

# DNA Analysis Enron Monthly 01

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## 2 Introduction

Here using the Enron Email data with timestamps from John Hopkins, I will attempt to recreate a dynamic network at monthly level of granularity. Previously I conducted this analysis at the yearly level.

For this analysis we use some typical attributes in addition to novel ones.

### 2.1 The traditional attributes used are:

- Degree Centrality
- Closeness Centrality
- Betweenness Centrality
- Eigenvector Centrality
- Katz Centrality

- Load Centrality
- Density
- Clustering Coefficient

## 2.2 Novel attributes introduced in this analysis:

- Instantaneous Phase
- Instantaneous Amplitude
- Instantaneous Frequency
- Gaussian Curvature
- Energy Envelope
- First Derivative of Energy Envelope
- Second Derivative of Energy Envelope

## 2.3 Attributes implemented from other authors:

- Persistence & Emergence (ref: CMU)
- Resistance Distance (ref: Klein 93)

## 2.4 Novel Visualisation suggested in this analysis:

- Frequency vs Wavenumber Plots
- Radon Domain plots

## 3 Import Libraries

```
In [386]: import pandas as pd
import numpy as np
import networkx as nx
import seaborn as sns
import matplotlib.pyplot as plt
import scipy as sc
%matplotlib inline
sns.set(style="whitegrid", color_codes=True, context='paper')
import random
random.seed(11111111111111)
plt.rc('axes', grid=False, titlesize='large', labelsizes='medium', labelweight='bold')
plt.rc('lines', linewidth=4)
plt.rc('font', family='serif', size=12, serif='Georgia')
plt.rc('figure', figsize = (15,6), titlesize='large', titleweight='heavy')
plt.rc('grid', linewidth=3)
sns.set_palette('cubehelix')
from scipy.signal import *
from numpy.linalg import *
```

## 4 Helper Functions

```
In [408]: def get_val(val):
          return sorted(set(val.values()))

In [409]: def avg_cent(cent):
          avg = sum(set(cent.values()))/len(cent)
          return avg

In [410]: def get_cent(net):
          degC = nx.degree_centrality(net)
          cloC = nx.closeness_centrality(net)
          betC = nx.betweenness_centrality(net)
          eigC = nx.eigenvector_centrality_numpy(net)
          commCC = nx.communicability_centrality(net)
          katzC = nx.katz_centrality_numpy(net)
          loadC = nx.load_centrality(net)

          return [degC, cloC, betC, eigC, commCC, katzC, loadC]

In [411]: def stationarity_ratio(G):
          #stationarity ratio with laplian
          L = nx.laplacian_matrix(G).todense()
          U = nx.laplacian_spectrum(G)
          C = np.cov(L)
          CF = np.dot(L, np.dot(np.dot(U.T, C), U))
          r = np.linalg.norm(np.diag(CF))/np.linalg.norm(CF)

          return [r]

In [412]: #cite: `klein1993resistance`
          def resistance_distance(net):
              M = nx.laplacian_matrix(net).todense()
              pseudo = pinv(M)
              N = M.shape[0]
              d = np.diag(pseudo)
              rd = np.kron(d, np.ones((N, 1))).T + np.kron(d, np.ones((N, 1))).T - pseudo

              return [rd, rd.mean()]

In [413]: def calc_seisatt(net):
          M = nx.laplacian_matrix(net).todense()
          Ht = hilbert(M)
          IA = np.real(np.nan_to_num(np.sqrt(np.dot(M, M) + np.dot(Ht, Ht))))
          IP = np.real(np.nan_to_num(np.arctan(Ht/M)))
          IF, _ = np.real(np.nan_to_num(np.asarray(np.gradient(IP))))
          E = np.real(np.sqrt(np.dot(M, M) + np.dot(Ht, Ht)))
          dE, _ = np.nan_to_num(np.asarray(np.gradient(E)))
          dEe, _ = np.nan_to_num(np.asarray(np.gradient(dE)))
```

```

att_globalval = pd.DataFrame([IA.mean(), IP.mean(), IF.mean(), \
                             E.mean(), dE.mean(), dEe.mean()]).T
att_globalval.columns = ['InstAmp', 'InstPhase', 'InstFreq.', 'EnergyEnv']

return [IA, IP, IF, E, dE, dEe, att_globalval]

In [414]: def curvature(net):
    from skimage.feature import hessian_matrix, hessian_matrix_det, hessi
    M = nx.laplacian_matrix(net).todense()
    M = np.float64(M)
    fx, fy = np.gradient(M)
    Hxx, Hxy, Hyy = hessian_matrix(M)
    K = np.divide((np.dot(Hxx, Hxy) - np.dot(Hxy, Hxy)), \
                  (1 + np.dot(fx, fx) + np.dot(fy, fy)))

    He1, He2 = hessian_matrix_eigvals(Hxx, Hxy, Hyy)
    mean_curv = np.trace(He1)
    s, a = np.linalg.slogdet(He1)
    conc = s * np.exp(a)
    Pmax = np.max(He1)
    Pmin = np.min(He1)

    return [K, mean_curv, conc]

In [708]: def cal_avgstat(net):
    #calculate all attributes from previously defined functions here
    degC, cloC, betC, eigC, _C, katzC, loadC = get_cent(net)
    _, meanK, _ = curvature(net)
    IA, IP, IF, E, dE, dEe, att_globalval = calc_seisatt(net)
    _, norm_rd = resistance_distance(net)
    r = stationarity_ratio(net)
    den = nx.density(net)
    clustcof = nx.clustering(net)

    #create attribute volume here
    stat_df = pd.DataFrame([avg_cent(degC), avg_cent(cloC), avg_cent(betC),
                           avg_cent(katzC), avg_cent(loadC), meanK, den, avg_cent(clustcof)])
    stat_df.columns = ['AvgDeg', 'AvgCloseness', 'AvgBet', 'AvgEig', 'AvgKatz', 'AvgLoad', 'AvgMeanK', 'AvgDen', 'AvgClustcof']
    stat_df = stat_df.join(att_globalval)
    stat_df['MeanResistanceDist'] = norm_rd
    stat_df['StatRat'] = r

    return stat_df

In [709]: def std_klpca_ratio(net):
    from sklearn.decomposition import KernelPCA

```

```

M = nx.laplacian_matrix(net)
kpca =KernelPCA(n_components=3, kernel='rbf')
eigv = kpca.fit_transform(M)
pc1_std = eigv[:,0] - eigv[:,0].mean() /eigv[:,0].std()
pc2_std = eigv[:,1] - eigv[:,1].mean() /eigv[:,1].std()
pc3_std = eigv[:,2] - eigv[:,2].mean() /eigv[:,2].std()
klpca_ratio_std = pc1_std - pc3_std/pc1_std - pc2_std

return klpca_ratio_std

```

## 5 Data Analysis Monthly Aggregation

### 5.1 Load Preprocessed Data

```
In [710]: data = pd.read_excel("../Data/data 03.2.xlsx")
```

```
In [711]: data.head()
```

```

Out[711]:
      timestamp  to  from  year  month
0  1979-12-31 21:00:00   24   153  1979     12
1  1979-12-31 21:00:00   24   153  1979     12
2  1979-12-31 21:00:00   29    29  1979     12
3  1979-12-31 21:00:00   29    29  1979     12
4  1979-12-31 21:00:00   29    29  1979     12

```

### 5.2 Check Year Labels

```
In [712]: set(data.year)
```

```
Out[712]: {1979, 1998, 1999, 2000, 2001, 2002}
```

```
In [713]: data[data.year==1979].count()
```

```

Out[713]: timestamp    174
          to           174
          from         174
          year         174
          month        174
          dtype: int64

```

```
In [714]: data.shape
```

```
Out[714]: (125409, 5)
```

```

In [715]: #total % of mislabelled 1979 entries
          (data[data.year==1979].count()/data.shape[0]) * 100

```

```

Out[715]: timestamp    0.138746
          to           0.138746
          from         0.138746
          year         0.138746
          month        0.138746
          dtype: float64

```

### 5.3 Split data into yearly slices

The entries labelled 1979 are mislabelled hence they will be excluded from analysis. As we see that they are a tiny fraction of the dataset anyway

```
In [716]: data = data[data.year!= 1979]
```

```
In [717]: years = sorted(set(data.year))
```

```
In [718]: years
```

```
Out[718]: [1998, 1999, 2000, 2001, 2002]
```

```
In [719]: df_98 = data[data.year==years[0]]
          df_99 = data[data.year==years[1]]
          df_2k = data[data.year==years[2]]
          df_2k1 = data[data.year==years[3]]
          df_2k2 = data[data.year==years[4]]
```

```
In [720]: df_98.head()
```

```
Out[720]:
```

		timestamp	to	from	year	month
174	1998-11-13	09:07:00	114	169	1998	11
175	1998-11-13	09:07:00	114	169	1998	11
176	1998-11-19	12:19:00	114	123	1998	11
177	1998-11-19	12:19:00	114	123	1998	11
178	1998-11-19	13:24:00	114	123	1998	11

```
In [721]: df_98.describe()
```

```
Out[721]:
```

		to	from	year	month
count	82.000000	82.000000	82.0	82.000000	
mean	114.292683	119.000000	1998.0	11.634146	
std	2.051725	48.393449	0.0	0.484633	
min	112.000000	11.000000	1998.0	11.000000	
25%	114.000000	110.000000	1998.0	11.000000	
50%	114.000000	123.000000	1998.0	12.000000	
75%	114.000000	169.000000	1998.0	12.000000	
max	123.000000	169.000000	1998.0	12.000000	

```
In [722]: df_99.head()
```

```
Out[722]:
```

		timestamp	to	from	year	month
256	1999-01-04	07:21:00	114	65	1999	1
257	1999-01-04	07:21:00	114	65	1999	1
258	1999-01-04	09:11:00	114	169	1999	1
259	1999-01-04	09:11:00	114	169	1999	1
260	1999-01-07	13:42:00	114	112	1999	1

```
In [723]: df_2k.head()
```

```
Out [723]:
```

	timestamp	to	from	year	month
3971	2000-01-03 06:47:00	82	51	2000	1
3972	2000-01-03 06:47:00	82	51	2000	1
3973	2000-01-03 06:47:00	82	51	2000	1
3974	2000-01-03 06:47:00	82	51	2000	1
3975	2000-01-03 06:47:00	82	51	2000	1

```
In [724]: df_99.describe()
```

```
Out [724]:
```

	to	from	year	month
count	3715.000000	3715.000000	3715.0	3715.000000
mean	116.191386	115.949933	1999.0	9.725707
std	56.692443	43.626945	0.0	2.648675
min	11.000000	2.000000	1999.0	1.000000
25%	65.000000	88.000000	1999.0	8.000000
50%	145.000000	114.000000	1999.0	10.000000
75%	169.000000	156.000000	1999.0	12.000000
max	178.000000	178.000000	1999.0	12.000000

```
In [725]: df_2k1.head()
```

```
Out [725]:
```

	timestamp	to	from	year	month
48030	2001-01-01 13:36:00	78	82	2001	1
48031	2001-01-01 13:36:00	78	82	2001	1
48032	2001-01-01 13:36:00	78	82	2001	1
48033	2001-01-01 13:55:00	78	127	2001	1
48034	2001-01-01 13:55:00	78	127	2001	1

```
In [726]: df_2k2.head()
```

```
Out [726]:
```

	timestamp	to	from	year	month
116918	2002-01-01 17:27:27	0	9	2002	1
116919	2002-01-01 17:27:27	0	48	2002	1
116920	2002-01-01 20:12:31	0	20	2002	1
116921	2002-01-01 21:27:27	0	9	2002	1
116922	2002-01-01 21:27:27	0	48	2002	1

## 6 Network Year 1998

```
In [727]: nov_98 = df_98[df_98.month==11]
          dec_98= df_98[df_98.month==12]
```

```
In [728]: def create_graph(df):
          tmp = df.values[:,1:3]
          G= nx.Graph()
          G = nx.from_edgelist(tmp)

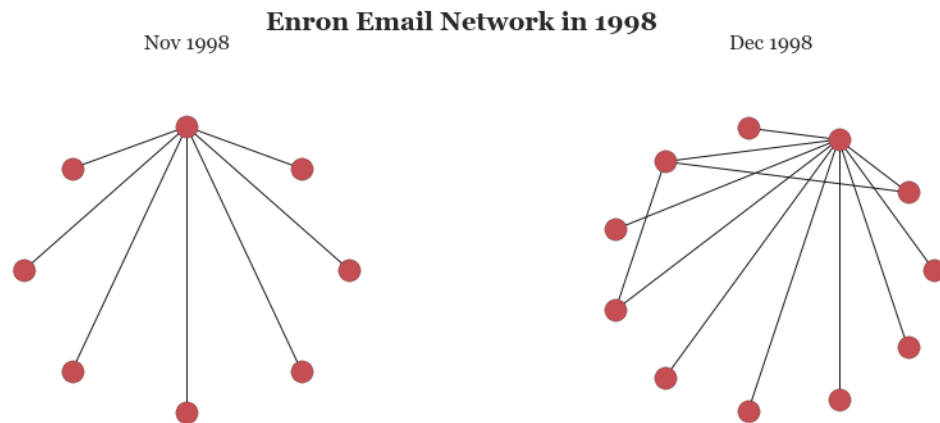
          return G
```

```

In [729]: G_nov98 = create_graph(nov_98)
          G_dec98 = create_graph(dec_98)

In [730]: plt.figure()
          plt.suptitle("Enron Email Network in 1998", fontsize=20)
          plt.subplot(121)
          plt.title("Nov 1998", fontsize=16)
          nx.draw_circular(G_nov98)
          plt.subplot(122)
          plt.title("Dec 1998", fontsize=16)
          nx.draw_circular(G_dec98)

```



```

In [731]: stat_nov98 = cal_avgstat(G_nov98)
          stat_dec98 = cal_avgstat(G_dec98)

```

```

In [732]: stat_nov98

```

```

Out [732]:      AvgDeg  AvgCloseness  AvgBet  AvgEig  AvgKatz  AvgLoad  MeanKurv
0  0.142857    0.192308    0.125  0.255427  0.185435    0.125 -0.186746

      Density  AvgClustCoef  InstAmp  InstPhase  InstFreq.  EnergyEnv  \
0      0.25          0.0  1.673053    0.402325    0.148795    1.673053

      dEnergyEnv  d2EnergyEnv  MeanResistanceDist  StatRat
0      0.005621    0.018756          1.53125    0.894427

```

```

In [733]: stat_dec98

```

```

Out [733]:      AvgDeg  AvgCloseness  AvgBet  AvgEig  AvgKatz  AvgLoad  MeanKurv
0  0.145455    0.242737  0.086869  0.25114  0.175111  0.086869 -0.21192

```



	Density	AvgClustCoef	InstAmp	InstPhase	InstFreq.	EnergyEnv	\
0	0.218182	0.155556	1.84759	0.249824	0.154485	1.84759	

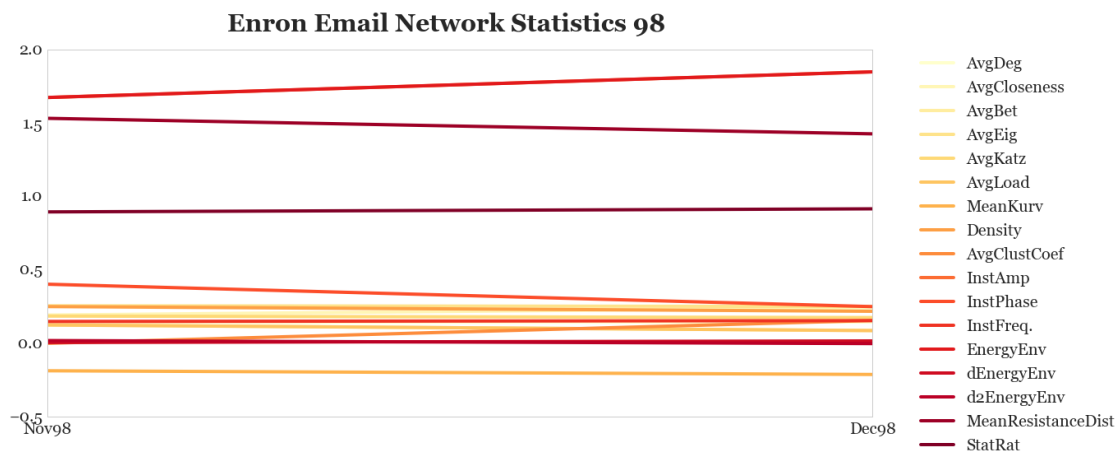
	dEnergyEnv	d2EnergyEnv	MeanResistanceDist	StatRat
0	0.016204	-0.001828	1.42562	0.915335

```
In [734]: stat98= stat_nov98.append(stat_dec98).T
```

```
In [735]: stat98.columns = ['Nov98', 'Dec98']
```

```
In [798]: stat98.T.plot(fontsize=20, figsize=(18,8),cmap='YlOrRd')
plt.suptitle("Enron Email Network Statistics 98", fontsize=30)
plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
```

```
Out[798]: <matplotlib.legend.Legend at 0x1488cd48978>
```



## 7 Network Year 1999

```
In [737]: df_99.describe()
```

```
Out[737]:
```

	to	from	year	month
count	3715.000000	3715.000000	3715.0	3715.000000
mean	116.191386	115.949933	1999.0	9.725707
std	56.692443	43.626945	0.0	2.648675
min	11.000000	2.000000	1999.0	1.000000
25%	65.000000	88.000000	1999.0	8.000000
50%	145.000000	114.000000	1999.0	10.000000
75%	169.000000	156.000000	1999.0	12.000000
max	178.000000	178.000000	1999.0	12.000000

```
In [738]: jan_99=df_99[df_99.month==1]
feb_99=df_99[df_99.month==2]
```

```

mar_99=df_99[df_99.month==3]
apr_99=df_99[df_99.month==4]
may_99=df_99[df_99.month==5]
jun_99=df_99[df_99.month==6]
jul_99=df_99[df_99.month==7]
aug_99=df_99[df_99.month==8]
sep_99=df_99[df_99.month==9]
oct_99=df_99[df_99.month==10]
nov_99=df_99[df_99.month==11]
dec_99=df_99[df_99.month==12]

```

```

G_jan_99=create_graph(jan_99)
G_feb_99=create_graph(feb_99)
G_mar_99=create_graph(mar_99)
G_apr_99=create_graph(apr_99)
G_may_99=create_graph(may_99)
G_jun_99=create_graph(jun_99)
G_jul_99=create_graph(jul_99)
G_aug_99=create_graph(aug_99)
G_sep_99=create_graph(sep_99)
G_oct_99=create_graph(oct_99)
G_nov_99=create_graph(nov_99)
G_dec_99=create_graph(dec_99)

```

```

In [739]: plt.figure(figsize=(32,18))
plt.suptitle("Enron Email Network in 99", fontsize=40)

plt.subplot(331)
plt.title("Jan 99", fontsize=25)
nx.draw_circular(G_jan_99)

plt.subplot(332)
plt.title("Feb 99", fontsize=25)
nx.draw_circular(G_feb_99)

plt.subplot(333)
plt.title("Mar 99", fontsize=25)
nx.draw_circular(G_mar_99)

plt.subplot(334)
plt.title("Apr 99", fontsize=25)
nx.draw_circular(G_apr_99)

plt.subplot(335)
plt.title("May 99", fontsize=25)
nx.draw_circular(G_may_99)

```

```
plt.subplot(336)
plt.title("Jun 99", fontsize=25)
nx.draw_circular(G_jun_99)

plt.subplot(337)
plt.title("Jul 99", fontsize=25)
nx.draw_circular(G_jul_99)

plt.subplot(338)
plt.title("Aug 99", fontsize=25)
nx.draw_circular(G_aug_99)

plt.subplot(339)
plt.title("Sep 99", fontsize=25)
nx.draw_circular(G_sep_99)

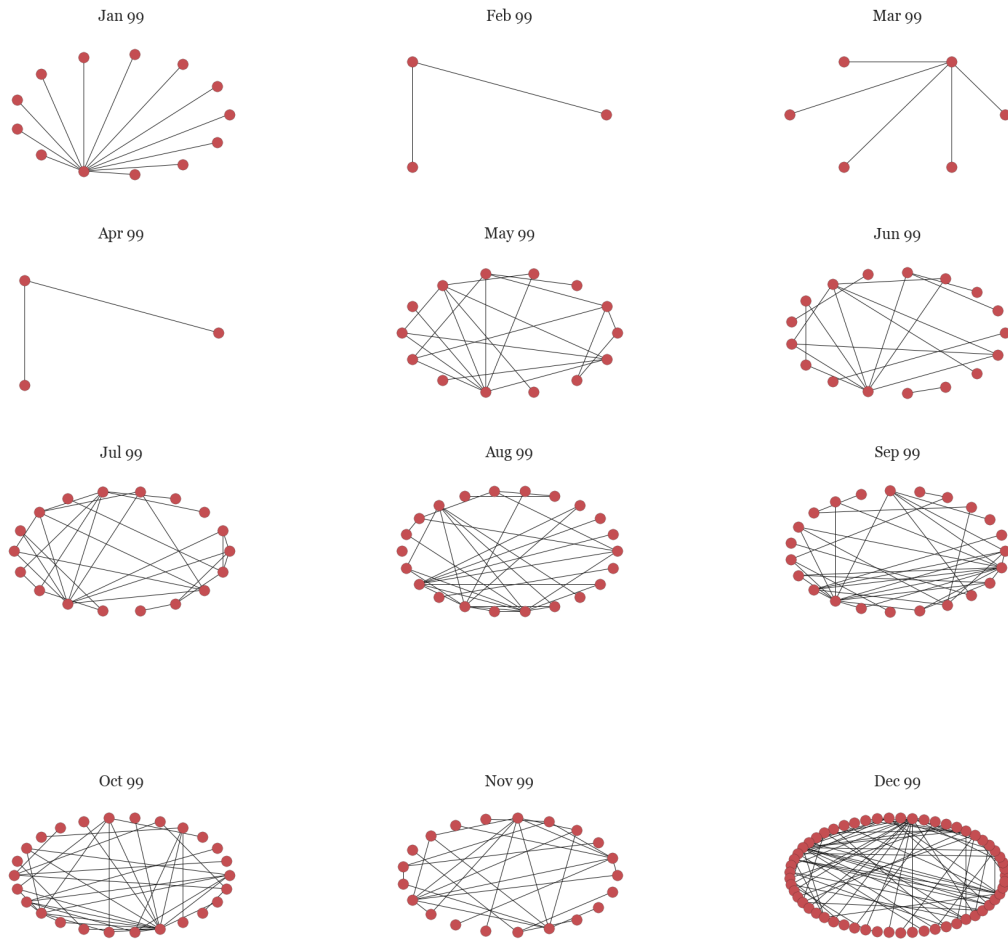
plt.figure(figsize=(32,5))
#plt.suptitle("Enron Email Network in 99", fontsize=40)

plt.subplot(131)
plt.title("Oct 99", fontsize=25)
nx.draw_circular(G_oct_99)

plt.subplot(132)
plt.title("Nov 99", fontsize=25)
nx.draw_circular(G_nov_99)

plt.subplot(133)
plt.title("Dec 99", fontsize=25)
nx.draw_circular(G_dec_99)
```

## Enron Email Network in 99

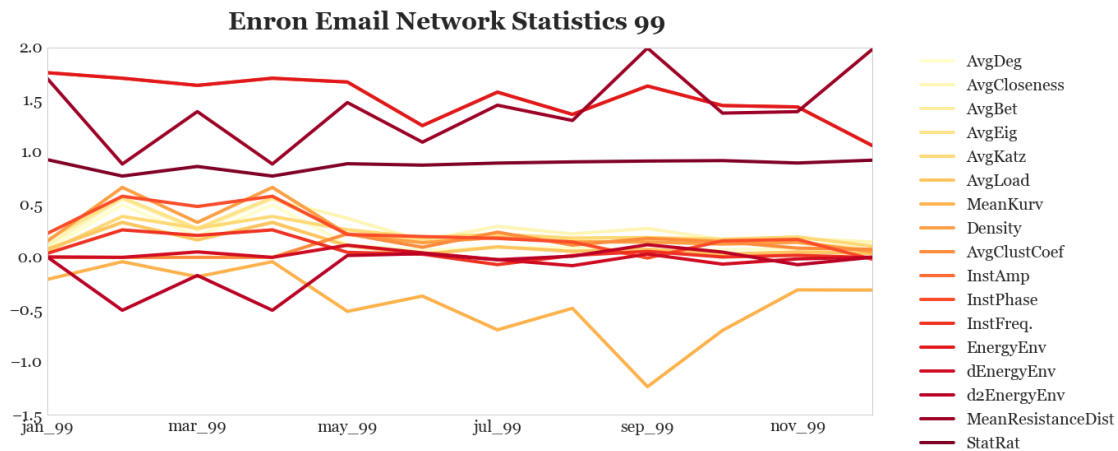


```
In [740]: stat_jan_99=cal_avgstat(G_jan_99)
stat_feb_99=cal_avgstat(G_feb_99)
stat_mar_99=cal_avgstat(G_mar_99)
stat_apr_99=cal_avgstat(G_apr_99)
stat_may_99=cal_avgstat(G_may_99)
stat_jun_99=cal_avgstat(G_jun_99)
stat_jul_99=cal_avgstat(G_jul_99)
stat_aug_99=cal_avgstat(G_aug_99)
stat_sep_99=cal_avgstat(G_sep_99)
stat_oct_99=cal_avgstat(G_oct_99)
stat_nov_99=cal_avgstat(G_nov_99)
stat_dec_99=cal_avgstat(G_dec_99)

In [741]: stat_99 = stat_jan_99.append(stat_feb_99).append(stat_mar_99).append(stat_apr_99)
stat_99 = stat_99.append(stat_jun_99).append(stat_jul_99).append(stat_aug_99)
stat_99 = stat_99.append(stat_sep_99).append(stat_oct_99).append(stat_nov_99).append(stat_dec_99)
stat_99.columns = ['jan_99', 'feb_99', 'mar_99', 'apr_99', 'may_99', 'jun_99', 'jul_99', 'aug_99', 'sep_99', 'oct_99', 'nov_99', 'dec_99']
```

```
In [795]: stat_99.T.plot(fontsize=20, figsize=(18,8), cmap='YlOrRd')
plt.suptitle("Enron Email Network Statistics 99", fontsize=30)
plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
```

```
Out[795]: <matplotlib.legend.Legend at 0x14882a87f60>
```



## 8 Network Year 2000

```
In [742]: df_2k.month.describe()
```

```
Out[742]: count      44059.000000
mean          8.183163
std           3.169912
min           1.000000
25%           6.000000
50%           9.000000
75%          11.000000
max           12.000000
Name: month, dtype: float64
```

```
In [743]: jan_2k=df_2k[df_2k.month==1]
feb_2k=df_2k[df_2k.month==2]
mar_2k=df_2k[df_2k.month==3]
apr_2k=df_2k[df_2k.month==4]
may_2k=df_2k[df_2k.month==5]
jun_2k=df_2k[df_2k.month==6]
jul_2k=df_2k[df_2k.month==7]
aug_2k=df_2k[df_2k.month==8]
sep_2k=df_2k[df_2k.month==9]
oct_2k=df_2k[df_2k.month==10]
nov_2k=df_2k[df_2k.month==11]
```

```
dec_2k=df_2k[df_2k.month==12]
```

```
G_jan_2k=create_graph(jan_2k)
G_feb_2k=create_graph(feb_2k)
G_mar_2k=create_graph(mar_2k)
G_apr_2k=create_graph(apr_2k)
G_may_2k=create_graph(may_2k)
G_jun_2k=create_graph(jun_2k)
G_jul_2k=create_graph(jul_2k)
G_aug_2k=create_graph(aug_2k)
G_sep_2k=create_graph(sep_2k)
G_oct_2k=create_graph(oct_2k)
G_nov_2k=create_graph(nov_2k)
G_dec_2k=create_graph(dec_2k)
```

```
In [744]: plt.figure(figsize=(32,18))
plt.suptitle("Enron Email Network in 2k", fontsize=40)

plt.subplot(331)
plt.title("Jan 2k", fontsize=25)
nx.draw_circular(G_jan_2k)

plt.subplot(332)
plt.title("Feb 2k", fontsize=25)
nx.draw_circular(G_feb_2k)

plt.subplot(333)
plt.title("Mar 2k", fontsize=25)
nx.draw_circular(G_mar_2k)

plt.subplot(334)
plt.title("Apr 2k", fontsize=25)
nx.draw_circular(G_apr_2k)

plt.subplot(335)
plt.title("May 2k", fontsize=25)
nx.draw_circular(G_may_2k)

plt.subplot(336)
plt.title("Jun 2k", fontsize=25)
nx.draw_circular(G_jun_2k)

plt.subplot(337)
plt.title("Jul 2k", fontsize=25)
nx.draw_circular(G_jul_2k)

plt.subplot(338)
```

```

plt.title("Aug 2k", fontsize=25)
nx.draw_circular(G_aug_2k)

plt.subplot(339)
plt.title("Sep 2k", fontsize=25)
nx.draw_circular(G_sep_2k)

plt.figure(figsize=(32,5))
#plt.suptitle("Enron Email Network in 2k", fontsize=40)

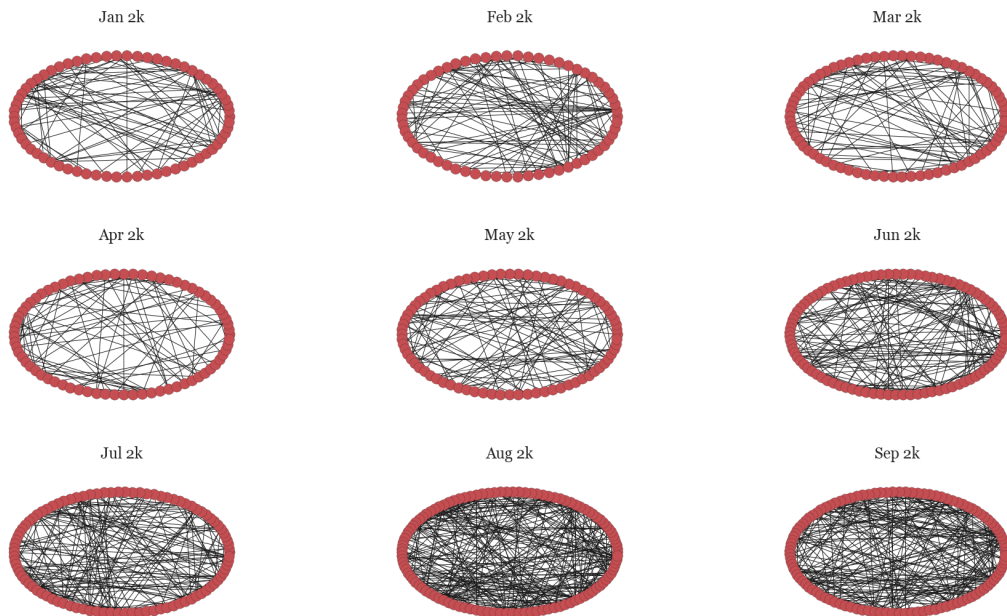
plt.subplot(131)
plt.title("Oct 2k", fontsize=25)
nx.draw_circular(G_oct_2k)

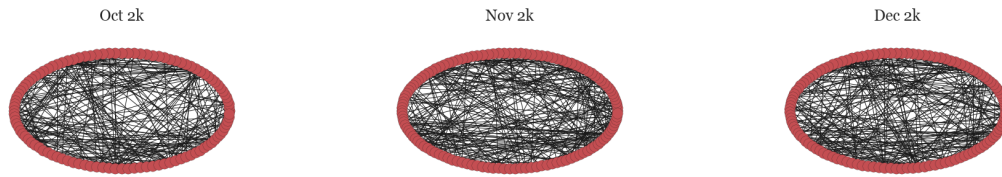
plt.subplot(132)
plt.title("Nov 2k", fontsize=25)
nx.draw_circular(G_nov_2k)

plt.subplot(133)
plt.title("Dec 2k", fontsize=25)
nx.draw_circular(G_dec_2k)

```

### Enron Email Network in 2k





```
In [746]: stat_jan_2k=cal_avgstat(G_jan_2k)
stat_feb_2k=cal_avgstat(G_feb_2k)
stat_mar_2k=cal_avgstat(G_mar_2k)
stat_apr_2k=cal_avgstat(G_apr_2k)
stat_may_2k=cal_avgstat(G_may_2k)
stat_jun_2k=cal_avgstat(G_jun_2k)
stat_jul_2k=cal_avgstat(G_jul_2k)
stat_aug_2k=cal_avgstat(G_aug_2k)
stat_sep_2k=cal_avgstat(G_sep_2k)
stat_oct_2k=cal_avgstat(G_oct_2k)
stat_nov_2k=cal_avgstat(G_nov_2k)
stat_dec_2k=cal_avgstat(G_dec_2k)
```

```
In [747]: stat_2k = stat_jan_2k.append(stat_feb_2k).append(stat_mar_2k).append(stat_apr_2k).append(stat_may_2k).append(stat_jun_2k).append(stat_jul_2k).append(stat_aug_2k).append(stat_sep_2k).append(stat_oct_2k).append(stat_nov_2k).append(stat_dec_2k)
stat_2k.columns = ['jan_2k', 'feb_2k', 'mar_2k', 'apr_2k', 'may_2k', 'jun_2k', 'jul_2k', 'aug_2k', 'sep_2k', 'oct_2k', 'nov_2k', 'dec_2k']
```

```
In [748]: stat_2k.head()
```

```
Out[748]:
```

	jan_2k	feb_2k	mar_2k	apr_2k	may_2k	jun_2k
AvgDeg	0.022378	0.026626	0.014809	0.010464	0.014035	0.010867
AvgCloseness	0.165628	0.208593	0.157985	0.113773	0.164460	0.192022
AvgBet	0.034948	0.035326	0.046531	0.042264	0.033091	0.027877
AvgEig	0.069247	0.084514	0.065003	0.053698	0.072435	0.068502
AvgKatz	0.099556	0.104652	0.097354	0.103359	0.104188	0.091210

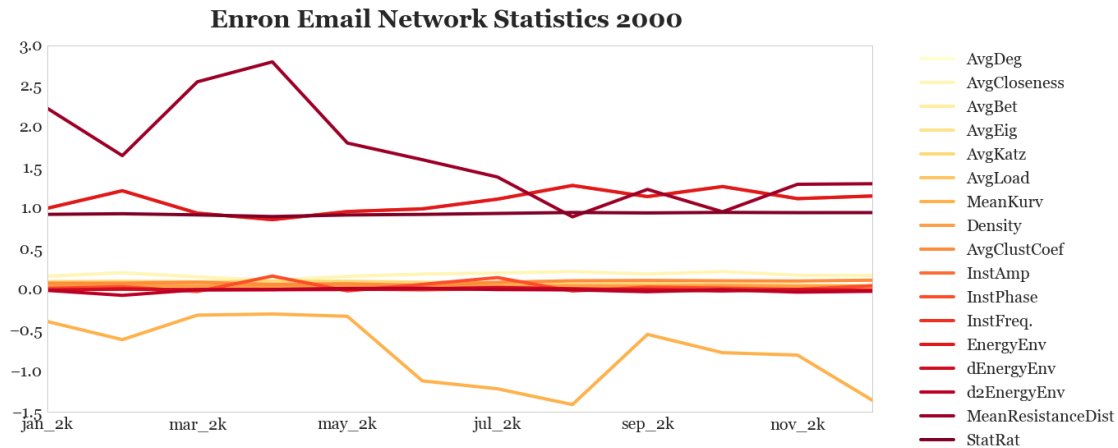
  

	jul_2k	aug_2k	sep_2k	oct_2k	nov_2k	dec_2k
AvgDeg	0.012697	0.016514	0.012147	0.018086	0.010884	0.013887
AvgCloseness	0.206035	0.223131	0.192691	0.222388	0.177589	0.174585
AvgBet	0.026787	0.017815	0.019417	0.019777	0.017789	0.019199
AvgEig	0.068311	0.062979	0.062015	0.062333	0.054194	0.051451
AvgKatz	0.086094	0.068437	0.073747	0.064164	0.058620	0.038943

```
In [800]: stat_2k.T.plot(fontsize=20, figsize=(18,8), cmap='YlOrRd')
plt.suptitle("Enron Email Network Statistics 2000", fontsize=30)
plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
```

```
Out[800]: <matplotlib.legend.Legend at 0x1488c068780>
```





## 9 Network Year 2k1

```
In [749]: df_2k1.month.describe()
```

```
Out [749]: count      68888.000000
           mean         6.125073
           std          3.537309
           min          1.000000
           25%          3.000000
           50%          5.000000
           75%         10.000000
           max         12.000000
           Name: month, dtype: float64
```

```
In [750]: jan_2k1=df_2k1[df_2k1.month==1]
           feb_2k1=df_2k1[df_2k1.month==2]
           mar_2k1=df_2k1[df_2k1.month==3]
           apr_2k1=df_2k1[df_2k1.month==4]
           may_2k1=df_2k1[df_2k1.month==5]
           jun_2k1=df_2k1[df_2k1.month==6]
           jul_2k1=df_2k1[df_2k1.month==7]
           aug_2k1=df_2k1[df_2k1.month==8]
           sep_2k1=df_2k1[df_2k1.month==9]
           oct_2k1=df_2k1[df_2k1.month==10]
           nov_2k1=df_2k1[df_2k1.month==11]
           dec_2k1=df_2k1[df_2k1.month==12]
```

```
G_jan_2k1=create_graph(jan_2k1)
G_feb_2k1=create_graph(feb_2k1)
G_mar_2k1=create_graph(mar_2k1)
```

```

G_apr_2k1=create_graph(apr_2k1)
G_may_2k1=create_graph(may_2k1)
G_jun_2k1=create_graph(jun_2k1)
G_jul_2k1=create_graph(jul_2k1)
G_aug_2k1=create_graph(aug_2k1)
G_sep_2k1=create_graph(sep_2k1)
G_oct_2k1=create_graph(oct_2k1)
G_nov_2k1=create_graph(nov_2k1)
G_dec_2k1=create_graph(dec_2k1)

plt.figure(figsize=(32,18))
plt.suptitle("Enron Email Network in 2k1", fontsize=40)

plt.subplot(331)
plt.title("Jan 2k1", fontsize=25)
nx.draw_circular(G_jan_2k1)

plt.subplot(332)
plt.title("Feb 2k1", fontsize=25)
nx.draw_circular(G_feb_2k1)

plt.subplot(333)
plt.title("Mar 2k1", fontsize=25)
nx.draw_circular(G_mar_2k1)

plt.subplot(334)
plt.title("Apr 2k1", fontsize=25)
nx.draw_circular(G_apr_2k1)

plt.subplot(335)
plt.title("May 2k1", fontsize=25)
nx.draw_circular(G_may_2k1)

plt.subplot(336)
plt.title("Jun 2k1", fontsize=25)
nx.draw_circular(G_jun_2k1)

plt.subplot(337)
plt.title("Jul 2k1", fontsize=25)
nx.draw_circular(G_jul_2k1)

plt.subplot(338)
plt.title("Aug 2k1", fontsize=25)
nx.draw_circular(G_aug_2k1)

plt.subplot(339)
plt.title("Sep 2k1", fontsize=25)
nx.draw_circular(G_sep_2k1)

```

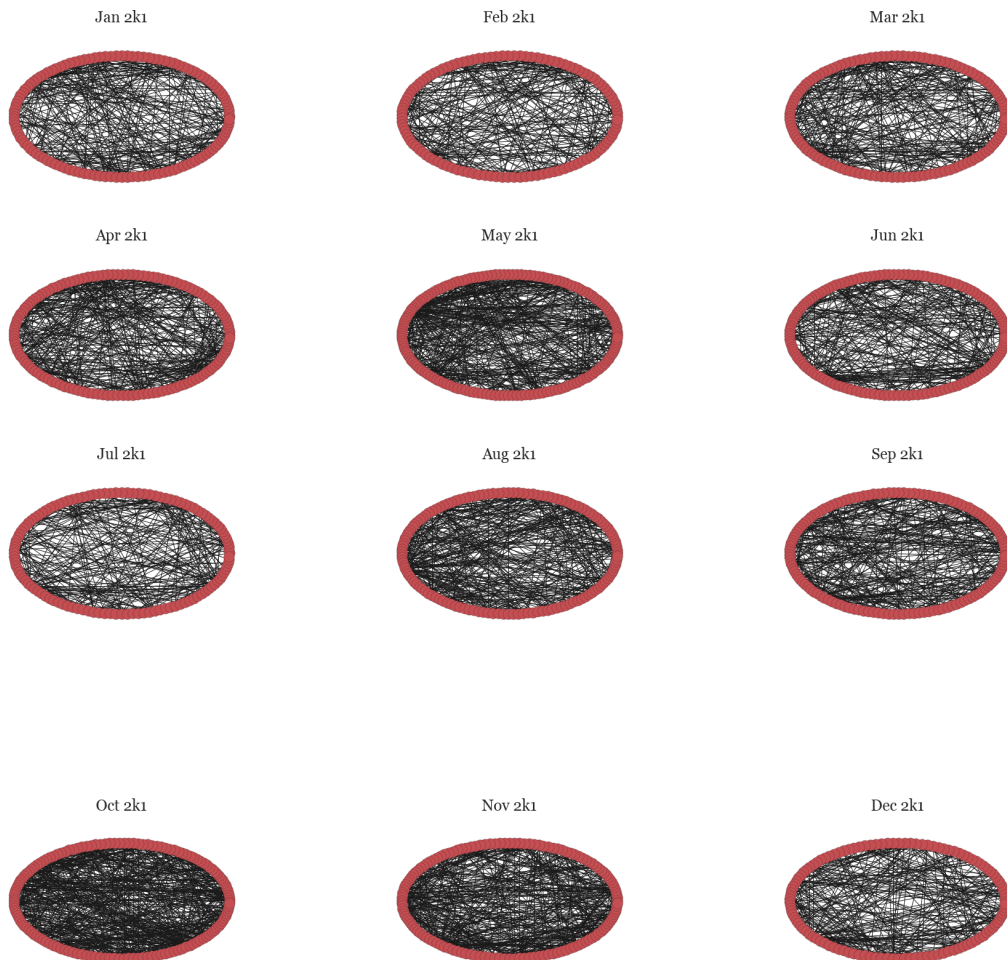
```
plt.figure(figsize=(32,5))
#plt.suptitle("Enron Email Network in 2k1", fontsize=40)

plt.subplot(131)
plt.title("Oct 2k1", fontsize=25)
nx.draw_circular(G_oct_2k1)

plt.subplot(132)
plt.title("Nov 2k1", fontsize=25)
nx.draw_circular(G_nov_2k1)

plt.subplot(133)
plt.title("Dec 2k1", fontsize=25)
nx.draw_circular(G_dec_2k1)
```

### Enron Email Network in 2k1



```

In [751]: stat_jan_2k1=cal_avgstat(G_jan_2k1)
          stat_feb_2k1=cal_avgstat(G_feb_2k1)
          stat_mar_2k1=cal_avgstat(G_mar_2k1)
          stat_apr_2k1=cal_avgstat(G_apr_2k1)
          stat_may_2k1=cal_avgstat(G_may_2k1)
          stat_jun_2k1=cal_avgstat(G_jun_2k1)
          stat_jul_2k1=cal_avgstat(G_jul_2k1)
          stat_aug_2k1=cal_avgstat(G_aug_2k1)
          stat_sep_2k1=cal_avgstat(G_sep_2k1)
          stat_oct_2k1=cal_avgstat(G_oct_2k1)
          stat_nov_2k1=cal_avgstat(G_nov_2k1)
          stat_dec_2k1=cal_avgstat(G_dec_2k1)

          stat_2k1 = stat_jan_2k1.append(stat_feb_2k1).append(stat_mar_2k1).append(
          stat_2k1 = stat_2k1.append(stat_jun_2k1).append(stat_jul_2k1).append(stat
          stat_2k1 = stat_2k1.append(stat_oct_2k1).append(stat_nov_2k1).append(stat
          stat_2k1.columns = ['jan_2k1', 'feb_2k1', 'mar_2k1', 'apr_2k1', 'may_2k1', 'ju

In [752]: stat_2k1.head()

Out[752]:
```

	jan_2k1	feb_2k1	mar_2k1	apr_2k1	may_2k1	jun_2k1
AvgDeg	0.013179	0.012052	0.012982	0.015607	0.012401	0.008815
AvgCloseness	0.191559	0.208446	0.186307	0.214056	0.221824	0.209019
AvgBet	0.017710	0.021867	0.017525	0.017928	0.011548	0.019936
AvgEig	0.055170	0.064597	0.058794	0.051706	0.053974	0.048988
AvgKatz	0.048244	0.074620	0.057339	0.002995	0.022108	0.059015

	jul_2k1	aug_2k1	sep_2k1	oct_2k1	nov_2k1	dec_2k1
AvgDeg	0.016036	0.016639	0.012930	0.019379	0.016612	0.011789
AvgCloseness	0.199306	0.247621	0.195912	0.232163	0.241737	0.202481
AvgBet	0.020539	0.015186	0.021407	0.012743	0.017048	0.022688
AvgEig	0.053507	0.051803	0.051587	0.058752	0.056969	0.065293
AvgKatz	0.046579	0.000057	0.021531	0.003949	0.002392	0.074214

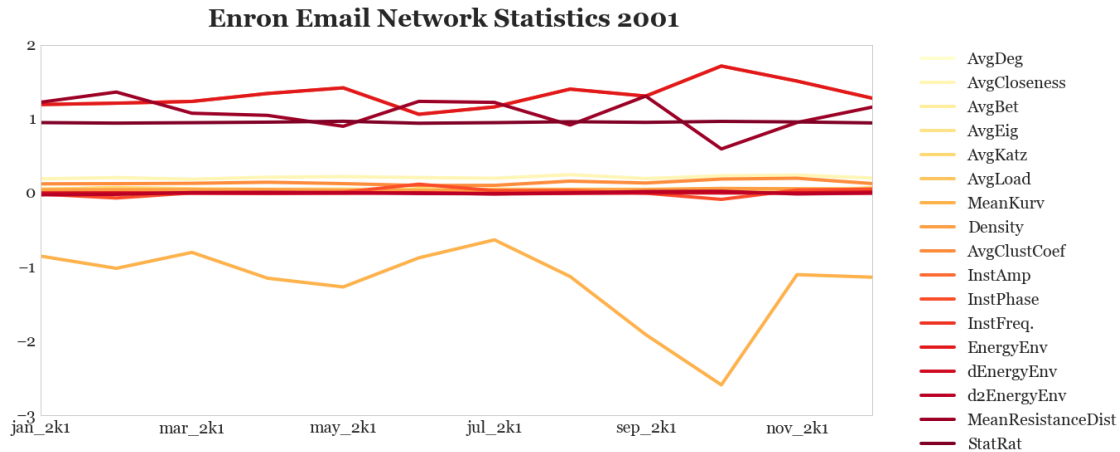
  

```

In [801]: stat_2k1.T.plot(fontsize=20, figsize=(18,8), cmap='YlOrRd')
          plt.suptitle("Enron Email Network Statistics 2001", fontsize=30)
          plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.

Out[801]: <matplotlib.legend.Legend at 0x1488c8ba400>

```



## 10 Network Year 2k2

```
In [753]: df_2k2.month.describe()
```

```
Out[753]: count      8491.000000
          mean         1.758921
          std          0.807467
          min          1.000000
          25%          1.000000
          50%          2.000000
          75%          2.000000
          max          6.000000
          Name: month, dtype: float64
```

```
In [754]: jan_2k2=df_2k2[df_2k2.month==1]
          feb_2k2=df_2k2[df_2k2.month==2]
          mar_2k2=df_2k2[df_2k2.month==3]
          apr_2k2=df_2k2[df_2k2.month==4]
          may_2k2=df_2k2[df_2k2.month==5]
          jun_2k2=df_2k2[df_2k2.month==6]
          jul_2k2=df_2k2[df_2k2.month==7]
          aug_2k2=df_2k2[df_2k2.month==8]
          sep_2k2=df_2k2[df_2k2.month==9]
          oct_2k2=df_2k2[df_2k2.month==10]
          nov_2k2=df_2k2[df_2k2.month==11]
          dec_2k2=df_2k2[df_2k2.month==12]
```

```
G_jan_2k2=create_graph(jan_2k2)
G_feb_2k2=create_graph(feb_2k2)
G_mar_2k2=create_graph(mar_2k2)
```

```

G_apr_2k2=create_graph(apr_2k2)
G_may_2k2=create_graph(may_2k2)
G_jun_2k2=create_graph(jun_2k2)
G_jul_2k2=create_graph(jul_2k2)
G_aug_2k2=create_graph(aug_2k2)
G_sep_2k2=create_graph(sep_2k2)
G_oct_2k2=create_graph(oct_2k2)
G_nov_2k2=create_graph(nov_2k2)
G_dec_2k2=create_graph(dec_2k2)

plt.figure(figsize=(32,18))
plt.suptitle("Enron Email Network in 2k2", fontsize=40)

plt.subplot(331)
plt.title("Jan 2k2", fontsize=25)
nx.draw_circular(G_jan_2k2)

plt.subplot(332)
plt.title("Feb 2k2", fontsize=25)
nx.draw_circular(G_feb_2k2)

plt.subplot(333)
plt.title("Mar 2k2", fontsize=25)
nx.draw_circular(G_mar_2k2)

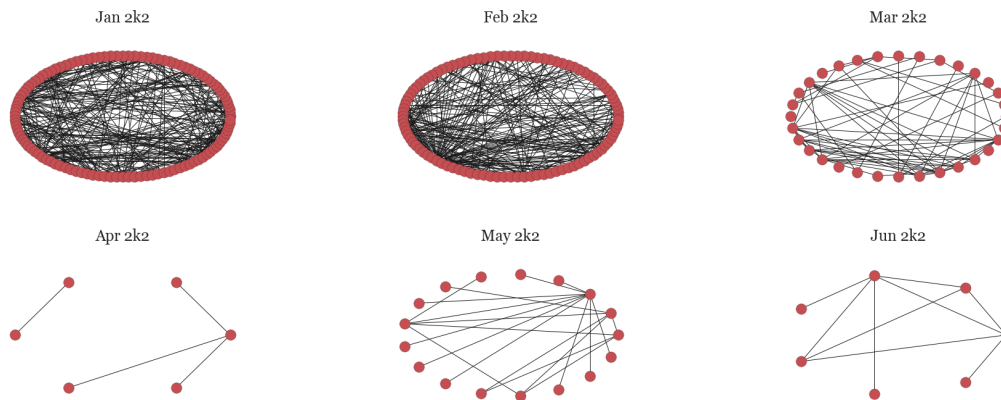
plt.subplot(334)
plt.title("Apr 2k2", fontsize=25)
nx.draw_circular(G_apr_2k2)

plt.subplot(335)
plt.title("May 2k2", fontsize=25)
nx.draw_circular(G_may_2k2)

plt.subplot(336)
plt.title("Jun 2k2", fontsize=25)
nx.draw_circular(G_jun_2k2)

```

## Enron Email Network in 2k2



```
In [755]: stat_jan_2k2=cal_avgstat(G_jan_2k2)
stat_feb_2k2=cal_avgstat(G_feb_2k2)
stat_mar_2k2=cal_avgstat(G_mar_2k2)
stat_apr_2k2=cal_avgstat(G_apr_2k2)
stat_may_2k2=cal_avgstat(G_may_2k2)
stat_jun_2k2=cal_avgstat(G_jun_2k2)
```

```
stat_2k2 = stat_jan_2k2.append(stat_feb_2k2).append(stat_mar_2k2).append(
stat_2k2.columns = ['jan_2k2', 'feb_2k2', 'mar_2k2', 'apr_2k2', 'may_2k2', 'jun_2k2']
```

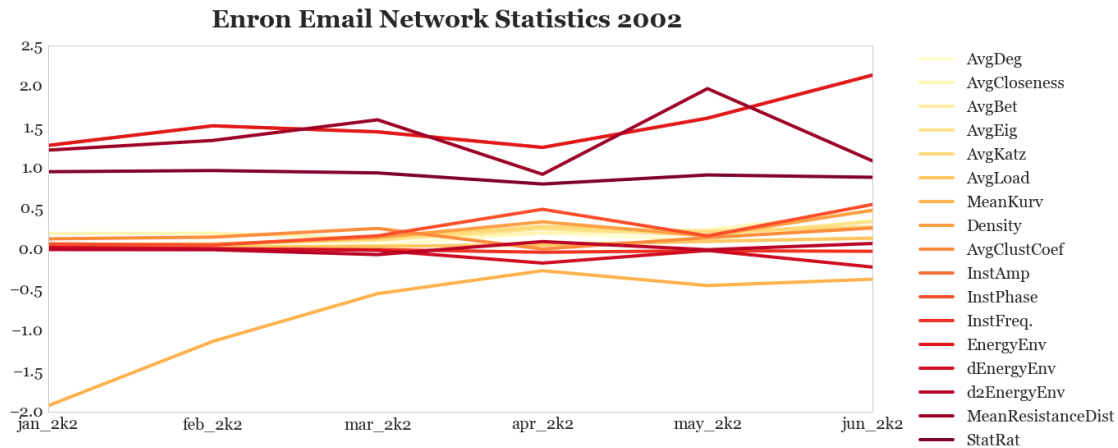
```
In [756]: stat_2k2.head()
```

```
Out[756]:
```

	jan_2k2	feb_2k2	mar_2k2	apr_2k2	may_2k2	jun_2k2
AvgDeg	0.018018	0.021818	0.049395	0.200000	0.106618	0.357143
AvgCloseness	0.191514	0.193146	0.133727	0.193333	0.234243	0.462193
AvgBet	0.022664	0.022488	0.033737	0.050000	0.094118	0.133333
AvgEig	0.055595	0.065003	0.108396	0.248878	0.168015	0.336275
AvgKatz	0.042359	0.046586	0.131887	0.277666	0.212198	0.273203

```
In [802]: stat_2k2.T.plot(fontsize=20, figsize=(18,8), cmap='YlOrRd')
plt.suptitle("Enron Email Network Statistics 2002", fontsize=30)
plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
```

```
Out[802]: <matplotlib.legend.Legend at 0x1488c1a7e48>
```

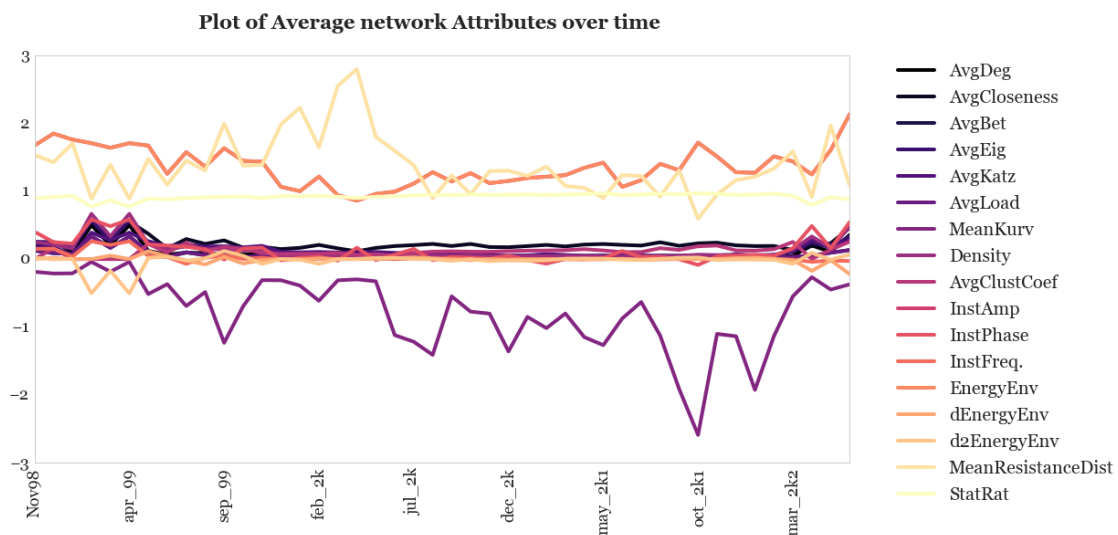


# Network Attributes over full time range

```
In [757]: stat_all = stat98.join(stat_99).join(stat_2k).join(stat_2k1).join(stat_2k2)
```

```
In [940]: stat_all.plot(figsize=(16,8), fontsize=18, rot=90, cmap='magma')
plt.suptitle("Plot of Average network Attributes over time", fontsize=22)
plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
```

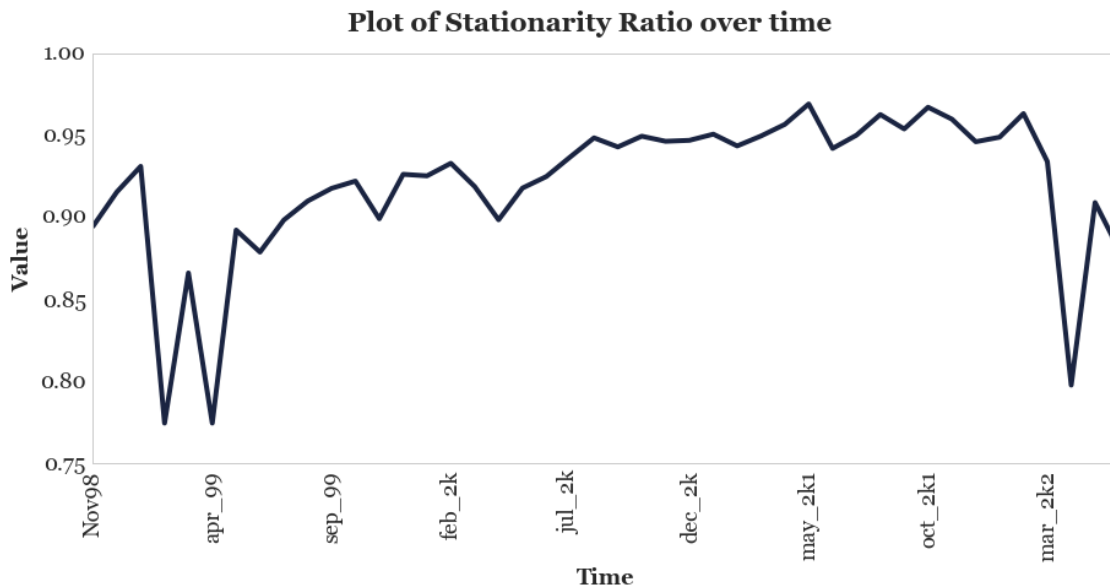
```
Out [940]: <matplotlib.legend.Legend at 0x148a4791da0>
```



```
In [759]: stat_all.StatRat.plot(fontsize=18, rot=90)
plt.suptitle('Plot of Stationarity Ratio over time', fontsize=22)
plt.xlabel("Time", fontsize=18)
plt.ylabel("Value", fontsize=18)
```



Out [759]: <matplotlib.text.Text at 0x1488c264278>



```
In [760]: stat_all_std = stat_all - stat_all.mean() / stat_all.std()
```

```
In [761]: stat_all_std.head()
```

```
Out [761]:
```

	AvgDeg	AvgCloseness	AvgBet	AvgEig	AvgKatz	AvgLoad	
Nov98	-0.501559	-2.190459	-0.683674	-0.811947	-1.049629	-0.683703	
Dec98	-0.498961	-2.140030	-0.721805	-0.816233	-1.059953	-0.721834	
jan_99	-0.561083	-2.265710	-0.731751	-0.903068	-1.177372	-0.731780	
feb_99	-0.144416	-1.827211	-0.475340	-0.498338	-0.844841	-0.475370	
mar_99	-0.444416	-2.123508	-0.642007	-0.791408	-0.957380	-0.642036	

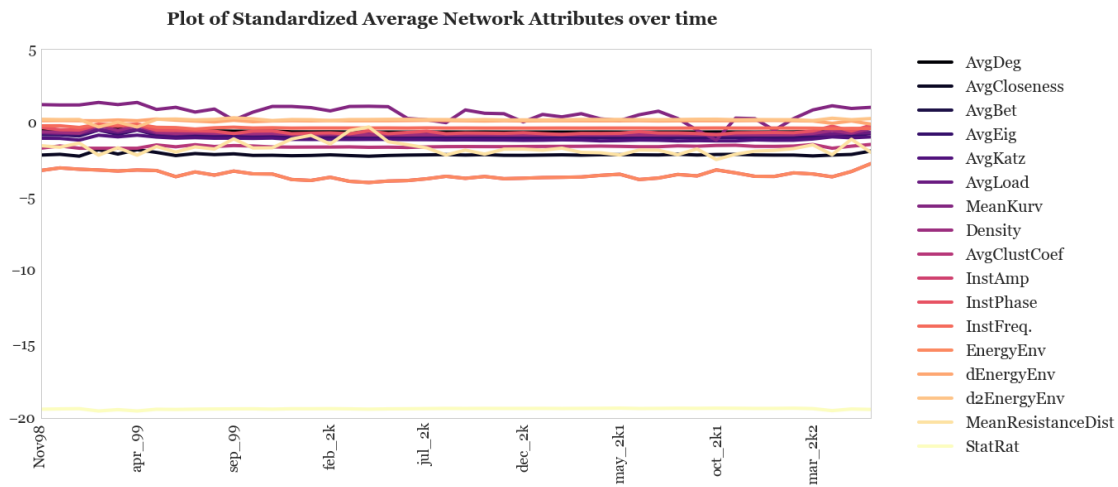
	MeanKurv	Density	AvgClustCoef	InstAmp	InstPhase	InstFreq.	
Nov98	1.223986	-0.614423	-1.728109	-3.234486	-0.322693	-0.213067	
Dec98	1.198805	-0.646242	-1.572554	-3.059949	-0.475194	-0.207377	
jan_99	1.201116	-0.710577	-1.728109	-3.148094	-0.497733	-0.324490	
feb_99	1.369452	-0.197757	-1.728109	-3.200872	-0.142786	-0.100063	
mar_99	1.225887	-0.531090	-1.728109	-3.269003	-0.240465	-0.152871	

	EnergyEnv	dEnergyEnv	d2EnergyEnv	MeanResistanceDist	StatRat	
Nov98	-3.234486	0.136924	0.231110	-1.554605	-19.387492	
Dec98	-3.059949	0.147507	0.210526	-1.660235	-19.366584	
jan_99	-3.148094	0.135652	0.216238	-1.381713	-19.350969	
feb_99	-3.200872	0.131303	-0.291969	-2.196966	-19.507322	
mar_99	-3.269003	0.183316	0.040216	-1.696966	-19.415893	

```
In [942]: stat_all_std.plot(figsize=(18,8), cmap='magma', fontsize=18, rot=90)
plt.suptitle("Plot of Standardized Average Network Attributes over time",
plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
```

Out [942]: <matplotlib.legend.Legend at 0x148a49ef438>

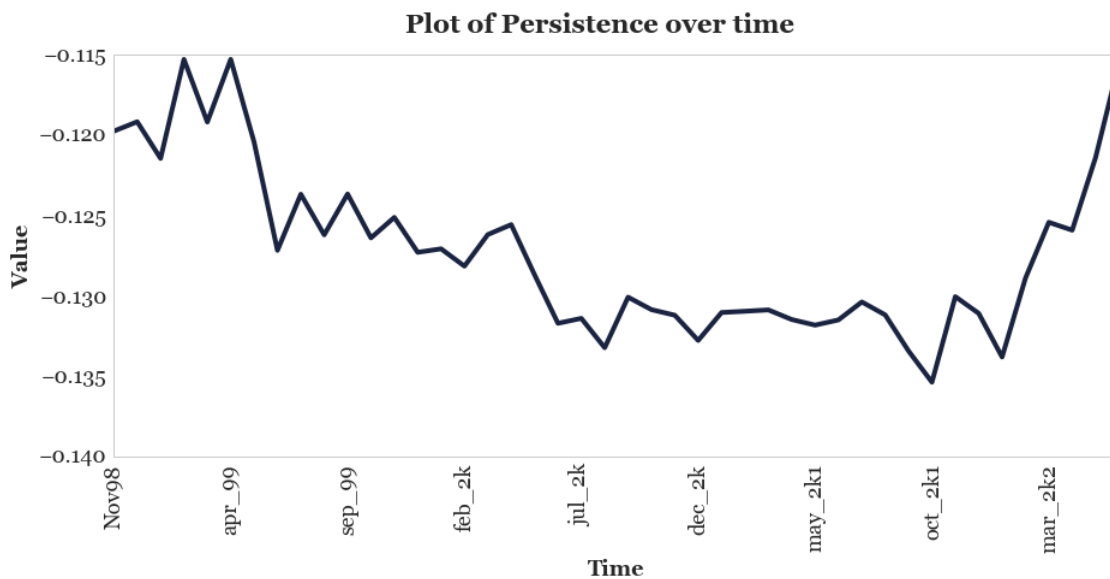


## 11 Persistence and Emergence

```
In [763]: persistence = stat_all_std.T.mean()/stat_all_std.T.shape[0]
```

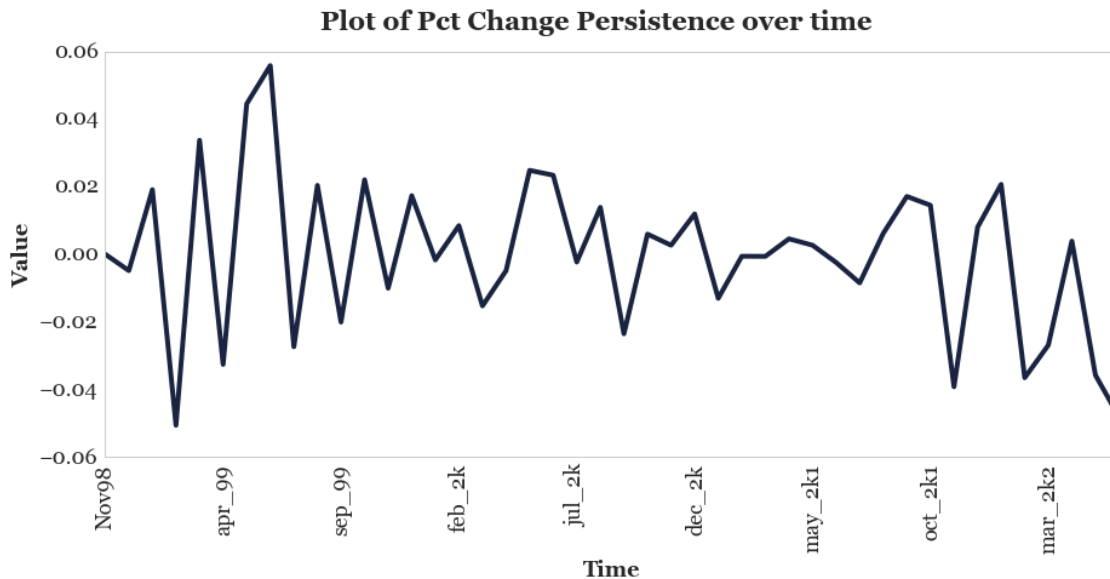
```
In [764]: persistence.plot(fontsize=18, rot=90)
plt.suptitle("Plot of Persistence over time", fontsize=22)
plt.xlabel("Time", fontsize=18)
plt.ylabel("Value", fontsize=18)
```

Out [764]: <matplotlib.text.Text at 0x1488d9fc518>



```
In [765]: persistence.pct_change().fillna(0).plot(fontsize=18, rot=90)
plt.suptitle("Plot of Pct Change Persistence over time", fontsize=22)
plt.xlabel("Time", fontsize=18)
plt.ylabel("Value", fontsize=18)
```

Out[765]: <matplotlib.text.Text at 0x1488d1b6dd8>



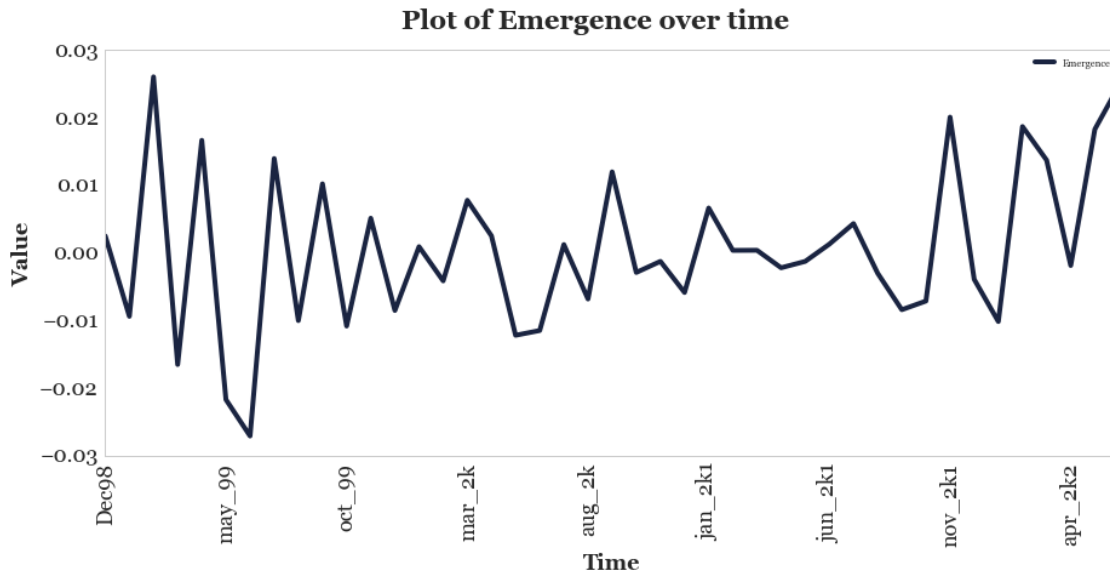
```
In [766]: def emergence(a,b):
           e = (a-b)/ (np.linalg.norm(a)+np.linalg.norm(b))
           return e

In [767]: emerg = []
           for i in range(0,persistence.shape[0]-1):
               x = int(i)
               y = x +1
               emerg.append(emergence(persistence.values[y],persistence.values[x]))

In [768]: emerg_df = pd.DataFrame(emerg).T
           emerg_df.columns = list(stat_all.T.columns)[1:]
           emerg_df = emerg_df.T
           emerg_df.columns = ['Emergence']

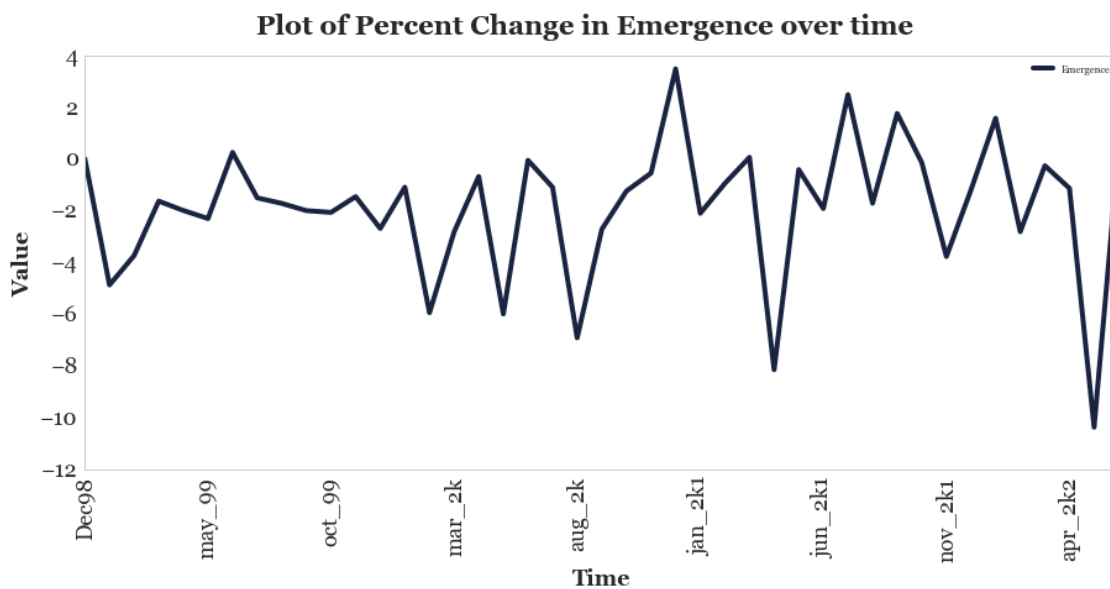
In [769]: emerg_df.plot(fontsize=18, rot=90)
plt.suptitle("Plot of Emergence over time", fontsize=22)
plt.xlabel("Time", fontsize=18)
plt.ylabel("Value", fontsize=18)
```

Out [769]: <matplotlib.text.Text at 0x14882b09cc0>



```
In [770]: emerg_df.pct_change().fillna(0).plot(fontsize=18, rot=90)
plt.suptitle("Plot of Percent Change in Emergence over time", fontsize=22)
plt.xlabel("Time", fontsize=18)
plt.ylabel("Value", fontsize=18)
```

Out [770]: <matplotlib.text.Text at 0x1488d05b710>

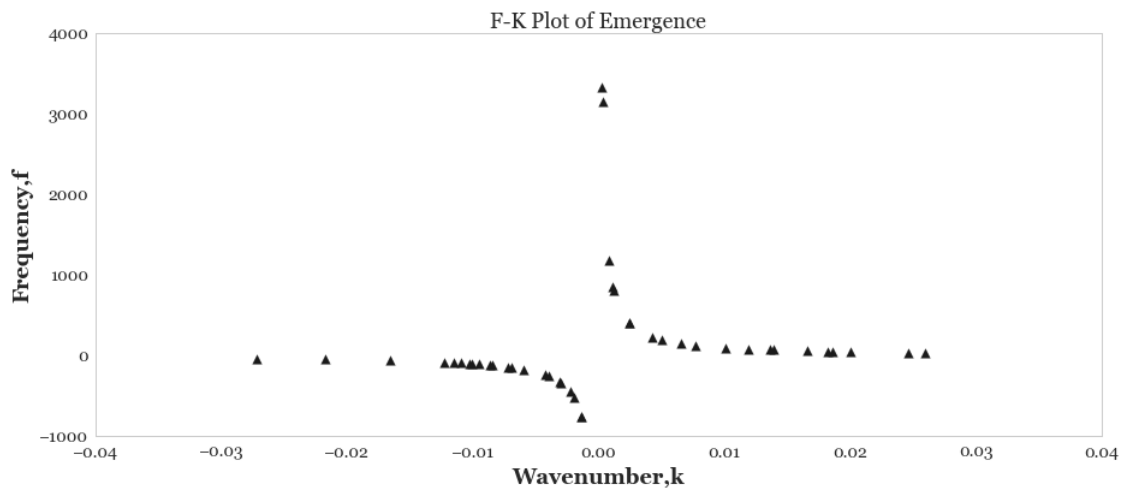


```
In [771]: freq = sc.fft(emerg_df)
          wavnumber = 1/freq
```

```
plt.scatter(freq, wavnumber, s=60, marker='^', c='k')
plt.title("F-K Plot of Emergence", fontsize=18)
plt.xticks(fontsize=14)
plt.yticks(fontsize=14)
plt.xlabel("Wavenumber,k", fontsize=18)
plt.ylabel("Frequency,f", fontsize=18)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:533: ComplexWarning:
  return array(a, dtype, copy=False, order=order, subok=True)
```

```
Out[771]: <matplotlib.text.Text at 0x1488e0f9208>
```



```
In [772]: print("Maximum Frequency :\n", emerg_df.iloc[np.argmax(freq)])
          print("-----\n")
          print("Minimum Frequency :\n", emerg_df.iloc[np.argmin(freq)])
```

```
Maximum Frequency :
Emergence      0.025951
Name: feb_99, dtype: float64
-----
```

```
Minimum Frequency :
Emergence     -0.027137
Name: jun_99, dtype: float64
```

```
In [773]: def plot_radon(m, name):
          from skimage.transform import radon
```

```

theta = np.linspace(0., 180., max(m.shape), endpoint=False)
sinogram = radon(m, theta=theta, circle=True)

fig, ax = plt.subplots(1,2)

ax[0].scatter(sinogram[0], sinogram[1], s=60, marker='^', c='k')
ax[0].set_title("Radon Transform Plot of "+str(name), fontsize=14)

ax[1].scatter(1/sinogram[0], sinogram[1], s=60, marker='^', c='r')
ax[1].set_title("1/Radon[0] vs Radon[1] Plot of "+str(name), fontsize=14)

plt.show()

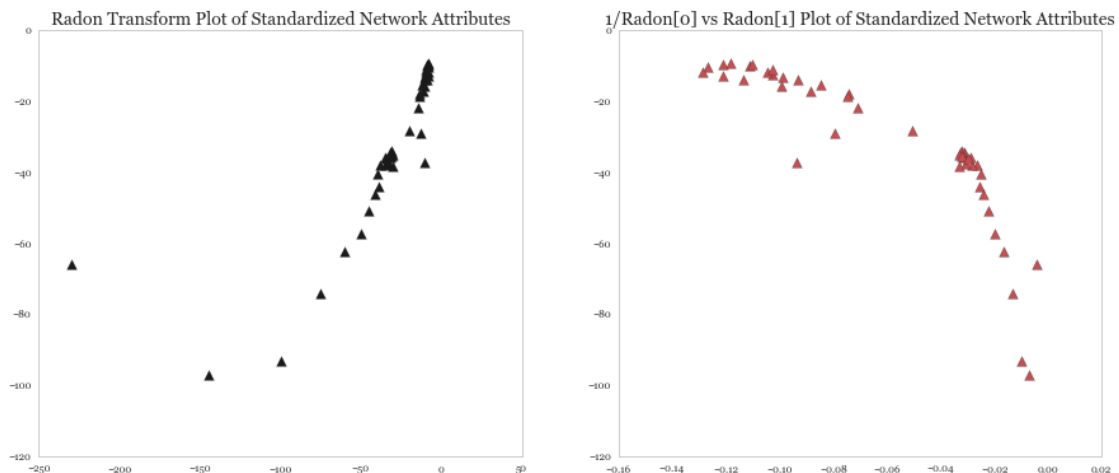
```

```
In [774]: plot_radon(stat_all_std.values, 'Standardized Network Attributes')
```

```

C:\Users\arsha_000\Anaconda3\lib\site-packages\skimage\transform\radon_transform.py
warn('Radon transform: image must be zero outside the '

```

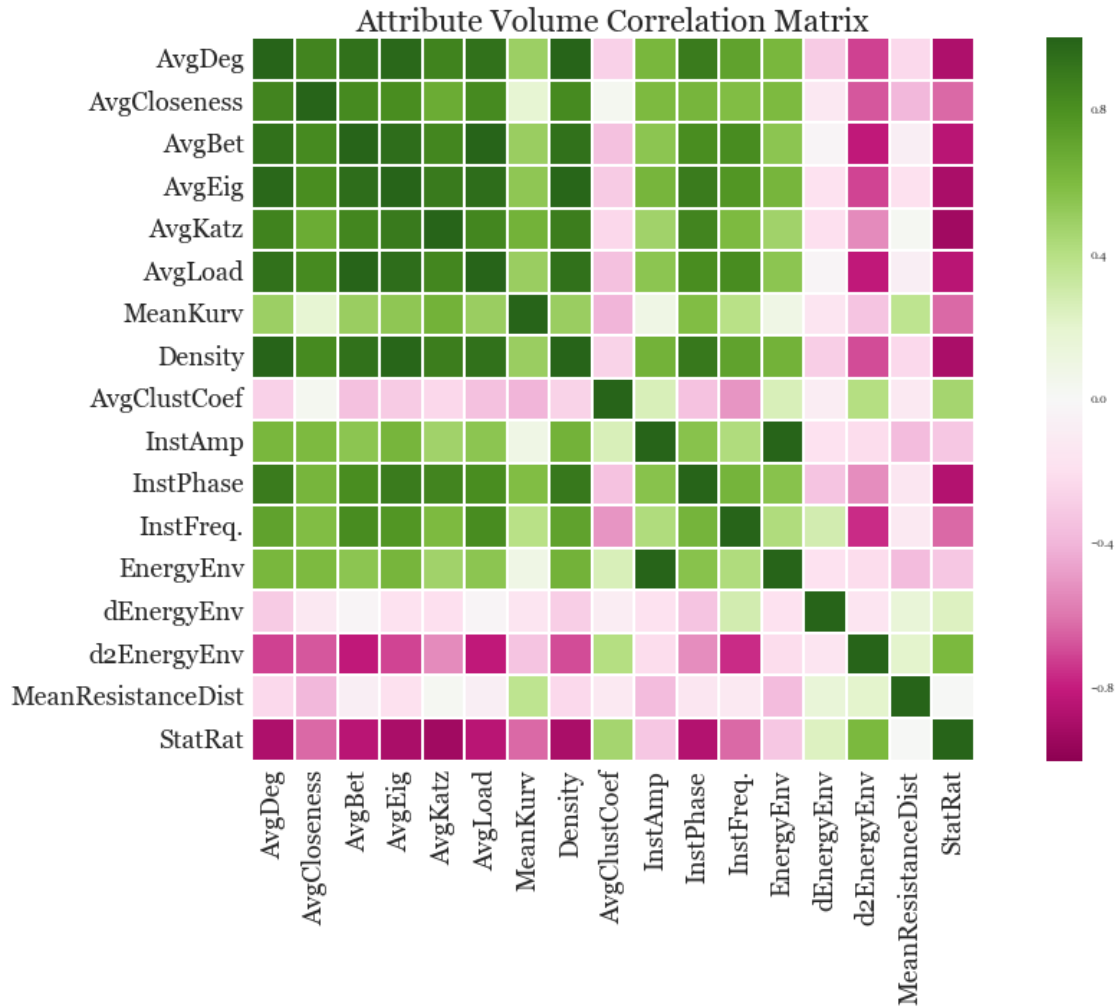


```

In [908]: plt.figure(figsize=(18,9))
          sns.heatmap(stat_all_std.corr(), cmap='PiYG', robust=True, fmt='d', linewidths=1)
          plt.title("Attribute Volume Correlation Matrix", fontsize=22)
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)

Out[908]: (array([ 0.5,  1.5,  2.5,  3.5,  4.5,  5.5,  6.5,  7.5,  8.5,
                    9.5, 10.5, 11.5, 12.5, 13.5, 14.5, 15.5, 16.5]),
          <a list of 17 Text yticklabel objects>)

```



## 12 Subgraph Stationarity

```
In [815]: def pad_shape(x, ref, offset=0):
           result = np.zeros_like(ref)
           result[0:x.shape[0]+0, 0:x.shape[1]+0] = x

           return result
```

```
In [838]: all_graphs = tuple([G_nov98,G_dec98,G_jan_99,G_feb_99,G_mar_99,G_apr_99,G_may_99,  
    G_nov_99,G_dec_99,G_jan_2k,G_feb_2k,G_mar_2k,G_apr_2k,G_may_2k,G_jun_2k,G_jul_2k,  
    G_oct_2k,G_nov_2k,G_dec_2k,G_jan_2k1,G_feb_2k1,G_mar_2k1,G_apr_2k1,G_may_2k1,  
    G_jun_2k1,G_jul_2k1,G_aug_2k1,G_sep_2k1,G_oct_2k1,G_nov_2k1,G_dec_2k1,G_jan_2k2])
```

```
In [909]: all_graphs[:5]
```

```
Out[909]: (<networkx.classes.graph.Graph at 0x1489f391ba8>,
           <networkx.classes.graph.Graph at 0x1489f386f28>)
```

```

<networkx.classes.graph.Graph at 0x148965122b0>,
<networkx.classes.graph.Graph at 0x1489f4ef4e0>,
<networkx.classes.graph.Graph at 0x148f102b0b8>)

```

```

In [913]: def subgraph_stat(net1,net2):
            net1_int_net2 = net1.copy()
            net1_int_net2.remove_nodes_from(n for n in net1 if n not in net2)
            net1_u_net2 = nx.disjoint_union(net1, net2)
            int_adjmat = nx.adjacency_matrix(net1_int_net2).todense()
            uni_adjmat = nx.adjacency_matrix(net1_u_net2).todense()
            int_adjmat_pad = pad_shape(int_adjmat,uni_adjmat)

            Ct = np.divide(norm(int_adjmat_pad),norm(uni_adjmat))

            return Ct

```

```

In [923]: Ct = []
            for i in range(0,len(all_graphs)-1):
                x = int(i)
                y = x +1
                Ct.append(subgraph_stat(all_graphs[x],all_graphs[y]))

```

```

In [972]: Ct_df = pd.DataFrame(Ct, columns=['Subgraph Autocorr']).T
            Ct_df.columns = list(stat_all.T.columns)[1:]
            Ct_df = Ct_df.T

            Ct_df.plot(fontsize=18, cmap='viridis', rot = 90)
            plt.title("Plot of Subgraph Autocorrelation  $C(t)$ ", fontsize=22)
            plt.xlabel("Timestep,  $t$ ", fontsize=18)
            plt.ylabel(" $C(t)$  Value", fontsize=18)
            plt.legend(fontsize=16, loc=2)
            plt.xticks(fontsize=18)
            plt.yticks(fontsize=18)

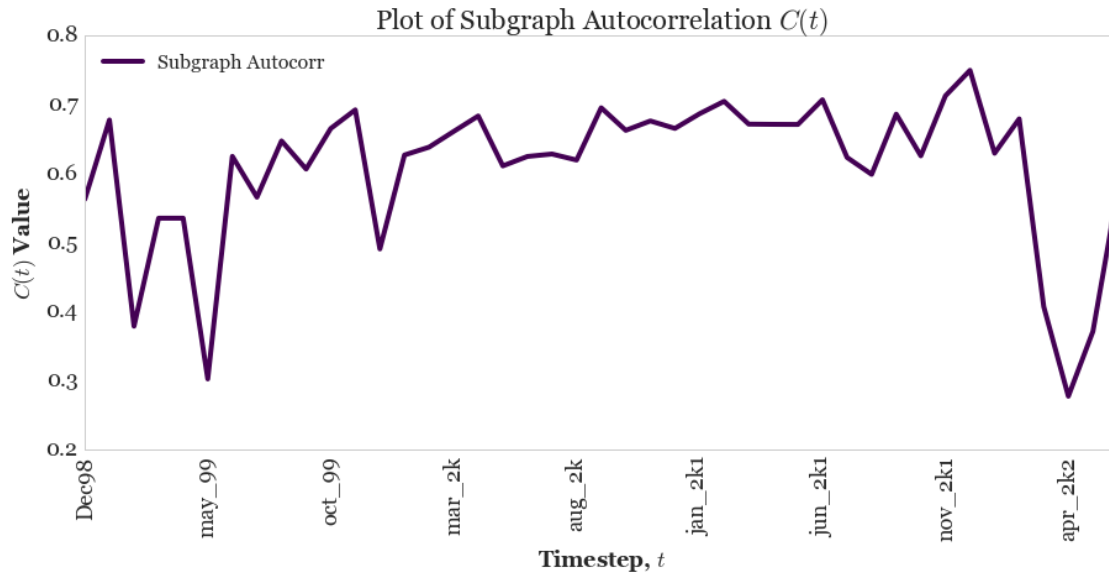
```

```

Out[972]: (array([ 0.2,  0.3,  0.4,  0.5,  0.6,  0.7,  0.8]),
            <a list of 7 Text yticklabel objects>)

```





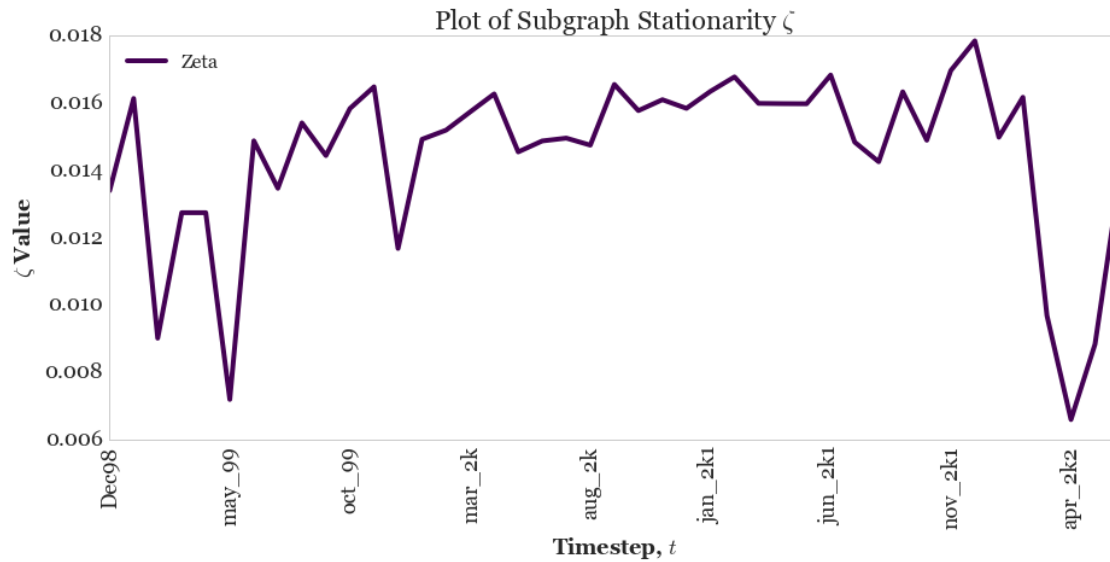
```
In [968]: zeta = Ct_df.cumsum(axis=1) / (Ct_df.shape[0]-1)
          zeta = zeta.T
          zeta.columns = list(stat_all.T.columns)[1:]
          zeta = zeta.T
          zeta.columns = ['Zeta']
          zeta.head()
```

```
Out[968]:
```

	Zeta
Dec98	0.013380
jan_99	0.016119
feb_99	0.008999
mar_99	0.012727
apr_99	0.012727

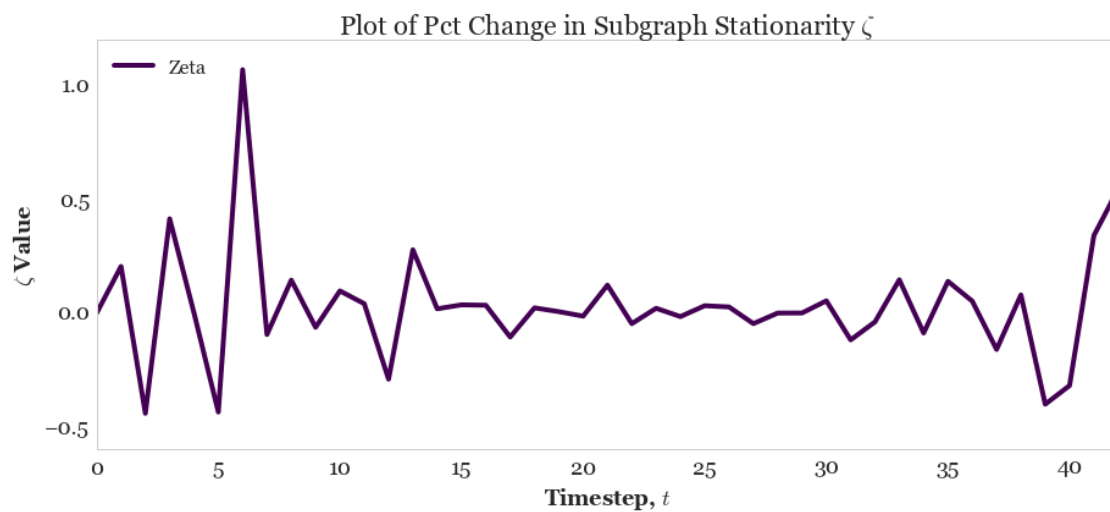
```
In [970]: zeta.plot(fontsize=18, cmap='viridis', rot =90)
          plt.title("Plot of Subgraph Stationarity $\zeta$", fontsize=22)
          plt.xlabel("Timestep, $t$", fontsize=18)
          plt.ylabel("$\zeta$ Value", fontsize=18)
          plt.legend(fontsize=16, loc=2)
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)
```

```
Out[970]: (array([ 0.006,  0.008,  0.01 ,  0.012,  0.014,  0.016,  0.018,  0.02 ]),
          <a list of 8 Text yticklabel objects>)
```



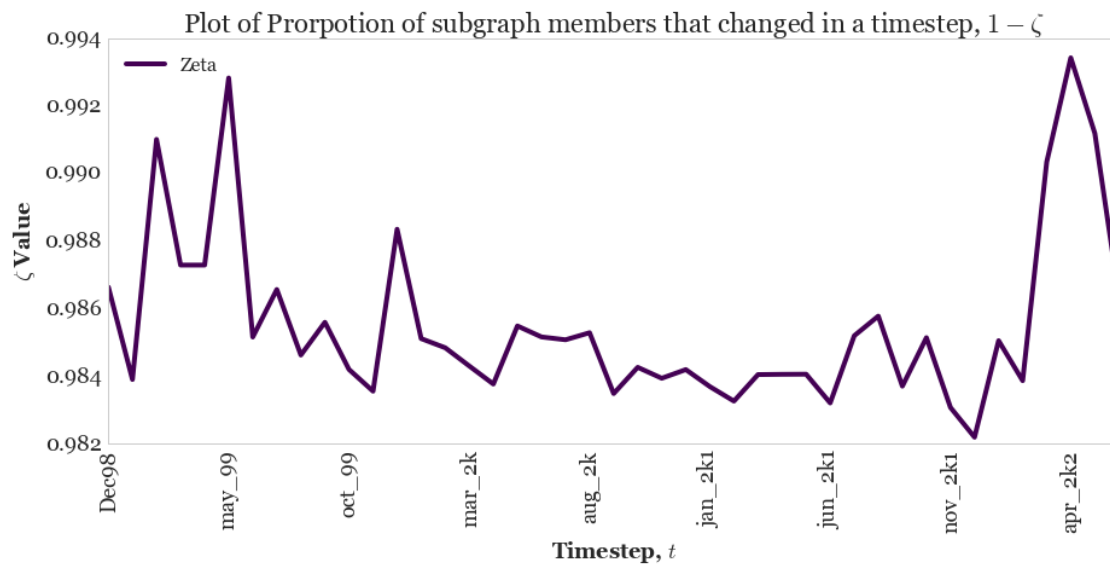
```
In [962]: zeta.pct_change().fillna(0).plot(fontsize=18, cmap='viridis')
plt.title("Plot of Pct Change in Subgraph Stationarity  $\zeta$ ", fontsize=18)
plt.xlabel("Timestep,  $t$ ", fontsize=18)
plt.ylabel(" $\zeta$  Value", fontsize=18)
plt.legend(fontsize=16, loc=2)
plt.xticks(fontsize=18)
plt.yticks(fontsize=18)
```

```
Out[962]: (array([-1. , -0.5,  0. ,  0.5,  1. ,  1.5]),
<a list of 6 Text yticklabel objects>)
```



```
In [978]: (1-zeta).plot(fontsize=18, cmap='viridis', rot =90)
plt.title("Plot of Prorportion of subgraph members that changed in a times
plt.xlabel("Timestep, $t$", fontsize=18)
plt.ylabel("$\zeta$ Value", fontsize=18)
plt.legend(fontsize=16, loc=2)
plt.xticks(fontsize=18)
plt.yticks(fontsize=18)

Out[978]: (array([ 0.98 ,  0.982,  0.984,  0.986,  0.988,  0.99 ,  0.992,  0.994]),
<a list of 8 Text yticklabel objects>)
```



```
In [988]: np.mean(np.correlate(std_klpca_ratio(G_nov98), std_klpca_ratio(G_dec98)))

Out[988]: -0.7282589594178992
```

```
In [990]: def avg_klpca_corr(net1,net2):
          avg = np.mean(np.correlate(std_klpca_ratio(net1),std_klpca_ratio(net2)))
          return avg
```

```
In [991]: avg_klpca_corr(G_nov98,G_dec98)
```

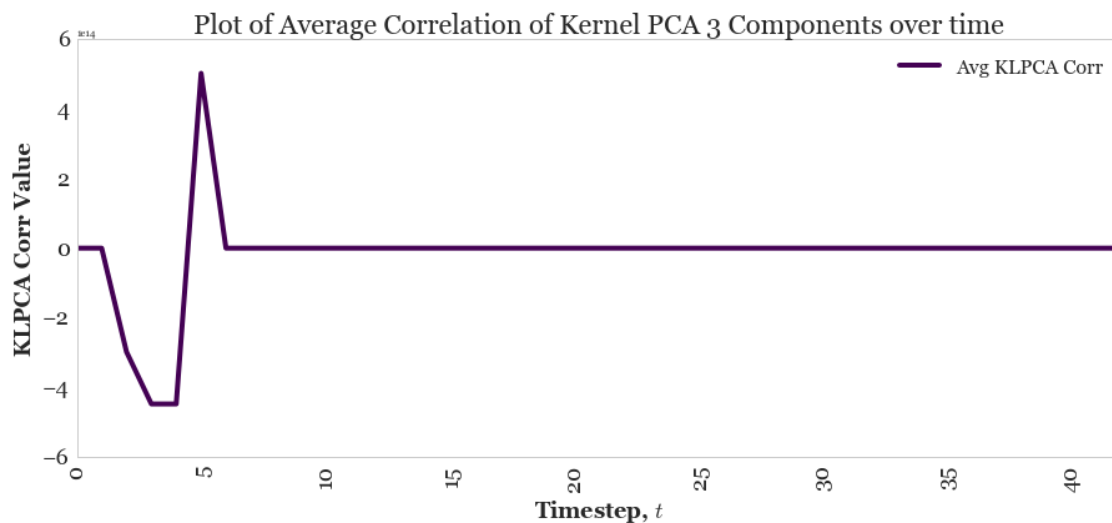
```
Out[991]: -0.7282589594178992
```

```
In [992]: avg_klpca3_corr = []
          for i in range(0, len(all_graphs)-1):
              x = int(i)
              y = x + 1
              avg_klpca3_corr.append(avg_klpca_corr(all_graphs[x], all_graphs[y]))
```

```
In [995]: klpca = pd.DataFrame(avg_klpca3_corr, columns=['Avg KLPCA Corr'])
```

```
In [998]: klpca.plot(fontsize=18, cmap='viridis', rot =90)
plt.title("Plot of Average Correlation of Kernel PCA 3 Components over ti
plt.xlabel("Timestep, $t$", fontsize=18)
plt.ylabel("KLPCA Corr Value", fontsize=18)
plt.legend(fontsize=16, loc=1)
plt.xticks(fontsize=18)
plt.yticks(fontsize=18)
```

```
Out[998]: (array([ -6.00000000e+14,  -4.00000000e+14,  -2.00000000e+14,
                    0.00000000e+00,   2.00000000e+14,   4.00000000e+14,
                    6.00000000e+14]), <a list of 7 Text yticklabel objects>)
```



```
In [1000]: klpca=klpca.T
klpca.columns = list(stat_all.T.columns)[1:]
klpca=klpca.T
klpca.head()
```

```
Out[1000]:
```

	Avg KLPCA Corr
Dec98	-7.282590e-01
jan_99	2.600181e+00
feb_99	-2.979981e+14
mar_99	-4.476561e+14
apr_99	-4.476561e+14

```
In [1004]: (klpca - klpca.mean())/klpca.std()
```

```
Out[1004]:
```

	Avg KLPCA Corr
Dec98	0.121836

jan_99	0.121836
feb_99	-2.137343
mar_99	-3.271929
apr_99	-3.271929
may_99	3.929607
jun_99	0.121836
jul_99	0.121836
aug_99	0.121836
sep_99	0.121836
oct_99	0.121836
nov_99	0.121836
dec_99	0.121836
jan_2k	0.121836
feb_2k	0.121836
mar_2k	0.121836
apr_2k	0.121836
may_2k	0.121836
jun_2k	0.121836
jul_2k	0.121836
aug_2k	0.121836
sep_2k	0.121836
oct_2k	0.121836
nov_2k	0.121836
dec_2k	0.121836
jan_2k1	0.121836
feb_2k1	0.121836
mar_2k1	0.121836
apr_2k1	0.121836
may_2k1	0.121836
jun_2k1	0.121836
jul_2k1	0.121836
aug_2k1	0.121836
sep_2k1	0.121836
oct_2k1	0.121836
nov_2k1	0.121836
dec_2k1	0.121836
jan_2k2	0.121836
feb_2k2	0.121836
mar_2k2	0.121836
apr_2k2	0.121836
may_2k2	0.121836
jun_2k2	0.121836

In [ ]: