layout()
plt.savefig('images/mds_plot.png')

```

Non-Metric Multi Dimensional Scaling of Attribute Volume



In [244]: tsne = TSNE(metric='euclidean', random_state=0)

```

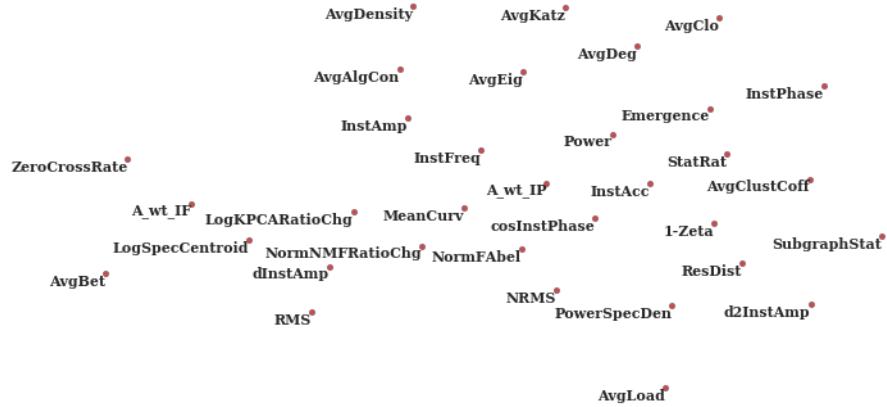
y_tsne = tsne.fit_transform(final_attvol_m)

plt.figure(figsize=(12, 6))
plt.axis('off')
plt.suptitle("TSNE Plot of Attribute Volume with Euclidean Distance", for
for i, txt in enumerate(final_attvol_m.columns):
    plt.scatter(y_tsne[:, 1][i], y_tsne[:, 0][i], c='r')
    plt.annotate(txt, (y_tsne[:, 1][i], y_tsne[:, 0][i])), horizontalalignm

plt.autoscale()
plt.tight_layout()
plt.savefig('images/tsne_plot_euc.png')

```

TSNE Plot of Attribute Volume with Euclidean Distance



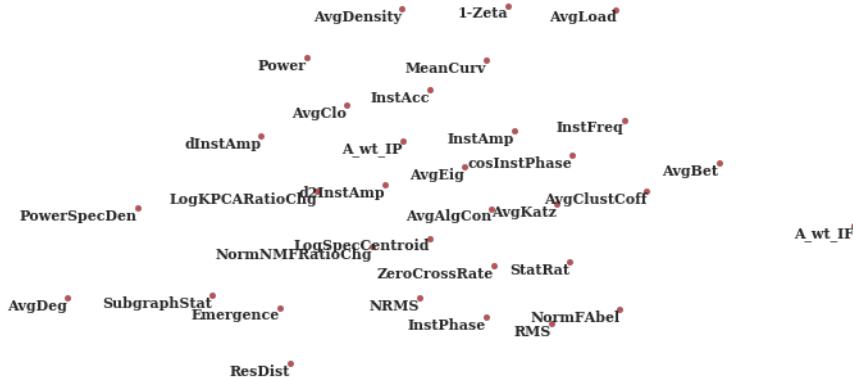
```
In [243]: tsne = TSNE(metric='canberra', random_state=0)

y_tsne = tsne.fit_transform(final_attvol_m)

plt.figure(figsize=(12, 6))
plt.axis('off')
plt.suptitle("TSNE Plot of Attribute Volume with Canberra Distance", fontweight='bold')
for i, txt in enumerate(final_attvol_m.columns):
    plt.scatter(y_tsne[:, 1][i], y_tsne[:, 0][i], c='r')
    plt.annotate(txt, (y_tsne[:, 1][i], y_tsne[:, 0][i]), horizontalalignment='left', verticalalignment='bottom', fontweight='bold', color='black', size=10)
    plt.annotate(txt, (y_tsne[:, 1][i], y_tsne[:, 0][i]), horizontalalignment='right', verticalalignment='top', fontweight='bold', color='red', size=8)

plt.autoscale()
plt.tight_layout()
plt.savefig('images/tsne_plot.png')
```

TSNE Plot of Attribute Volume with Canberra Distance



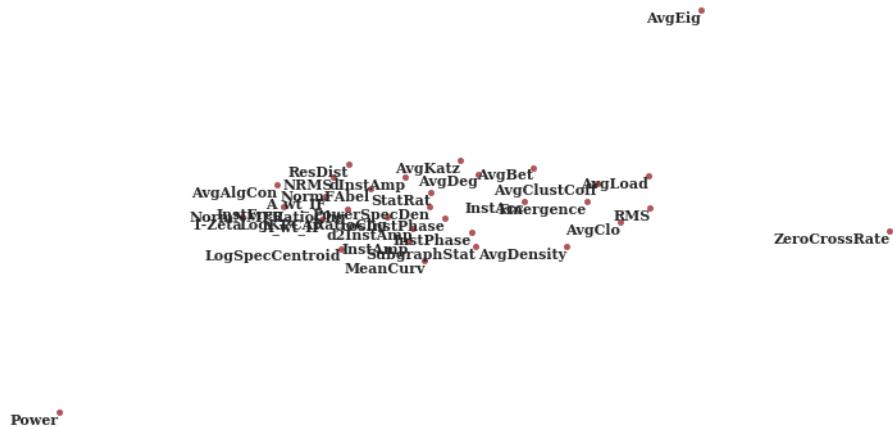
```
In [242]: tsne = TSNE(metric='correlation', random_state=0)

y_tsne = tsne.fit_transform(final_attvol_m)

plt.figure(figsize=(12, 6))
plt.axis('off')
plt.suptitle("TSNE Plot of Attribute Volume with Correlation Distance", f
for i, txt in enumerate(final_attvol_m.columns):
    plt.scatter(y_tsne[:, 1][i], y_tsne[:, 0][i], c='r')
    plt.annotate(txt, (y_tsne[:, 1][i], y_tsne[:, 0][i]), horizontalalignn

plt.autoscale()
plt.tight_layout()
plt.savefig('images/tsne_plot_corr.png')
```

TSNE Plot of Attribute Volume with Correlation Distance



11 FK and Radon Plot

```
In [85]: def radon(m, name):
    from skimage.transform import radon
    theta = np.linspace(0., 180., max(m.shape), endpoint=False)
    sinogram = radon(m, theta=theta, circle=True)
    return sinogram

In [86]: def fk_plot(f, name):
    freq = sc.fft(f)
    wavnum = 1/freq

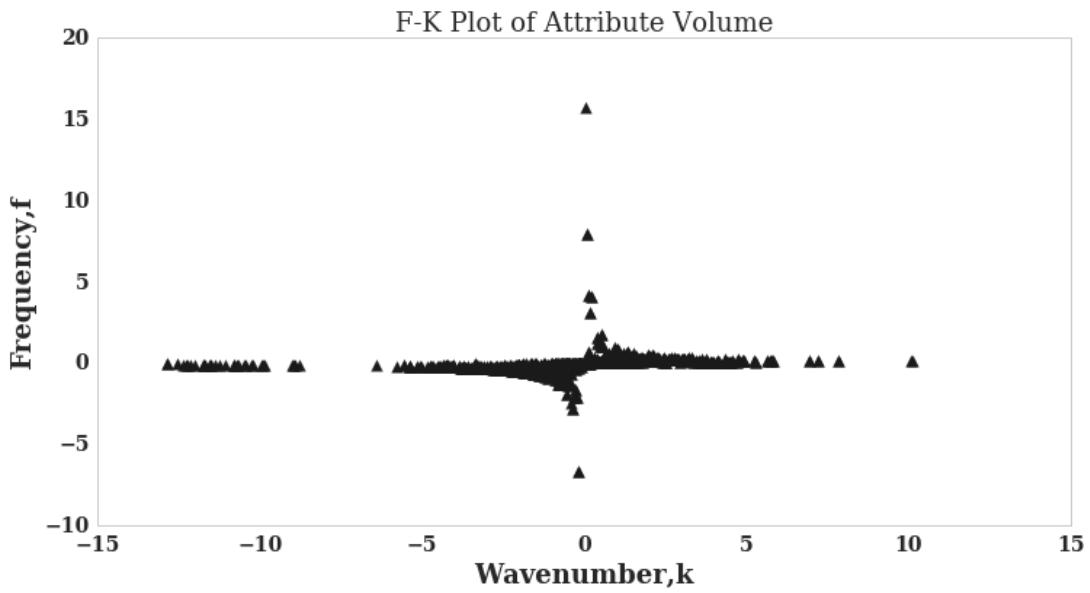
    plt.scatter(freq,wavnum, s=60, marker='^', c='k')
    plt.title("F-K Plot of "+str(name), fontsize=18)
    plt.xticks(fontsize=14)
    plt.yticks(fontsize=14)
    plt.xlabel("Wavenumber,k", fontsize=18)
    plt.ylabel("Frequency,f", fontsize=18)

    plt.show()

    return [freq, wavnum]

In [87]: f,k, = fk_plot(final_attvol_m,'Attribute Volume')
plt.savefig('images/fkplot.png')

/home/arshad/anaconda3/lib/python3.5/site-packages/numpy/core/numeric.py:533: Compl
return array(a, dtype, copy=False, order=order, subok=True)
```



```
<matplotlib.figure.Figure at 0x7fcacf190a0b8>
```

```
In [88]: sgram = radon(final_attvol_m.values, 'Attribute Volume')

/home/arshad/anaconda3/lib/python3.5/site-packages/skimage/transform/radon_transform
warn('Radon transform: image must be zero outside the '


In [102]: plt.figure(figsize=(12, 8))

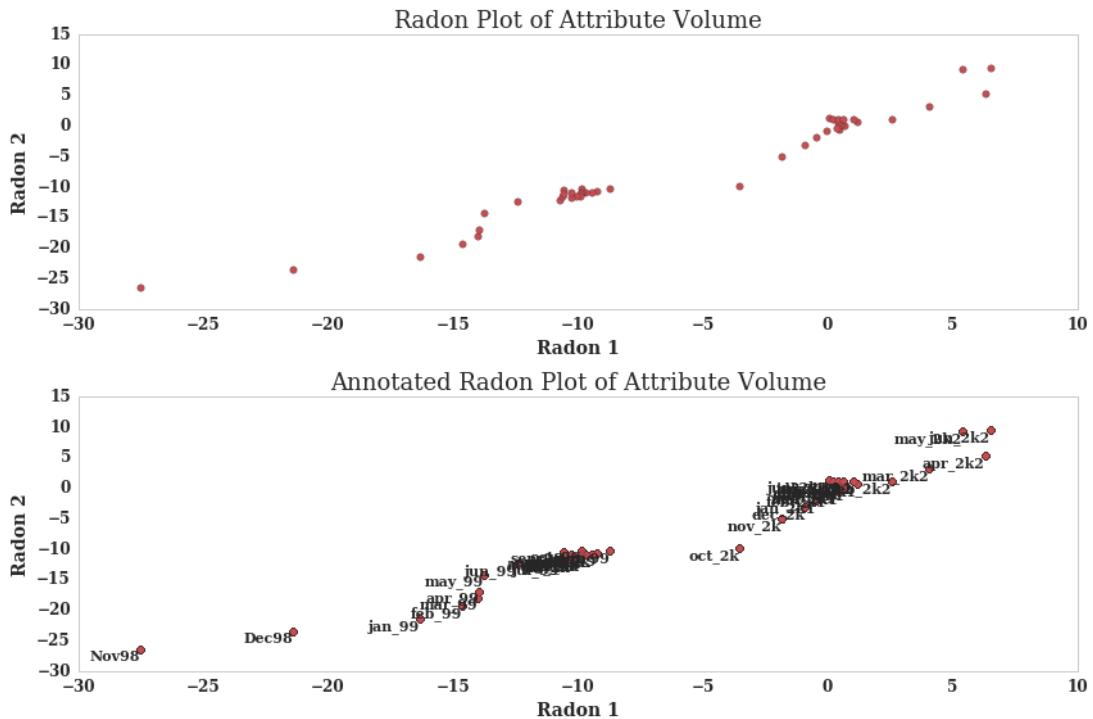
    plt.subplot(211)
    plt.scatter(sgram[0], sgram[1], c='r', s=30)
    plt.xticks(fontsize=12)
    plt.yticks(fontsize=12)
    plt.xlabel("Radon 1", fontsize=14)
    plt.ylabel("Radon 2", fontsize=14)
    plt.title("Radon Plot of Attribute Volume", fontsize=18)
    plt.tight_layout()

    plt.subplot(212)
    plt.title("Annotated Radon Plot of Attribute Volume", fontsize=18)
    for i, txt in enumerate(months):
        plt.scatter(sgram[0], sgram[1], c='r', s=30)
        plt.annotate(txt, (sgram[0][i], sgram[1][i]), horizontalalignment='right')
    plt.xticks(fontsize=12)
    plt.yticks(fontsize=12)
    plt.xlabel("Radon 1", fontsize=14)
```

```

plt.ylabel("Radon 2", fontsize=14)
plt.tight_layout()
plt.savefig('images/radonplot.png')

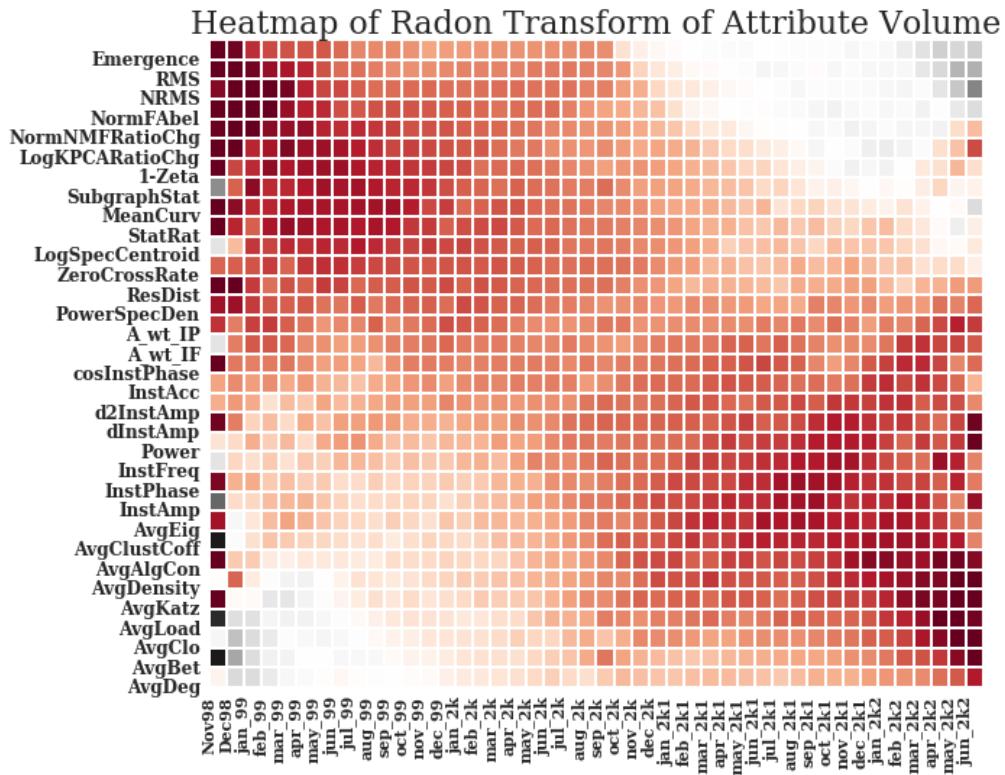
```



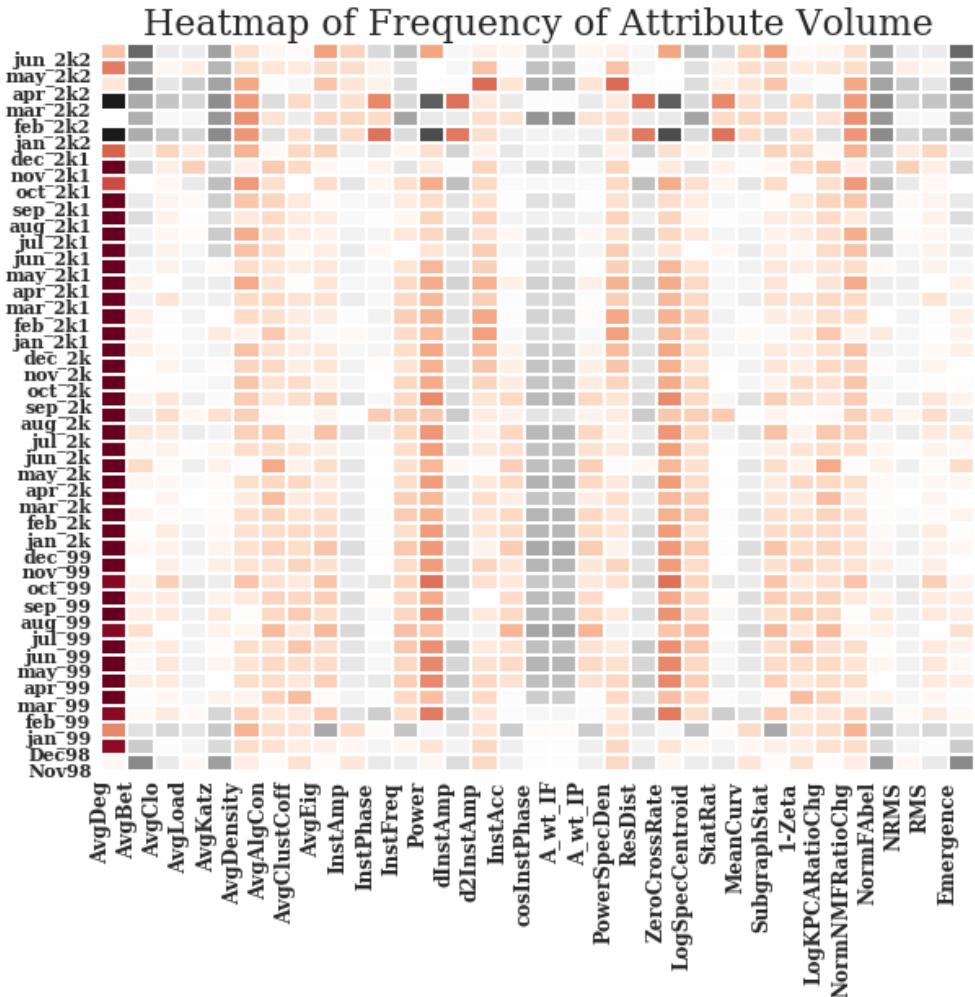
```

In [103]: plt.figure(figsize=(12, 8))
sns.heatmap(sgram, cmap='RdGy', robust=True, cbar_kws={'orientation': 'vertical'})
plt.xticks(np.arange(len(months)), months, fontsize=11, rotation=90);
plt.yticks(np.arange(len(final_attvol_m.columns))), final_attvol_m.columns
plt.title("Heatmap of Radon Transform of Attribute Volume", fontsize=22)
plt.savefig('images/radon_heat.png')

```

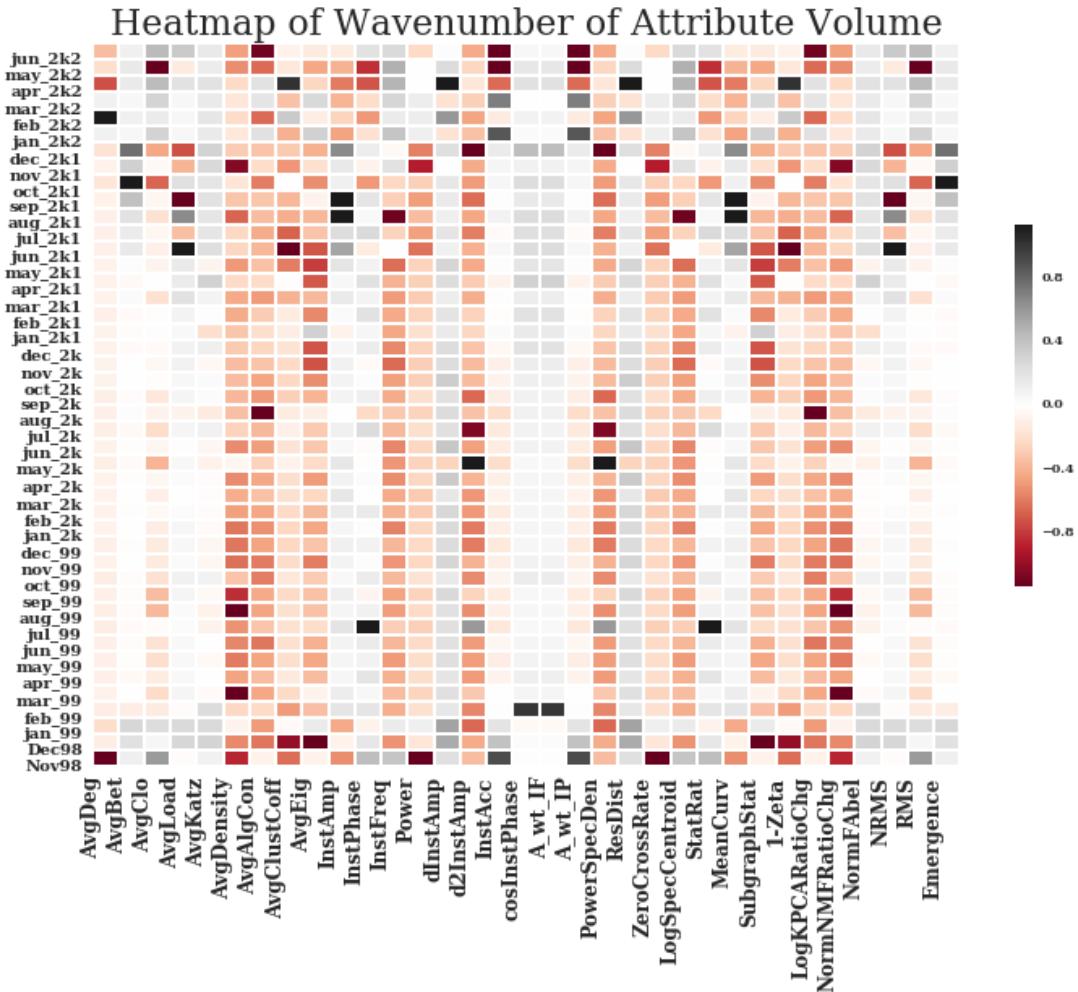


```
In [104]: plt.figure(figsize=(12, 8))
sns.heatmap(np.real(f), cmap='RdGy', robust=True, cbar_kws={'orientation':
plt.yticks(np.arange(len(months)), months, fontsize=11, rotation=360);
plt.xticks(np.arange(len(final_attvol_m.columns)), final_attvol_m.columns)
plt.title("Heatmap of Frequency of Attribute Volume", fontsize=22)
plt.savefig('images/freq_heat.png')
```



```
In [105]: plt.figure(figsize=(12,8))
sns.heatmap(k, cmap='RdGy', robust=True, cbar_kws={'orientation':'vertical'}
plt.yticks(np.arange(len(months)), months, fontsize=10, rotation=360);
plt.xticks(np.arange(len(final_attvol_m.columns)), final_attvol_m.columns)
plt.title("Heatmap of Wavenumber of Attribute Volume", fontsize=22)
plt.savefig('images/wavnum_heat.png')
```

```
/home/arshad/anaconda3/lib/python3.5/site-packages/numpy/core.py:3095: ComplexWarning:
output = self._data.astype(newtype).view(type(self))
```



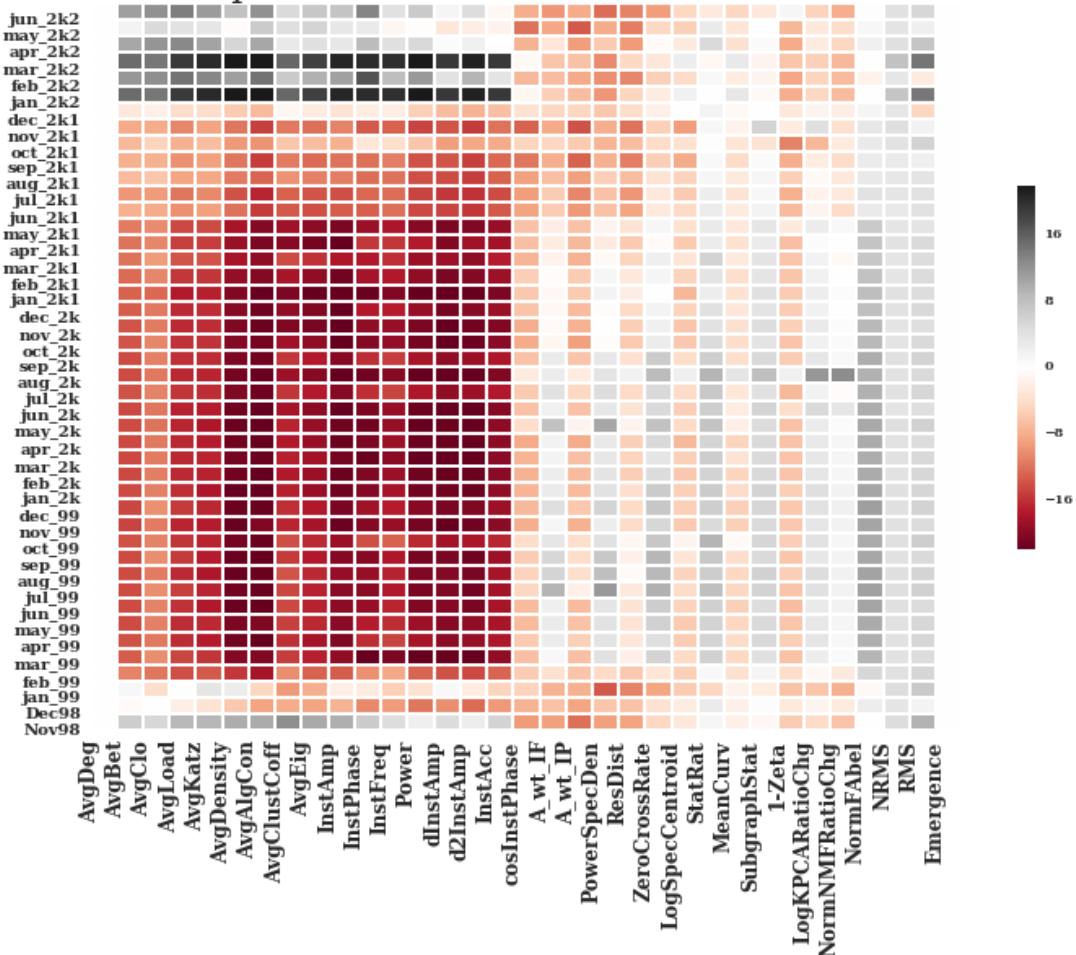
12 Exotic Transforms

```
In [106]: fabel = abel.Transform(final_attvol_m.values, direction='forward', method=
```

```
In [107]: plt.figure(figsize=(12,8))
sns.heatmap(fabel, cmap='RdGy', robust=True, cbar_kws={'orientation': 'vertical'})
plt.yticks(np.arange(len(months)), months, fontsize=10, rotation=360);
plt.xticks(np.arange(len(final_attvol_m.columns)), final_attvol_m.columns)
plt.title("Heatmap of Abel Transform of Attribute Volume", fontsize=22)
```

```
Out[107]: <matplotlib.text.Text at 0x7fcacf1538c18>
```

Heatmap of Abel Transform of Attribute Volume



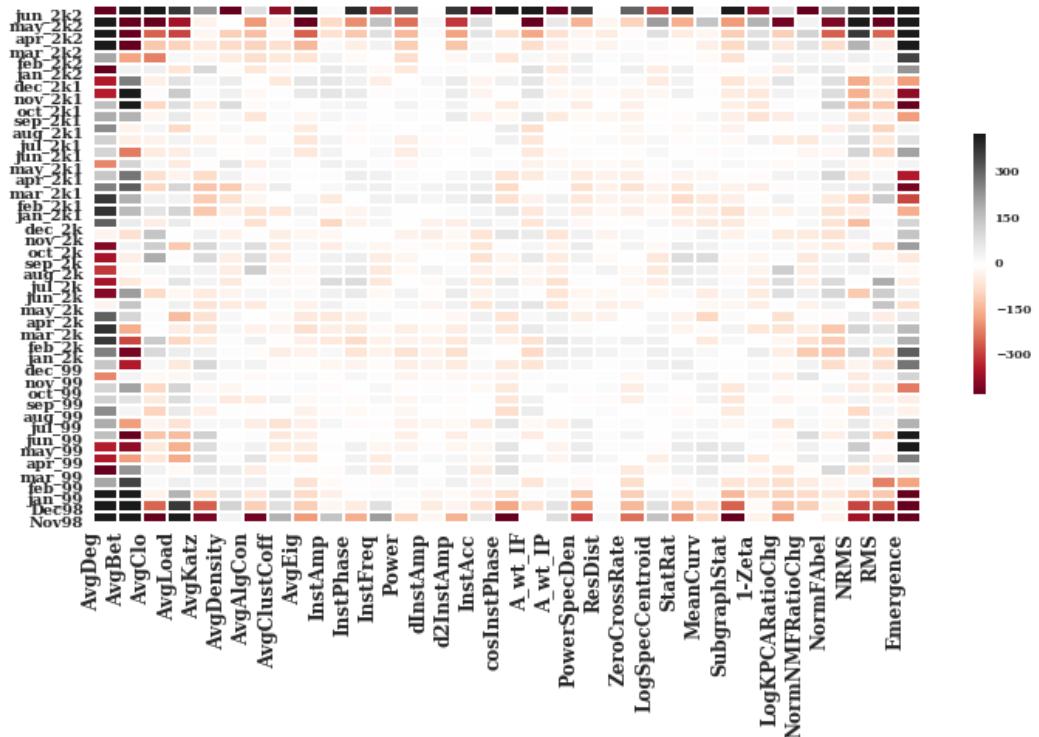
```
In [108]: H = sc.fftpack.fft2(fabel)
```

```
In [109]: sns.heatmap(H, cmap='RdGy', robust=True, cbar_kws={'orientation': 'vertical'}
plt.yticks(np.arange(len(months)), months, fontsize=10, rotation=360);
plt.xticks(np.arange(len(final_attvol_m.columns)), final_attvol_m.columns)
plt.suptitle("Heatmap of Hankel Transform of Attribute Volume", fontsize=10)
```

```
/home/arshad/anaconda3/lib/python3.5/site-packages/numpy/ma/core.py:3095: ComplexWarning
  output = self._data.astype(newtype).view(type(self))
```

```
Out[109]: <matplotlib.text.Text at 0x7fcf05830f0>
```

Heatmap of Hankel Transform of Attribute Volume



13 Which nodes are common to all years?

```
In [110]: int_0 = np.intersect1d(Gt0.nodes(), Gt1.nodes())
int_1 = np.intersect1d(int_0, Gt2.nodes())
int_2 = np.intersect1d(int_1, Gt3.nodes())
int_all = np.intersect1d(int_2, Gt4.nodes())
int_all
```

```
Out[110]: array([ 38, 110, 123, 155, 169])
```

```
In [111]: def get_cent(net):
    degC = nx.degree_centrality(net)
    cloC = nx.closeness_centrality(net)
    betC = nx.betweenness_centrality(net)
    eigC = nx.eigenvector_centrality_numpy(net)
    katzC = nx.katz_centrality_numpy(net)
    loadC = nx.load_centrality(net)

    return [degC,cloC,betC,eigC,katzC, loadC]
```

```
In [112]: degC0, cloC0, betC0, eigC0, katzC0, loadC0 = get_cent(Gt0)
degC1, cloC1, betC1, eigC1, katzC1, loadC1 = get_cent(Gt1)
```

```

degC2, cloC2, betC2, eigC2, katzC2, loadC2 = get_cent(Gt2)
degC3, cloC3, betC3, eigC3, katzC3, loadC3 = get_cent(Gt3)
degC4, cloC4, betC4, eigC4, katzC4, loadC4 = get_cent(Gt4)

```

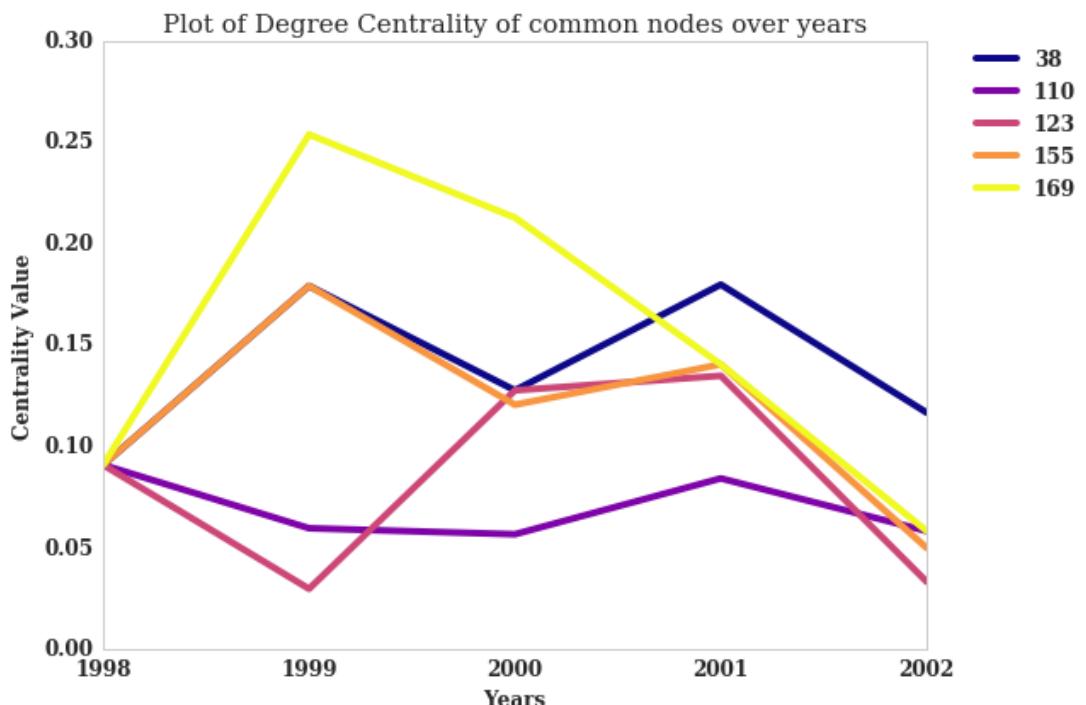
```

In [113]: node_deg = pd.DataFrame.from_dict(degC0, orient='index').\
join(pd.DataFrame.from_dict(degC1, orient='index'), rsuffix = 'Gt1', lsuffix='Gt0')
join(pd.DataFrame.from_dict(degC2, orient='index'), rsuffix = 'Gt2', lsuffix='Gt1')
join(pd.DataFrame.from_dict(degC3, orient='index'), rsuffix = 'Gt3', lsuffix='Gt2')
join(pd.DataFrame.from_dict(degC4, orient='index'), rsuffix = 'Gt4', lsuffix='Gt3')

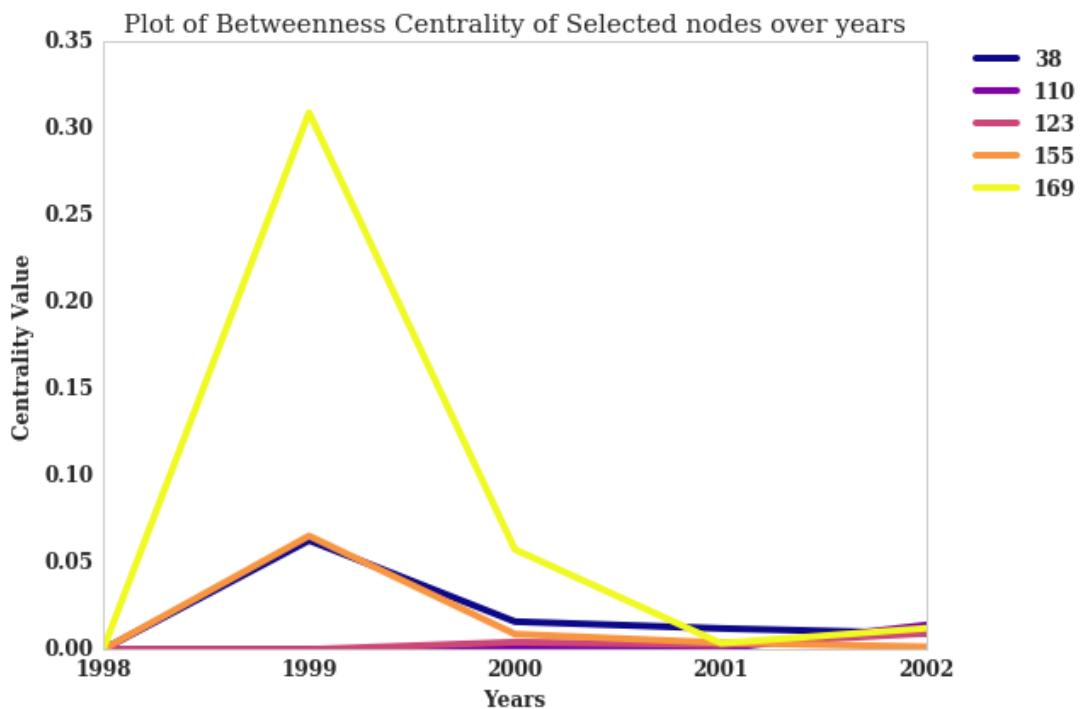
node_deg = node_deg.T
node_deg.columns = years
node_deg = node_deg.T

node_deg.plot.line(figsize=(8,6), cmap='plasma')
plt.title("Plot of Degree Centrality of common nodes over years", fontsize=14)
plt.xlabel("Years", fontsize=12)
plt.ylabel("Centrality Value", fontsize=12)
plt.legend(fontsize=12, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0)
plt.xticks(years, [i for i in years], fontsize=12)
plt.yticks(fontsize=12)
plt.tight_layout()
plt.savefig('images/node_deg.png')

```



```
In [114]: node_bet = pd.DataFrame.from_dict(betC0, orient='index').\
join(pd.DataFrame.from_dict(betC1,orient='index'), rsuffix = 'Gt1', lsuffix='Gt0')
join(pd.DataFrame.from_dict(betC2,orient='index'), rsuffix = 'Gt2', lsuffix='Gt0')
join(pd.DataFrame.from_dict(betC3,orient='index'), rsuffix = 'Gt3', lsuffix='Gt0')
join(pd.DataFrame.from_dict(betC4,orient='index'), rsuffix = 'Gt4', lsuffix='Gt0')
node_bet = node_bet.T
node_bet.columns = years
node_bet = node_bet.T
node_bet.plot(figsize=(8,6), cmap='plasma')
plt.title("Plot of Betweenness Centrality of Selected nodes over years",
plt.xlabel("Years", fontsize=12)
plt.ylabel("Centrality Value", fontsize=12)
plt.legend(fontsize=12, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.5)
plt.xticks(years,[i for i in years],fontsize=12)
plt.yticks(fontsize=12)
plt.tight_layout()
plt.savefig('images/node_bet.png')
```

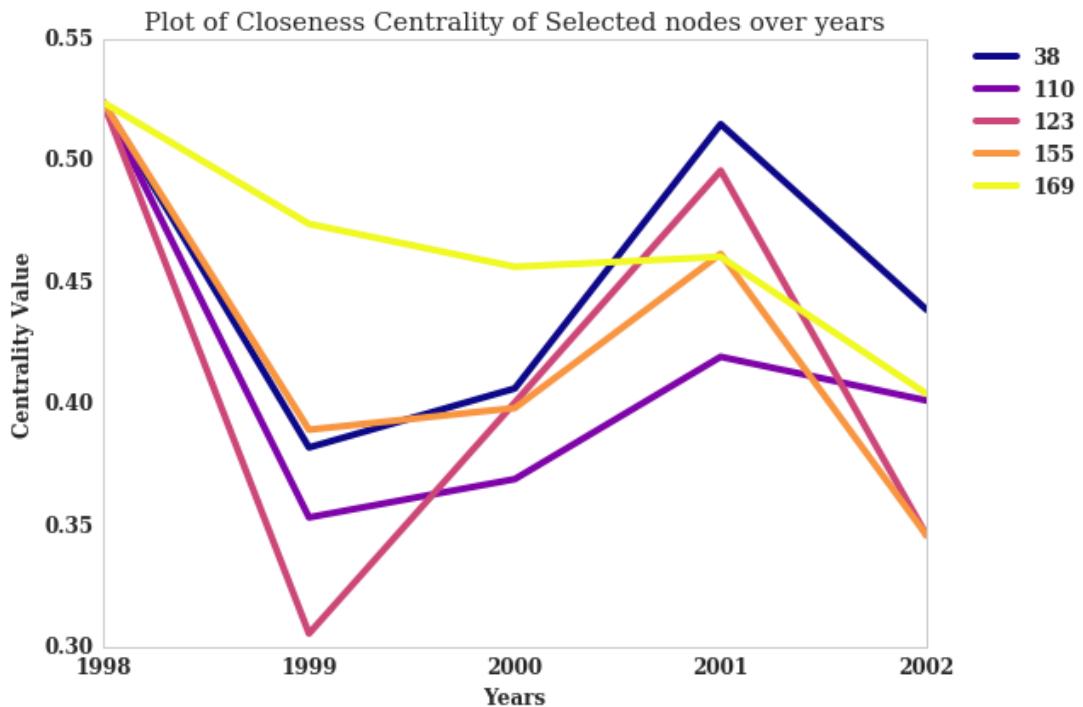


```
In [115]: node_clo = pd.DataFrame.from_dict(cloC0, orient='index').\
join(pd.DataFrame.from_dict(cloC1,orient='index'), rsuffix = 'Gt1', lsuffix='Gt0')
join(pd.DataFrame.from_dict(cloC2,orient='index'), rsuffix = 'Gt2', lsuffix='Gt0')
join(pd.DataFrame.from_dict(cloC3,orient='index'), rsuffix = 'Gt3', lsuffix='Gt0')
join(pd.DataFrame.from_dict(cloC4,orient='index'), rsuffix = 'Gt4', lsuffix='Gt0')
node_clo = node_clo.T
```

```

node_clo.columns = years
node_clo = node_clo.T
node_clo.plot(figsize=(8,6), cmap='plasma')
plt.title("Plot of Closeness Centrality of Selected nodes over years", fontweight='bold')
plt.xlabel("Years", fontsize=12)
plt.ylabel("Centrality Value", fontsize=12)
plt.legend(fontsize=12, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0)
plt.xticks(years, [i for i in years], fontsize=12)
plt.yticks(fontsize=12)
plt.tight_layout()
plt.savefig('images/node_clo.png')

```



```

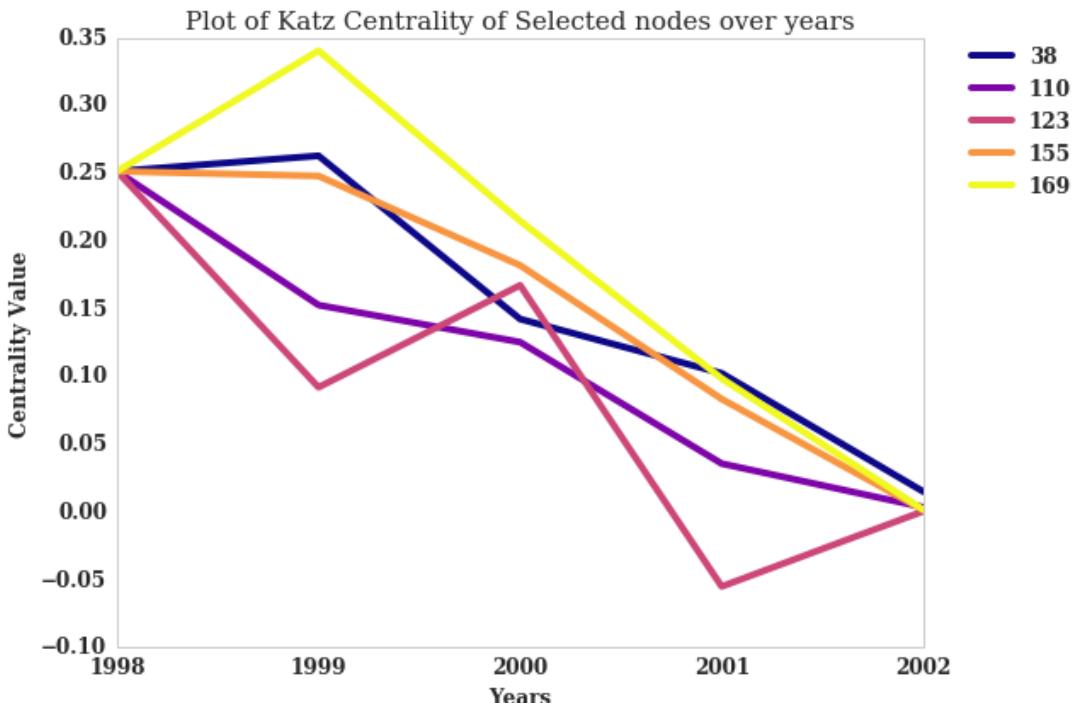
In [116]: node_katz = pd.DataFrame.from_dict(katzC0, orient='index').\
join(pd.DataFrame.from_dict(katzC1,orient='index'), rsuffix = 'Gt1', lsuffix = 'Gt0')
join(pd.DataFrame.from_dict(katzC2,orient='index'), rsuffix = 'Gt2', lsuffix = 'Gt1')
join(pd.DataFrame.from_dict(katzC3,orient='index'), rsuffix = 'Gt3', lsuffix = 'Gt2')
join(pd.DataFrame.from_dict(katzC4,orient='index'), rsuffix = 'Gt4', lsuffix = 'Gt3')
node_katz = node_katz.T
node_katz.columns = years
node_katz = node_katz.T
node_katz.plot(figsize=(8,6), cmap='plasma')
plt.title("Plot of Katz Centrality of Selected nodes over years", fontweight='bold')
plt.xlabel("Years", fontsize=12)
plt.ylabel("Centrality Value", fontsize=12)

```

```

plt.legend(fontsize=12, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0)
plt.xticks(years,[i for i in years],fontsize=12)
plt.yticks(fontsize=12)
plt.tight_layout()
plt.savefig('images/node_katz.png')

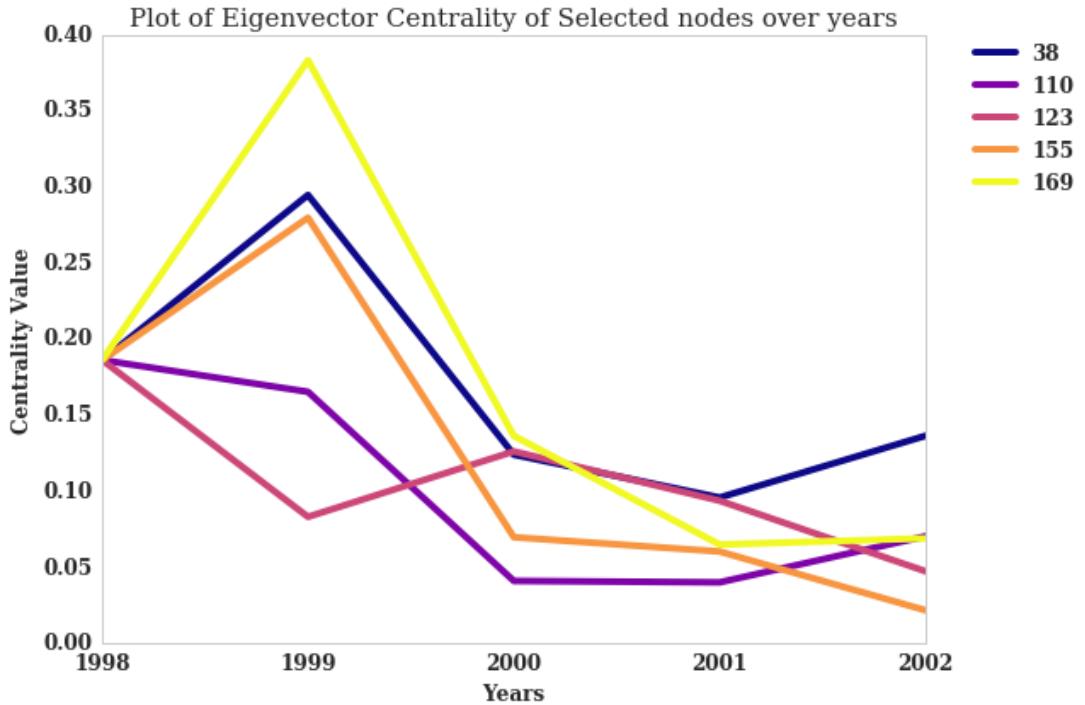
```



```

In [117]: node_eig = pd.DataFrame.from_dict(eigC0, orient='index').\
join(pd.DataFrame.from_dict(eigC1,orient='index'), rsuffix = 'Gt1', lsuffix='C0')
join(pd.DataFrame.from_dict(eigC2,orient='index'), rsuffix = 'Gt2', lsuffix='C1')
join(pd.DataFrame.from_dict(eigC3,orient='index'), rsuffix = 'Gt3', lsuffix='C2')
join(pd.DataFrame.from_dict(eigC4,orient='index'), rsuffix = 'Gt4', lsuffix='C3')
node_eig = node_eig.T
node_eig.columns = years
node_eig = node_eig.T
node_eig.plot(figsize=(8,6), cmap='plasma')
plt.title("Plot of Eigenvector Centrality of Selected nodes over years",
plt.xlabel("Years", fontsize=12)
plt.ylabel("Centrality Value", fontsize=12)
plt.legend(fontsize=12, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0)
plt.xticks(years,[i for i in years],fontsize=12)
plt.yticks(fontsize=12)
plt.tight_layout()
plt.savefig('images/node_eig.png')

```



```
In [118]: node_load = pd.DataFrame.from_dict(loadC0, orient='index').\
join(pd.DataFrame.from_dict(loadC1,orient='index'), rsuffix = 'Gt1', lsuffix='Gt1')
join(pd.DataFrame.from_dict(loadC2,orient='index'), rsuffix = 'Gt2', lsuffix='Gt2')
join(pd.DataFrame.from_dict(loadC3,orient='index'), rsuffix = 'Gt3', lsuffix='Gt3')
join(pd.DataFrame.from_dict(loadC4,orient='index'), rsuffix = 'Gt4', lsuffix='Gt4')
node_load = node_load.T
node_load.columns = years
node_load = node_load.T
node_load.plot(figsize=(8,6),cmap='plasma')
plt.title("Plot of Load Centrality of Selected nodes over years", fontsize=12)
plt.xlabel("Years", fontsize=12)
plt.ylabel("Centrality Value", fontsize=12)
plt.legend(fontsize=12, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0)
plt.xticks(years,[i for i in years],fontsize=12)
plt.yticks(fontsize=12)
plt.tight_layout()
plt.savefig('images/node_load.png')
```