DNA02

August 3, 2016

1 Table of Contents

2 Import Libraries

In [2]:
```
import pandas as pd
import numpy as np
import networkx as nx
import seaborn as sns
import matplotlib.pyplot as plt
import scipy as sc
import random
from scipy.signal import *
from numpy.linalg import *
#plotting parameters
%matplotlib inline
sns.set(style="whitegrid", color_codes=True, context='paper')
plt.rc('axes', grid=False, titlesize='large', labelsize='large',labelweight
plt.rc('lines', linewidth=4)
plt.rc('font',family='serif',size=16, serif='Georgia Pro')
plt.rc('figure', figsize = (15,6),titlesize='large',titleweight='heavy')
plt.rc('grid',linewidth=5)
from sklearn.decomposition import *
```

Set color palette

In [3]:
```
sns.palplot(sns.husl_palette(20, h=0.8,l=0.5))
```

In [4]:
```
sns.set_palette(sns.husl_palette(20, h=0.8,l=0.5))
```

3 Helper Functions

In [5]:
```
def get_val(val):
    return sorted(set(val.values()))
```

In [6]:
```
def avg_cent(cent):
    avg = sum(set(cent.values()))/len(cent)
    return avg
```

In [7]:
```
def get_cent(net):
    degC = nx.degree_centrality(net)
    cloC = nx.closeness_centrality(net)
    betC = nx.betweenness_centrality(net)
    eigC = nx.eigenvector_centrality_numpy(net)
    katzC = nx.katz_centrality_numpy(net)
    loadC = nx.load_centrality(net)

    #cent_df = pd.Data

    return [degC,cloC,betC,eigC,katzC, loadC]
```

In [8]:
```
def stationarity_ratio(G):
    #stationarity ratio with laplian
    L = nx.normalized_laplacian_matrix(G).todense()
    U = eigvals(L)
    C =np.cov(L)
    CF = np.dot(L,np.dot(np.dot(U.T,C),U))
    r = np.linalg.norm(np.diag(CF))/np.linalg.norm(CF)

    return [r]
```

In [9]:
```
#cite:`klein1993resistance`
def resistance_distance(net):
    M = nx.normalized_laplacian_matrix(net).todense()
    pseudo = pinv(M)
    N = M.shape[0]
    d = np.diag(pseudo)
    rd  = np.kron(d,np.ones((N,1))).T+np.kron(d,np.ones((N,1))).T - pseudo

    return [rd, rd.mean()]
```

In [10]:
```
def calc_seisatt(net):
    M = nx.normalized_laplacian_matrix(net).todense()
    Ht = hilbert(M)

    #Basic attributes IA, IP, IF
    IA = np.real(np.nan_to_num(np.sqrt(np.dot(M,M)+np.dot(Ht,Ht))))
    IP = np.real(np.nan_to_num(np.arctan(Ht/M)))
    IF,_ = np.real(np.nan_to_num(np.asarray(np.gradient(IP))))
```

```
        #Derivatives
        dIA,_ = np.nan_to_num(np.asarray(np.gradient(IA)))
        d2IA,_ = np.nan_to_num(np.asarray(np.gradient(dIA)))
        IAcc,_ =np.nan_to_num(np.asarray(np.gradient(IF)))

        #Derived Attributes
        cosIP = np.cos(IP)
        IA_weit_IF = IA * IF
        IA_weit_IP = IA*IP

        att_globalval = pd.DataFrame([IA.mean(),IP.mean(),IF.mean(), dIA.mean(
                                      IAcc.mean(),cosIP.mean(),IA_weit_IF.mean
        att_globalval.columns =['InstAmp','InstPhase','InstFreq','dInstAmp','d
                                'IAwIP']

        return [IA,IP,IF,dIA,d2IA,IAcc,cosIP, IA_weit_IF,IA_weit_IP, att_globa
```

In [11]:
```
def curvature(net):
    from skimage.feature import hessian_matrix, hessian_matrix_det, hessia
    M = nx.normalized_laplacian_matrix(net).todense()
    M = np.float64(M)
    fx, fy = np.gradient(M)
    Hxx, Hxy, Hyy = hessian_matrix(M)
    K = np.divide((np.dot(Hxx,Hxy)-np.dot(Hxy,Hxy)),\
                  (1+np.dot(fx,fx)+np.dot(fy,fy)))

    He1,_ = hessian_matrix_eigvals(Hxx,Hxy,Hyy)
    mean_curv = np.trace(He1)
    s, a = np.linalg.slogdet(He1)
    conc = s * np.exp(a)
    Pmax = np.max(He1)
    Pmin = np.min(He1)

    return [K,mean_curv]
```

In [12]:
```
def cal_avgstat(net):
    #calculate all attributes from previously defined functions here
    degC,cloC,betC,eigC,katzC, loadC = get_cent(net)
    _,meanK = curvature(net)
    _,_,_,_,_,_,_,_,_, att_globalval = calc_seisatt(net)
    _,norm_rd = resistance_distance(net)
    r = stationarity_ratio(net)
    den = nx.density(net)
    clustcof = nx.average_clustering(net)
    algcon = nx.algebraic_connectivity(net)

    #create attribute volume here
```

```
        stat_df = pd.DataFrame([avg_cent(degC),avg_cent(cloC), avg_cent(betC),
                avg_cent(katzC), avg_cent(loadC), den, clustcof,algcon, meanK,
        stat_df.columns= ['AvgDeg','AvgCloseness','AvgBet','AvgEig','AvgKatz',
                          'AlgebraicConnect','MeanKurv',]
        stat_df = stat_df.join(att_globalval)
        stat_df['MeanResistanceDist']= norm_rd
        stat_df['StatRat']=r


        return stat_df

In [13]: def get_top_keys(dictionary, top):
        items = dictionary.items()
        items = sorted(items,reverse=True, key=lambda x: x[1])
        obj = map(lambda x: x[0], items[:top])
        for i in obj:
            print(i)
        #return map(lambda x: x[0], items[:top])

In [14]: def create_graph(df):
        tmp = df.values[:,1:3]
        G= nx.Graph()
        G = nx.from_edgelist(tmp)

        return G

In [15]: def rms(a, axis=None):
        from numpy import mean, sqrt,square
        rms = sqrt(mean(square(a), axis=axis))
        return rms

    def nrms(a,b):
        nrms = 200*rms(a-b)/(rms(a)+ rms(b))
        return nrms

In [16]: def get_centhistval(cent_dict, title=None):
        from collections import Counter
        vals = list(Counter(cent_dict.values()))
        freq = list(Counter(cent_dict.values()).values())
        plt.figure()
        pd.DataFrame.from_dict(cent_dict, orient='index').plot(kind='hist', le
        #sns.distplot(vals,hist=True,kde=False, color='#6F4E37')
        plt.title("Distribution of " + str(title), fontsize=20)
        plt.xlabel("Centrality Value", fontsize=18)
        plt.ylabel("No. of Nodes", fontsize=18)
        plt.xticks(fontsize=18)
        plt.yticks(fontsize=18)
        plt.tight_layout()
```

```
            plt.show()

In [17]: def pad_shape(x,ref,offset=0):
            result = np.zeros_like(ref)
            result[0:x.shape[0]+0,0:x.shape[1]+0] = x

            return result

In [18]: def subgraph_stat(net1,net2):
            net1_int_net2 = net1.copy()
            net1_int_net2.remove_nodes_from(n for n in net1 if n not in net2)
            net1_u_net2 = nx.disjoint_union(net1, net2)
            int_adjmat = nx.adjacency_matrix(net1_int_net2).todense()
            uni_adjmat = nx.adjacency_matrix(net1_u_net2).todense()
            int_adjmat_pad = pad_shape(int_adjmat,uni_adjmat)

            Ct = np.divide(norm(int_adjmat_pad),norm(uni_adjmat))

            return Ct

In [19]: def fk_plot(f,name):
            freq = sc.fft(f)
            wavnum = 1/freq

            plt.scatter(freq, wavnum, s=60, marker='^', c='k')
            plt.title("F-K Plot of "+str(name), fontsize=18)
            plt.xticks(fontsize=14)
            plt.yticks(fontsize=14)
            plt.xlabel("Wavenumber,k", fontsize=18)
            plt.ylabel("Frequency,f", fontsize=18)

            plt.show()

            return [freq,wavnum]

In [20]: def plot_radon(m, name):
            from skimage.transform import radon
            theta = np.linspace(0., 180., max(m.shape), endpoint=False)
            sinogram = radon(m, theta=theta, circle=True)

            fig, ax = plt.subplots(1,2)

            ax[0].scatter(sinogram[0], sinogram[1], s=60, marker='^', c='k')
            ax[0].set_title("Radon Transform Plot of "+str(name), fontsize=14)


            ax[1].scatter(1/sinogram[0], sinogram[1], s=60, marker='^', c='r')
            ax[1].set_title("1/Radon[0] vs Radon[1] Plot of "+str(name), fontsize=
```

```
        plt.show()

        return sinogram
```

4 Data Import

```
In [21]: data = pd.read_excel("../Data/data 03.2.xlsx")

In [22]: data.head()
```

Out[22]:

	timestamp	to	from	year	month
0	1979-12-31 21:00:00	24	153	1979	12
1	1979-12-31 21:00:00	24	153	1979	12
2	1979-12-31 21:00:00	29	29	1979	12
3	1979-12-31 21:00:00	29	29	1979	12
4	1979-12-31 21:00:00	29	29	1979	12

```
In [23]: years = sorted(set(data.year))
         years
```

```
Out[23]: [1979, 1998, 1999, 2000, 2001, 2002]

In [24]: #total % of mislabelled 1979 entries
         (data[data.year==1979].count()/data.shape[0]) * 100
```

```
Out[24]: timestamp    0.138746
         to           0.138746
         from         0.138746
         year         0.138746
         month        0.138746
         dtype: float64
```

4.1 Split into years

```
In [25]: df_98 = data[data.year==years[1]]
         df_99 = data[data.year==years[2]]
         df_2k = data[data.year==years[3]]
         df_2k1 = data[data.year==years[4]]
         df_2k2 = data[data.year==years[5]]

In [26]: df_2k.head()
```

Out[26]:

	timestamp	to	from	year	month
3971	2000-01-03 06:47:00	82	51	2000	1
3972	2000-01-03 06:47:00	82	51	2000	1
3973	2000-01-03 06:47:00	82	51	2000	1
3974	2000-01-03 06:47:00	82	51	2000	1
3975	2000-01-03 06:47:00	82	51	2000	1

4.2 Create yearly graphs

```
In [27]: Gt0 = create_graph(df_98)
         Gt1 = create_graph(df_99)
         Gt2 = create_graph(df_2k)
         Gt3 = create_graph(df_2k1)
         Gt4 = create_graph(df_2k2)
```

4.3 Split into months and create monthly graphs

```
In [28]: nov_98 = df_98[df_98.month==11]
         dec_98= df_98[df_98.month==12]

         G_nov98 = create_graph(nov_98)
         G_dec98 = create_graph(dec_98)
```

```
In [29]: jan_99=df_99[df_99.month==1]
         feb_99=df_99[df_99.month==2]
         mar_99=df_99[df_99.month==3]
         apr_99=df_99[df_99.month==4]
         may_99=df_99[df_99.month==5]
         jun_99=df_99[df_99.month==6]
         jul_99=df_99[df_99.month==7]
         aug_99=df_99[df_99.month==8]
         sep_99=df_99[df_99.month==9]
         oct_99=df_99[df_99.month==10]
         nov_99=df_99[df_99.month==11]
         dec_99=df_99[df_99.month==12]


         G_jan_99=create_graph(jan_99)
         G_feb_99=create_graph(feb_99)
         G_mar_99=create_graph(mar_99)
         G_apr_99=create_graph(apr_99)
         G_may_99=create_graph(may_99)
         G_jun_99=create_graph(jun_99)
         G_jul_99=create_graph(jul_99)
         G_aug_99=create_graph(aug_99)
         G_sep_99=create_graph(sep_99)
         G_oct_99=create_graph(oct_99)
         G_nov_99=create_graph(nov_99)
         G_dec_99=create_graph(dec_99)
```

```
In [30]: jan_2k=df_2k[df_2k.month==1]
         feb_2k=df_2k[df_2k.month==2]
         mar_2k=df_2k[df_2k.month==3]
         apr_2k=df_2k[df_2k.month==4]
         may_2k=df_2k[df_2k.month==5]
         jun_2k=df_2k[df_2k.month==6]
```

```
jul_2k=df_2k[df_2k.month==7]
aug_2k=df_2k[df_2k.month==8]
sep_2k=df_2k[df_2k.month==9]
oct_2k=df_2k[df_2k.month==10]
nov_2k=df_2k[df_2k.month==11]
dec_2k=df_2k[df_2k.month==12]


G_jan_2k=create_graph(jan_2k)
G_feb_2k=create_graph(feb_2k)
G_mar_2k=create_graph(mar_2k)
G_apr_2k=create_graph(apr_2k)
G_may_2k=create_graph(may_2k)
G_jun_2k=create_graph(jun_2k)
G_jul_2k=create_graph(jul_2k)
G_aug_2k=create_graph(aug_2k)
G_sep_2k=create_graph(sep_2k)
G_oct_2k=create_graph(oct_2k)
G_nov_2k=create_graph(nov_2k)
G_dec_2k=create_graph(dec_2k)
```

In [31]:
```
jan_2k1=df_2k1[df_2k1.month==1]
feb_2k1=df_2k1[df_2k1.month==2]
mar_2k1=df_2k1[df_2k1.month==3]
apr_2k1=df_2k1[df_2k1.month==4]
may_2k1=df_2k1[df_2k1.month==5]
jun_2k1=df_2k1[df_2k1.month==6]
jul_2k1=df_2k1[df_2k1.month==7]
aug_2k1=df_2k1[df_2k1.month==8]
sep_2k1=df_2k1[df_2k1.month==9]
oct_2k1=df_2k1[df_2k1.month==10]
nov_2k1=df_2k1[df_2k1.month==11]
dec_2k1=df_2k1[df_2k1.month==12]


G_jan_2k1=create_graph(jan_2k1)
G_feb_2k1=create_graph(feb_2k1)
G_mar_2k1=create_graph(mar_2k1)
G_apr_2k1=create_graph(apr_2k1)
G_may_2k1=create_graph(may_2k1)
G_jun_2k1=create_graph(jun_2k1)
G_jul_2k1=create_graph(jul_2k1)
G_aug_2k1=create_graph(aug_2k1)
G_sep_2k1=create_graph(sep_2k1)
G_oct_2k1=create_graph(oct_2k1)
G_nov_2k1=create_graph(nov_2k1)
G_dec_2k1=create_graph(dec_2k1)
```

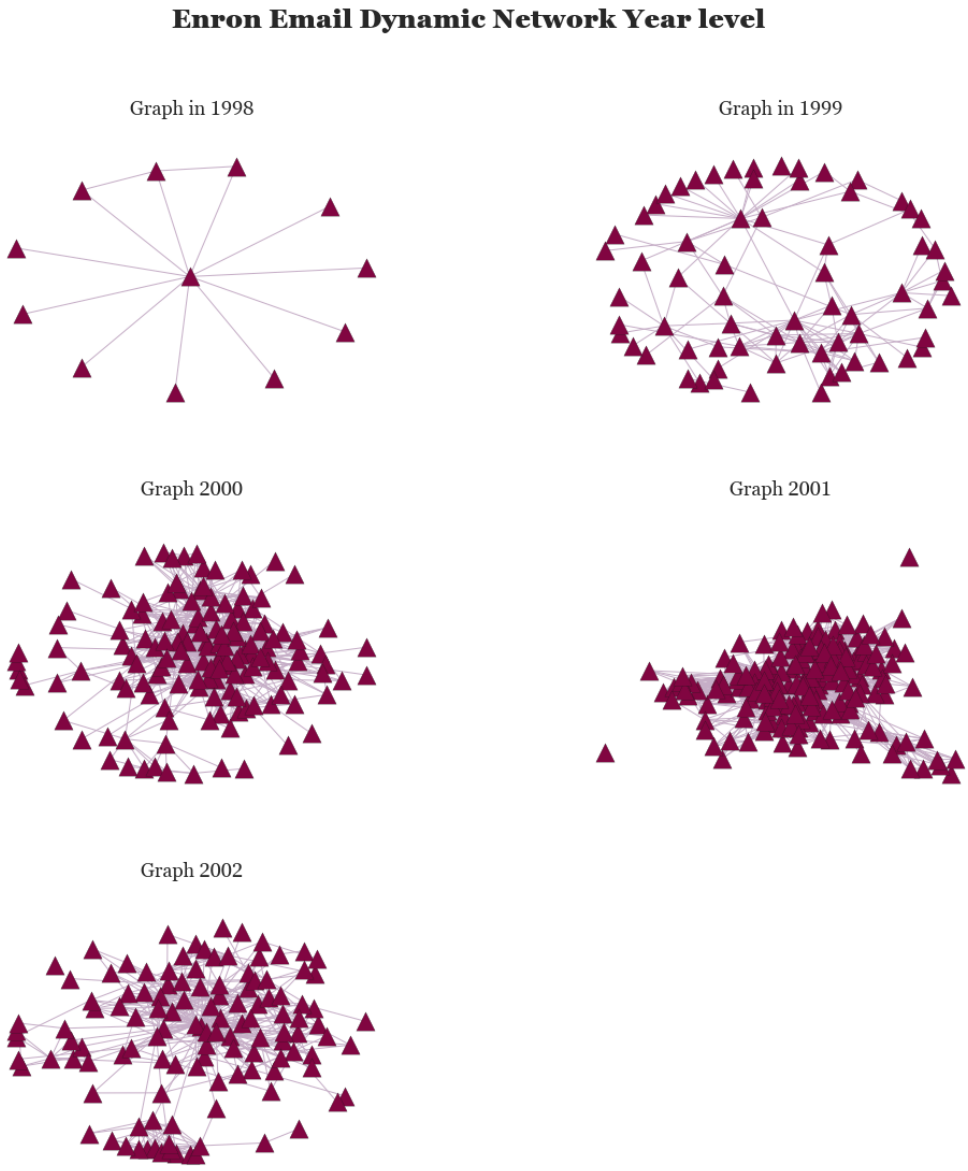
In [32]:
```
jan_2k2=df_2k2[df_2k2.month==1]
```

```
feb_2k2=df_2k2[df_2k2.month==2]
mar_2k2=df_2k2[df_2k2.month==3]
apr_2k2=df_2k2[df_2k2.month==4]
may_2k2=df_2k2[df_2k2.month==5]
jun_2k2=df_2k2[df_2k2.month==6]
jul_2k2=df_2k2[df_2k2.month==7]
aug_2k2=df_2k2[df_2k2.month==8]
sep_2k2=df_2k2[df_2k2.month==9]
oct_2k2=df_2k2[df_2k2.month==10]
nov_2k2=df_2k2[df_2k2.month==11]
dec_2k2=df_2k2[df_2k2.month==12]


G_jan_2k2=create_graph(jan_2k2)
G_feb_2k2=create_graph(feb_2k2)
G_mar_2k2=create_graph(mar_2k2)
G_apr_2k2=create_graph(apr_2k2)
G_may_2k2=create_graph(may_2k2)
G_jun_2k2=create_graph(jun_2k2)
G_jul_2k2=create_graph(jul_2k2)
G_aug_2k2=create_graph(aug_2k2)
G_sep_2k2=create_graph(sep_2k2)
G_oct_2k2=create_graph(oct_2k2)
G_nov_2k2=create_graph(nov_2k2)
G_dec_2k2=create_graph(dec_2k2)
```

4.4 Visualise Networks

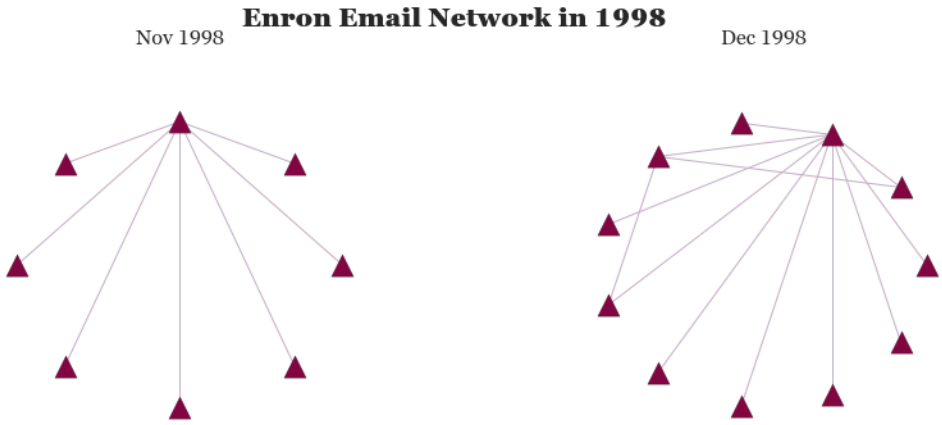
```
In [33]: plt.figure(figsize=(18,18))
         plt.suptitle('Enron Email Dynamic Network Year level', fontsize=24)
         plt.subplot(321)
         nx.draw_spring(Gt0,  node_color='#810541',node_shape='^',edge_color ='#C6A
         plt.title("Graph in 1998", fontsize=18)

         plt.subplot(322)
         nx.draw_spring(Gt1,  node_color='#810541',node_shape='^',edge_color ='#C6A
         plt.title("Graph in 1999", fontsize=18)

         plt.subplot(323)
         nx.draw_spring(Gt2,  node_color='#810541',node_shape='^',edge_color ='#C6A
         plt.title("Graph 2000", fontsize=18)

         plt.subplot(324)
         nx.draw_spring(Gt3,  node_color='#810541',node_shape='^',edge_color ='#C6A
         plt.title("Graph 2001", fontsize=18)

         plt.subplot(325)
         nx.draw_spring(Gt4,  node_color='#810541',node_shape='^',edge_color ='#C6A
```

```
plt.title("Graph 2002", fontsize=18)

plt.show()
```



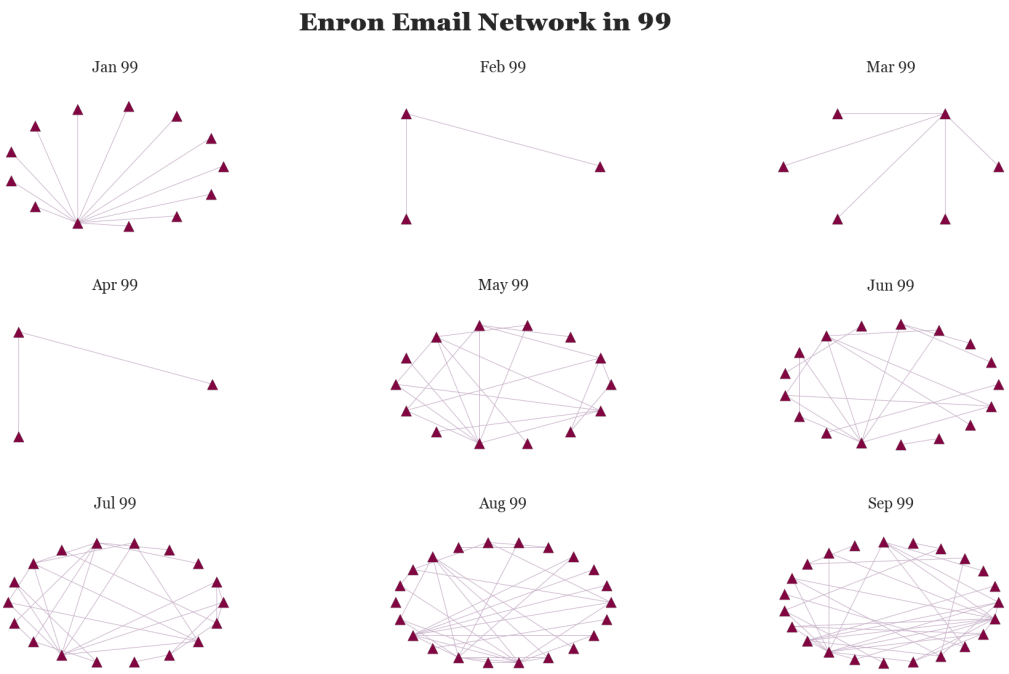
In [34]:
```
plt.figure()
plt.suptitle("Enron Email Network in 1998", fontsize=20)
plt.subplot(121)
plt.title("Nov 1998", fontsize=16)
nx.draw_circular(G_nov98, node_color='#810541',node_shape='^',edge_color =
plt.subplot(122)
```

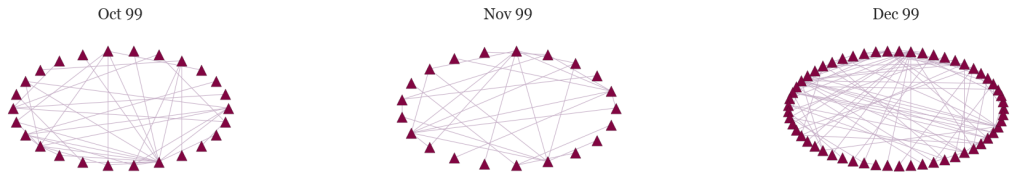
```
plt.title("Dec 1998", fontsize=16)
nx.draw_circular(G_dec98, node_color='#810541',node_shape='^',edge_color =
```



In [35]:
```
plt.figure(figsize=(32,18))
plt.suptitle("Enron Email Network in 99", fontsize=40)

plt.subplot(331)
plt.title("Jan 99", fontsize=25)
nx.draw_circular(G_jan_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(332)
plt.title("Feb 99", fontsize=25)
nx.draw_circular(G_feb_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(333)
plt.title("Mar 99", fontsize=25)
nx.draw_circular(G_mar_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(334)
plt.title("Apr 99", fontsize=25)
nx.draw_circular(G_apr_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(335)
plt.title("May 99", fontsize=25)
nx.draw_circular(G_may_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(336)
plt.title("Jun 99", fontsize=25)
nx.draw_circular(G_jun_99, node_color='#810541',node_shape='^',edge_color
```

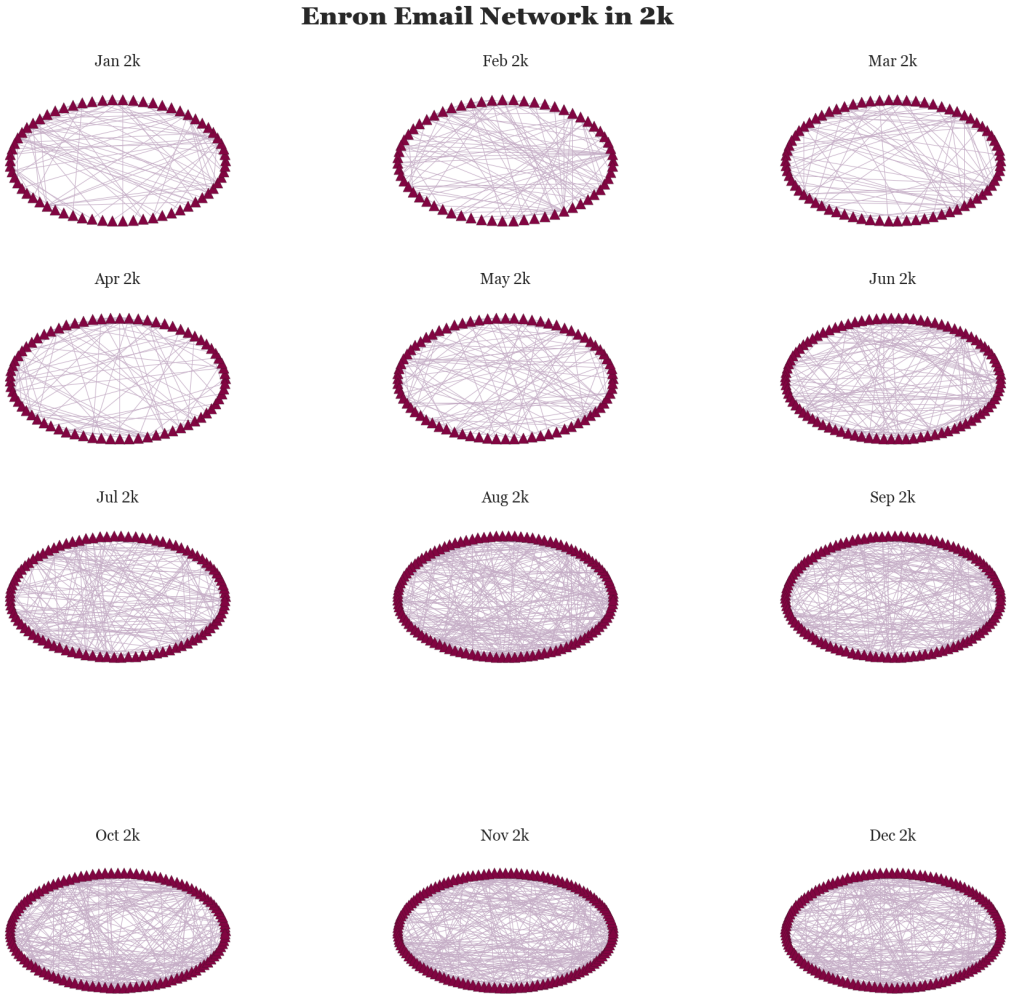
```
plt.subplot(337)
plt.title("Jul 99", fontsize=25)
nx.draw_circular(G_jul_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(338)
plt.title("Aug 99", fontsize=25)
nx.draw_circular(G_aug_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(339)
plt.title("Sep 99", fontsize=25)
nx.draw_circular(G_sep_99, node_color='#810541',node_shape='^',edge_color

plt.figure(figsize=(32,5))
#plt.suptitle("Enron Email Network in 99", fontsize=40)

plt.subplot(131)
plt.title("Oct 99", fontsize=25)
nx.draw_circular(G_oct_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(132)
plt.title("Nov 99", fontsize=25)
nx.draw_circular(G_nov_99, node_color='#810541',node_shape='^',edge_color

plt.subplot(133)
plt.title("Dec 99", fontsize=25)
nx.draw_circular(G_dec_99, node_color='#810541',node_shape='^',edge_color
```





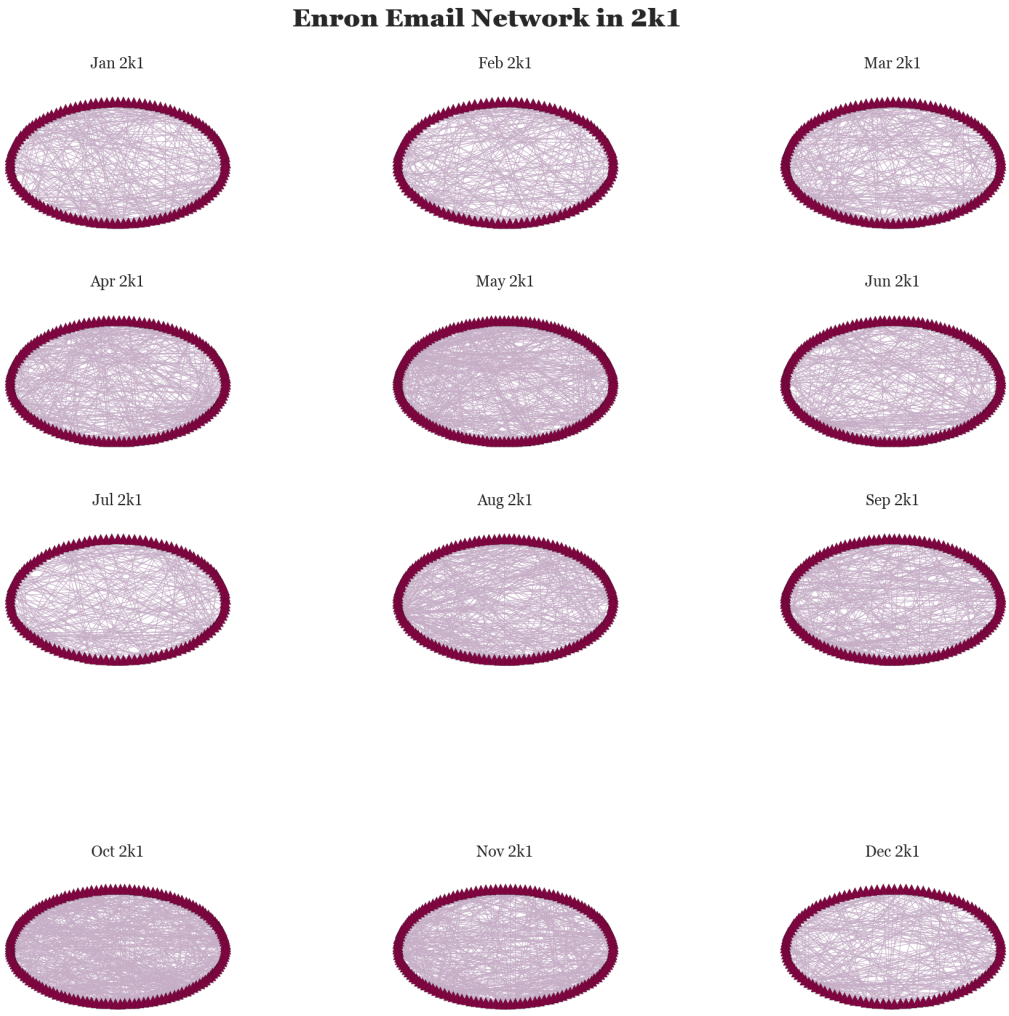
In [36]:
```
plt.figure(figsize=(32,18))
plt.suptitle("Enron Email Network in 2k", fontsize=40)

plt.subplot(331)
plt.title("Jan 2k", fontsize=25)
nx.draw_circular(G_jan_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(332)
plt.title("Feb 2k", fontsize=25)
nx.draw_circular(G_feb_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(333)
plt.title("Mar 2k", fontsize=25)
nx.draw_circular(G_mar_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(334)
plt.title("Apr 2k", fontsize=25)
nx.draw_circular(G_apr_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(335)
plt.title("May 2k", fontsize=25)
nx.draw_circular(G_may_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(336)
plt.title("Jun 2k", fontsize=25)
nx.draw_circular(G_jun_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(337)
plt.title("Jul 2k", fontsize=25)
nx.draw_circular(G_jul_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(338)
plt.title("Aug 2k", fontsize=25)
nx.draw_circular(G_aug_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(339)
plt.title("Sep 2k", fontsize=25)
```

```
nx.draw_circular(G_sep_2k, node_color='#810541',node_shape='^',edge_color

plt.figure(figsize=(32,5))
#plt.suptitle("Enron Email Network in 2k", fontsize=40)

plt.subplot(131)
plt.title("Oct 2k", fontsize=25)
nx.draw_circular(G_oct_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(132)
plt.title("Nov 2k", fontsize=25)
nx.draw_circular(G_nov_2k, node_color='#810541',node_shape='^',edge_color

plt.subplot(133)
plt.title("Dec 2k", fontsize=25)
nx.draw_circular(G_dec_2k, node_color='#810541',node_shape='^',edge_color
```



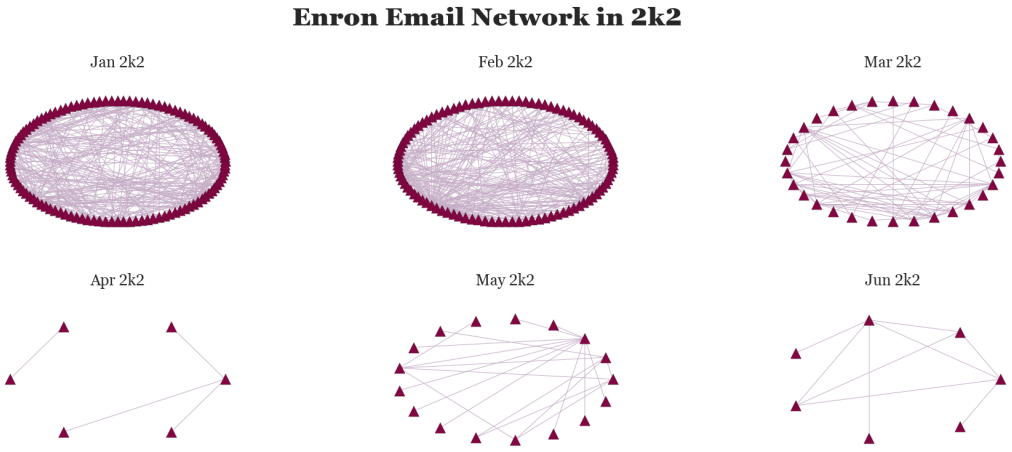
In [37]:
```
plt.figure(figsize=(32,18))
plt.suptitle("Enron Email Network in 2k1", fontsize=40)

plt.subplot(331)
plt.title("Jan 2k1", fontsize=25)
nx.draw_circular(G_jan_2k1, node_color='#810541',node_shape='^',edge_color

plt.subplot(332)
plt.title("Feb 2k1", fontsize=25)
nx.draw_circular(G_feb_2k1, node_color='#810541',node_shape='^',edge_color

plt.subplot(333)
plt.title("Mar 2k1", fontsize=25)
nx.draw_circular(G_mar_2k1, node_color='#810541',node_shape='^',edge_color

plt.subplot(334)
plt.title("Apr 2k1", fontsize=25)
nx.draw_circular(G_apr_2k1, node_color='#810541',node_shape='^',edge_color

plt.subplot(335)
plt.title("May 2k1", fontsize=25)
nx.draw_circular(G_may_2k1, node_color='#810541',node_shape='^',edge_color

plt.subplot(336)
plt.title("Jun 2k1", fontsize=25)
nx.draw_circular(G_jun_2k1, node_color='#810541',node_shape='^',edge_color

plt.subplot(337)
plt.title("Jul 2k1", fontsize=25)
nx.draw_circular(G_jul_2k1, node_color='#810541',node_shape='^',edge_color

plt.subplot(338)
plt.title("Aug 2k1", fontsize=25)
nx.draw_circular(G_aug_2k1, node_color='#810541',node_shape='^',edge_color

plt.subplot(339)
plt.title("Sep 2k1", fontsize=25)
nx.draw_circular(G_sep_2k1, node_color='#810541',node_shape='^',edge_color

plt.figure(figsize=(32,5))
#plt.suptitle("Enron Email Network in 2k1", fontsize=40)

plt.subplot(131)
plt.title("Oct 2k1", fontsize=25)
nx.draw_circular(G_oct_2k1, node_color='#810541',node_shape='^',edge_color

plt.subplot(132)
plt.title("Nov 2k1", fontsize=25)
```

```
nx.draw_circular(G_nov_2k1, node_color='#810541',node_shape='^',edge_color

plt.subplot(133)
plt.title("Dec 2k1", fontsize=25)
nx.draw_circular(G_dec_2k1, node_color='#810541',node_shape='^',edge_color
```



In [38]:
```
plt.figure(figsize=(32,18))
plt.suptitle("Enron Email Network in 2k2", fontsize=40)

plt.subplot(331)
plt.title("Jan 2k2", fontsize=25)
nx.draw_circular(G_jan_2k2, node_color='#810541',node_shape='^',edge_color

plt.subplot(332)
plt.title("Feb 2k2", fontsize=25)
```

```
nx.draw_circular(G_feb_2k2, node_color='#810541',node_shape='^',edge_color

plt.subplot(333)
plt.title("Mar 2k2", fontsize=25)
nx.draw_circular(G_mar_2k2, node_color='#810541',node_shape='^',edge_color

plt.subplot(334)
plt.title("Apr 2k2", fontsize=25)
nx.draw_circular(G_apr_2k2, node_color='#810541',node_shape='^',edge_color

plt.subplot(335)
plt.title("May 2k2", fontsize=25)
nx.draw_circular(G_may_2k2, node_color='#810541',node_shape='^',edge_color

plt.subplot(336)
plt.title("Jun 2k2", fontsize=25)
nx.draw_circular(G_jun_2k2, node_color='#810541',node_shape='^',edge_color
```



5 Centrality and Assortativity Analysis

We look at some traditional network measures in this section for the networks both at the year and monthly level.

Analysis Methodology

1. Analyse network at the yearly level using centrality measures and other measures
   - Find top 5 nodes for each centrality measure across the years
2. Analyse network across years using averaged measures
3. Analyse network in monthly intervals using average centrality and other measures
4. Compare yearly and monthly trends

The specific measures used are:

Degree Centrality

Degree Centrality is defined as the number of links incident upon a node (i.e., the number of ties that a node has). The degree can be interpreted in terms of the immediate risk of a node for catching whatever is flowing through the network (such as a virus, or some information).

Closeness Centrality

The farness of a node x is defined as the sum of its distances from all other nodes, and its closeness was defined by the reciprocal of the farness.

Betweenness Centrality

Betweenness centrality quantifies the number of times a node acts as a bridge along the shortest path between two other nodes

Eigenvector Centrality

Eigenvector centrality (also called eigencentrality) is a measure of the influence of a node in a network. It assigns relative scores to all nodes in the network based on the concept that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes.

Katz Centrality

Katz centrality measures the number of all nodes that can be connected through a path, while the contributions of distant nodes are penalized.

Load Centrality

The load centrality of a node is the fraction of all shortest paths that pass through that node.

Density

The ratio of the number of edges and the number of possible edges.

Clustering Coefficient

The local clustering of each node in a graph is the fraction of triangles that actually exist over all possible triangles in its neighborhood. The average clustering coefficient of a graph is the mean of local clusterings.

Algebraic Connectivity

By the definition of algebraic connectivity, the second smallest eigenvalue of the graph Laplacian is a lower bound on the node connectivity. Increases in algebraic connectivity actually correspond to a decrease in node connectivity. This means that the networks are actually less robust with respect to node-connectivity as the algebraic connectivity increases.

References:

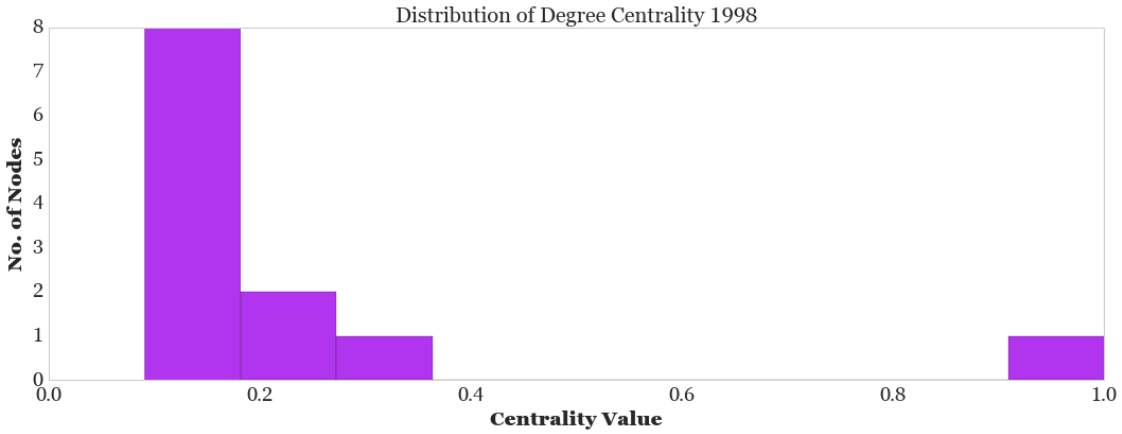
1. Centrality
2. Algebraic Connectivity

5.1 Centrality Analysis: Year

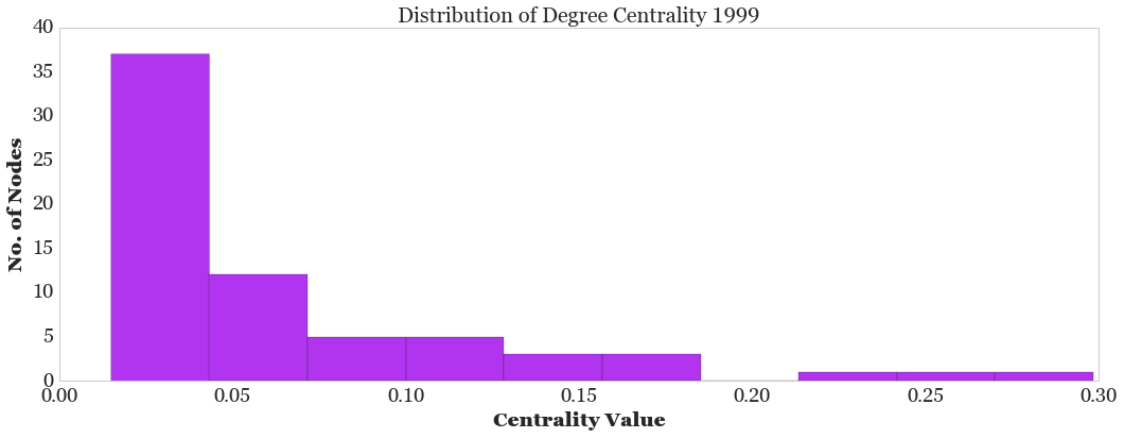
```
In [39]: degC0, cloC0, betC0, eigC0,  katzC0, loadC0 = get_cent(Gt0)
         degC1, cloC1, betC1, eigC1,  katzC1, loadC1 = get_cent(Gt1)
         degC2, cloC2, betC2, eigC2,  katzC2, loadC2 = get_cent(Gt2)
         degC3, cloC3, betC3, eigC3,  katzC3, loadC3 = get_cent(Gt3)
         degC4, cloC4, betC4, eigC4,  katzC4, loadC4 = get_cent(Gt4)

In [40]: get_centhistval(degC0,'Degree Centrality 1998')
         get_centhistval(degC1,'Degree Centrality 1999')
         get_centhistval(degC2,'Degree Centrality 2000')
```

```
get_centhistval(degC3,'Degree Centrality 2001')
get_centhistval(degC4,'Degree Centrality 2002')
```

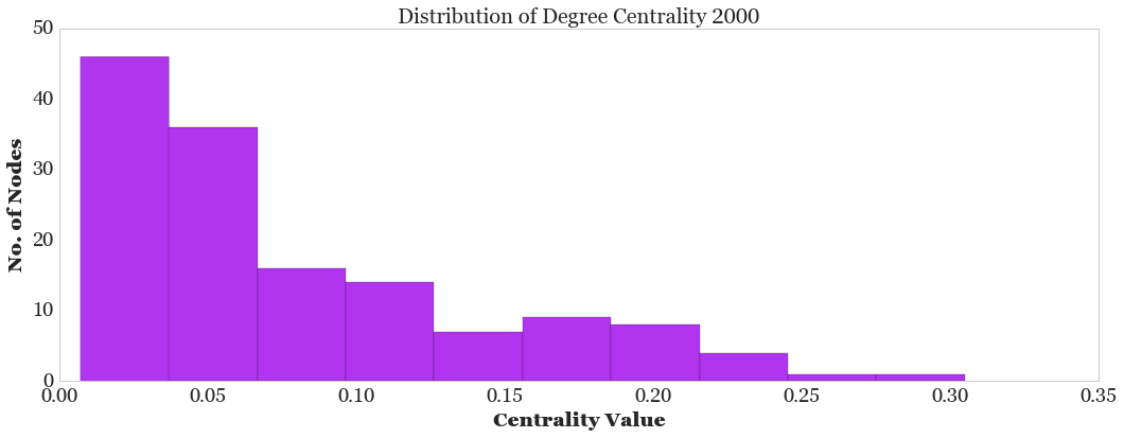
```
<matplotlib.figure.Figure at 0x29c4eb31e80>
```



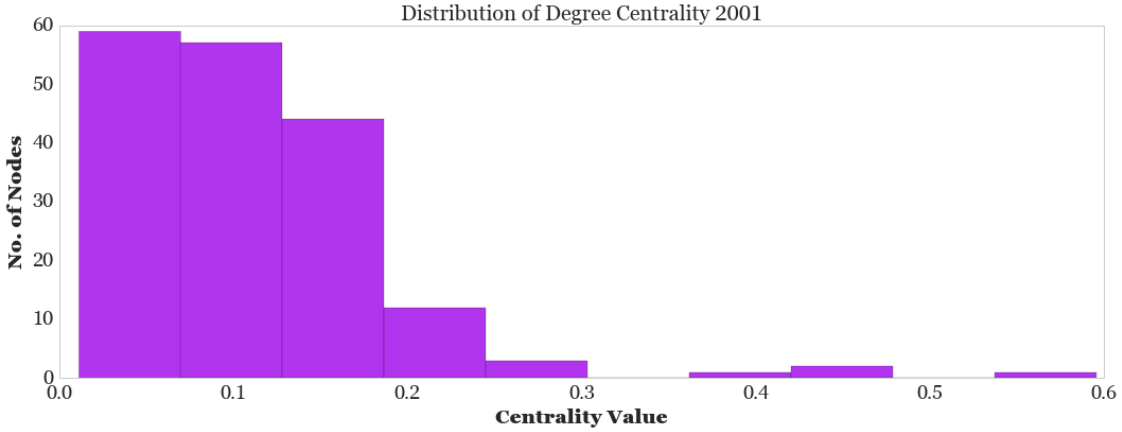

```
<matplotlib.figure.Figure at 0x29c4e4b9438>
```



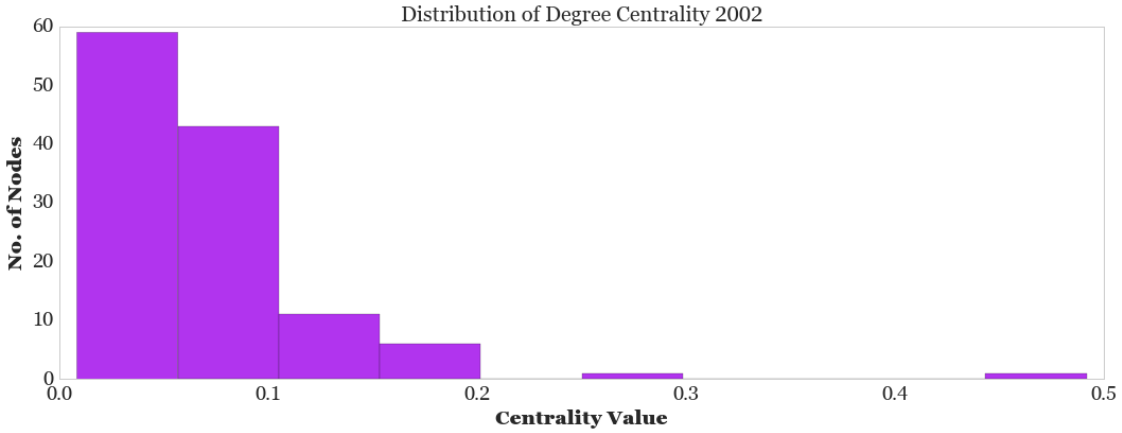

```
<matplotlib.figure.Figure at 0x29c4c967e48>
```




```
<matplotlib.figure.Figure at 0x29c4e4c95c0>
```



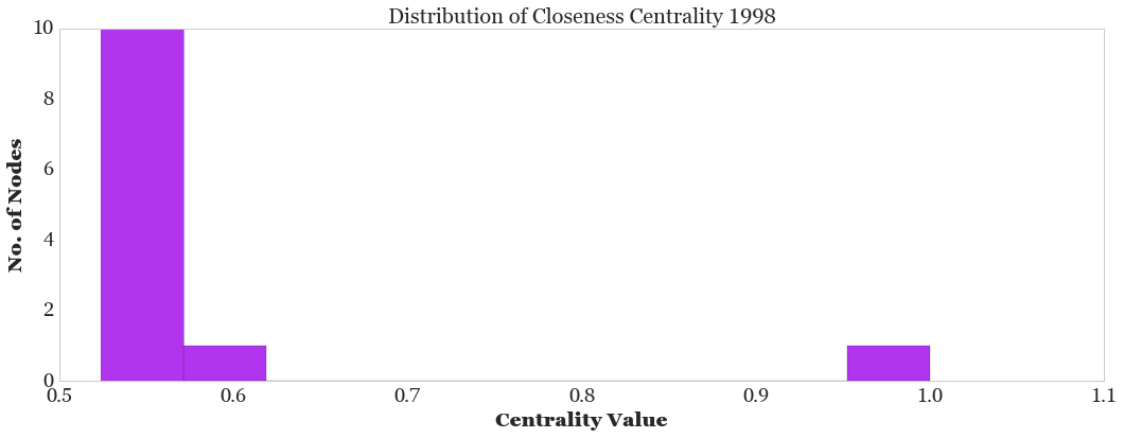

```
<matplotlib.figure.Figure at 0x29c4b5396d8>
```



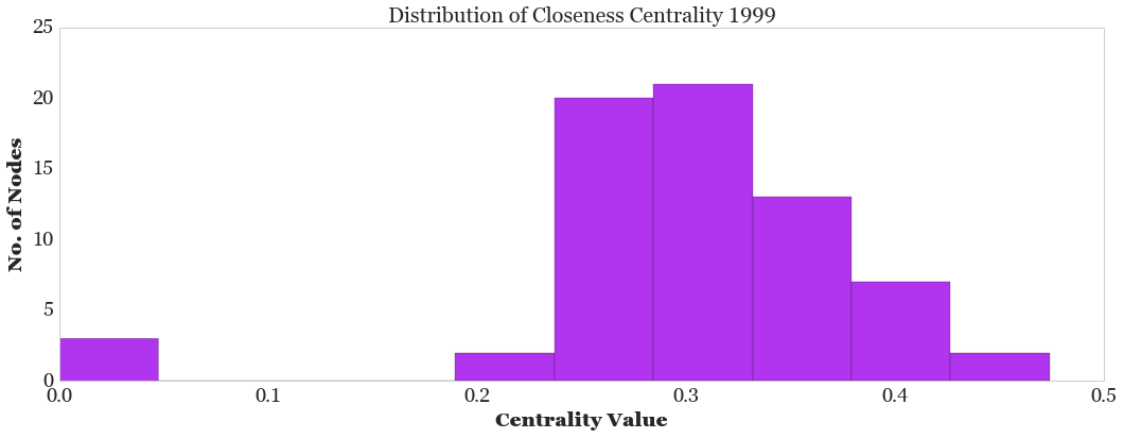
```
In [41]: print("Top 5 Degree Centrality Nodes year 98")
         get_top_keys(degC0,5)
         print("Top 5 Degree Centrality Nodes year 99")
         get_top_keys(degC1,5)
         print("Top 5 Degree Centrality Nodes year 2k")
         get_top_keys(degC2,5)
         print("Top 5 Degree Centrality Nodes year 2k1")
         get_top_keys(degC3,5)
         print("Top 5 Degree Centrality Nodes year 2k2")
         get_top_keys(degC4,5)

Top 5 Degree Centrality Nodes year 98
114
112
65
145
160
Top 5 Degree Centrality Nodes year 99
153
169
114
155
38
Top 5 Degree Centrality Nodes year 2k
82
27
140
157
78
Top 5 Degree Centrality Nodes year 2k1
82
153
107
94
126
Top 5 Degree Centrality Nodes year 2k2
105
107
82
90
159


In [42]: get_centhistval(cloC0,'Closeness Centrality 1998')
         get_centhistval(cloC1,'Closeness Centrality 1999')
```

```
        get_centhistval(cloC2,'Closeness Centrality 2000')
        get_centhistval(cloC3,'Closeness Centrality 2001')
        get_centhistval(cloC4,'Closeness Centrality 2002')
```

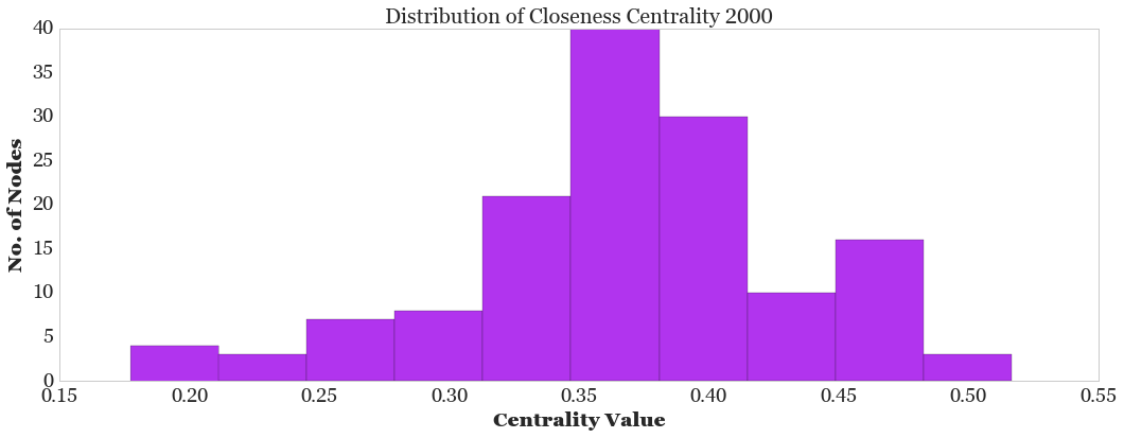
```
<matplotlib.figure.Figure at 0x29c4eb31b70>
```



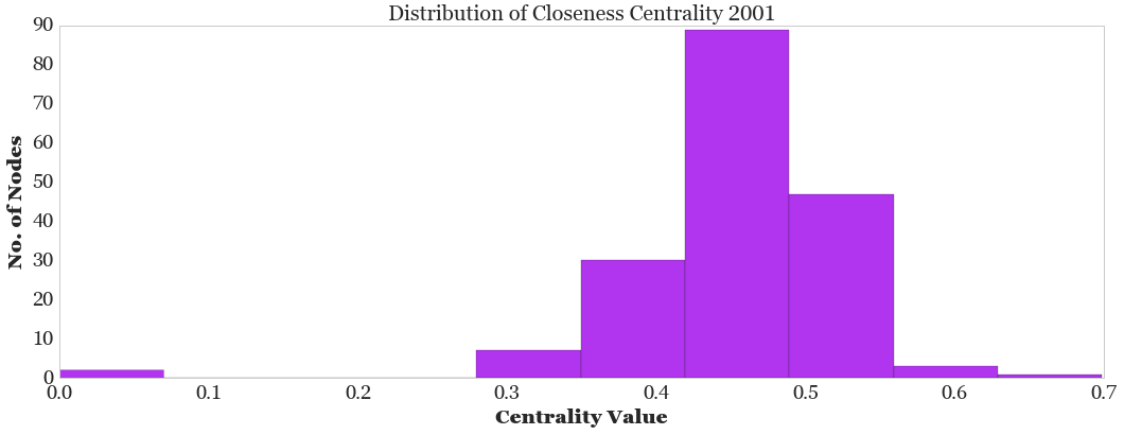

```
<matplotlib.figure.Figure at 0x29c4ce59278>
```



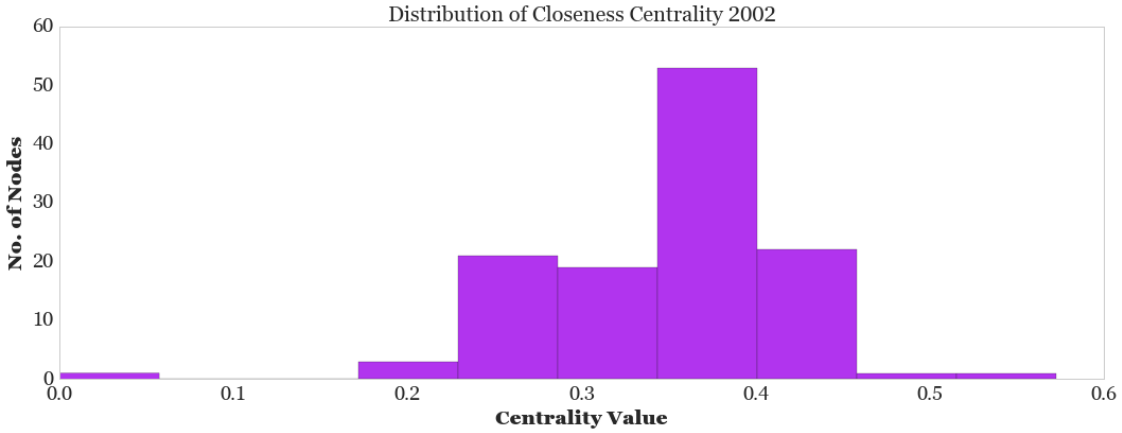

```
<matplotlib.figure.Figure at 0x29c4e90eb00>
```




```
<matplotlib.figure.Figure at 0x29c4eef3b70>
```



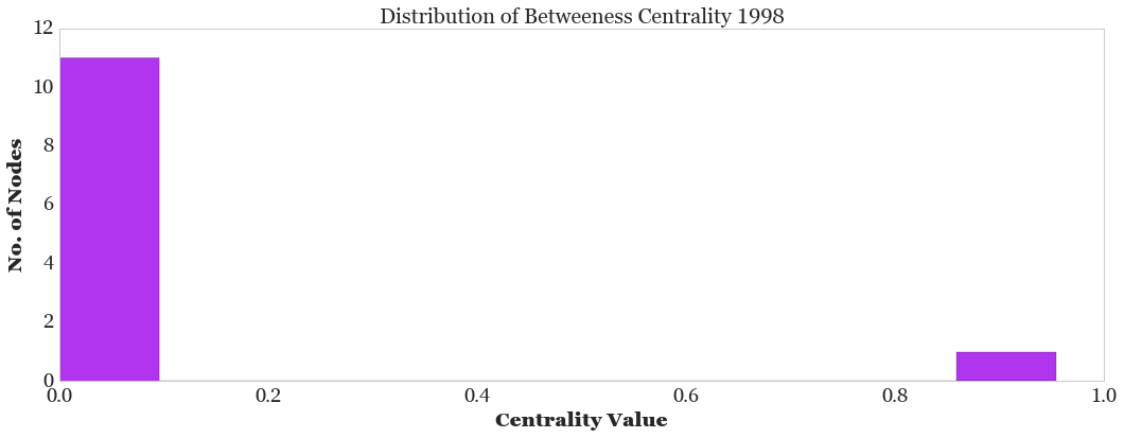

```
<matplotlib.figure.Figure at 0x29c4e4cad30>
```



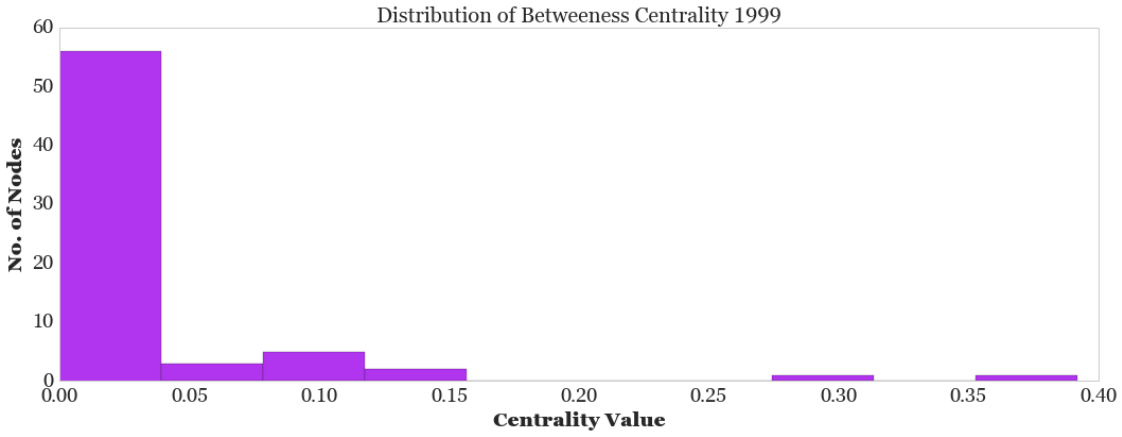
```
In [43]: print("Top 5 Closeness Centrality Nodes year 98")
         get_top_keys(cloC0,5)
         print("Top 5 Closeness Centrality Nodes year 99")
         get_top_keys(cloC1,5)
         print("Top 5 Closeness Centrality Nodes year 2k")
         get_top_keys(cloC2,5)
         print("Top 5 Closeness Centrality Nodes year 2k1")
         get_top_keys(cloC3,5)
         print("Top 5 Closeness Centrality Nodes year 2k2")
         get_top_keys(cloC4,5)

Top 5 Closeness Centrality Nodes year 98
114
112
65
145
160
Top 5 Closeness Centrality Nodes year 99
169
153
114
22
145
Top 5 Closeness Centrality Nodes year 2k
82
27
140
145
163
Top 5 Closeness Centrality Nodes year 2k1
82
153
107
94
126
Top 5 Closeness Centrality Nodes year 2k2
105
107
34
82
133


In [44]: get_centhistval(betC0,'Betweeness Centrality 1998')
         get_centhistval(betC1,'Betweeness Centrality 1999')
```

```
get_centhistval(betC2,'Betweeness Centrality 2000')
get_centhistval(betC3,'Betweeness Centrality 2001')
get_centhistval(betC4,'Betweeness Centrality 2002')
```

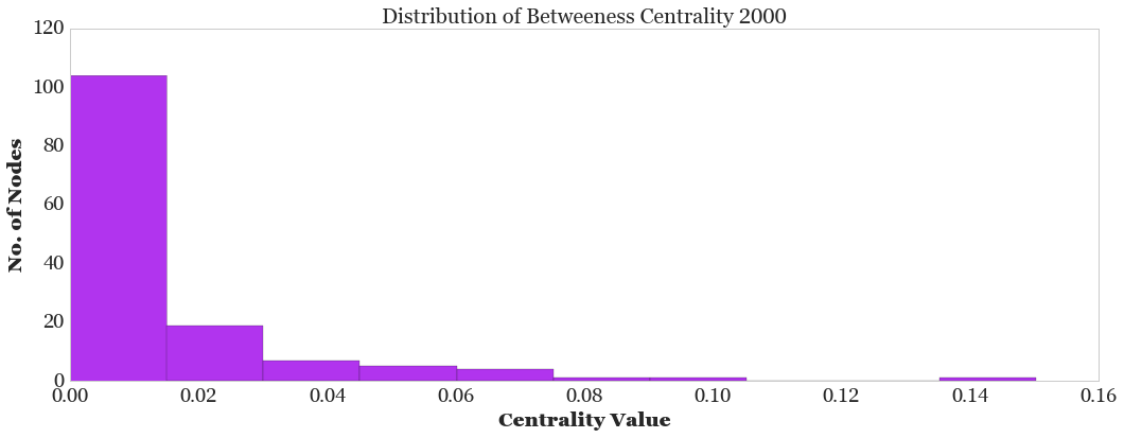
```
<matplotlib.figure.Figure at 0x29c4eb31978>
```



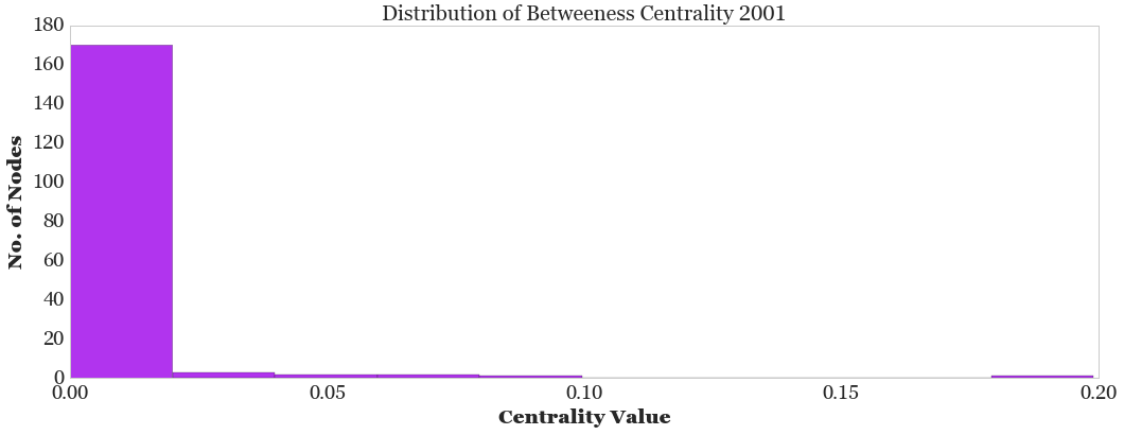

```
<matplotlib.figure.Figure at 0x29c4ecfbba8>
```



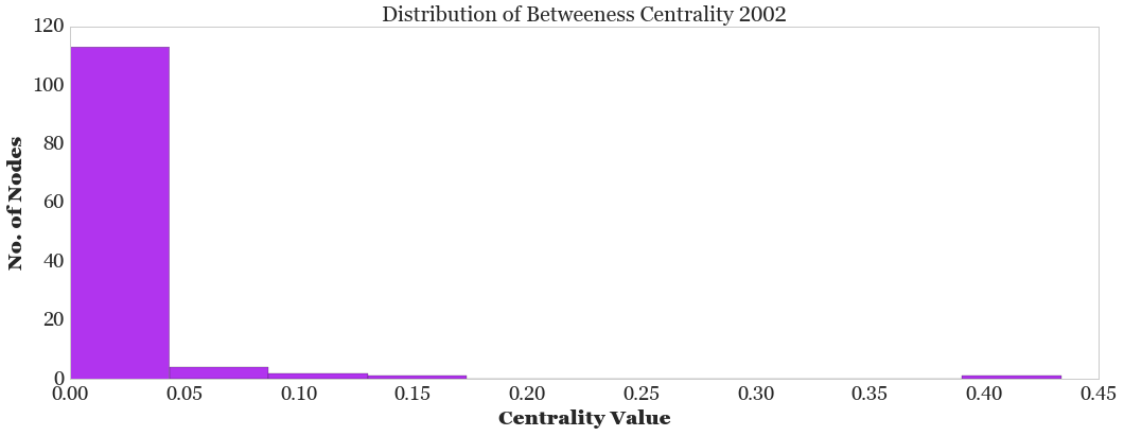

```
<matplotlib.figure.Figure at 0x29c4ef4c2b0>
```




```
<matplotlib.figure.Figure at 0x29c4e7ff9e8>
```



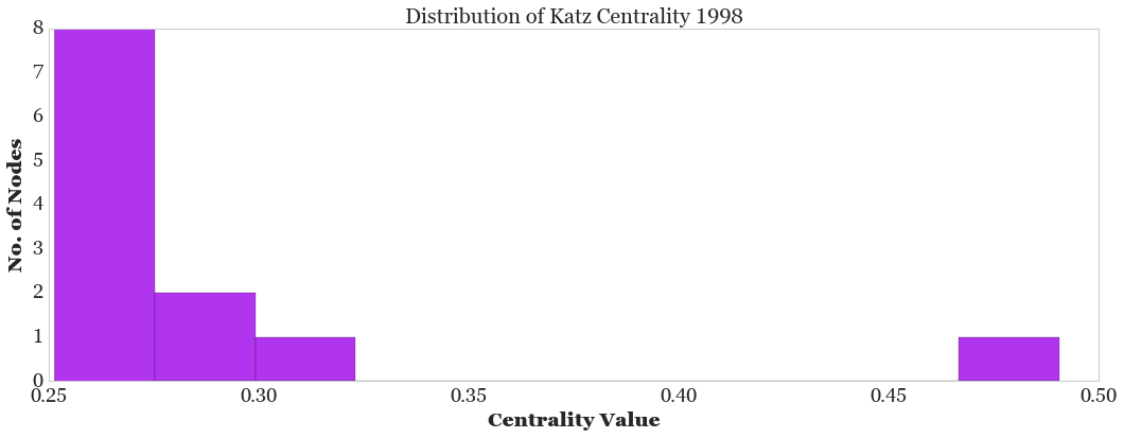

```
<matplotlib.figure.Figure at 0x29c4b5f2470>
```



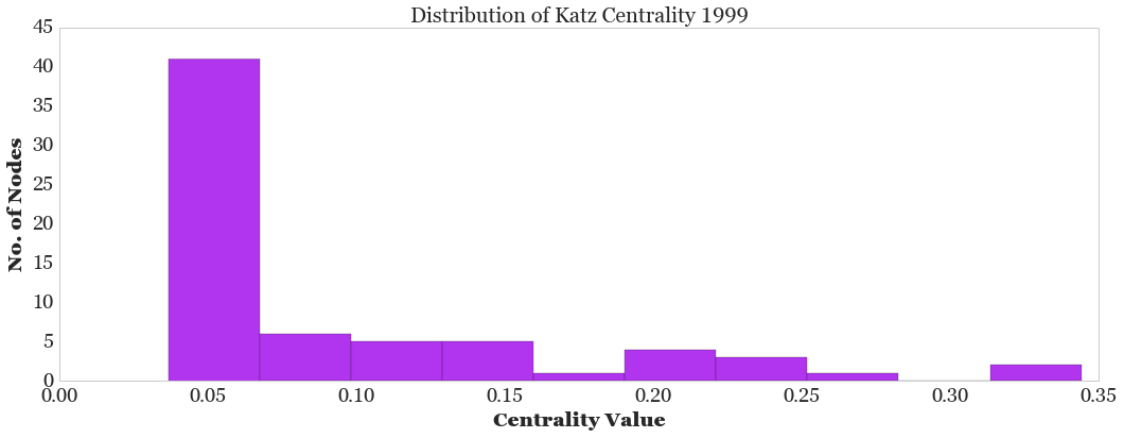
```
In [45]: print("Top 5 Betweeness Centrality Nodes year 98")
         get_top_keys(betC0,5)
         print("Top 5 Betweeness Centrality Nodes year 99")
         get_top_keys(betC1,5)
         print("Top 5 Betweeness Centrality Nodes year 2k")
         get_top_keys(betC2,5)
         print("Top 5 Betweeness Centrality Nodes year 2k1")
         get_top_keys(betC3,5)
         print("Top 5 Betweeness Centrality Nodes year 2k2")
         get_top_keys(betC4,5)

Top 5 Betweeness Centrality Nodes year 98
114
112
160
65
155
Top 5 Betweeness Centrality Nodes year 99
153
169
17
163
22
Top 5 Betweeness Centrality Nodes year 2k
82
63
140
157
79
Top 5 Betweeness Centrality Nodes year 2k1
82
153
94
107
9
Top 5 Betweeness Centrality Nodes year 2k2
105
34
107
158
17


In [46]: get_centhistval(katzC0,'Katz Centrality 1998')
         get_centhistval(katzC1,'Katz Centrality 1999')
```

```
get_centhistval(katzC2,'Katz Centrality 2000')
get_centhistval(katzC3,'Katz Centrality 2001')
get_centhistval(katzC4,'Katz Centrality 2002')
```

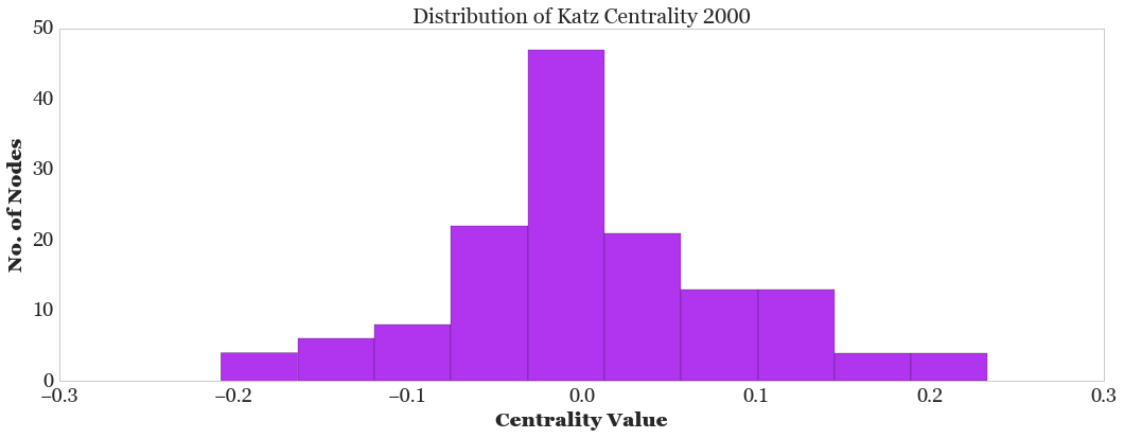
```
<matplotlib.figure.Figure at 0x29c4ba184e0>
```



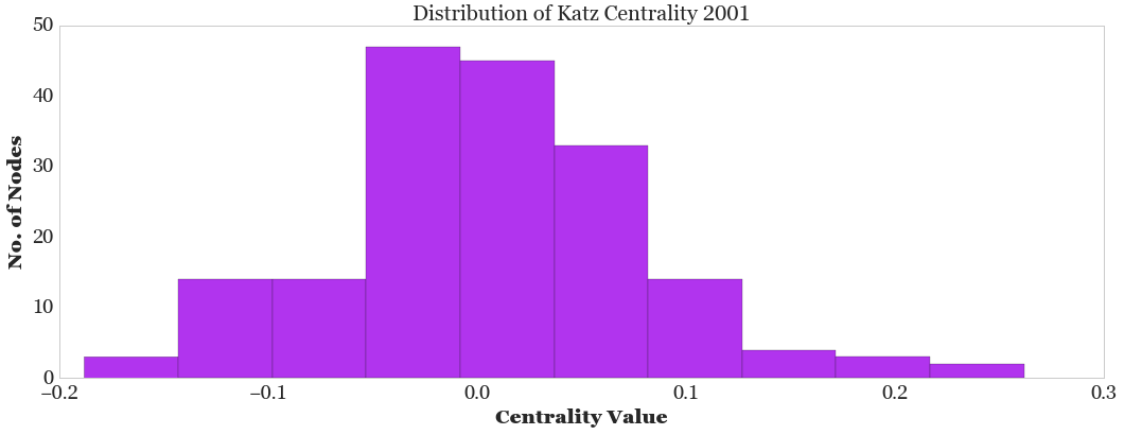

```
<matplotlib.figure.Figure at 0x29c4ecb3e10>
```



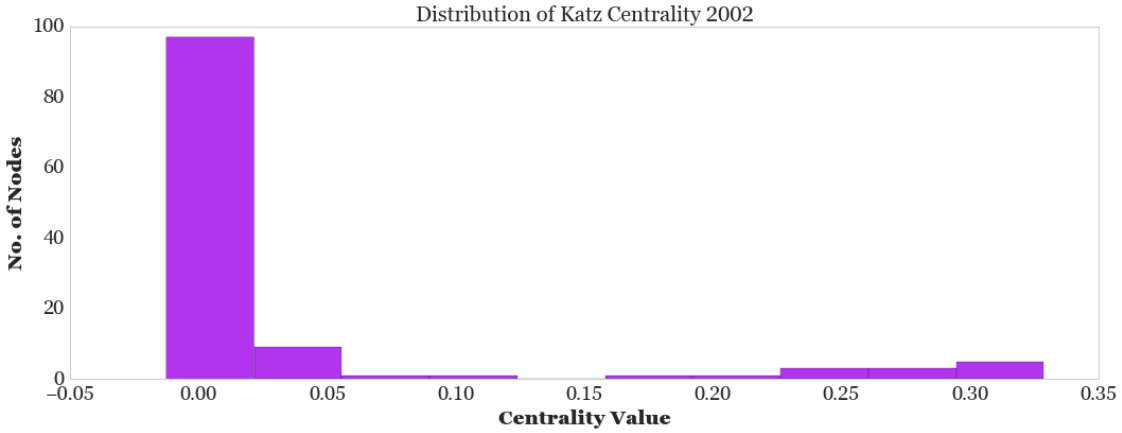

```
<matplotlib.figure.Figure at 0x29c4eccd0f0>
```




```
<matplotlib.figure.Figure at 0x29c4ebc2e10>
```



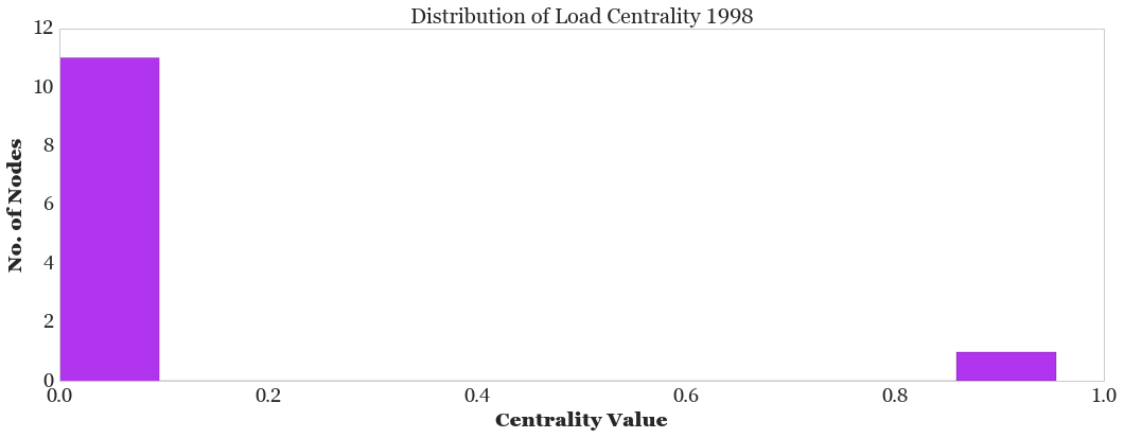

```
<matplotlib.figure.Figure at 0x29c4e4cd710>
```



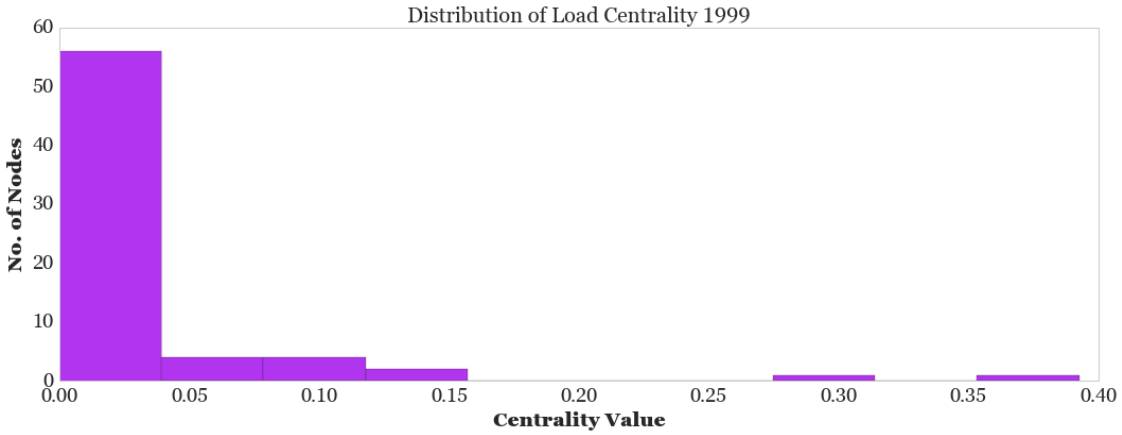
```
In [47]: print("Top 5 Katz Centrality Nodes year 98")
         get_top_keys(katzC0,5)
         print("Top 5 Katz Centrality Nodes year 99")
         get_top_keys(katzC1,5)
         print("Top 5 Katz Centrality Nodes year 2k")
         get_top_keys(katzC2,5)
         print("Top 5 Katz Centrality Nodes year 2k1")
         get_top_keys(katzC3,5)
         print("Top 5 Katz Centrality Nodes year 2k2")
         get_top_keys(katzC4,5)

Top 5 Katz Centrality Nodes year 98
114
112
65
145
155
Top 5 Katz Centrality Nodes year 99
114
169
38
155
112
Top 5 Katz Centrality Nodes year 2k
114
169
33
158
155
Top 5 Katz Centrality Nodes year 2k1
167
133
32
153
21
Top 5 Katz Centrality Nodes year 2k2
98
103
124
158
108


In [48]: get_centhistval(loadC0,'Load Centrality 1998')
         get_centhistval(loadC1,'Load Centrality 1999')
```

```
get_centhistval(loadC2,'Load Centrality 2000')
get_centhistval(loadC3,'Load Centrality 2001')
get_centhistval(loadC4,'Load Centrality 2002')
```

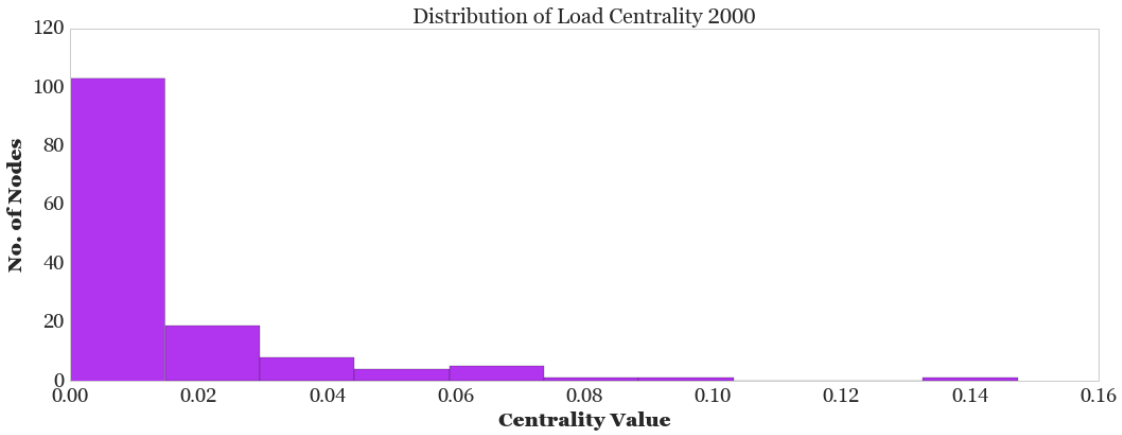
```
<matplotlib.figure.Figure at 0x29c4b5a24a8>
```



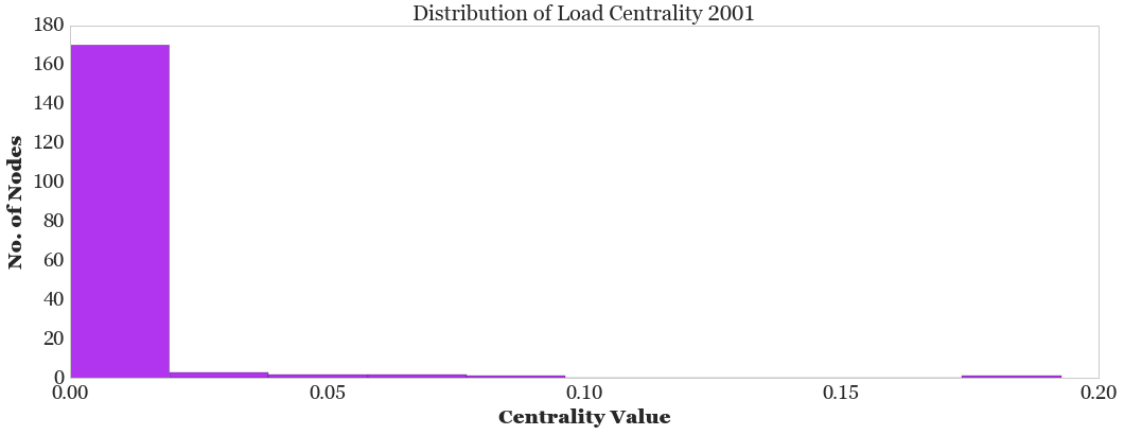

```
<matplotlib.figure.Figure at 0x29c4c955e48>
```



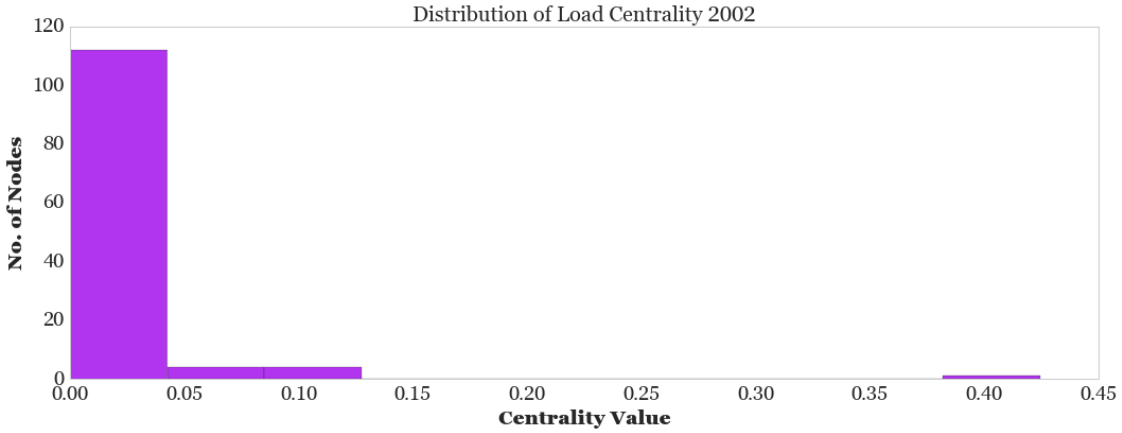

```
<matplotlib.figure.Figure at 0x29c4ce1aac8>
```




```
<matplotlib.figure.Figure at 0x29c4ebe4dd8>
```



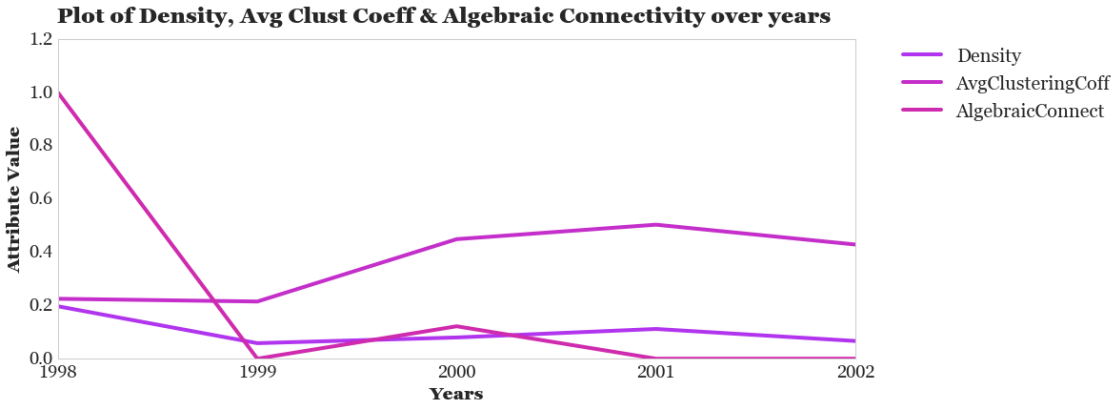

```
<matplotlib.figure.Figure at 0x29c4b5edf60>
```



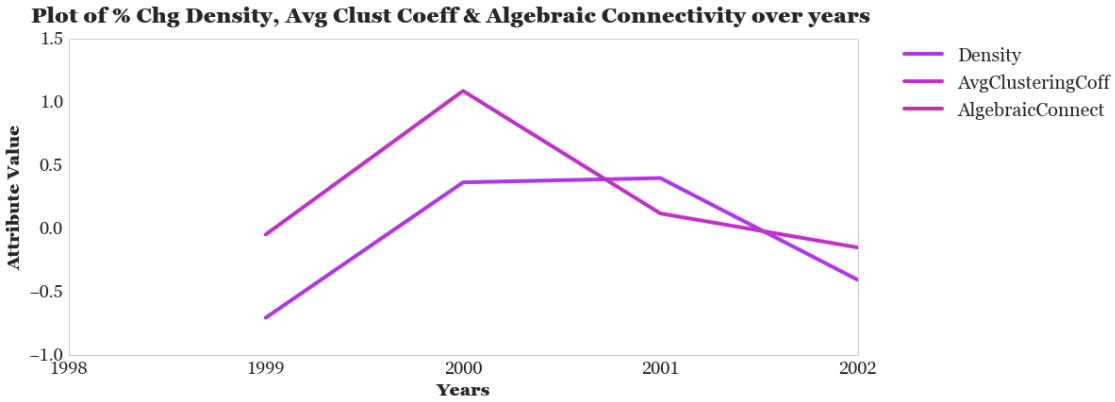
```
In [49]: print("Top 5 Load Centrality Nodes year 98")
         get_top_keys(loadC0,5)
         print("Top 5 Load Centrality Nodes year 99")
         get_top_keys(loadC1,5)
         print("Top 5 Load Centrality Nodes year 2k")
         get_top_keys(loadC2,5)
         print("Top 5 Load Centrality Nodes year 2k1")
         get_top_keys(loadC3,5)
         print("Top 5 Load Centrality Nodes year 2k2")
         get_top_keys(loadC4,5)

Top 5 Load Centrality Nodes year 98
114
112
160
65
155
Top 5 Load Centrality Nodes year 99
153
169
17
163
22
Top 5 Load Centrality Nodes year 2k
82
63
140
79
125
Top 5 Load Centrality Nodes year 2k1
82
153
94
107
9
Top 5 Load Centrality Nodes year 2k2
105
34
107
158
17


In [50]: all_year_G  =tuple([Gt0,Gt1,Gt2,Gt3,Gt4])
         all_year_G[:5]
```

```
Out[50]: (<networkx.classes.graph.Graph at 0x29c4b5ed588>,
          <networkx.classes.graph.Graph at 0x29c4b591780>,
          <networkx.classes.graph.Graph at 0x29c4b591390>,
          <networkx.classes.graph.Graph at 0x29c4b5faa20>,
          <networkx.classes.graph.Graph at 0x29c4b5fa8d0>)

In [51]: density_year = []
         avclustcoff_year = []
         algebconn_year = []

         for i in all_year_G:
             density_year.append(nx.density(i))
             avclustcoff_year.append(nx.average_clustering(i))
             algebconn_year.append(nx.algebraic_connectivity(i))

In [52]: density_year

Out[52]: [0.19696969696969696,
          0.058384547848990345,
          0.07981220657276995,
          0.11179461427405687,
          0.06666666666666667]

In [53]: avclustcoff_year

Out[53]: [0.22525252525252523,
          0.21497313000340684,
          0.4493480384358036,
          0.5037725950614431,
          0.4292464564453092]

In [54]: algebconn_year

Out[54]: [1.0000000000000011, 0.0, 0.12190804467954702, 0.0, 0.0]

In [55]: assorstat_year = pd.DataFrame([density_year,avclustcoff_year,algebconn_yea
         assorstat_year.columns = ['Density','AvgClusteringCoff','AlgebraicConnect'
         assorstat_year = assorstat_year.T
         assorstat_year.columns = years[1:]
         assorstat_year = assorstat_year.T
         assorstat_year

Out[55]:        Density  AvgClusteringCoff  AlgebraicConnect
         1998  0.196970           0.225253          1.000000
         1999  0.058385           0.214973          0.000000
         2000  0.079812           0.449348          0.121908
         2001  0.111795           0.503773          0.000000
         2002  0.066667           0.429246          0.000000
```

```
In [56]: assorstat_year.plot(fontsize=18, use_index=True)
         plt.suptitle("Plot of Density, Avg Clust Coeff & Algebraic Connectivity ov
         plt.xlabel("Years", fontsize=18)
         plt.ylabel("Attribute Value", fontsize=18)
         plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=18)
         plt.yticks(fontsize=18)

Out[56]: (array([ 0. ,  0.2,  0.4,  0.6,  0.8,  1. ,  1.2,  1.4]),
          <a list of 8 Text yticklabel objects>)
```



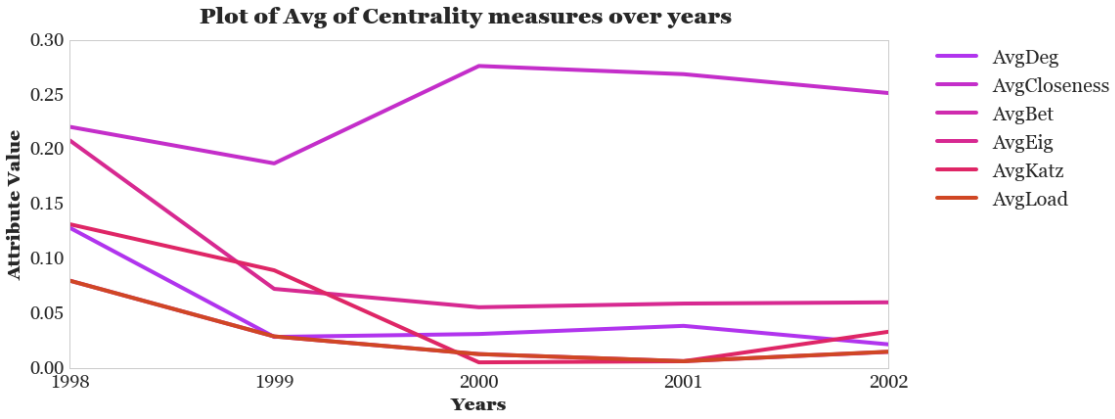

```
In [57]: assorstat_year.pct_change().plot(fontsize=18, use_index=True)
         plt.suptitle("Plot of % Chg Density, Avg Clust Coeff & Algebraic Connectiv
         plt.xlabel("Years", fontsize=18)
         plt.ylabel("Attribute Value", fontsize=18)
         plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=18)
         plt.yticks(fontsize=18)

Out[57]: (array([-1. , -0.5,  0. ,  0.5,  1. ,  1.5]),
          <a list of 6 Text yticklabel objects>)
```



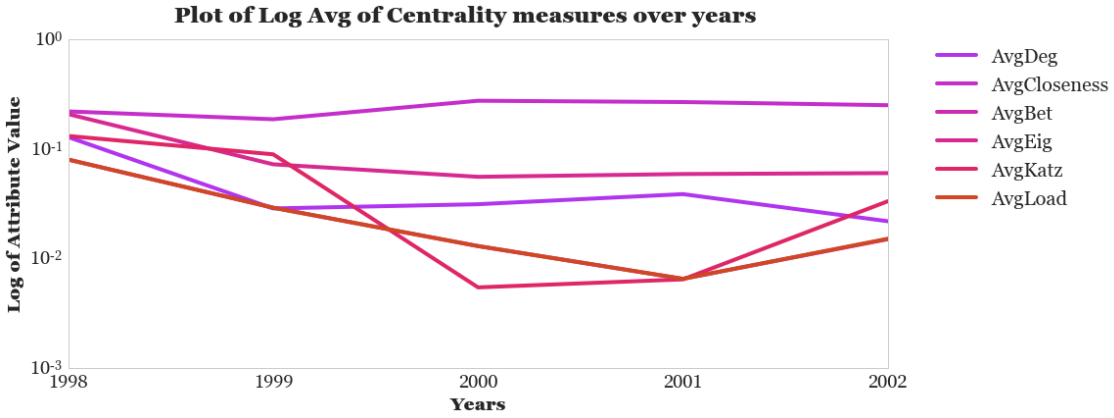
```
In [58]: print("No. of edges 2000 and 2001: \n", nx.number_of_edges(Gt2),nx.number_
         print("No. of nodes 2000 and 2001: \n",  nx.number_of_nodes(Gt2),nx.numbe

No. of edges 2000 and 2001:
 799 1781
No. of nodes 2000 and 2001:
 142 179
```

5.2 Calculate average of attributes for yearly networks

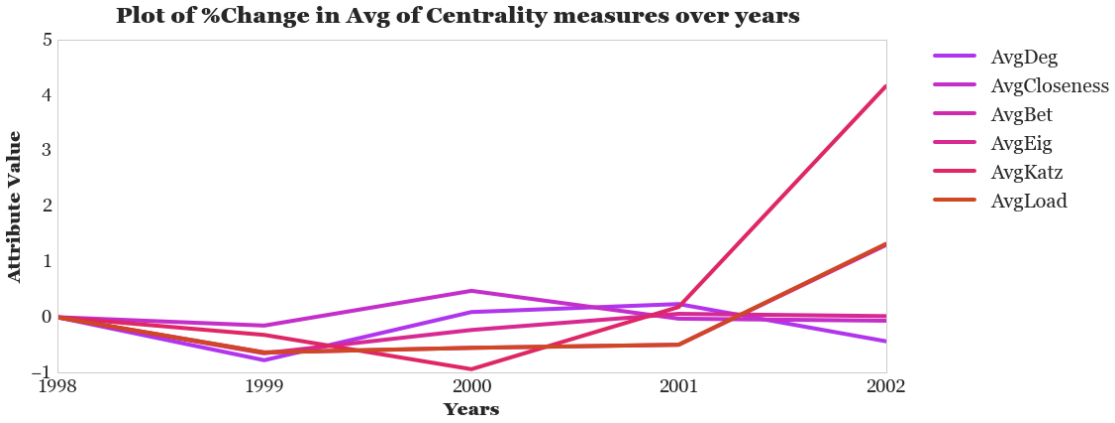
```
In [59]: y_avgst0 = cal_avgstat(Gt0)
         y_avgst1 = cal_avgstat(Gt1)
         y_avgst2 = cal_avgstat(Gt2)
         y_avgst3 = cal_avgstat(Gt3)
         y_avgst4 = cal_avgstat(Gt4)

         y_avgst_all = y_avgst0.append(y_avgst1).append(y_avgst2).append(y_avgst3).
         y_avgst_all = y_avgst_all.T
         y_avgst_all.columns = years[1:]
         y_avgst_all = y_avgst_all.T

In [60]: y_avgst_all.iloc[:,:6].plot(fontsize=18)
         plt.suptitle("Plot of Avg of Centrality measures over years", fontsize=22)
         plt.xlabel("Years", fontsize=18)
         plt.ylabel("Attribute Value", fontsize=18)
         plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=18)
         plt.yticks(fontsize=18)

Out[60]: (array([ 0.  ,  0.05,  0.1 ,  0.15,  0.2 ,  0.25,  0.3 ,  0.35]),
          <a list of 8 Text yticklabel objects>)
```



```
In [61]: y_avgst_all.iloc[:,:6].plot(fontsize=18, logy=True)
         plt.suptitle("Plot of Log Avg of Centrality measures over years", fontsize
         plt.xlabel("Years", fontsize=18)
         plt.ylabel("Log of Attribute Value", fontsize=18)
         plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=18)
         plt.yticks(fontsize=18)

Out[61]: (array([  1.00000000e-04,   1.00000000e-03,   1.00000000e-02,
                   1.00000000e-01,   1.00000000e+00,   1.00000000e+01]),
          <a list of 6 Text yticklabel objects>)
```




```
In [62]: y_avgst_all.iloc[:,:6].pct_change().fillna(0).plot(fontsize=18)
         plt.suptitle("Plot of %Change in Avg of Centrality measures over years", f
         plt.xlabel("Years", fontsize=18)
         plt.ylabel("Attribute Value", fontsize=18)
         plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=18)
         plt.yticks(fontsize=18)

Out[62]: (array([-1.,  0.,  1.,  2.,  3.,  4.,  5.]),
          <a list of 7 Text yticklabel objects>)
```



5.3 Observations

The algebraic connectivity > 0 indicates that the network is connected so excluding the first network the only other time that this measure indicates that the network is connected is in 2000. This is the first signal that emerges from these set of measures. From the percentage change plot we see that the year 2000 experiences the largest increase in density and clustering coefficient. The density implies that more edges that could be formed are formed in the network in 2000 as opposed to the other years. While the density is largely similar between 2000 and 2001 the local average clustering coefficient decreases between 2000 and 2001 this could largely be down to the larger network size. This means that although the network has grown the number of triangles has decreased due to much higher number of edges between nodes.

From the average of the centrality measures we see a similar signal across the years. We see that 2000 again experiences a big change compared to the previous years. The Closeness Centrality dominates due to its scale but in the log plot we see that the Closeness Centrality changes the most for this year before almost flat lining for the remaining years. The Degree centrality confirms the observationsregarding the density because we see only marginal change in the Degree Centrality between 2000 and 2001. The Eigenvector Centrality tells us that the power structure or the influence structure in the network did not change much within the network over this time period. However, the Katz centrality and Load centrality are much more interesting. The Katz centrality is an indicator of node importance and from 2000 it seems that there was an increase in the number average number of influential nodes and the path between them decreased as indicated by the falling Load centrality. They coincide in 2001 just at the end of the Golden age of Enron. The rising Katz and Load Centrality can be interepreted as the involvement of outside actors as the accounting scandal unfolded leading to more influential players in the network. Hence Load and Katz Centrality experience a largest percentage increase during this time.

5.4 Monthly Analysis

Calculate average statistics for monthly networks

```
In [63]: stat_nov98 = cal_avgstat(G_nov98)
         stat_dec98 = cal_avgstat(G_dec98)
```

```
stat98= stat_nov98.append(stat_dec98).T
stat98.columns = ['Nov98','Dec98']
stat98.head()
```

Out[63]:

	Nov98	Dec98
AvgDeg	0.142857	0.145455
AvgCloseness	0.192308	0.242737
AvgBet	0.125000	0.086869
AvgEig	0.222019	0.233760
AvgKatz	0.185435	0.175111

```
In [64]: stat_jan_99=cal_avgstat(G_jan_99)
stat_feb_99=cal_avgstat(G_feb_99)
stat_mar_99=cal_avgstat(G_mar_99)
stat_apr_99=cal_avgstat(G_apr_99)
stat_may_99=cal_avgstat(G_may_99)
stat_jun_99=cal_avgstat(G_jun_99)
stat_jul_99=cal_avgstat(G_jul_99)
stat_aug_99=cal_avgstat(G_aug_99)
stat_sep_99=cal_avgstat(G_sep_99)
stat_oct_99=cal_avgstat(G_oct_99)
stat_nov_99=cal_avgstat(G_nov_99)
stat_dec_99=cal_avgstat(G_dec_99)

stat_99 = stat_jan_99.append(stat_feb_99).append(stat_mar_99).append(stat_
stat_99 = stat_99.append(stat_jun_99).append(stat_jul_99).append(stat_aug_
stat_99 = stat_99.append(stat_oct_99).append(stat_nov_99).append(stat_dec_
stat_99.columns = ['jan_99','feb_99','mar_99','apr_99','may_99','jun_99','
stat_99.head()
```

Out[64]:

	jan_99	feb_99	mar_99	apr_99	may_99	jun_99
AvgDeg	0.083333	0.500000	0.200000	0.500000	0.120879	0.066176
AvgCloseness	0.117057	0.555556	0.259259	0.555556	0.372644	0.157004
AvgBet	0.076923	0.333333	0.166667	0.333333	0.110806	0.028922
AvgEig	0.132902	0.569036	0.328670	0.569036	0.224884	0.166311
AvgKatz	0.057692	0.390223	0.277684	0.390223	0.263068	0.191024

	jul_99	aug_99	sep_99	oct_99	nov_99	dec_99
AvgDeg	0.094771	0.088745	0.071146	0.070769	0.085714	0.025689
AvgCloseness	0.295189	0.223778	0.275119	0.169032	0.179564	0.147367
AvgBet	0.100490	0.057143	0.077734	0.028803	0.051128	0.038836
AvgEig	0.189606	0.142963	0.152945	0.127862	0.147848	0.069662
AvgKatz	0.221168	0.183912	0.189669	0.172447	0.194931	0.106213

```
In [65]: stat_jan_2k=cal_avgstat(G_jan_2k)
stat_feb_2k=cal_avgstat(G_feb_2k)
stat_mar_2k=cal_avgstat(G_mar_2k)
stat_apr_2k=cal_avgstat(G_apr_2k)
stat_may_2k=cal_avgstat(G_may_2k)
```

```
stat_jun_2k=cal_avgstat(G_jun_2k)
stat_jul_2k=cal_avgstat(G_jul_2k)
stat_aug_2k=cal_avgstat(G_aug_2k)
stat_sep_2k=cal_avgstat(G_sep_2k)
stat_oct_2k=cal_avgstat(G_oct_2k)
stat_nov_2k=cal_avgstat(G_nov_2k)
stat_dec_2k=cal_avgstat(G_dec_2k)
```

In [66]:
```
stat_2k = stat_jan_2k.append(stat_feb_2k).append(stat_mar_2k).append(stat_
stat_2k = stat_2k.append(stat_jun_2k).append(stat_jul_2k).append(stat_aug_
stat_2k = stat_2k.append(stat_oct_2k).append(stat_nov_2k).append(stat_dec_
stat_2k.columns = ['jan_2k','feb_2k','mar_2k','apr_2k','may_2k','jun_2k','
stat_2k.head()
```

Out[66]:

	jan_2k	feb_2k	mar_2k	apr_2k	may_2k	jun_2k
AvgDeg	0.022378	0.026626	0.014809	0.010464	0.014035	0.010867
AvgCloseness	0.165628	0.208593	0.157985	0.113773	0.164460	0.192022
AvgBet	0.034948	0.035326	0.046531	0.042264	0.033091	0.027877
AvgEig	0.066998	0.084335	0.065003	0.053698	0.071458	0.068502
AvgKatz	0.099556	0.104652	0.097354	0.103359	0.104188	0.091210

	jul_2k	aug_2k	sep_2k	oct_2k	nov_2k	dec_2k
AvgDeg	0.012697	0.016514	0.012147	0.018086	0.010884	0.013887
AvgCloseness	0.206035	0.223131	0.192691	0.222388	0.177589	0.174585
AvgBet	0.026787	0.017815	0.019417	0.019777	0.017789	0.019199
AvgEig	0.068311	0.063361	0.062015	0.062537	0.054661	0.051451
AvgKatz	0.086094	0.068437	0.073747	0.064164	0.058620	0.038943

In [67]:
```
stat_jan_2k1=cal_avgstat(G_jan_2k1)
stat_feb_2k1=cal_avgstat(G_feb_2k1)
stat_mar_2k1=cal_avgstat(G_mar_2k1)
stat_apr_2k1=cal_avgstat(G_apr_2k1)
stat_may_2k1=cal_avgstat(G_may_2k1)
stat_jun_2k1=cal_avgstat(G_jun_2k1)
stat_jul_2k1=cal_avgstat(G_jul_2k1)
stat_aug_2k1=cal_avgstat(G_aug_2k1)
stat_sep_2k1=cal_avgstat(G_sep_2k1)
stat_oct_2k1=cal_avgstat(G_oct_2k1)
stat_nov_2k1=cal_avgstat(G_nov_2k1)
stat_dec_2k1=cal_avgstat(G_dec_2k1)

stat_2k1 = stat_jan_2k1.append(stat_feb_2k1).append(stat_mar_2k1).append(s
stat_2k1 = stat_2k1.append(stat_jun_2k1).append(stat_jul_2k1).append(stat_
stat_2k1 = stat_2k1.append(stat_oct_2k1).append(stat_nov_2k1).append(stat_
stat_2k1.columns = ['jan_2k1','feb_2k1','mar_2k1','apr_2k1','may_2k1','jun
stat_2k1.head()
```

Out[67]:

	jan_2k1	feb_2k1	mar_2k1	apr_2k1	may_2k1	jun_2k1
AvgDeg	0.013179	0.012052	0.012982	0.015607	0.012401	0.008815

```
AvgCloseness  0.191559  0.208446  0.186307  0.214056  0.221824  0.209019
AvgBet        0.017710  0.021867  0.017525  0.017928  0.011548  0.019936
AvgEig        0.055170  0.064686  0.058794  0.051914  0.052995  0.049033
AvgKatz       0.048244  0.074620  0.057339  0.002995  0.022108  0.059015

                jul_2k1   aug_2k1   sep_2k1   oct_2k1   nov_2k1   dec_2k1
AvgDeg        0.016036  0.016639  0.012930  0.019379  0.016612  0.011789
AvgCloseness  0.199306  0.247621  0.195912  0.232163  0.241737  0.202481
AvgBet        0.020539  0.015186  0.021407  0.012743  0.017048  0.022688
AvgEig        0.053660  0.051803  0.051584  0.058570  0.056969  0.065293
AvgKatz       0.046579  0.000057  0.021531  0.003949  0.002392  0.074214
```

In [68]:
```
stat_jan_2k2=cal_avgstat(G_jan_2k2)
stat_feb_2k2=cal_avgstat(G_feb_2k2)
stat_mar_2k2=cal_avgstat(G_mar_2k2)
stat_apr_2k2=cal_avgstat(G_apr_2k2)
stat_may_2k2=cal_avgstat(G_may_2k2)
stat_jun_2k2=cal_avgstat(G_jun_2k2)

stat_2k2 = stat_jan_2k2.append(stat_feb_2k2).append(stat_mar_2k2).append(s
stat_2k2.columns = ['jan_2k2','feb_2k2','mar_2k2','apr_2k2','may_2k2','jun
stat_2k2.head()
```

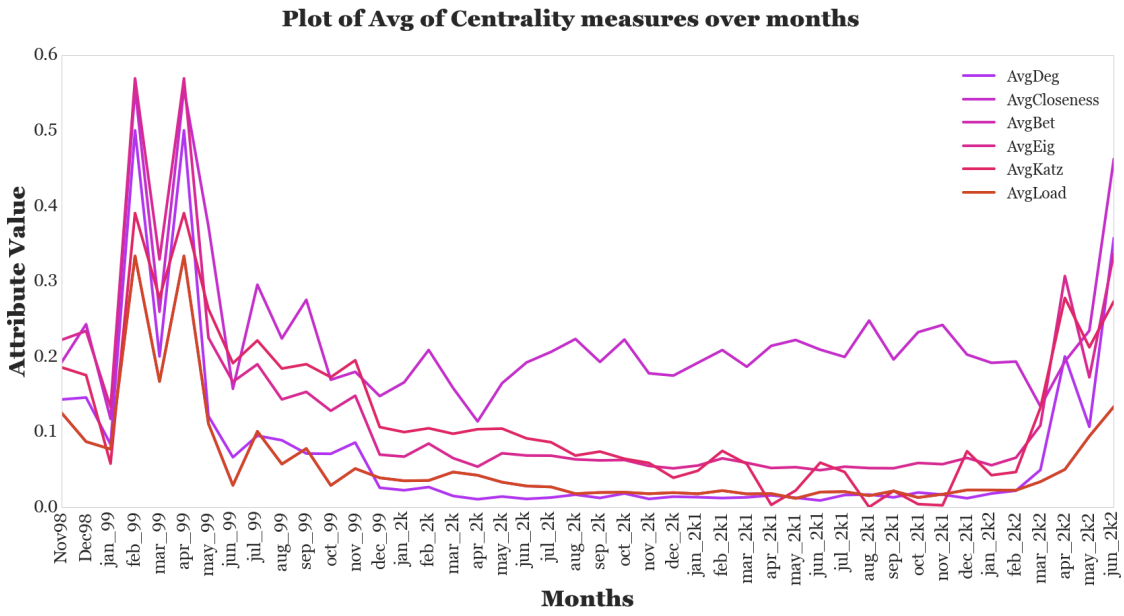
Out[68]:
```
                jan_2k2   feb_2k2   mar_2k2   apr_2k2   may_2k2   jun_2k2
AvgDeg        0.018018  0.021818  0.049395  0.200000  0.106618  0.357143
AvgCloseness  0.191514  0.193146  0.133727  0.193333  0.234243  0.462193
AvgBet        0.022664  0.022488  0.033737  0.050000  0.094118  0.133333
AvgEig        0.055595  0.065612  0.108396  0.306719  0.172235  0.336275
AvgKatz       0.042359  0.046586  0.131887  0.277666  0.212198  0.273203
```

In [69]:
```
stat_all = stat98.join(stat_99).join(stat_2k).join(stat_2k1).join(stat_2k2
```

In [70]:
```
months = list(stat_all.index)
```

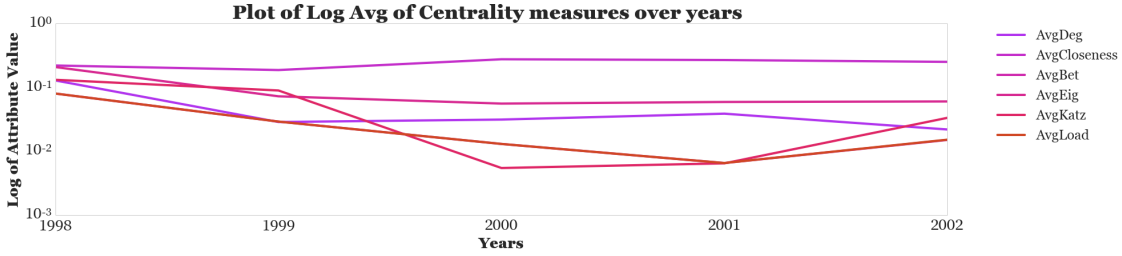
In [71]:
```
stat_all.iloc[:,:6].plot(fontsize=22, rot=90, figsize=(28,12))
plt.suptitle("Plot of Avg of Centrality measures over months", fontsize=33
plt.xlabel("Months", fontsize=33)
plt.ylabel("Attribute Value", fontsize=33)
plt.legend(fontsize=23, loc=1)
#plt.legend(fontsize=20, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xticks(np.arange(len(months)), months,fontsize=26)
plt.yticks(fontsize=26)
```

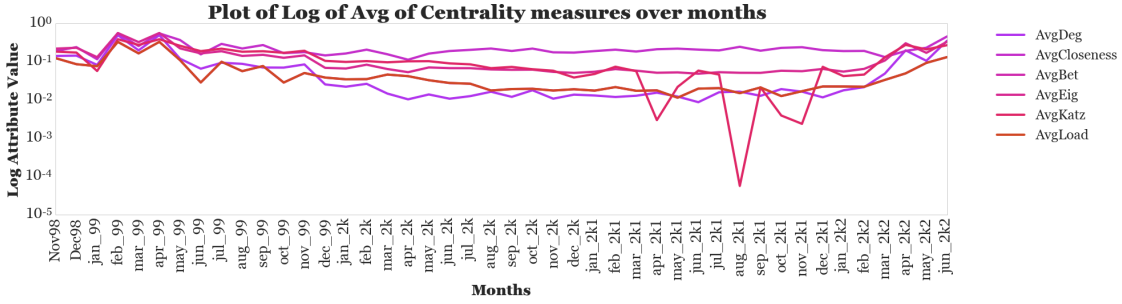
Out[71]:
```
(array([ 0. ,  0.1,  0.2,  0.3,  0.4,  0.5,  0.6,  0.7]),
 <a list of 8 Text yticklabel objects>)
```



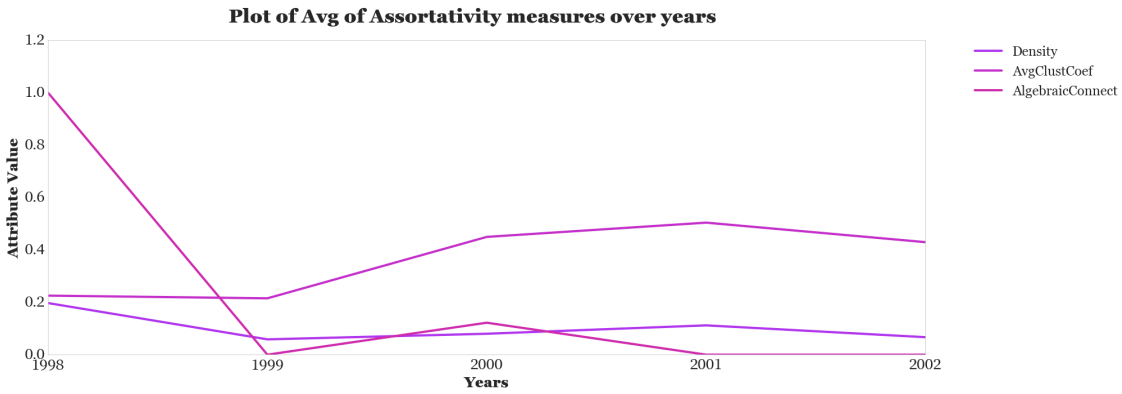
5.5 Comparison of Yearly and Monthly trends in Centrality

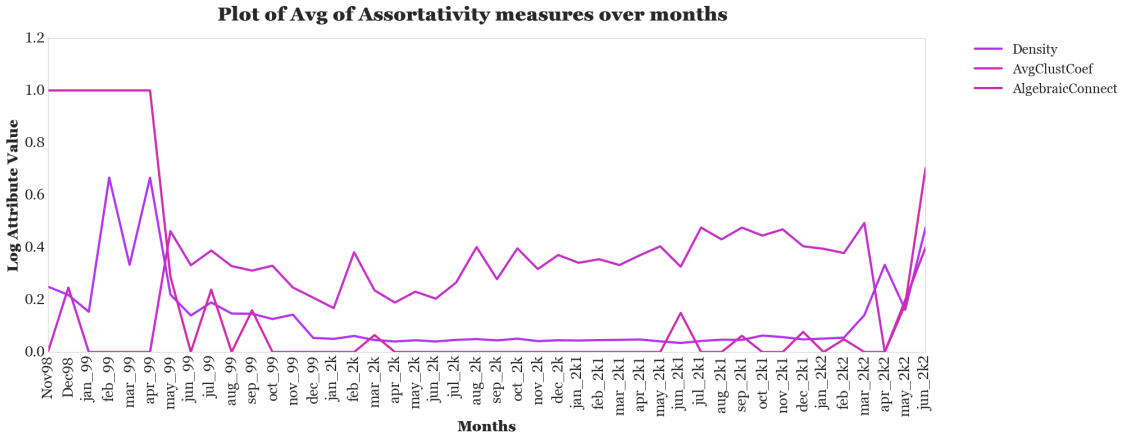
```
In [72]: y_avgst_all.iloc[:,:6].plot(fontsize=22, figsize=(28,6),logy=True)
         plt.suptitle("Plot of Log Avg of Centrality measures over years", fontsize
         plt.xlabel("Years", fontsize=25)
         plt.ylabel("Log of Attribute Value", fontsize=25)
         plt.legend(fontsize=24, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
         plt.yticks(fontsize=26)

         stat_all.iloc[:,:6].plot(fontsize=22, rot=90, figsize=(28,6), logy=True)
         plt.suptitle("Plot of Log of Avg of Centrality measures over months", font
         plt.xlabel("Months", fontsize=25)
         plt.ylabel("Log Attribute Value", fontsize=25)
         plt.legend(fontsize=24, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
         plt.xticks(np.arange(len(months)), months,fontsize=26)
         plt.yticks(fontsize=26)

Out[72]: (array([  1.00000000e-06,   1.00000000e-05,   1.00000000e-04,
                   1.00000000e-03,   1.00000000e-02,   1.00000000e-01,
                   1.00000000e+00,   1.00000000e+01]),
          <a list of 8 Text yticklabel objects>)
```






```
In [73]: y_avgst_all.iloc[:,6:9].plot(fontsize=22, figsize=(28,10))
         plt.suptitle("Plot of Avg of Assortativity measures over years", fontsize=
         plt.xlabel("Years", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=24, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
         plt.yticks(fontsize=26)

         stat_all.iloc[:,6:9].plot(fontsize=22, rot=90, figsize=(28,10))
         plt.suptitle("Plot of Avg of Assortativity measures over months", fontsize
         plt.xlabel("Months", fontsize=25)
         plt.ylabel("Log Attribute Value", fontsize=25)
         plt.legend(fontsize=24, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
         plt.xticks(np.arange(len(months)), months,fontsize=26)
         plt.yticks(fontsize=26)

Out[73]: (array([ 0. ,  0.2,  0.4,  0.6,  0.8,  1. ,  1.2,  1.4]),
          <a list of 8 Text yticklabel objects>)
```





6 Attributes Analysis on Graphs

Here I implement novel attributes taken from seismic data analysis.

The complex trace is defined as

$$C(t) = S(t) + iH(t)$$

Where:

$C(t)$= Complex Trace

$S(t)$= Real Data

$H(t)$= Hilbert Transform of data

Complex Trace Attributes:

1. Instantaneous Amplitude, IA

$$\sqrt{S(t)^2 + H(t)^2}$$

2. Instantaneous Phase, IP

$$\arctan(\frac{H(t)}{S(t)})$$

3. Instantatneous Frequency, IF

$$\frac{\mathrm{d}}{\mathrm{d}t}(IP))$$

4. Derivative of IA

$$\frac{\mathrm{d}}{\mathrm{d}t}(IA)$$

5. 2nd Derivative of IA

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}(IA)$$

6. Instantaneous Acceleration, IAcc

$$\frac{\mathrm{d}}{\mathrm{d}t}(IF)$$

7. Amplitude Weighted Instantaneous Phase

$$IA * IP$$

8. Amplitude Weighted Instantaneous Frequency

$$IA * IF$$

Attributes from the normalised Graph Laplacian:

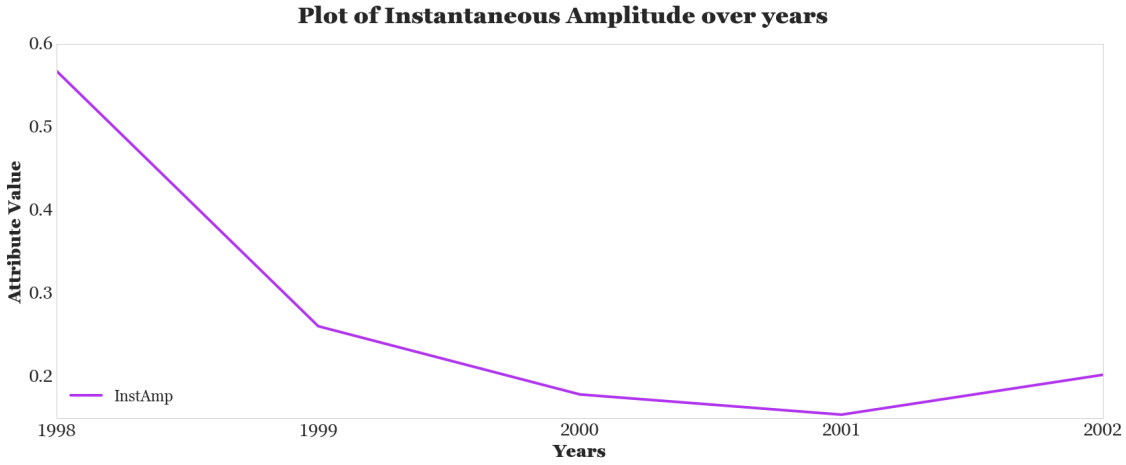
- Gaussian Curvature
- Kernel PCA 3 Component Ratio Change
- Resistane Distance
- Stationarity Ratio
- ICA 3 Component Ratio
- Norm NMF Ratio

6.1 Instantaneous Amplitude

Instantaneous Amplitude is widely used in traditional tectonic and stratigraphic interpretation. As one of the basic parameters of the amplitude attribute, it helps delineate the high- or low-amplitude anomaly (bright or dark spots). In this context this should highlight the bright and dark spots in the network when used as an attribute map and should show the highest and lowest points when used as a timeseries. As I show later this amplitude has a high correlation (> 0.7) with the traditional centrality measures so its behaviour is very similar to those. Therefore it is highly plausible that for this data it should suffice to look at this attribute instead of many different centrality measures.

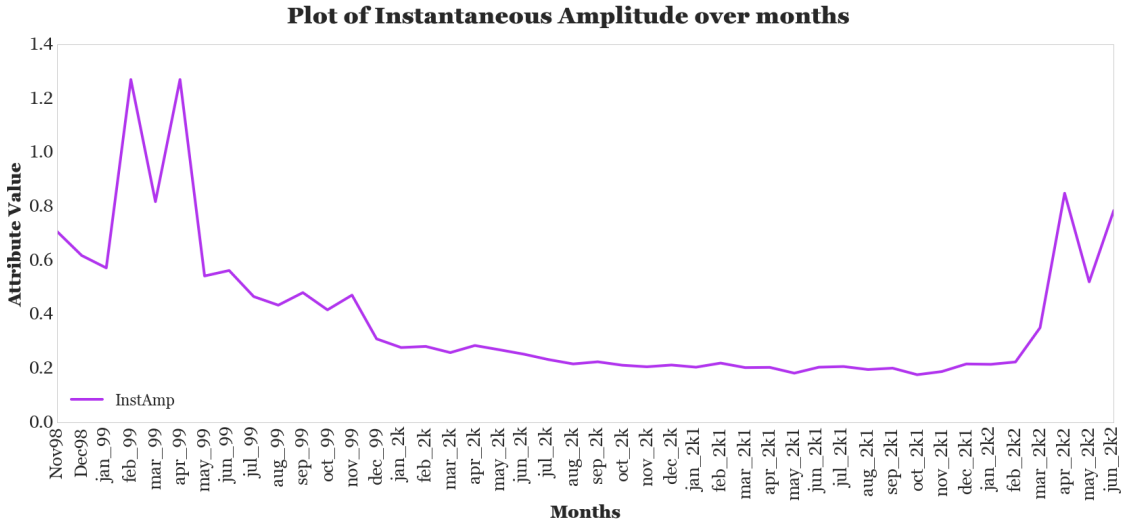
```
In [74]: y_avgst_all.InstAmp.plot(fontsize=22, figsize=(28,10))
         plt.suptitle("Plot of Instantaneous Amplitude over years", fontsize=33)
         plt.xlabel("Years", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
         plt.yticks(fontsize=26)
```

```
Out[74]: (array([ 0.1,  0.2,  0.3,  0.4,  0.5,  0.6,  0.7]),
          <a list of 7 Text yticklabel objects>)
```




```
In [75]: stat_all.InstAmp.plot(fontsize=22, rot=90, figsize=(28,10))
         plt.suptitle("Plot of Instantaneous Amplitude over months", fontsize=33)
         plt.xlabel("Months", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(np.arange(len(months)), months,fontsize=26)
         plt.yticks(fontsize=26)
```

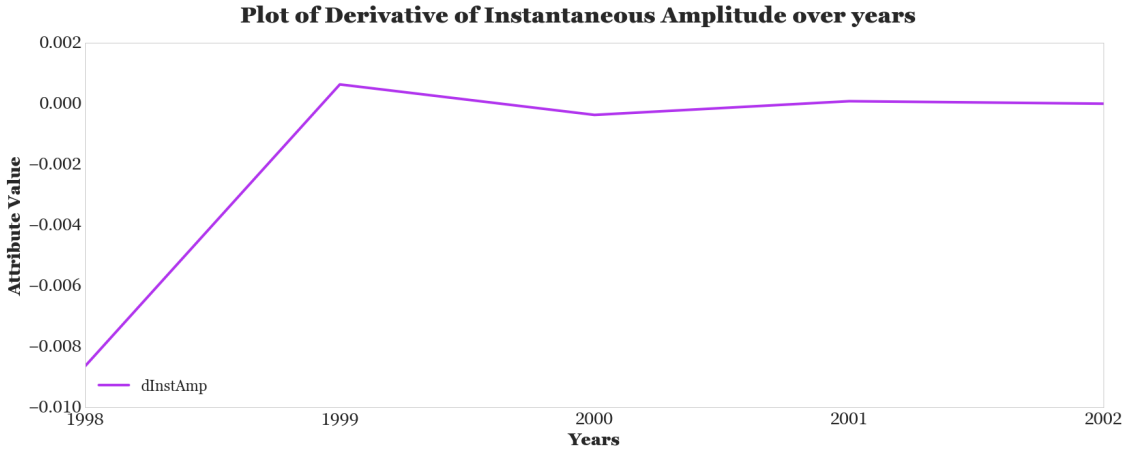
```
Out[75]: (array([ 0. ,  0.2,  0.4,  0.6,  0.8,  1. ,  1.2,  1.4,  1.6]),
          <a list of 9 Text yticklabel objects>)
```



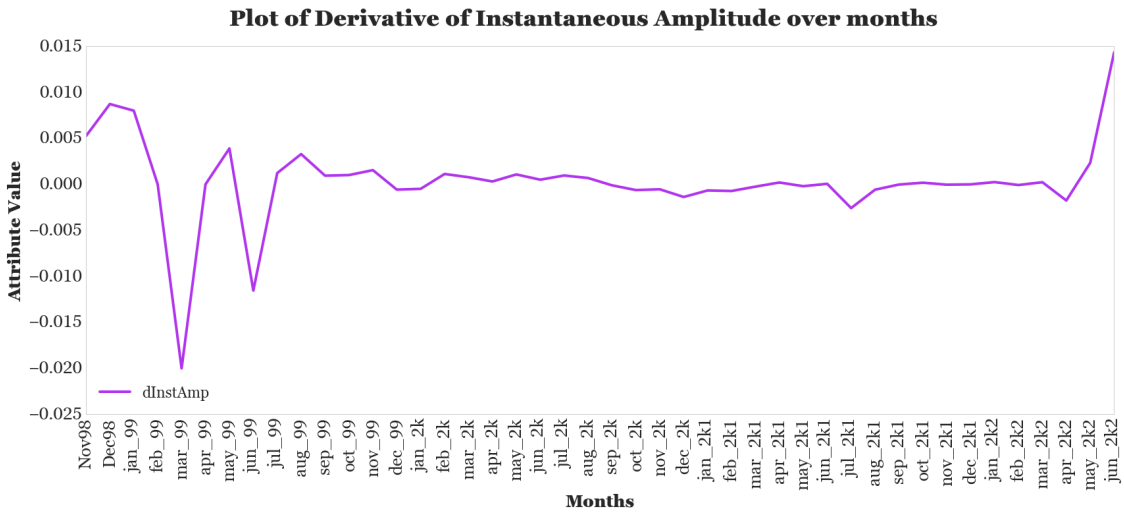
6.2 Derivative of Instantanteous Amplitue, $\frac{\mathrm{d}}{\mathrm{d}t}(IA)$

The derivative of IA highlights the change in reflectivity and shows sharp interfaces and discontinuities. Effectively this should highlight the big changes in the IA and the smaller changes should make the attribute smooth. This is what we observe because the attribute highlights the peaks observed in the IA plot and the rest of the signal is fairly smooth for the time range.

```
In [76]: y_avgst_all.dInstAmp.plot(fontsize=22, figsize=(28,10))
         plt.suptitle("Plot of Derivative of Instantaneous Amplitude over years", f
         plt.xlabel("Years", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
         plt.yticks(fontsize=26)

Out[76]: (array([-0.01 , -0.008, -0.006, -0.004, -0.002,  0.   ,  0.002]),
          <a list of 7 Text yticklabel objects>)
```



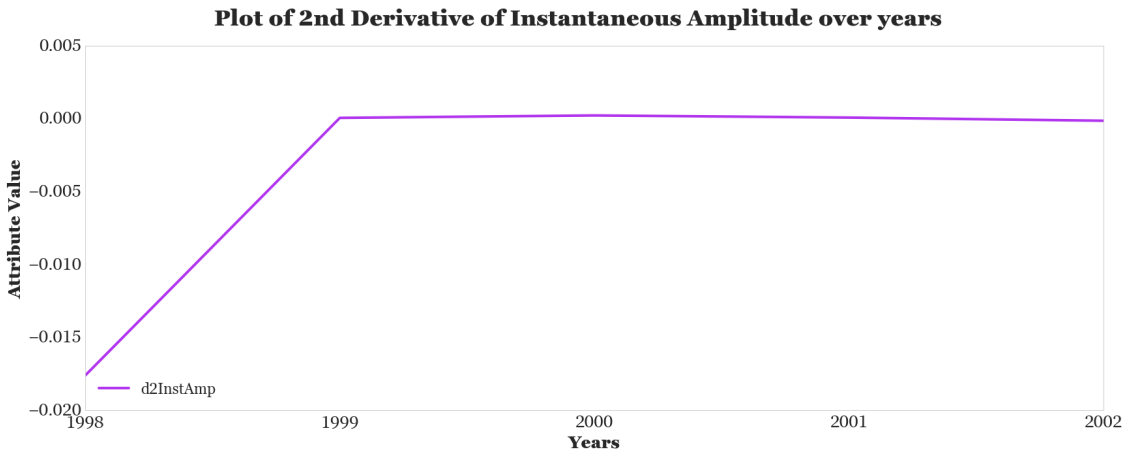

```
In [77]: stat_all.dInstAmp.plot(fontsize=22, rot=90, figsize=(28,10))
         plt.suptitle("Plot of Derivative of Instantaneous Amplitude over months",
         plt.xlabel("Months", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(np.arange(len(months)), months,fontsize=26)
         plt.yticks(fontsize=26)

Out[77]: (array([-0.03 , -0.025, -0.02 , -0.015, -0.01 , -0.005,  0.   ,  0.005,
                  0.01 ,  0.015]), <a list of 10 Text yticklabel objects>)
```



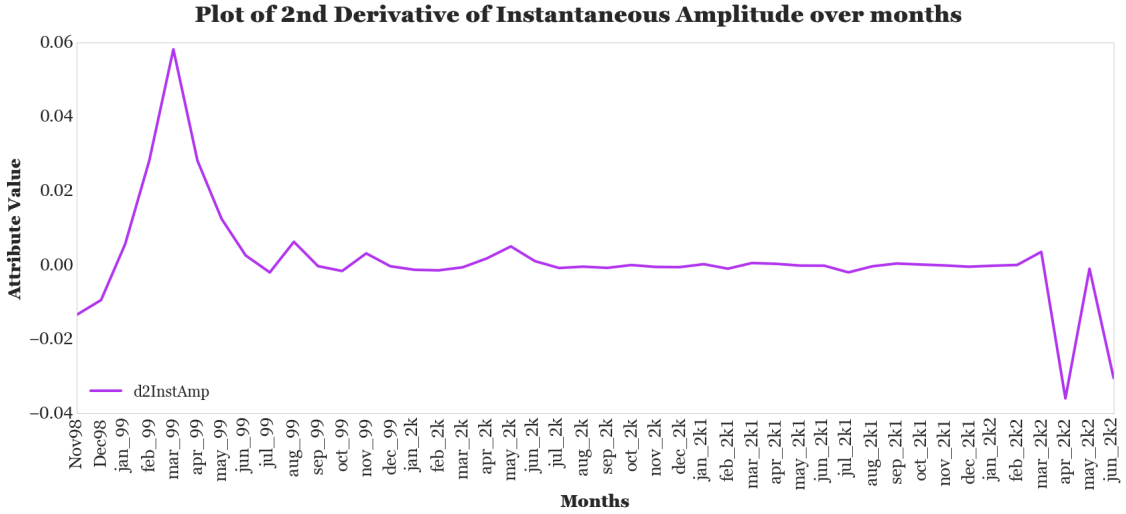
6.3 2nd Derivative of Instantaneous Amplitude $\frac{d^2}{dt^2}(IA)$

The second derivative of the IA highlights the interfaces very well - the places of change. This attribute is not too sensitive to the amplitude and can highlight even weak events.We see this to be the case where the first derivative highlights the individual peaks the second derivative smoothes the individual peaks and gives a smooth peak over the range of months where they occur and does a better job of highlighting the peaks towards the end of the timeseries than the first derivative.

```
In [78]: y_avgst_all.d2InstAmp.plot(fontsize=22, figsize=(28,10))
         plt.suptitle("Plot of 2nd Derivative of Instantaneous Amplitude over years
         plt.xlabel("Years", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
         plt.yticks(fontsize=26)

Out[78]: (array([-0.02 , -0.015, -0.01 , -0.005,  0.   ,  0.005,  0.01 ]),
          <a list of 7 Text yticklabel objects>)
```




```
In [79]: stat_all.d2InstAmp.plot(fontsize=22, rot=90, figsize=(28,10))
         plt.suptitle("Plot of 2nd Derivative of Instantaneous Amplitude over month
         plt.xlabel("Months", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(np.arange(len(months)), months,fontsize=26)
         plt.yticks(fontsize=26)

Out[79]: (array([-0.04, -0.02,  0.  ,  0.02,  0.04,  0.06,  0.08]),
          <a list of 7 Text yticklabel objects>)
```

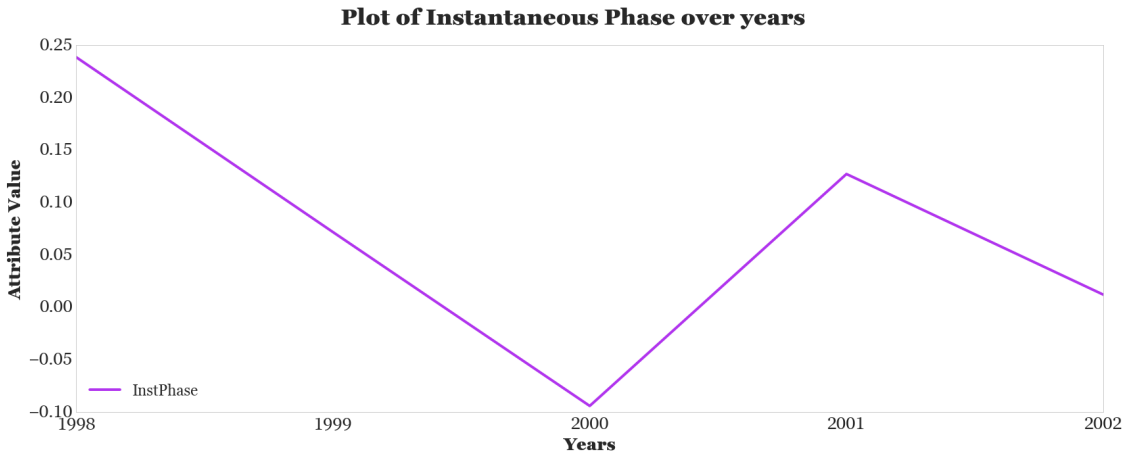


6.4 Instantaneous Phase

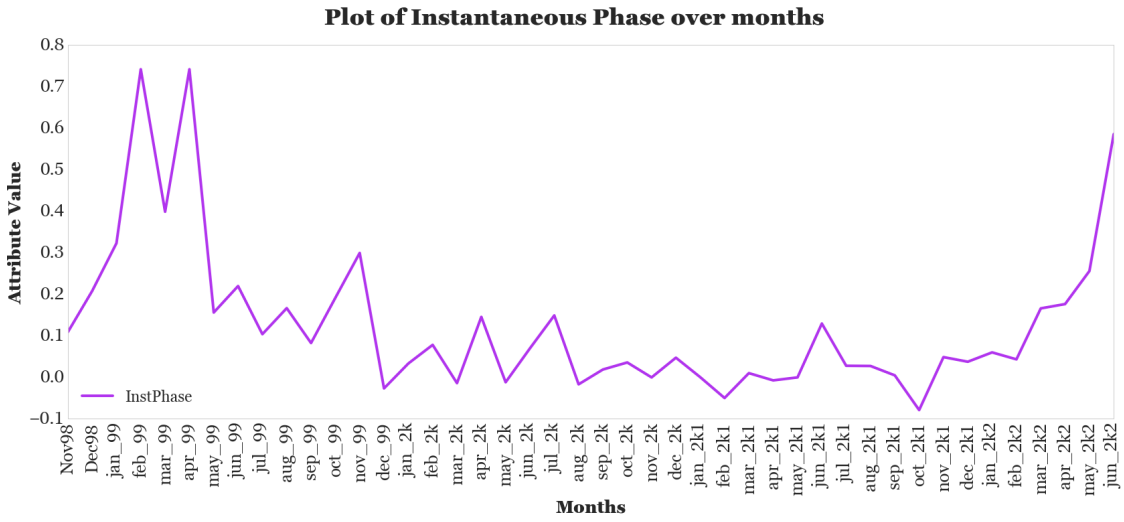
Instantaneous Phase is expressed in degrees or radians at the selected sampling point. Instantaneous phase helps strengthen weak reflections in the inner parts of reservoirs

and also strengthens the noise. Because the hydrocarbon accumulation often causes phase changes, this attribute can be used as a direct indication of hydrocarbons. The cosine of the instantaneous phase is derived from the instantaneous phase. It is commonly used to improve the variation display of the instantaneous phase because it has fixed boundary values (-1 to +1). In this context the change in the IP can be used as an indicator for the itervals which are the most interesting since they have a noticeable phase change. This is better illustrated by the cosine of IP especially the percentage change plot here we see the months with the greatest phase change really well and the peaks are well delineated in constrast to the just the IP plot.

```
In [80]: y_avgst_all.InstPhase.plot(fontsize=22, figsize=(28,10))
         plt.suptitle("Plot of Instantaneous Phase over years", fontsize=33)
         plt.xlabel("Years", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
         plt.yticks(fontsize=26)

Out[80]: (array([-0.1 , -0.05,  0.  ,  0.05,  0.1 ,  0.15,  0.2 ,  0.25,  0.3 ]),
          <a list of 9 Text yticklabel objects>)
```

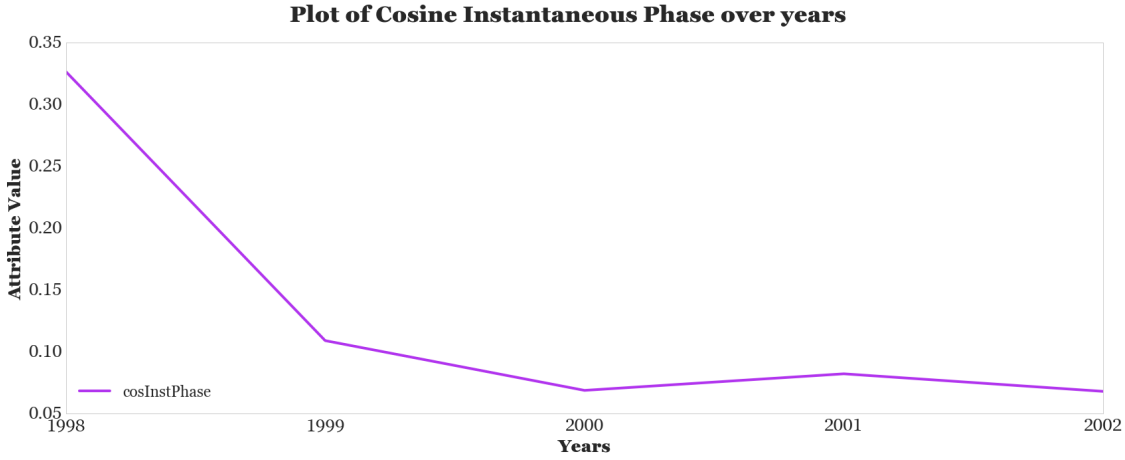



```
In [81]: stat_all.InstPhase.plot(fontsize=22, rot=90, figsize=(28,10))
         plt.suptitle("Plot of Instantaneous Phase over months", fontsize=33)
         plt.xlabel("Months", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(np.arange(len(months)), months,fontsize=26)
         plt.yticks(fontsize=26)

Out[81]: (array([-0.1,  0. ,  0.1,  0.2,  0.3,  0.4,  0.5,  0.6,  0.7,  0.8]),
          <a list of 10 Text yticklabel objects>)
```



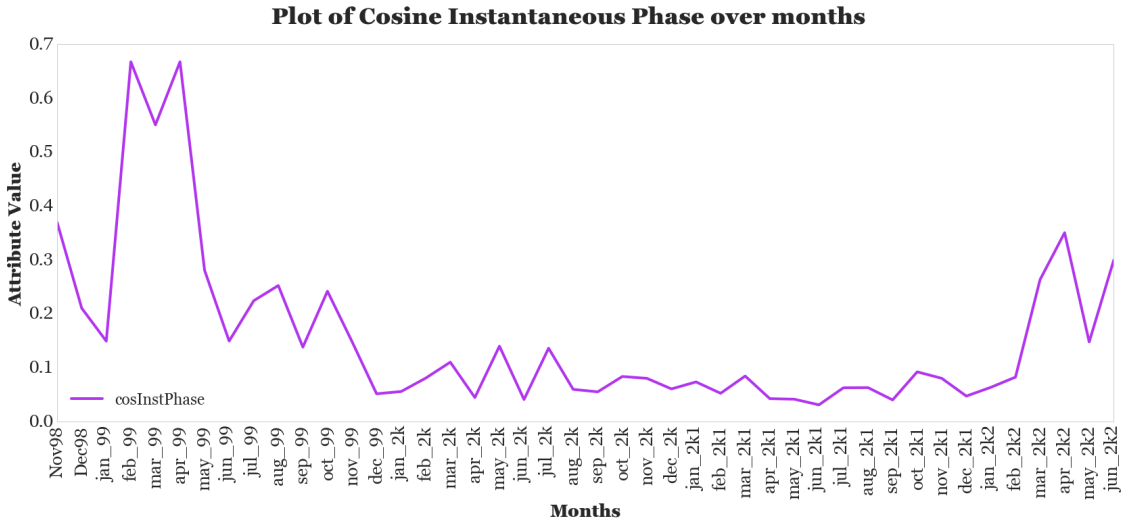
6.5 Cosine of IP

```
In [82]: y_avgst_all.cosInstPhase.plot(fontsize=22, figsize=(28,10))
         plt.suptitle("Plot of Cosine Instantaneous Phase over years", fontsize=33)
         plt.xlabel("Years", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
         plt.yticks(fontsize=26)

Out[82]: (array([ 0.05,  0.1 ,  0.15,  0.2 ,  0.25,  0.3 ,  0.35,  0.4 ]),
          <a list of 8 Text yticklabel objects>)
```



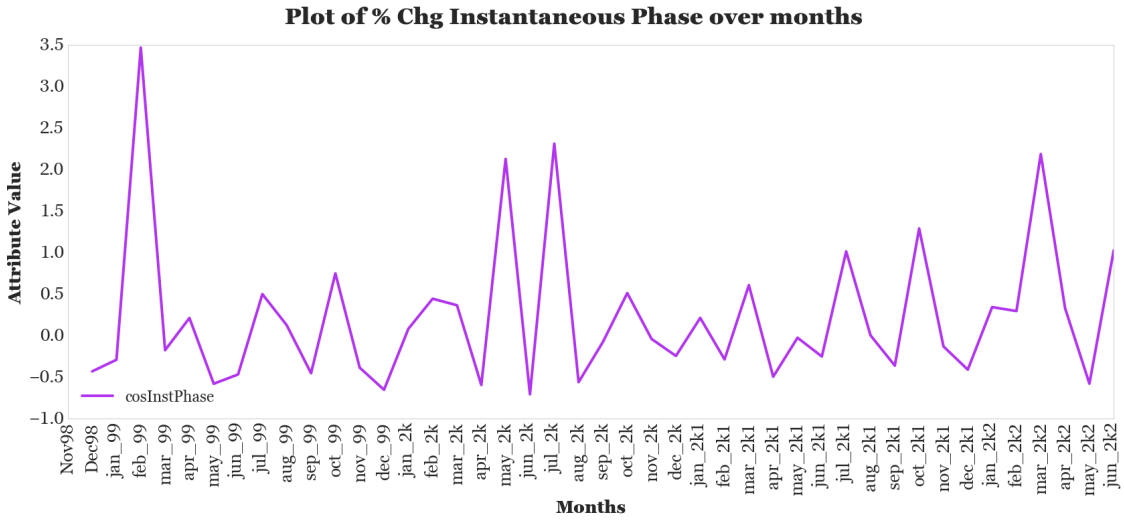
In [83]:
```
stat_all.cosInstPhase.plot(fontsize=22, rot=90, figsize=(28,10))
plt.suptitle("Plot of Cosine Instantaneous Phase over months", fontsize=33
plt.xlabel("Months", fontsize=25)
plt.ylabel("Attribute Value", fontsize=25)
plt.legend(fontsize=23, loc=3)
plt.xticks(np.arange(len(months)), months,fontsize=26)
plt.yticks(fontsize=26)
```

Out[83]:
```
(array([ 0. ,  0.1,  0.2,  0.3,  0.4,  0.5,  0.6,  0.7,  0.8]),
 <a list of 9 Text yticklabel objects>)
```



In [84]:
```
stat_all.cosInstPhase.pct_change().plot(fontsize=22, rot=90, figsize=(28,1
plt.suptitle("Plot of % Chg Instantaneous Phase over months", fontsize=33)
plt.xlabel("Months", fontsize=25)
plt.ylabel("Attribute Value", fontsize=25)
plt.legend(fontsize=23, loc=3)
plt.xticks(np.arange(len(months)), months,fontsize=26)
plt.yticks(fontsize=26)
```

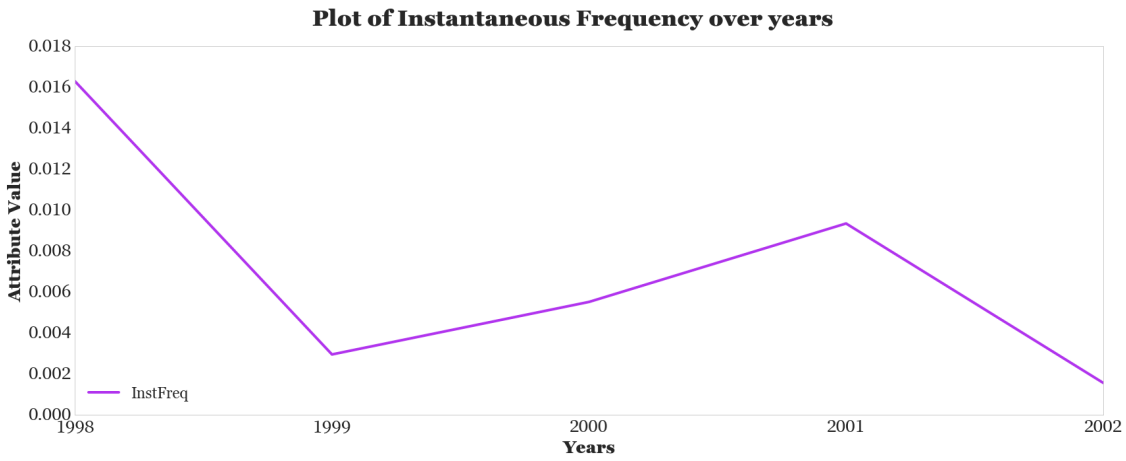
Out[84]:
```
(array([-1. , -0.5,  0. ,  0.5,  1. ,  1.5,  2. ,  2.5,  3. ,  3.5]),
 <a list of 10 Text yticklabel objects>)
```



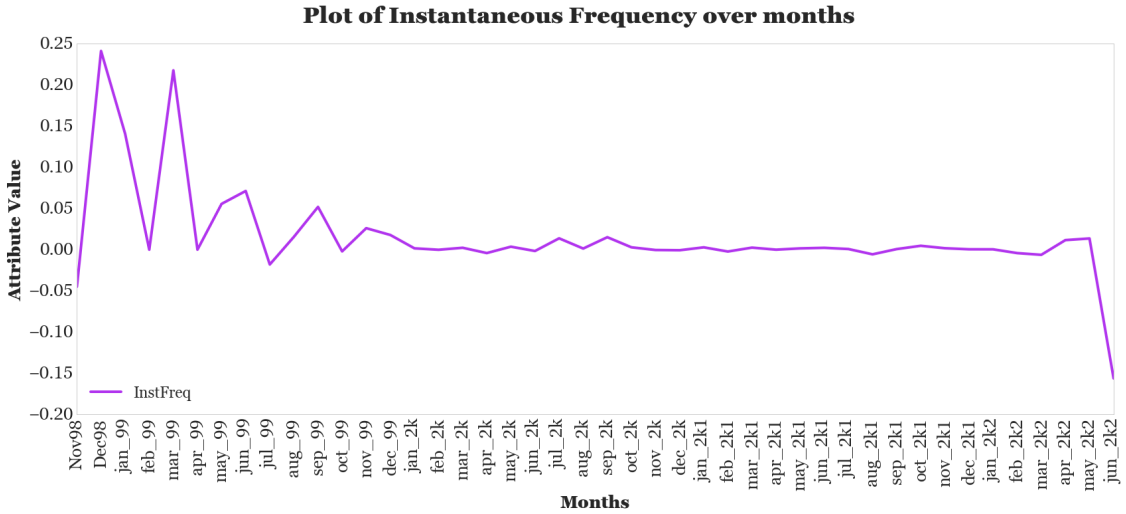
6.6 Instantaneous Frequency

Instantaneous frequency is defined as the time derivative of the instantaneous phase and it is used for estimating seismic attenuation. Oil and gas reservoirs often cause the attenuation of high-frequency components, so this attribute is also conducive to measuring stratigraphic periodic intervals. In this context I expect this to show clearly the peaks of the signal. This is exactly what we see especially in the monthly trend where the two large peaks are clearly delineated with a some smaller peaks highlighted the attribute is smooth over the rest of the series. So it has the property of finding the most significant peaks in our data.

```
In [85]: y_avgst_all.InstFreq.plot(fontsize=22, figsize=(28,10))
         plt.suptitle("Plot of Instantaneous Frequency over years", fontsize=33)
         plt.xlabel("Years", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
         plt.yticks(fontsize=26)

Out[85]: (array([ 0.   ,  0.002,  0.004,  0.006,  0.008,  0.01 ,  0.012,  0.014,
                  0.016,  0.018]), <a list of 10 Text yticklabel objects>)
```




```
In [86]: stat_all.InstFreq.plot(fontsize=22, rot=90, figsize=(28,10))
         plt.suptitle("Plot of Instantaneous Frequency over months", fontsize=33)
         plt.xlabel("Months", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(np.arange(len(months)), months,fontsize=26)
         plt.yticks(fontsize=26)

Out[86]: (array([-0.2 , -0.15, -0.1 , -0.05,  0.  ,  0.05,  0.1 ,  0.15,  0.2 ,  0.
          <a list of 10 Text yticklabel objects>)
```

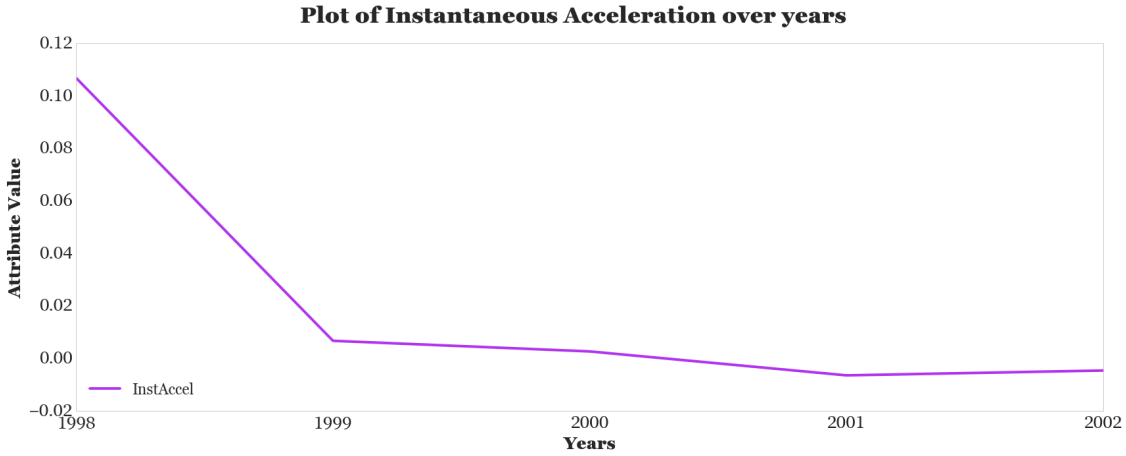


6.7 Instantaneous Acceleration

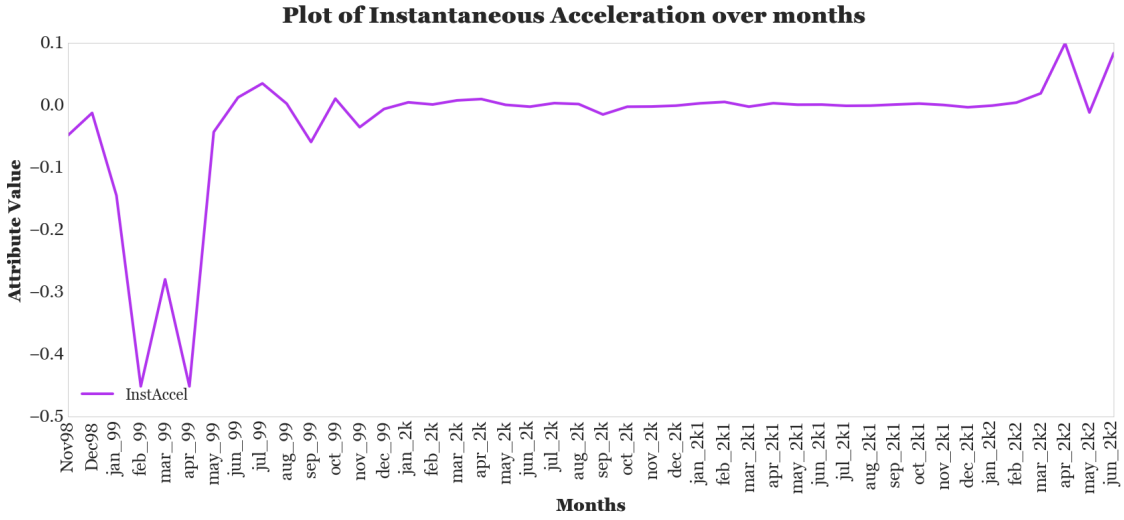
Instantaneous Acceleration is defined as the rate of change of the instantaneous frequency, which is often used to indicate the rate of attenuation and absorption. As gas

(or oil, or water) can cause the attenuation of seismic waves, this attribute can represent a fluid interface in the high-resolution data. As a deriative attribute I expect it to highlight the peaks and troughs very effectively and be smooth in areas of less pronounced change. This is what is observed as the attribute highlights the two large peaks but with the opposite polarity of IF.

```
In [87]: y_avgst_all.InstAccel.plot(fontsize=22, figsize=(28,10))
         plt.suptitle("Plot of Instantaneous Acceleration over years", fontsize=33)
         plt.xlabel("Years", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
         plt.yticks(fontsize=26)

Out[87]: (array([-0.02,  0.  ,  0.02,  0.04,  0.06,  0.08,  0.1 ,  0.12,  0.14]),
          <a list of 9 Text yticklabel objects>)
```



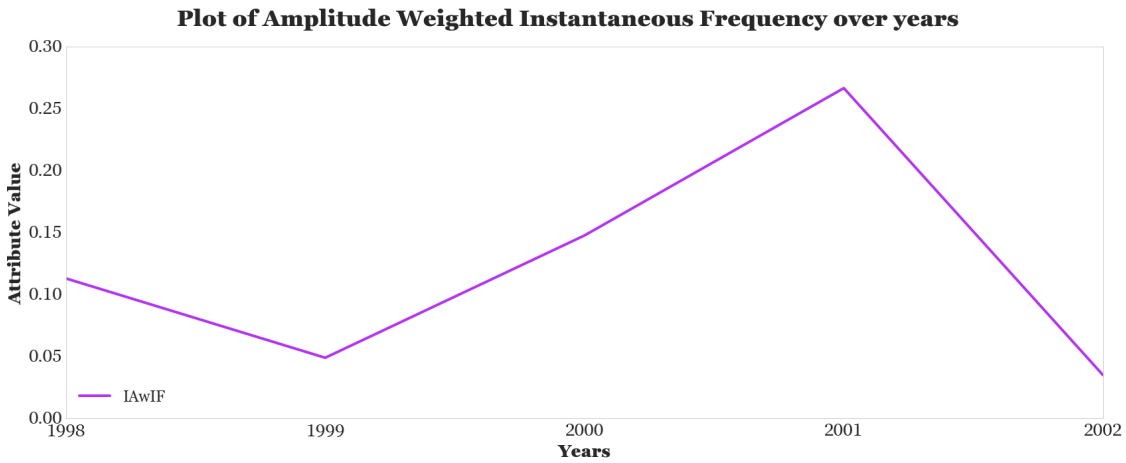

```
In [88]: stat_all.InstAccel.plot(fontsize=22, rot=90, figsize=(28,10))
         plt.suptitle("Plot of Instantaneous Acceleration over months", fontsize=33
         plt.xlabel("Months", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(np.arange(len(months)), months,fontsize=26)
         plt.yticks(fontsize=26)

Out[88]: (array([-0.5, -0.4, -0.3, -0.2, -0.1,  0. ,  0.1,  0.2]),
          <a list of 8 Text yticklabel objects>)
```



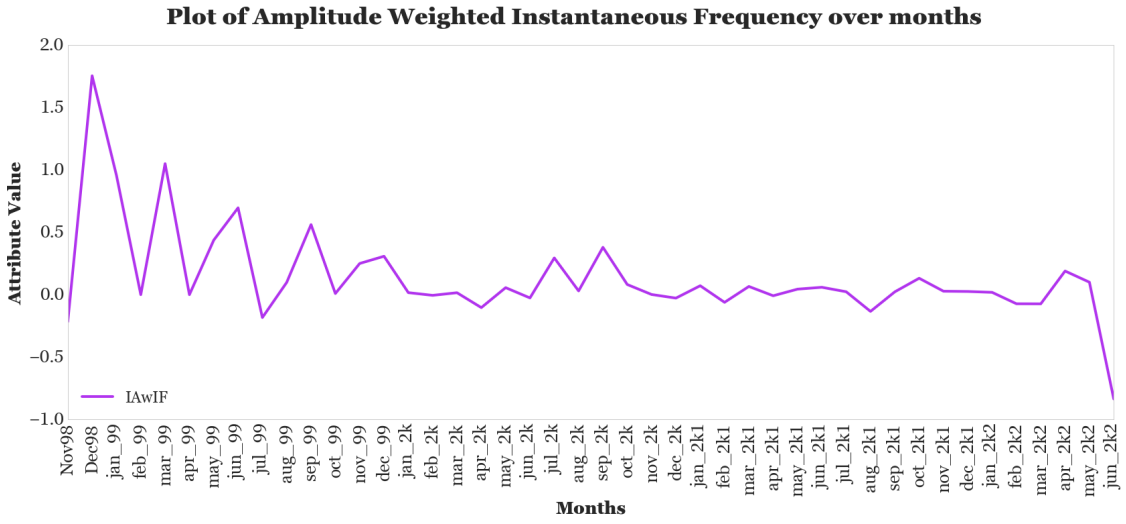
6.8 Amplitude Weighted IF

Amplitude weighted Instantaneous Frequency provides a reliable smooth instantaneous frequency estimation in order to reduce the interference damage. I expcect this to better highlight frequency anomalies in the data and suppress insignificant anomalies. We see that the weighted IF is sharper in places where the IF is fairly smooth. This highlights the major peaks as well as minor anomalies not immediately obvious from the IF alone.

```
In [89]: y_avgst_all.IAwIF.plot(fontsize=22, figsize=(28,10))
         plt.suptitle("Plot of Amplitude Weighted Instantaneous Frequency over year
         plt.xlabel("Years", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
         plt.yticks(fontsize=26)

Out[89]: (array([ 0.  ,  0.05,  0.1 ,  0.15,  0.2 ,  0.25,  0.3 ,  0.35]),
          <a list of 8 Text yticklabel objects>)
```




```
In [90]: stat_all.IAwIF.plot(fontsize=22, rot=90, figsize=(28,10))
         plt.suptitle("Plot of Amplitude Weighted Instantaneous Frequency over mont
         plt.xlabel("Months", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(np.arange(len(months)), months,fontsize=26)
         plt.yticks(fontsize=26)

Out[90]: (array([-1. , -0.5,  0. ,  0.5,  1. ,  1.5,  2. ]),
          <a list of 7 Text yticklabel objects>)
```

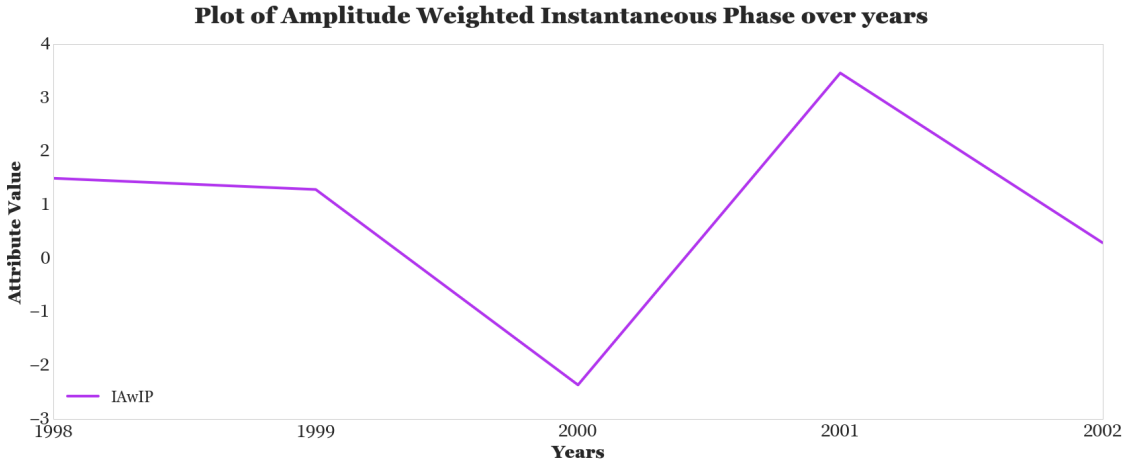


6.9 Amplitude weighted Instantaneous Phase

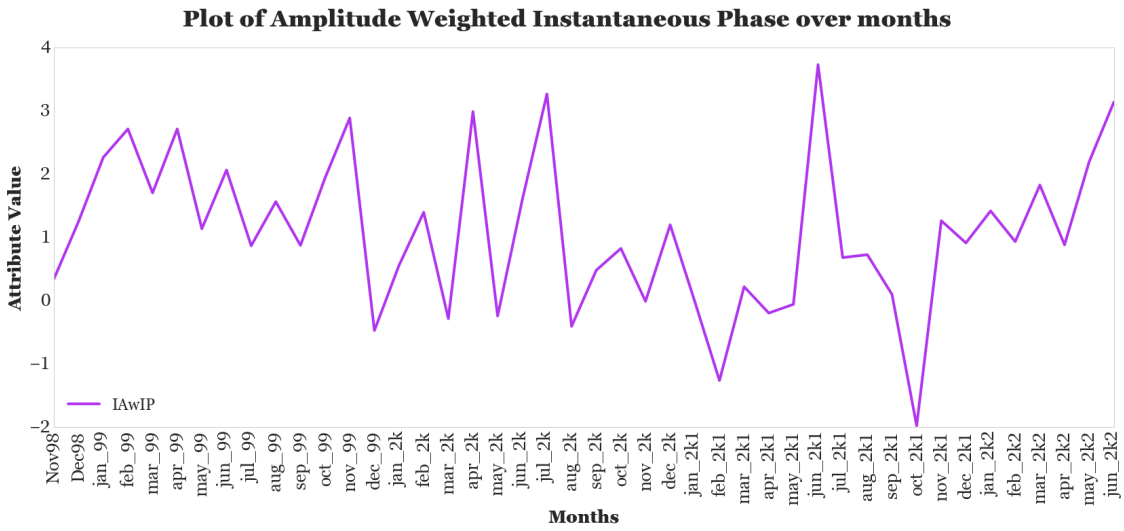
The amplitude weighted IP should cause the phase to become sharper with the peaks and troughs more accentuated. This has the effect of magnifying the signal as well as

the noise. However, in this case we see that the trend in IP weighted and regular IP plot are identical with the only difference being the magnitude of the peaks.

```
In [91]: y_avgst_all.IAwIP.plot(fontsize=22, figsize=(28,10))
         plt.suptitle("Plot of Amplitude Weighted Instantaneous Phase over years",
         plt.xlabel("Years", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
         plt.yticks(fontsize=26)

Out[91]: (array([-3., -2., -1.,  0.,  1.,  2.,  3.,  4.]),
          <a list of 8 Text yticklabel objects>)
```




```
In [92]: stat_all.IAwIP.plot(fontsize=22, rot=90, figsize=(28,10))
         plt.suptitle("Plot of Amplitude Weighted Instantaneous Phase over months",
         plt.xlabel("Months", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(np.arange(len(months)), months,fontsize=26)
         plt.yticks(fontsize=26)

Out[92]: (array([-2., -1.,  0.,  1.,  2.,  3.,  4.]),
          <a list of 7 Text yticklabel objects>)
```



6.10 Mean Curvature

There are many ways to define curvature I use the Gaussian Curvature which is derived from the Hessian Matrix. A surface might be curved upward in places, curve downward in places, or even be flat in places. Also, at some given point, the surface may be curved upward in some directions and downward in others. The curvature measure helps us detect such change. Gaussian curvature can be positive, negative, or zero. A useful property of the curvature attribute is that it is independent of orientation of the surface. If we place vectors on this surface the would indicates where the curve bends i.e., the vectors are either diverging, converging, or parallel. This can correspond to the negative, positive or zero value of curvature.

$$K = \frac{f_{xx}f_{yy} + f_{xy}f_{yx}}{(1 + f_x^2 + f_y^2)^2}$$

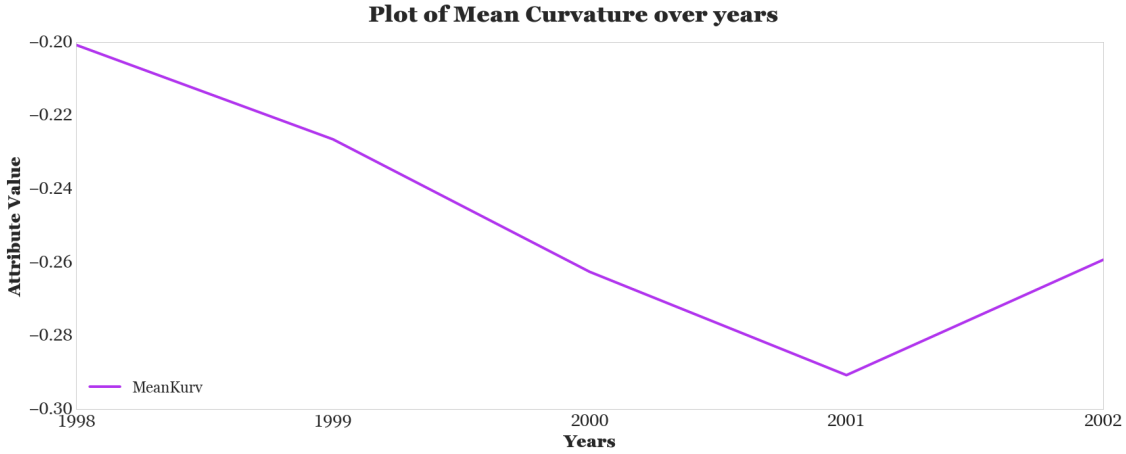
and the mean curvature is

$$K_{mean} = tr(\lambda_{hessian})$$

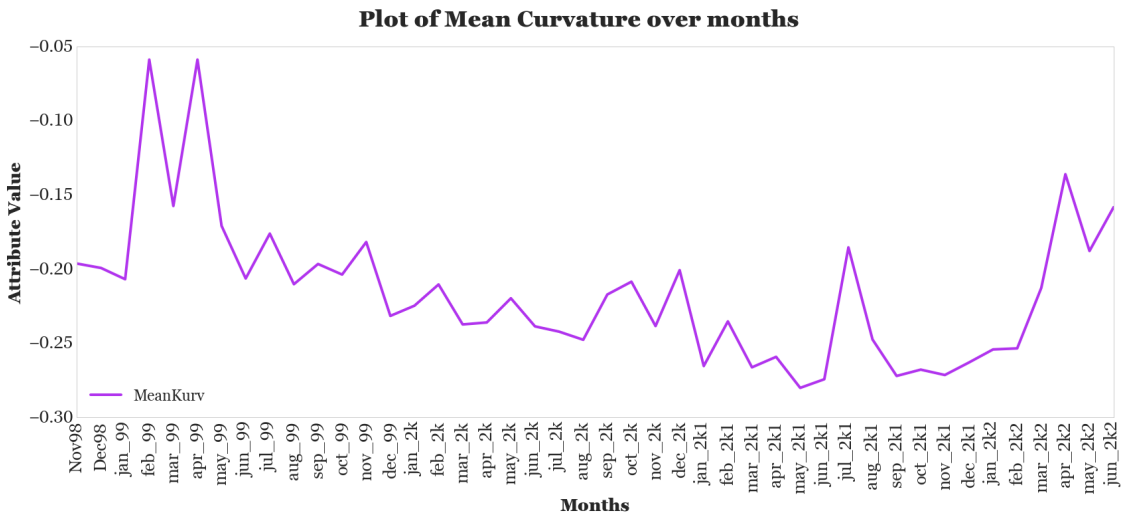
the trace of the eigenvalues of the Hessian Matrix.

The Mean Curvature for the network seems to be negative throughout this range which could mean that this network is diverging. This rate of divergence is captured on a yearly and monthly scale in this case by this attribute in its normal form. To get a better sense and interpret this attribute the percent change plot is more useful here we see that the two peaks are well highlighted and the remaining profile is smoothed indicating that the network is not changing very fast more most of the months but the divergence is greater in 1999 during its golden period and in 2002 when the firm went bankrupt. This ties well with what we know from reality and what the other attributes highlight. In contrast to the KPCA we have to note that this highlights the big features as opposed to more subtle features. Curvature is typically used to dilineate structures such as faults or other structures such as anticlines (+ curvature) or synclines (- curvature). In the network context it also seems to have a preference for large features and recovers the peaks of the signal very well.

```
In [93]: y_avgst_all.MeanKurv.plot(fontsize=22, figsize=(28,10))
         plt.suptitle("Plot of Mean Curvature over years", fontsize=33)
         plt.xlabel("Years", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
         plt.yticks(fontsize=26)

Out[93]: (array([-0.3 , -0.28, -0.26, -0.24, -0.22, -0.2 , -0.18]),
          <a list of 7 Text yticklabel objects>)
```

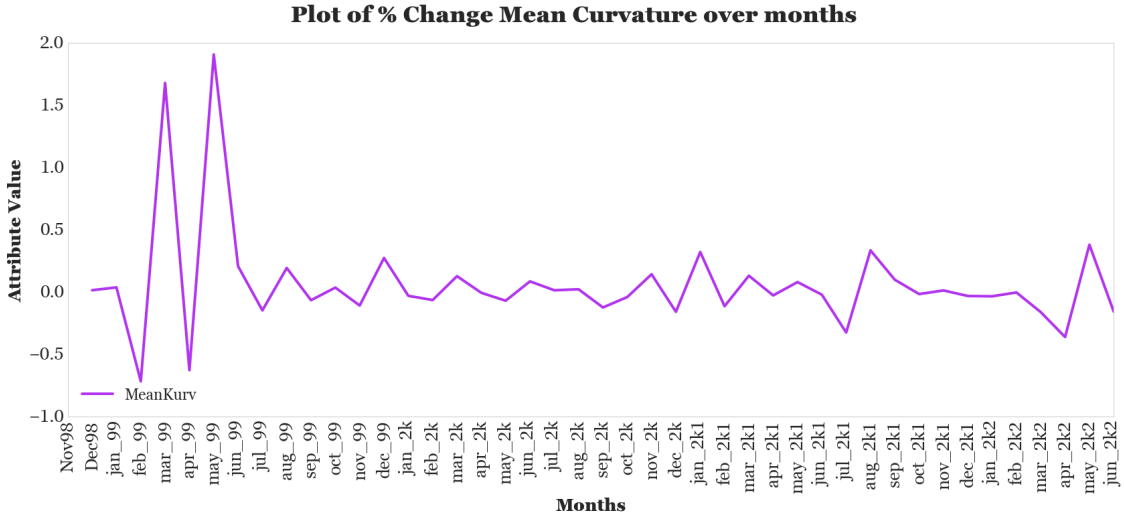



```
In [94]: stat_all.MeanKurv.plot(fontsize=22, rot=90, figsize=(28,10))
         plt.suptitle("Plot of Mean Curvature over months", fontsize=33)
         plt.xlabel("Months", fontsize=25)
         plt.ylabel("Attribute Value", fontsize=25)
         plt.legend(fontsize=23, loc=3)
         plt.xticks(np.arange(len(months)), months,fontsize=26)
         plt.yticks(fontsize=26)

Out[94]: (array([-0.35, -0.3 , -0.25, -0.2 , -0.15, -0.1 , -0.05]),
          <a list of 7 Text yticklabel objects>)
```



In [95]:
```
stat_all.MeanKurv.pct_change().plot(fontsize=22, rot=90, figsize=(28,10))
plt.suptitle("Plot of % Change Mean Curvature over months", fontsize=33)
plt.xlabel("Months", fontsize=25)
plt.ylabel("Attribute Value", fontsize=25)
plt.legend(fontsize=23, loc=3)
plt.xticks(np.arange(len(months)), months,fontsize=26)
plt.yticks(fontsize=26)
```

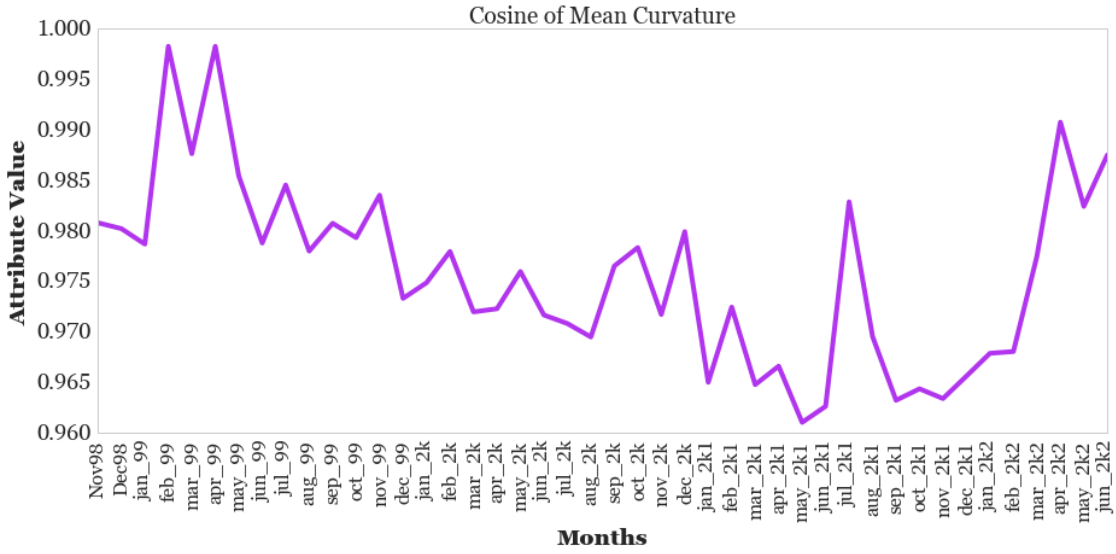
Out[95]:
```
(array([-1. , -0.5,  0. ,  0.5,  1. ,  1.5,  2. ]),
 <a list of 7 Text yticklabel objects>)
```



In [96]:
```
stat_all.MeanKurv.apply(lambda x: np.cos(x)).plot(title="Cosine of Mean Cu
plt.xticks(np.arange(len(months)), months,fontsize=16);
```

```
        plt.xlabel("Months", fontsize=18)
        plt.ylabel("Attribute Value", fontsize=18)
```

```
Out[96]: <matplotlib.text.Text at 0x29c4e6916a0>
```



6.11 Kernel PCA 3 Components Ratio

Here I calculate the Kernel PCA 3 Component ratio from the Normalised Graph Laplacian. This is the ratio of the main element difference between two networks. The reason a Kernel PCA is used is because of its ability to handle non-linearity through kernels. Here the RBF kernel is used to fit the Normalised Graph Laplacian and then the ratio is calculated as follows:

Step 1: Fit and transform the Normalised Graph Laplacian keeping only the first 3 compoenets: PC1, PC2 and PC3

Step 2: Calculate the Kernel PCA Ratio,

$$KPCA_r = \frac{PC1 - PC3}{PC1 - PC2}$$

Step 3: For all the networks calculate the change in the KPCA Ratio as

$$\delta K_r = \frac{K_{rt0}}{K_{rt1}}$$

recursively to derive a rolling measure of the change in KLPCA Ratio over time.

The KLPCA is one of the main element analysis methods in seismic attribute analysis where it is used to calculate the correlation in a multitrace window. A small value represents a degree of intermittent or no correlation of geological phenomena. It is also used to detect discontinuities, such as faults and unconformities. Here the purpose is to locate big discontinuities in our networks over time. This represents a scalalble and easy way to locate the biggest changes even when we are dealing with a large number of dynamic networks. Most of the values encountered

are relatively small. But from the monthly plot we see that there is a large discontinuity in June and July 1999 whose scale dominates the plot. Hence the log of this attribute is used and we see other smaller signals emerge as a result.

It is encouraging to see that the attribute picks up the Feb-Mar 99 peak like the other attributes but the the discontinuity it finds is June/July 99 which are relatively minor peaks according to the other attributes. This could be due to the fact that I use 3 components of the KLPCA to calculate the change as opposed to just the first component. The second and third KPCA components are typically used to find geological unconformities which is a discontinuity in rock sequence indicating interruption of sedimentation, commonly accompanied by erosion of rocks below the break i.e it represents an abrupt change. Hence it can be reasonably assumed that by substracting these components from the first component we are more likely to find these more abrupt breaks in the data that are no as clear looking at other attributes.

```
In [97]: all_month_G = tuple([G_nov98,G_dec98,G_jan_99,G_feb_99,G_mar_99,G_apr_99,G
                          G_nov_99,G_dec_99,G_jan_2k,G_feb_2k,G_mar_2k,G_apr_2k,G_may_
                          G_oct_2k,G_nov_2k,G_dec_2k,G_jan_2k1,G_feb_2k1,G_mar_2k1,G_a
                          G_aug_2k1,G_sep_2k1,G_oct_2k1,G_nov_2k1,G_dec_2k1,G_jan_2k2,

In [98]: def kpca_ratio(net):
             from sklearn.decomposition import KernelPCA
             kpca = KernelPCA(n_components=3,kernel='rbf')
             m = nx.normalized_laplacian_matrix(net).todense()
             X_kpca = kpca.fit_transform(m)
             pc1 = X_kpca[:,0]
             pc2 = X_kpca[:,1]
             pc3 = X_kpca[:,2]
             kpca_rat = norm(pc1-pc3/(pc1-pc2))
             return kpca_rat

In [99]: def kpca_att(net):
             kpca_chg = []
             for i in range(len(net)-1):
                 x = int(i)
                 y = 1+x
                 kpcar1= kpca_ratio(net[x])
                 kpcar2= kpca_ratio(net[y])
                 kpca_chg.append(np.divide(kpcar2,kpcar1))
             kpca_chgpad = np.zeros(len(kpca_chg)+1);
             kpca_chgpad[1:] = kpca_chg

             return kpca_chgpad

In [100]: kpca_chg_y= kpca_att(all_year_G)
          kpca_chg_m = kpca_att(all_month_G)

In [101]: kpca_chg_m

Out[101]: array([  0.00000000e+00,   4.88883814e-01,   3.45460170e+00,
                   8.87338190e-02,   4.98218569e+01,   2.00715120e-02,
```

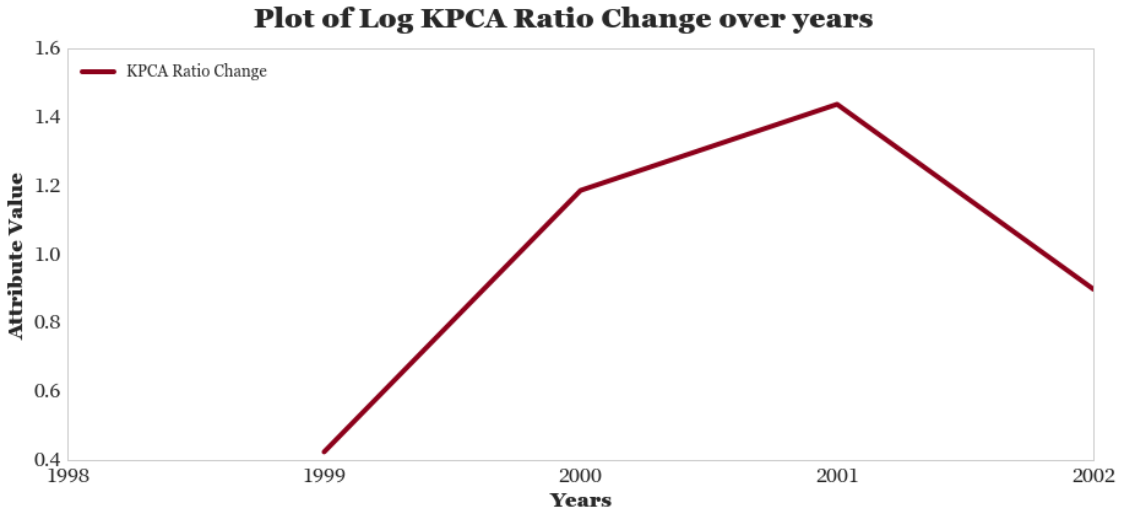
```
                1.10992264e+01,   1.17738962e+14,   2.32254840e-14,
                6.41831923e-01,   1.08075430e+01,   2.31947056e-02,
                1.67524790e+02,   5.02193494e+00,   1.46856694e-03,
                1.48814197e+01,   5.66322852e-01,   4.25853308e-01,
                1.12397367e+01,   1.39875121e+01,   3.09181415e-02,
                1.33596264e+01,   2.87383472e-01,   3.97053182e-01,
                1.16145289e+00,   7.21630828e-01,   3.22575646e-01,
                5.95254751e-01,   1.68793140e+00,   2.01282069e+01,
                3.26933404e-01,   1.86023362e-01,   2.57792631e+01,
                1.49918679e-01,   3.29429718e+01,   5.12458586e+00,
                8.46003304e-03,   3.97316868e-01,   1.62273256e+01,
                1.63227862e-01,   3.55140567e-01,   1.86324914e-02,
                1.95969629e+00,   2.01185382e+00])
```

```
In [102]: max(kpca_chg_m), min(kpca_chg_m)
```

```
Out[102]: (117738962094924.84, 0.0)
```

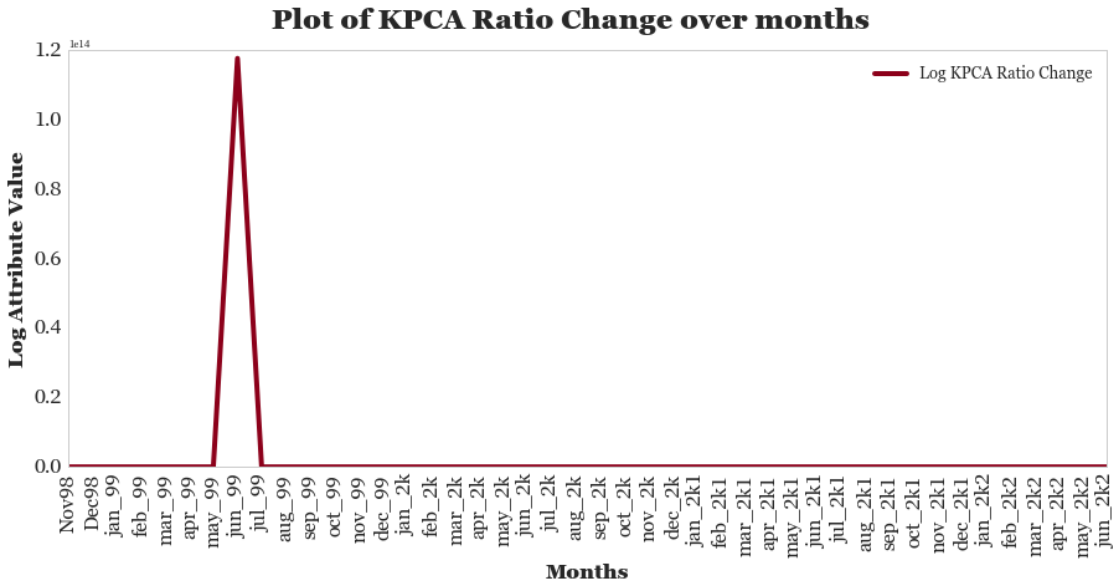
```
In [103]: plt.plot(np.log(kpca_chg_y), '#8C001A',label='KPCA Ratio Change')
          plt.suptitle("Plot of Log KPCA Ratio Change over years", fontsize=22)
          plt.xlabel("Years", fontsize=16)
          plt.ylabel("Attribute Value", fontsize=16)
          plt.legend(fontsize=13, loc=2)
          plt.xticks(np.arange(len(years[1:])), years[1:],fontsize=16)
          plt.yticks(fontsize=16)
```

```
Out[103]: (array([ 0.4,  0.6,  0.8,  1. ,  1.2,  1.4,  1.6]),
           <a list of 7 Text yticklabel objects>)
```

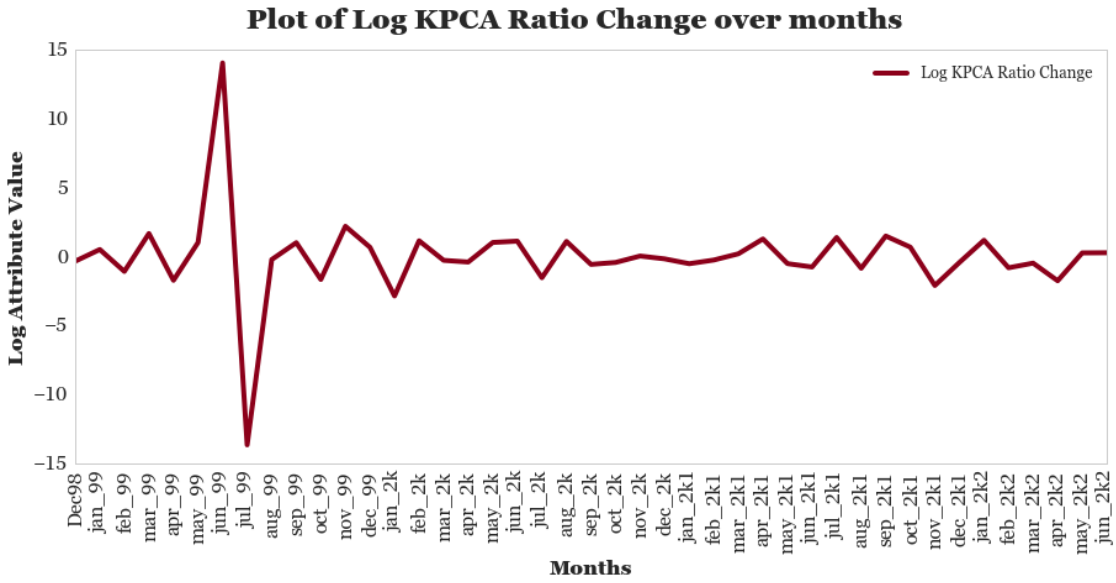



```
In [104]: plt.plot(kpca_chg_m, '#8C001A',label='Log KPCA Ratio Change')
          plt.suptitle("Plot of KPCA Ratio Change over months", fontsize=22)
```

```
plt.xlabel("Months", fontsize=16)
plt.ylabel("Log Attribute Value", fontsize=16)
plt.legend(fontsize=13, loc=1)
plt.xticks(np.arange(len(months)), months,fontsize=16, rotation=90)
plt.yticks(fontsize=16)
plt.autoscale()
```



In [105]:
```
plt.plot(np.log10(kpca_chg_m), '#8C001A',label='Log KPCA Ratio Change')
plt.suptitle("Plot of Log KPCA Ratio Change over months", fontsize=22)
plt.xlabel("Months", fontsize=16)
plt.ylabel("Log Attribute Value", fontsize=16)
plt.legend(fontsize=13, loc=1)
plt.xticks(np.arange(len(months)), months,fontsize=16, rotation=90)
plt.yticks(fontsize=16)
plt.autoscale()
```




```
In [106]: y_avgst_all['LogKPCARatio'] =np.log10(kpca_chg_y)
          stat_all['LogKPCARatio'] =np.log10(kpca_chg_m)

In [107]: stat_all.LogKPCARatio.iloc[0]=0
          y_avgst_all.LogKPCARatio.iloc[0]=0
```

6.12 ICA Ratio

Similarly to the KPCA ratio above I derive a ICA ratio for comparison. Since this another decomposition technique but assumes non-Gaussian structure it should find more local features in contrast to PCA which find smore global features. However, since we used a Kernel PCA with a RBF Kernel this should handle non-linearity much beter and I expect the ICA ratio in this case to behave similarly to the KPCA ratio but should highlight more local changes.

$$ICA_r = \frac{IC1 - IC3}{IC1 - IC2}$$

Then the change ratio is defined as:

$$\delta ICA_r = \frac{ICA_{rt0}}{ICA_{rt1}}$$

This is calculated the same way as the KPCA ratio above. As with the KPCA Ratio

The trends that the ICA ratio highlights are similar to what we have seen so far. But it also highlights other points of interest. Similarly to the KPCA ratio the values are large so there is a need to scale it to keep the values in a reasonable range. So the log of the ICA Ration is used.

```
In [108]: def fica_ratio(net):
              from sklearn.decomposition import FastICA
              fica = FastICA(n_components=3, max_iter=3000, tol=0.001, fun='exp')
```

```
            m = nx.normalized_laplacian_matrix(net).todense()
            X_fica = fica.fit_transform(m)
            fica_ratio = norm(X_fica[:,0]-X_fica[:,2]/(X_fica[:,0]-X_fica[:,1]))

            return fica_ratio
```

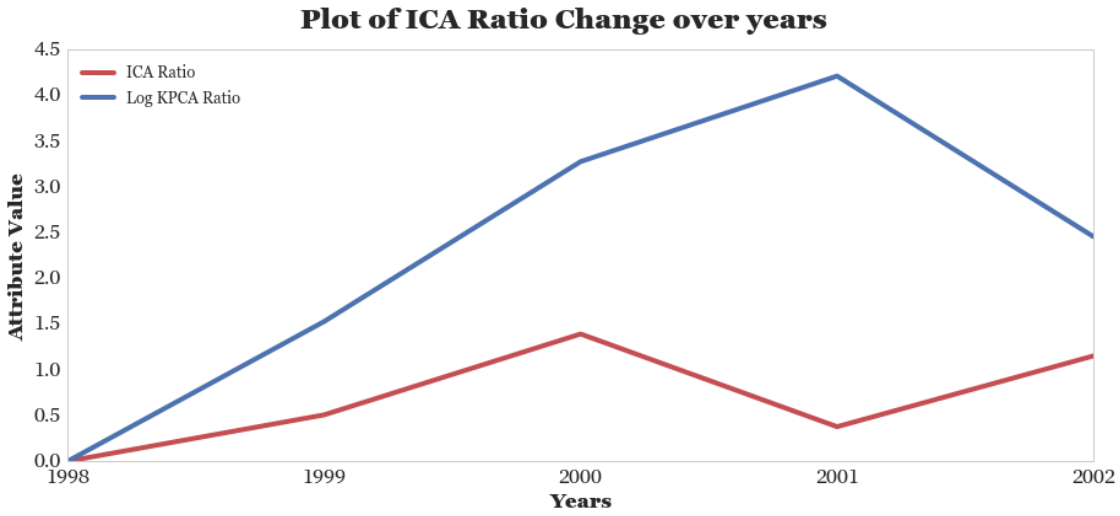
```
In [109]: def fica_att(net):
            fica_chg = []
            for i in range(len(net)-1):
                x = int(i)
                y = 1+x
                fica1= fica_ratio(net[x])
                fica2= fica_ratio(net[y])
                fica_chg.append(np.divide(fica2,fica1))
            fica_chgpad = np.zeros(len(fica_chg)+1);
            fica_chgpad[1:] = fica_chg

            return fica_chgpad
```

```
In [110]: fica_ratio_y = fica_att(all_year_G)
          fica_ratio_m = fica_att(all_month_G)
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\sklearn\decomposition\fastica_.py:11
  warnings.warn('FastICA did not converge. Consider increasing '
```

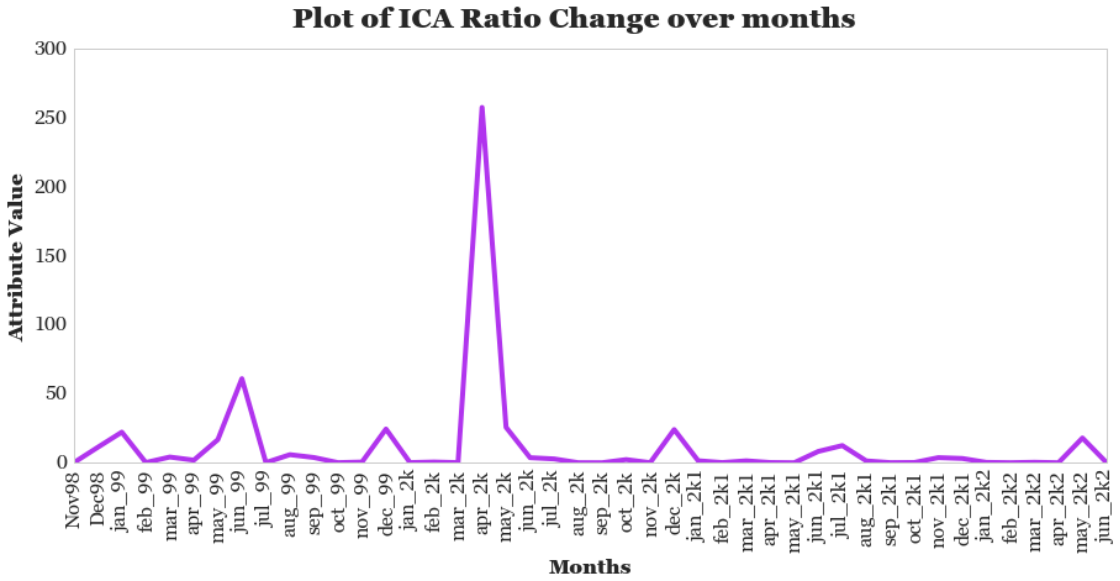
```
In [111]: plt.plot(fica_ratio_y,'r',label='ICA Ratio')
          plt.plot(kpca_chg_y, 'b', label='Log KPCA Ratio')

          plt.suptitle("Plot of ICA Ratio Change over years", fontsize=22)
          plt.xlabel("Years", fontsize=16)
          plt.ylabel("Attribute Value", fontsize=16)
          plt.legend(fontsize=13, loc=2)
          plt.xticks(np.arange(len(years[1:])), years[1:],fontsize=16)
          plt.yticks(fontsize=16)
```

```
Out[111]: (array([ 0. ,  0.5,  1. ,  1.5,  2. ,  2.5,  3. ,  3.5,  4. ,  4.5]),
           <a list of 10 Text yticklabel objects>)
```

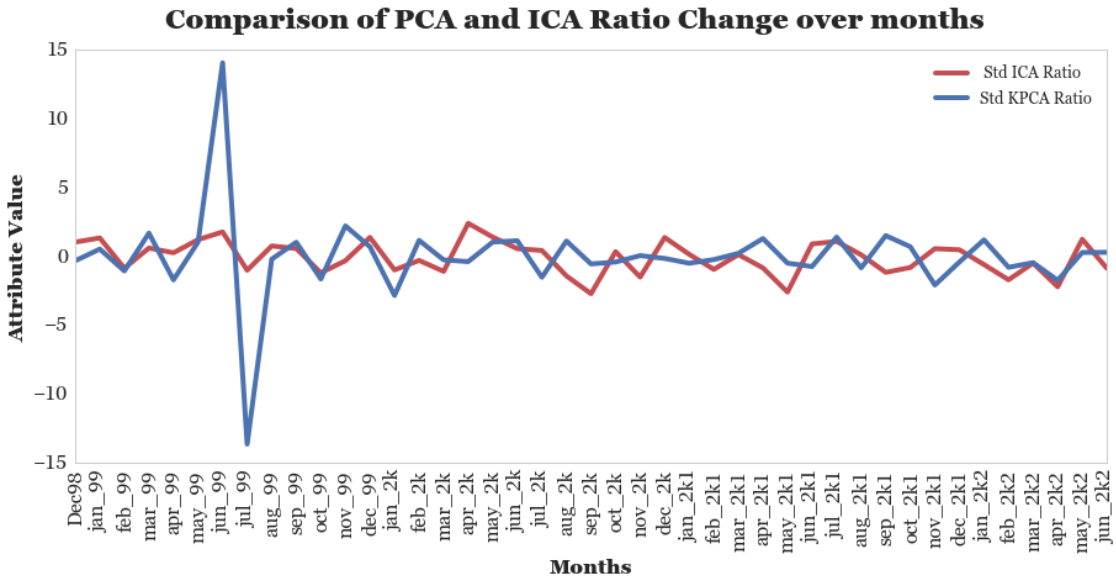



```
In [112]: plt.plot(fica_ratio_m)
          plt.suptitle("Plot of ICA Ratio Change over months", fontsize=22)
          plt.xlabel("Months", fontsize=16)
          plt.ylabel("Attribute Value", fontsize=16)
          plt.legend(fontsize=13, loc=1)
          plt.xticks(np.arange(len(months)), months,fontsize=16, rotation=90)
          plt.yticks(fontsize=16)
          plt.autoscale()
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\matplotlib\axes\_axes.py:519: UserWa
  warnings.warn("No labelled objects found. "
```



```
In [113]: plt.plot(np.log10(fica_ratio_m), 'r',label=' Std ICA Ratio')
          plt.plot(np.log10(kpca_chg_m),'b',label='Std KPCA Ratio')
          plt.suptitle("Comparison of PCA and ICA Ratio Change over months", fontsi
          plt.xlabel("Months", fontsize=16)
          plt.ylabel("Attribute Value", fontsize=16)
          plt.legend(fontsize=13, loc=1)
          plt.xticks(np.arange(len(months)), months,fontsize=16, rotation=90)
          plt.yticks(fontsize=16)
          plt.autoscale()
```



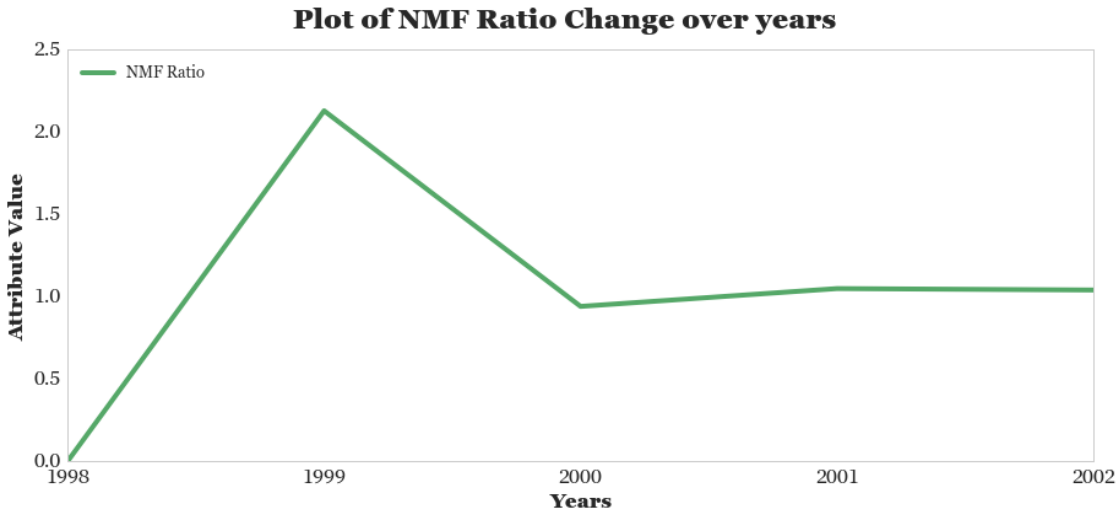

```
In [114]: np.log10(fica_ratio_y)

Out[114]: array([       -inf, -0.29457696,  0.14373978, -0.42278692,  0.06179354])

In [115]: np.log10(fica_ratio_m)

Out[115]: array([       -inf,  1.04887005,  1.34588595, -0.85364522,  0.61479656,
                  0.27009754,  1.22434315,  1.78523177, -1.00281088,  0.76629432,
                  0.57675788, -1.18834814, -0.30607838,  1.38816304, -0.97906323,
                 -0.28589781, -1.08268964,  2.41117601,  1.40976557,  0.56243163,
                  0.43972882, -1.41525851, -2.70338417,  0.3545017 , -1.49678312,
                  1.38151543,  0.15498694, -0.93455707,  0.1392276 , -0.82863389,
                 -2.58756808,  0.91368737,  1.09501346,  0.12183546, -1.15232183,
                 -0.80151033,  0.56683758,  0.48768383, -0.58740432, -1.69455934,
                 -0.48040407, -2.20627014,  1.25358685, -0.85421221])
```

```
In [116]: y_avgst_all['LogICARatio'] =np.log10(fica_ratio_y)
          stat_all['LogICARatio'] =np.nan_to_num(np.log10(fica_ratio_m))

In [117]: stat_all.LogICARatio.iloc[0]=0
          y_avgst_all.LogICARatio.iloc[0]=0
```

6.13 Norm NMF Ratio

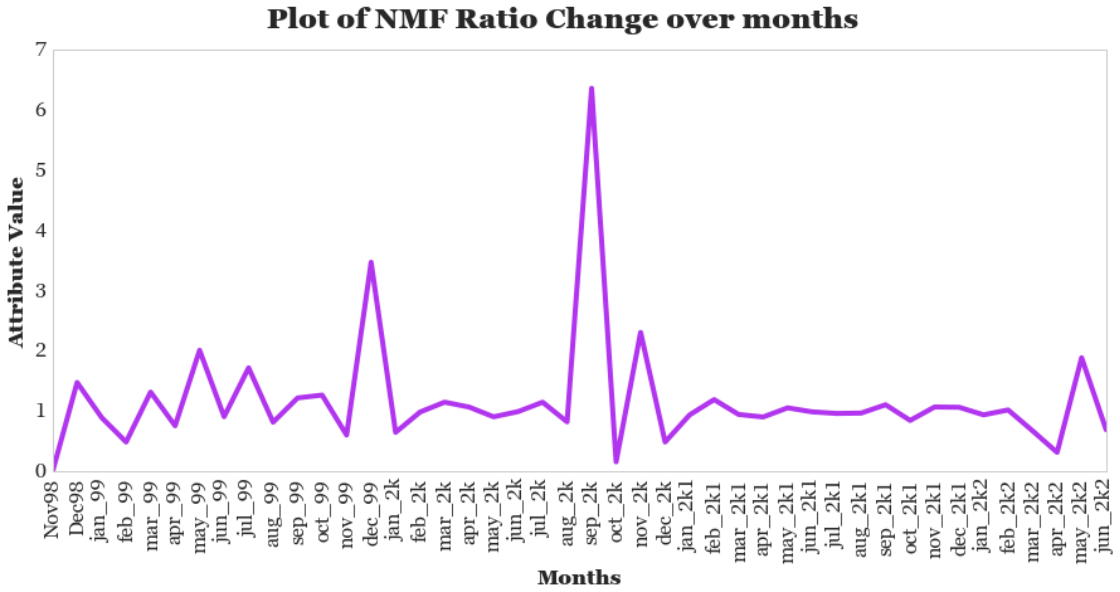
This is another metric derived from the Non-negative Matrix Factorisation of the normalised graph laplacian. Essentially the laplacian is decomposed and then I take the Frobenius norm of the result to derive a measure of comparison of the networks at different times.

```
In [118]: def nmf_att(net):
              nmf_chg = []
              nmf = NMF(init='nndsvda',solver='cd', random_state=0, l1_ratio=1)
              for i in range(len(net)-1):
                  x = int(i)
                  y = 1+x
                  m1= abs(nx.normalized_laplacian_matrix(net[x]).todense())
                  m2= abs(nx.normalized_laplacian_matrix(net[y]).todense())
                  nmf1 = norm(nmf.fit_transform(m1))
                  nmf2 = norm(nmf.fit_transform(m2))
                  nmf_chg.append(np.divide(nmf2,nmf1))
              nmf_chgpad = np.zeros(len(nmf_chg)+1);
              nmf_chgpad[1:] = nmf_chg

              return nmf_chgpad

In [119]: nmf_ratio_y = nmf_att(all_year_G)
          nmf_ratio_m = nmf_att(all_month_G)

In [120]: plt.plot(nmf_ratio_y,'g',label='NMF Ratio')

          plt.suptitle("Plot of NMF Ratio Change over years", fontsize=22)
          plt.xlabel("Years", fontsize=16)
          plt.ylabel("Attribute Value", fontsize=16)
          plt.legend(fontsize=13, loc=2)
          plt.xticks(np.arange(len(years[1:])), years[1:],fontsize=16)
          plt.yticks(fontsize=16)

Out[120]: (array([ 0. ,  0.5,  1. ,  1.5,  2. ,  2.5]),
           <a list of 6 Text yticklabel objects>)
```




```
In [121]: plt.plot(nmf_ratio_m)
          plt.suptitle("Plot of NMF Ratio Change over months", fontsize=22)
          plt.xlabel("Months", fontsize=16)
          plt.ylabel("Attribute Value", fontsize=16)
          plt.legend(fontsize=13, loc=1)
          plt.xticks(np.arange(len(months)), months,fontsize=16, rotation=90)
          plt.yticks(fontsize=16)
          plt.autoscale()
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\matplotlib\axes\_axes.py:519: UserWa
  warnings.warn("No labelled objects found. "
```



```
In [122]: y_avgst_all['NormNMFRatio'] =nmf_ratio_y
          stat_all['NormNMFRatio'] =nmf_ratio_m

In [123]: y_avgst_all.head()
```

Out[123]:

	AvgDeg	AvgCloseness	AvgBet	AvgEig	AvgKatz	AvgLoad	\
1998	0.128788	0.221063	0.080303	0.208801	0.131932	0.080303	
1999	0.028753	0.187570	0.029206	0.072719	0.089714	0.029206	
2000	0.031415	0.276721	0.013034	0.055911	0.005469	0.013034	
2001	0.038855	0.269216	0.006547	0.059285	0.006471	0.006546	
2002	0.021970	0.251953	0.015063	0.060401	0.033446	0.015204	

	Density	AvgClustCoef	AlgebraicConnect	MeanKurv	...	\
1998	0.196970	0.225253	1.000000	-0.200753	...	
1999	0.058385	0.214973	0.000000	-0.226467	...	
2000	0.079812	0.449348	0.121908	-0.262589	...	
2001	0.111795	0.503773	0.000000	-0.290741	...	
2002	0.066667	0.429246	0.000000	-0.259305	...	

	d2InstAmp	InstAccel	cosInstPhase	IAwIF	IAwIP	\
1998	-0.017605	0.106758	0.326620	0.113057	1.495918	
1999	0.000065	0.006731	0.109029	0.049019	1.286366	
2000	0.000233	0.002686	0.068788	0.147710	-2.361936	
2001	0.000087	-0.006425	0.082182	0.266621	3.461118	
2002	-0.000132	-0.004578	0.067986	0.034919	0.292512	

	MeanResistanceDist	StatRat	LogKPCARatio	LogICARatio	NormNMFRat
1998	1.825529	0.914026	0.000000	0.000000	0.0000
1999	2.818535	0.908905	0.184267	-0.294577	2.1299
2000	2.852828	0.952490	0.515577	0.143740	0.9411
2001	2.220536	0.974288	0.624672	-0.422787	1.0494
2002	3.086302	0.937605	0.390293	0.061794	1.0406

[5 rows x 24 columns]

```
In [124]: stat_all.head()
```

Out[124]:

	AvgDeg	AvgCloseness	AvgBet	AvgEig	AvgKatz	AvgLoad	\
Nov98	0.142857	0.192308	0.125000	0.222019	0.185435	0.125000	
Dec98	0.145455	0.242737	0.086869	0.233760	0.175111	0.086869	
jan_99	0.083333	0.117057	0.076923	0.132902	0.057692	0.076923	
feb_99	0.500000	0.555556	0.333333	0.569036	0.390223	0.333333	
mar_99	0.200000	0.259259	0.166667	0.328670	0.277684	0.166667	

	Density	AvgClustCoef	AlgebraicConnect	MeanKurv	...
Nov98	0.250000	0.000000	1.0	-0.196247	...

```
Dec98   0.218182      0.246465              1.0 -0.199212      ...
jan_99  0.153846      0.000000              1.0 -0.206795      ...
feb_99  0.666667      0.000000              1.0 -0.058750      ...
mar_99  0.333333      0.000000              1.0 -0.157405      ...

        d2InstAmp  InstAccel  cosInstPhase         IAwIF     IAwIP  \
Nov98   -0.013358  -0.048538      0.369891 -2.164452e-01  0.345301
Dec98   -0.009423  -0.012273      0.210647  1.753912e+00  1.255140
jan_99   0.005669  -0.144891      0.149447  9.562196e-01  2.265634
feb_99   0.028173  -0.451282      0.667721 -1.233581e-17  2.714095
mar_99   0.058205  -0.279494      0.550674  1.049833e+00  1.703481

        MeanResistanceDist   StatRat  LogKPCARatio  LogICARatio  NormNMFR
Nov98             1.603840  0.894427      0.000000     0.000000      0.00
Dec98             1.804050  0.908295     -0.310794     1.048870      1.47
jan_99            1.751267  0.930949      0.538398     1.345886      0.89
feb_99            0.990468  0.774597     -1.051911    -0.853645      0.48
mar_99            1.478780  0.866025      1.697420     0.614797      1.31

[5 rows x 24 columns]
```

6.14 Resistance Distance

The resistance distance between vertices i and j of a graph G is defined as the effective resistance between the two vertices (as when a battery is attached across them) when each graph edge is replaced by a unit resistor. This resistance distance is a metric on graphs.

I calculate the resistance distance as:

M = Normalised Graph Laplacian Matrix

N = Length of M

P = Moore-Penrose Pseudo Inverse of M

D = The diagonal of Pinv

$$R_d = (D \otimes (N,1))^T + (D \otimes (N,1))^T - P - P^T$$

Here $\otimes$ denotes the Kronecker or outer product
Ref:

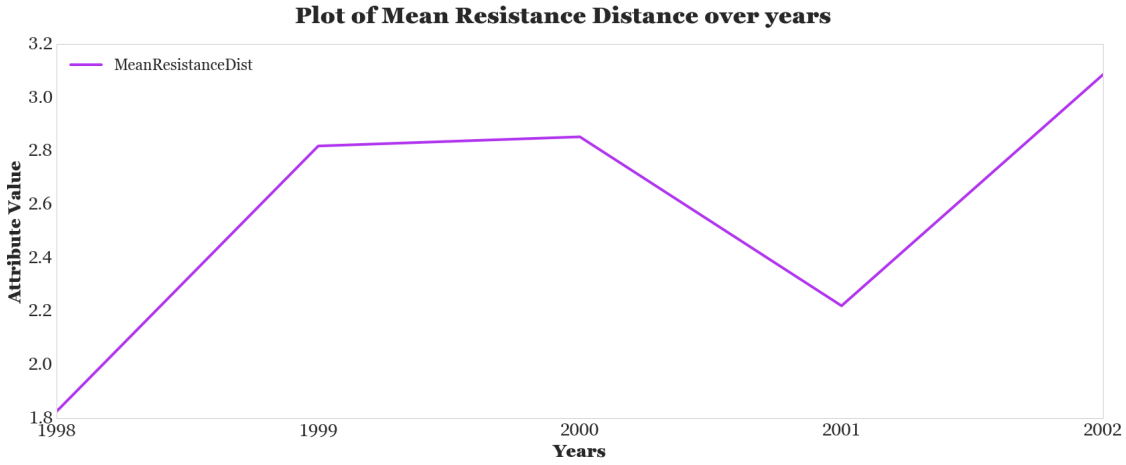
1. Weisstein, Eric W. "Resistance Distance." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/ResistanceDistance.html

2. Function implemented from here


```
In [125]: y_avgst_all.MeanResistanceDist.plot(fontsize=22, figsize=(28,10))
          plt.suptitle("Plot of Mean Resistance Distance over years", fontsize=33)
          plt.xlabel("Years", fontsize=25)
```

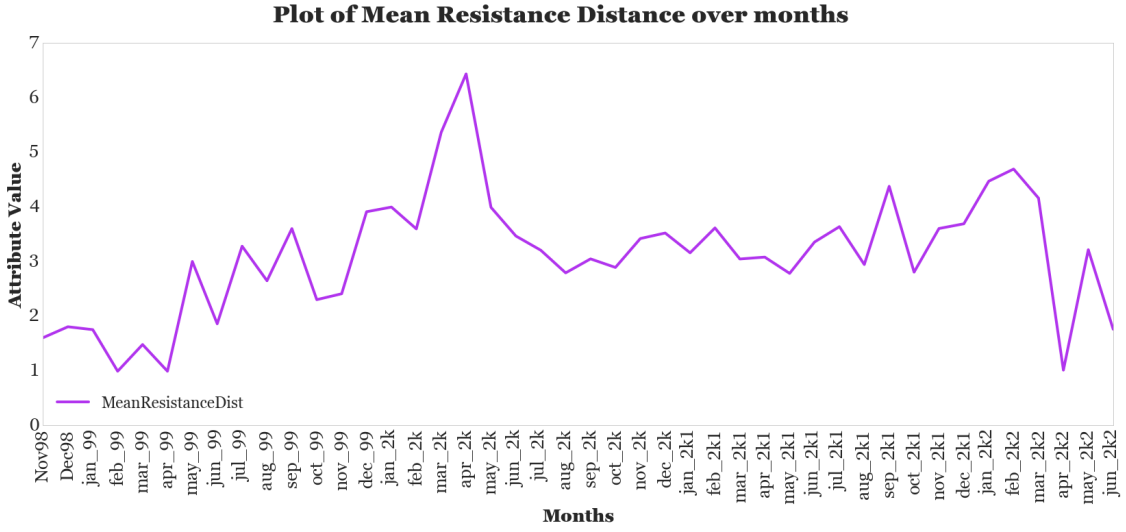
```
        plt.ylabel("Attribute Value", fontsize=25)
        plt.legend(fontsize=23, loc=2)
        plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
        plt.yticks(fontsize=26)
```

```
Out[125]: (array([ 1.6,  1.8,  2. ,  2.2,  2.4,  2.6,  2.8,  3. ,  3.2]),
           <a list of 9 Text yticklabel objects>)
```

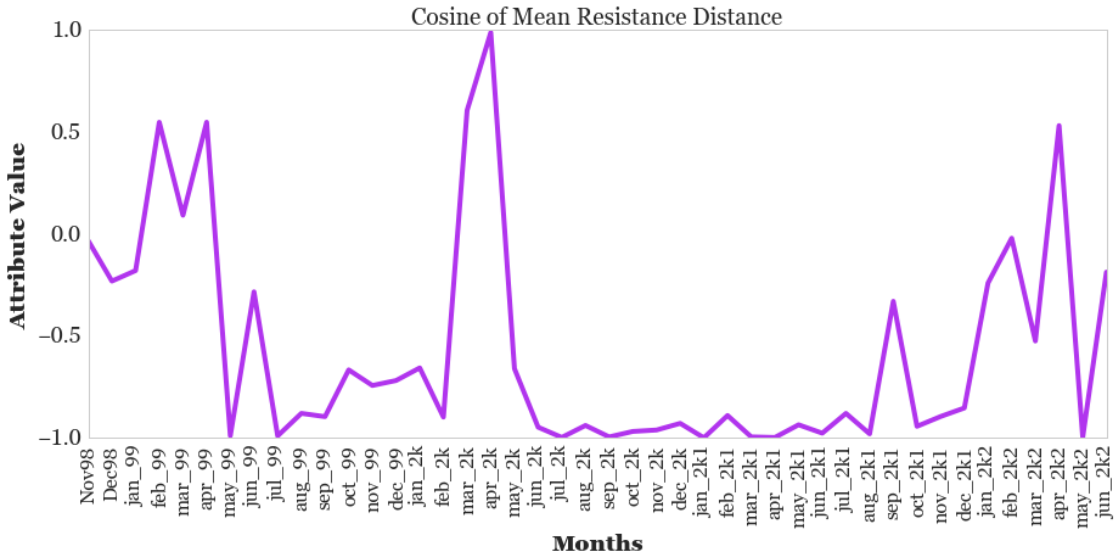



```
In [126]: stat_all.MeanResistanceDist.plot(fontsize=22, rot=90, figsize=(28,10))
        plt.suptitle("Plot of Mean Resistance Distance over months", fontsize=33)
        plt.xlabel("Months", fontsize=25)
        plt.ylabel("Attribute Value", fontsize=25)
        plt.legend(fontsize=23, loc=3)
        plt.xticks(np.arange(len(months)), months,fontsize=26)
        plt.yticks(fontsize=26)
```

```
Out[126]: (array([ 0.,  1.,  2.,  3.,  4.,  5.,  6.,  7.]),
           <a list of 8 Text yticklabel objects>)
```

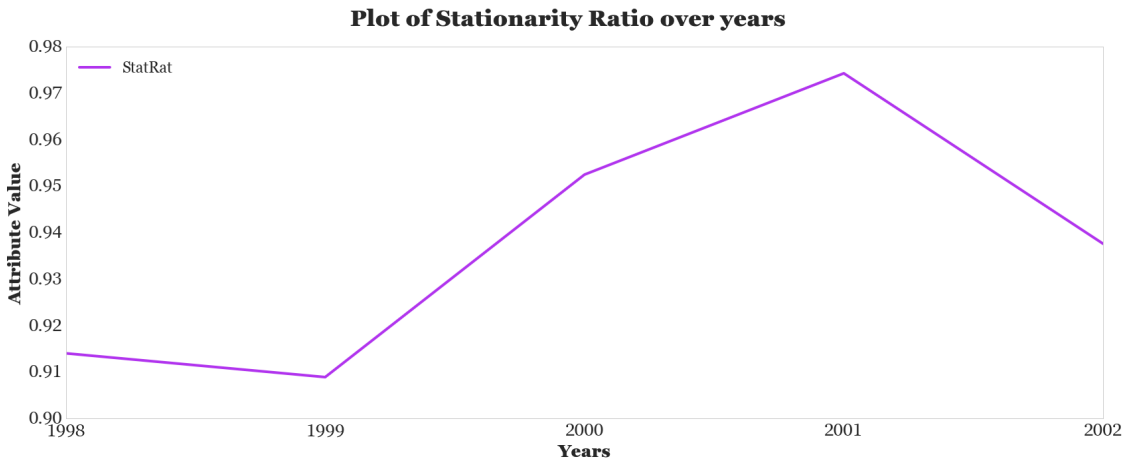


```
In [127]: stat_all.MeanResistanceDist.apply(lambda x: np.cos(x)).plot(title="Cosine
          plt.xticks(np.arange(len(months)), months,fontsize=16);
          plt.xlabel("Months", fontsize=18)
          plt.ylabel("Attribute Value", fontsize=18)

Out[127]: <matplotlib.text.Text at 0x29c5072a1d0>
```



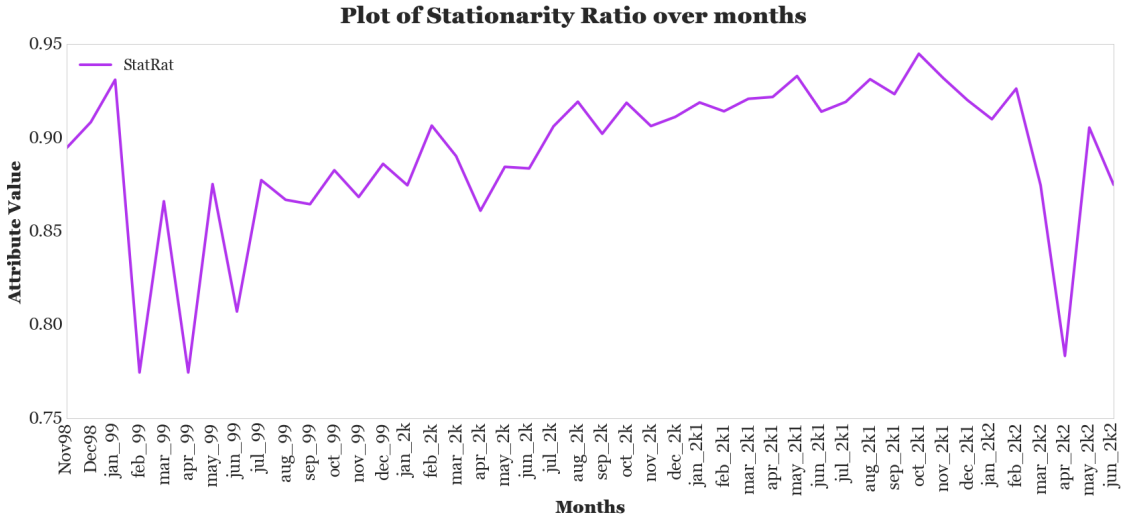
6.15 Stationarity Ratio

```
In [128]: y_avgst_all.StatRat.plot(fontsize=22, figsize=(28,10))
          plt.suptitle("Plot of Stationarity Ratio over years", fontsize=33)
          plt.xlabel("Years", fontsize=25)
          plt.ylabel("Attribute Value", fontsize=25)
          plt.legend(fontsize=23, loc=2)
          plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
          plt.yticks(fontsize=26)

Out[128]: (array([ 0.9 ,  0.91,  0.92,  0.93,  0.94,  0.95,  0.96,  0.97,  0.98]),
           <a list of 9 Text yticklabel objects>)
```



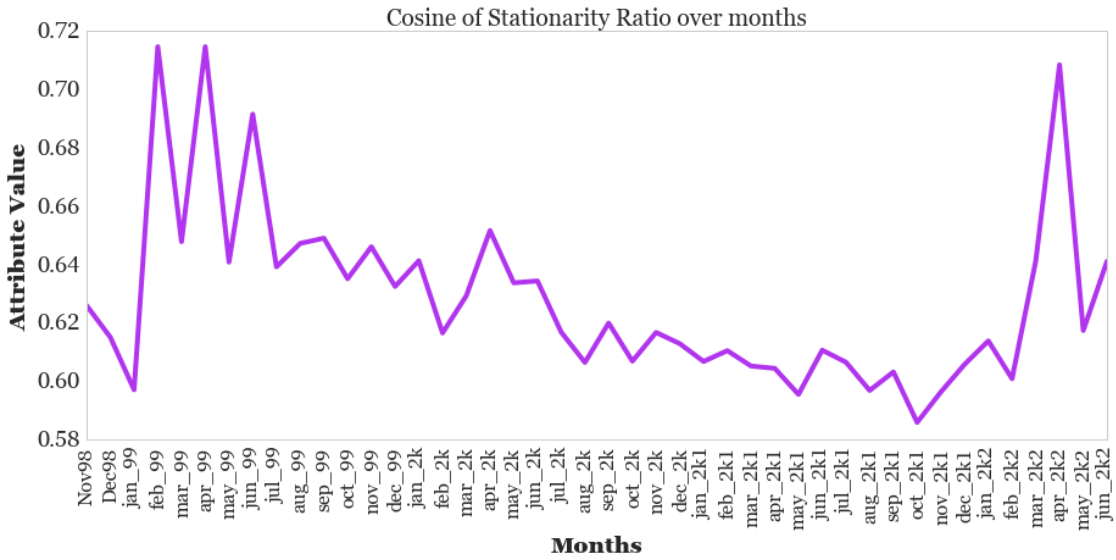

```
In [129]: stat_all.StatRat.plot(fontsize=22, rot=90, figsize=(28,10))
          plt.suptitle("Plot of Stationarity Ratio over months", fontsize=33)
          plt.xlabel("Months", fontsize=25)
          plt.ylabel("Attribute Value", fontsize=25)
          plt.legend(fontsize=23, loc=2)
          plt.xticks(np.arange(len(months)), months,fontsize=26)
          plt.yticks(fontsize=26)
```

```
Out[129]: (array([ 0.7 ,  0.75,  0.8 ,  0.85,  0.9 ,  0.95]),
           <a list of 6 Text yticklabel objects>)
```




```
In [130]: stat_all.StatRat.apply(lambda x: np.cos(x)).plot(title="Cosine of Station
          plt.xticks(np.arange(len(months)), months,fontsize=16);
          plt.xlabel("Months", fontsize=18)
          plt.ylabel("Attribute Value", fontsize=18)
```

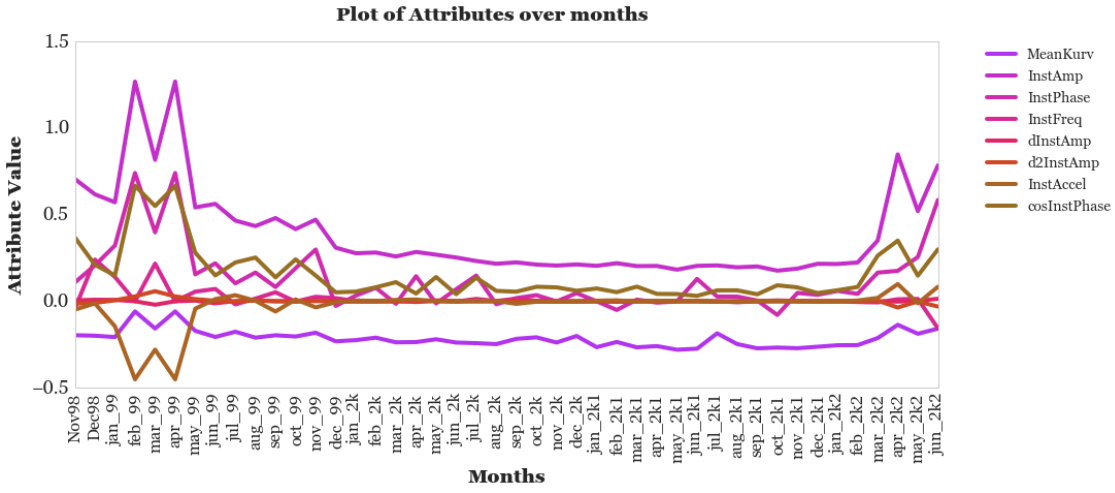
Out[130]: <matplotlib.text.Text at 0x29c5313d6a0>

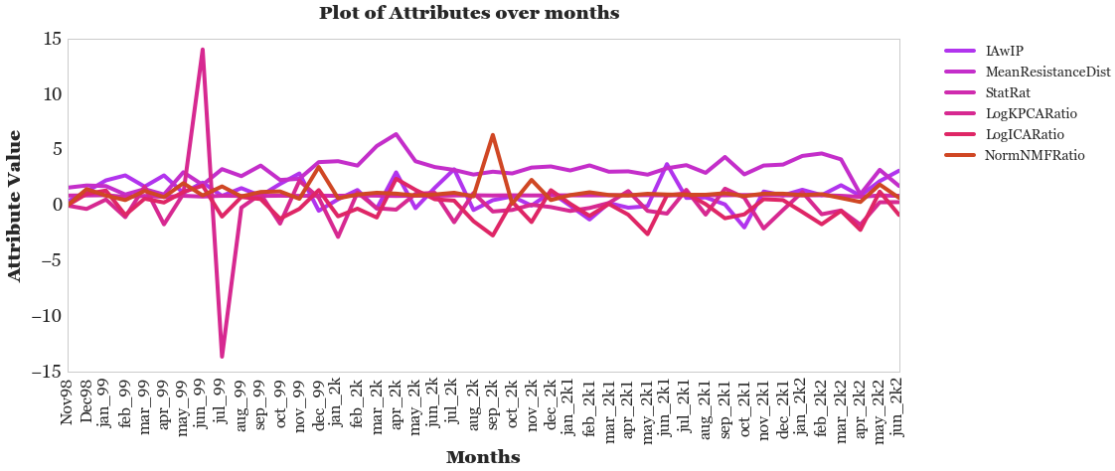


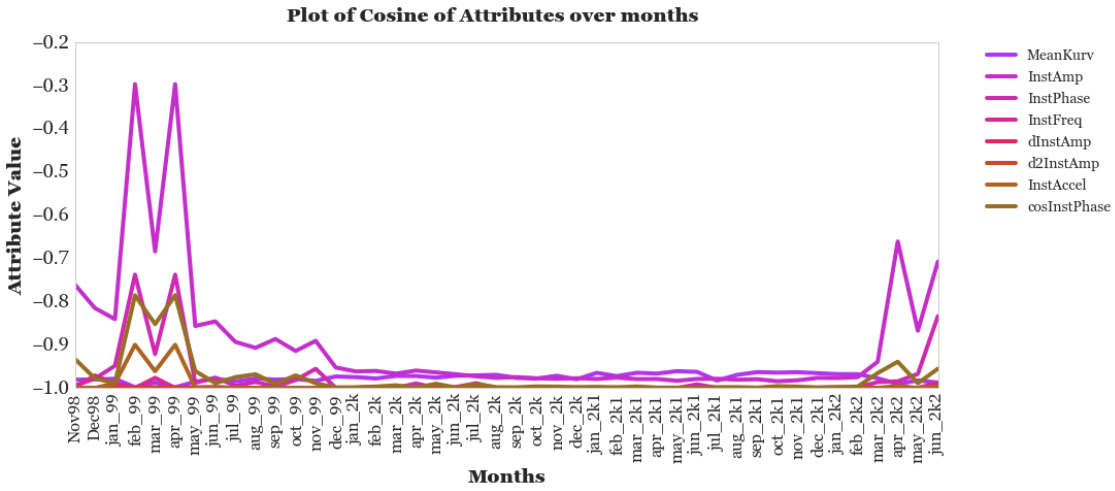
In [131]:
```
stat_all.iloc[:,9:17].plot(fontsize=18, rot =90)
plt.suptitle("Plot of Attributes over months", fontsize=18)
plt.xticks(np.arange(len(months)), months,fontsize=16);
plt.legend(fontsize=14, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xlabel("Months", fontsize=18)
plt.ylabel("Attribute Value", fontsize=18)

stat_all.iloc[:,18:].plot(fontsize=18, rot =90)
plt.suptitle("Plot of Attributes over months", fontsize=18)
plt.xticks(np.arange(len(months)), months,fontsize=16);
plt.legend(fontsize=14, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xlabel("Months", fontsize=18)
plt.ylabel("Attribute Value", fontsize=18)

stat_all.iloc[:,9:17].apply(lambda x: np.cos(x)*-1).plot(fontsize=18, rot
plt.suptitle("Plot of Cosine of Attributes over months", fontsize=18)
plt.xticks(np.arange(len(months)), months,fontsize=16);
plt.legend(fontsize=14, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xlabel("Months", fontsize=18)
plt.ylabel("Attribute Value", fontsize=18)

stat_all.iloc[:,18:].apply(lambda x: np.cos(x)*-1).plot(fontsize=18, rot
plt.suptitle("Plot of Cosine of Attributes over months", fontsize=18)
plt.xticks(np.arange(len(months)), months,fontsize=16);
plt.legend(fontsize=14, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xlabel("Months", fontsize=18)
plt.ylabel("Attribute Value", fontsize=18)
```

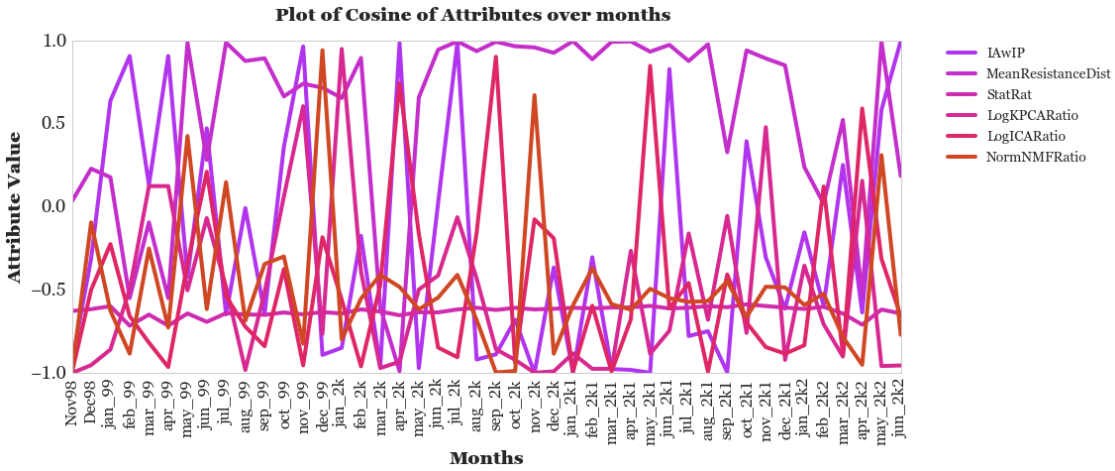
```
stat_all.iloc[:,:6].plot(fontsize=18, rot=90)
plt.suptitle("Plot of Avg of Centrality measures over months", fontsize=1
plt.xlabel("Months", fontsize=18)
plt.ylabel("Attribute Value", fontsize=18)
plt.legend(fontsize=14, loc=1)
plt.legend(fontsize=14, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xticks(np.arange(len(months)), months,fontsize=16)
plt.yticks(fontsize=16)

stat_all.iloc[:,:6].apply(lambda x: np.cos(x)*-1).plot(fontsize=18, rot=9
plt.suptitle("Plot of Cosine of Avg of Centrality measures over months",
plt.xlabel("Months", fontsize=18)
plt.ylabel("Attribute Value", fontsize=18)
plt.legend(fontsize=14, loc=1)
plt.legend(fontsize=14, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xticks(np.arange(len(months)), months,fontsize=16)
plt.yticks(fontsize=16)
```

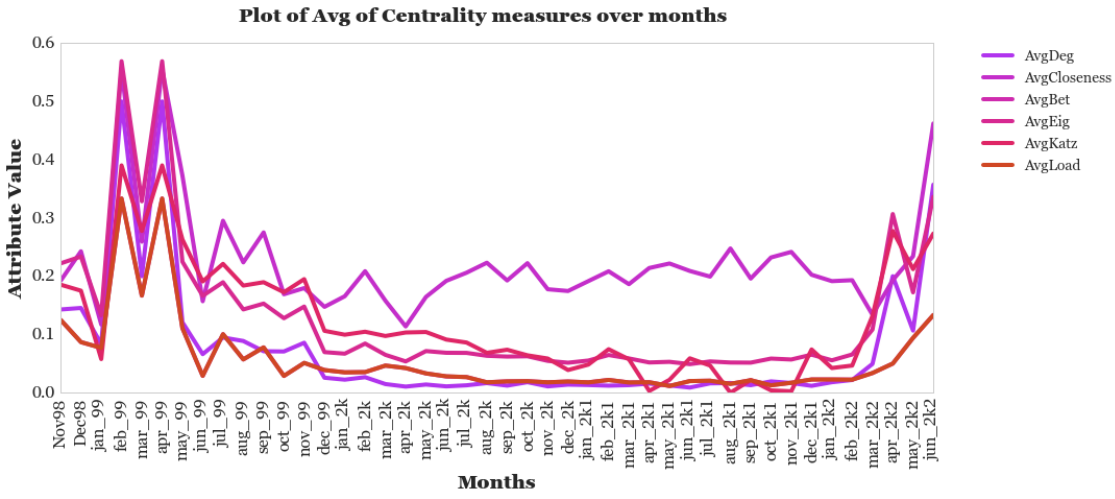
```
Out[131]: (array([-1.  , -0.98, -0.96, -0.94, -0.92, -0.9 , -0.88, -0.86, -0.84, -0
           <a list of 10 Text yticklabel objects>)
```

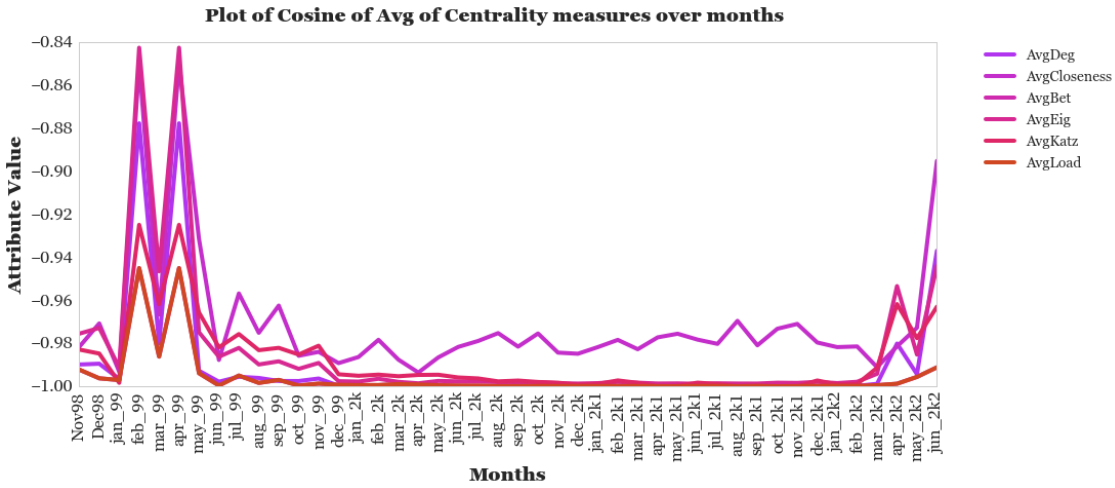



Plot of Attributes over months
Attribute Value
15
10
5
0
−5
−10
−15
Nov98
Dec98
jan_99
feb_99
mar_99
apr_99
may_99
jun_99
jul_99
aug_99
sep_99
oct_99
nov_99
dec_99
jan_2k
feb_2k
mar_2k
apr_2k
may_2k
jun_2k
jul_2k
aug_2k
sep_2k
oct_2k
nov_2k
dec_2k
jan_2k1
feb_2k1
mar_2k1
apr_2k1
may_2k1
jun_2k1
jul_2k1
aug_2k1
sep_2k1
oct_2k1
nov_2k1
dec_2k1
jan_2k2
feb_2k2
mar_2k2
apr_2k2
may_2k2
jun_2k2
Months
IAwIP
MeanResistanceDist
StatRat
LogKPCARatio
LogICARatio
NormNMFRatio


Plot of Cosine of Attributes over months
Attribute Value
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
−1.0
Nov98
Dec98
jan_99
feb_99
mar_99
apr_99
may_99
jun_99
jul_99
aug_99
sep_99
oct_99
nov_99
dec_99
jan_2k
feb_2k
mar_2k
apr_2k
may_2k
jun_2k
jul_2k
aug_2k
sep_2k
oct_2k
nov_2k
dec_2k
jan_2k1
feb_2k1
mar_2k1
apr_2k1
may_2k1
jun_2k1
jul_2k1
aug_2k1
sep_2k1
oct_2k1
nov_2k1
dec_2k1
jan_2k2
feb_2k2
mar_2k2
apr_2k2
may_2k2
jun_2k2
Months
MeanKurv
InstAmp
InstPhase
InstFreq
dInstAmp
d2InstAmp
InstAccel
cosInstPhase


Plot of Cosine of Attributes over months
Attribute Value
1.0
0.5
0.0
−0.5
−1.0
Nov98
Dec98
jan_99
feb_99
mar_99
apr_99
may_99
jun_99
jul_99
aug_99
sep_99
oct_99
nov_99
dec_99
jan_2k
feb_2k
mar_2k
apr_2k
may_2k
jun_2k
jul_2k
aug_2k
sep_2k
oct_2k
nov_2k
dec_2k
jan_2k1
feb_2k1
mar_2k1
apr_2k1
may_2k1
jun_2k1
jul_2k1
aug_2k1
sep_2k1
oct_2k1
nov_2k1
dec_2k1
jan_2k2
feb_2k2
mar_2k2
apr_2k2
may_2k2
jun_2k2
Months
IAwIP
MeanResistanceDist
StatRat
LogKPCARatio
LogICARatio
NormNMFRatio

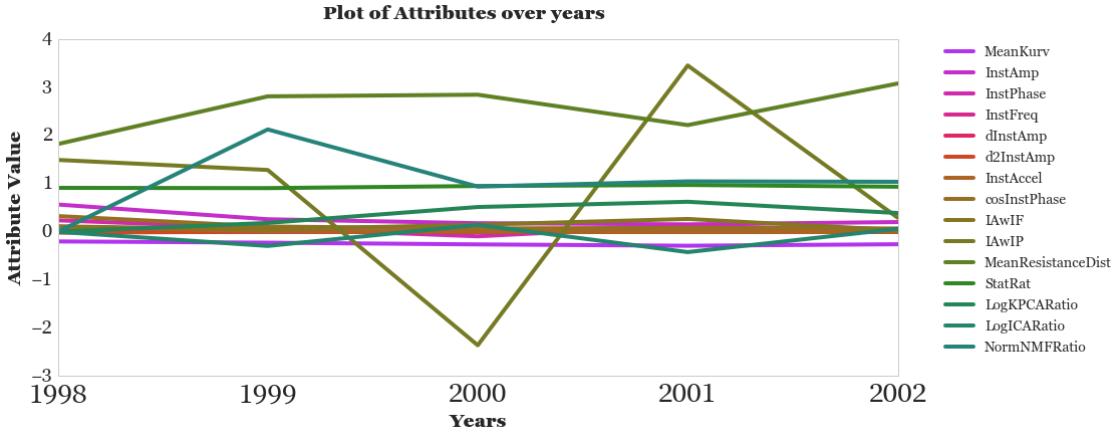


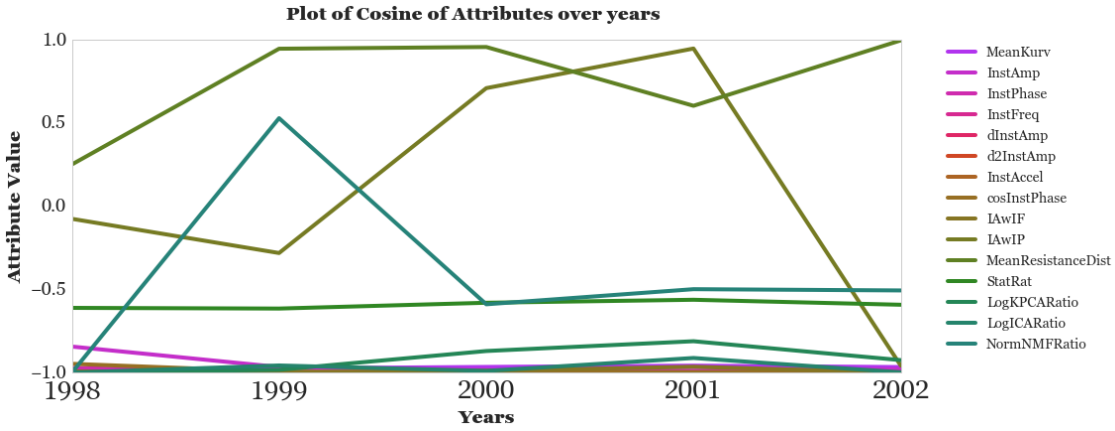


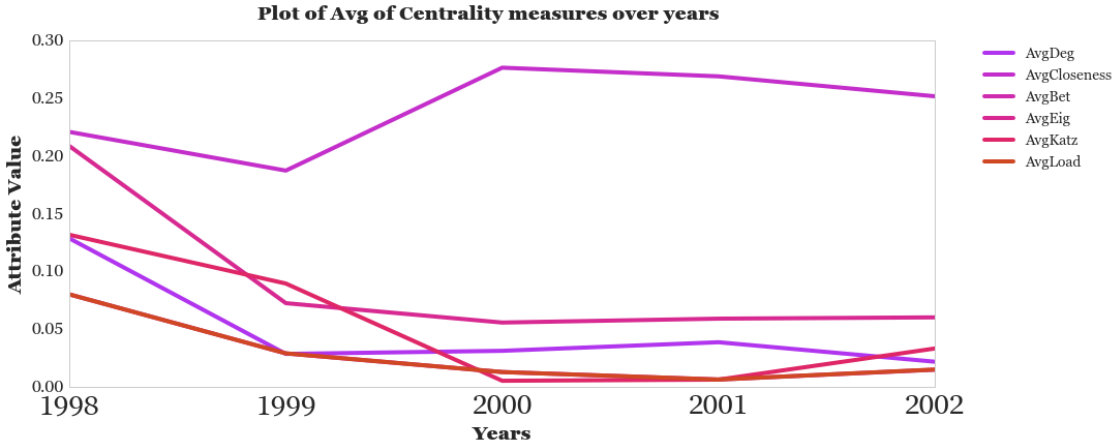

```
In [132]: y_avgst_all.iloc[:,9:].plot(fontsize=18)
          plt.suptitle("Plot of Attributes over years", fontsize=18)
          plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
          plt.legend(fontsize=14, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
          plt.xlabel("Years", fontsize=18)
          plt.ylabel("Attribute Value", fontsize=18)

          y_avgst_all.iloc[:,9:].apply(lambda x: np.cos(x)*-1).plot(fontsize=18)
          plt.suptitle("Plot of Cosine of Attributes over years", fontsize=18)
          plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
```

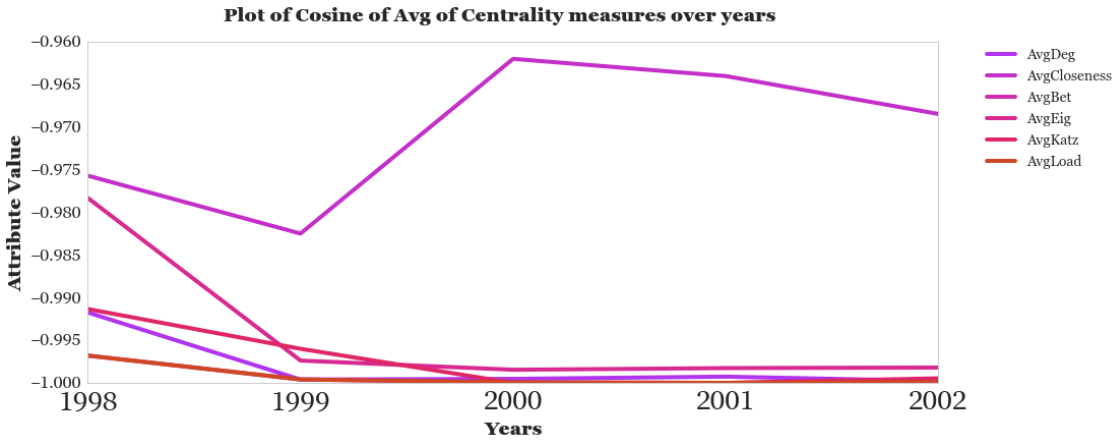
```
plt.legend(fontsize=14, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xlabel("Years", fontsize=18)
plt.ylabel("Attribute Value", fontsize=18)

y_avgst_all.iloc[:,:6].plot(fontsize=18)
plt.suptitle("Plot of Avg of Centrality measures over years", fontsize=18
plt.xlabel("Years", fontsize=18)
plt.ylabel("Attribute Value", fontsize=18)
plt.legend(fontsize=14, loc=1)
plt.legend(fontsize=14, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
plt.yticks(fontsize=16)

y_avgst_all.iloc[:,:6].apply(lambda x: np.cos(x)*-1).plot(fontsize=18)
plt.suptitle("Plot of Cosine of Avg of Centrality measures over years", f
plt.xlabel("Years", fontsize=18)
plt.ylabel("Attribute Value", fontsize=18)
plt.legend(fontsize=14, loc=1)
plt.legend(fontsize=14, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
plt.yticks(fontsize=16)
```

```
Out[132]: (array([-1.   , -0.995, -0.99 , -0.985, -0.98 , -0.975, -0.97 , -0.965,
                  -0.96 ]), <a list of 9 Text yticklabel objects>)
```




Plot of Cosine of Attributes over years
Attribute Value
1.0
0.5
0.0
−0.5
−1.0
1998
1999
2000
2001
2002
Years
MeanKurv
InstAmp
InstPhase
InstFreq
dInstAmp
d2InstAmp
InstAccel
cosInstPhase
IAwIF
IAwIP
MeanResistanceDist
StatRat
LogKPCARatio
LogICARatio
NormNMFRatio


Plot of Avg of Centrality measures over years
Attribute Value
0.30
0.25
0.20
0.15
0.10
0.05
0.00
1998
1999
2000
2001
2002
Years
AvgDeg
AvgCloseness
AvgBet
AvgEig
AvgKatz
AvgLoad


Plot of Cosine of Avg of Centrality measures over years
Attribute Value
−0.960
−0.965
−0.970
−0.975
−0.980
−0.985
−0.990
−0.995
−1.000
1998
1999
2000
2001
2002
Years
AvgDeg
AvgCloseness
AvgBet
AvgEig
AvgKatz
AvgLoad

7 Frequency Wavenumber (fk) and Radon Plots

I propose utilising the FK and Radon domain plots to visualise the high dimensional attribute volume. In both FK and Radon space the signal should be seperated from outliers. I then use the indices from the FK and Radon to sort the attribute volume and find the interesting areas.

The FK Plot is derived as follows: + I take the attribute volume and apply a Fourier Transform to the values thus converting them to a frequency component, f + I calculate the wavenumber, k as $\frac{1}{f}$ + Plot the f and k components to reduce the high dimensional volume into a 2 dimensional representation.

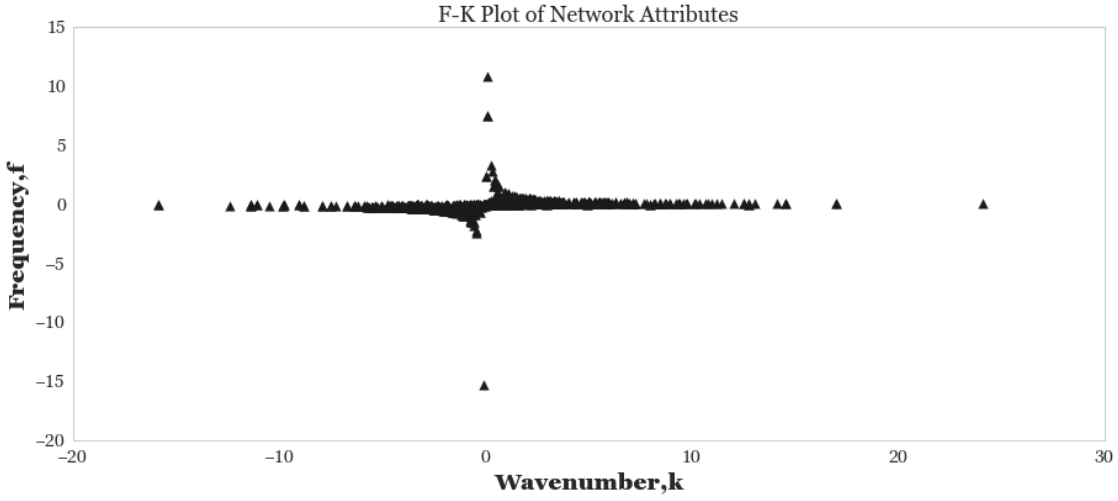
This plot allows for the identification of potential outliers or points of interest. These items can then be extracted or filtered based on the f,k values and a filtered attribute volume can be easily derived. For example we can filter k values > 15 in the plot below and look at them seperately.

The Radon Plot is based on the Radon Transform. The Radon transform is the integral transform which takes a function f defined on the plane to a function Rf defined on the (two-dimensional) space of lines in the plane, whose value at a particular line is equal to the line integral of the function over that line. Strictly speaking the Radon transform is a generic mathematical procedure in which the input data in the frequency domain are decomposed into a series of events in the RADON domain. Whichever curve type is chosen will map to a point. Note this is similar to Fourier decomposition but using more complex functions than sinusoids. Once the attribes are transformed into radon space these can be visualised like the FK as before. Also I plot the reciprocal of the first component of the radon matrix in order to simluate a p-trace mapping which is defined as dt/dx of the trace. This gives better seperation of signal and noise as evens are seperated along p-values.

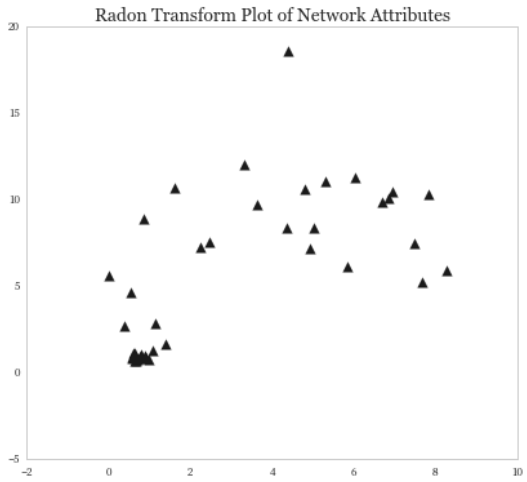
> Ref: + http://www.xsgeo.com/course/basic.htm#fk + http://scikit-image.org/docs/dev/auto_examples/plot_radon_transform.html + http://wiki.seg.org/wiki/Dictionary:Tau-p_mapping_(%CF%84-p)

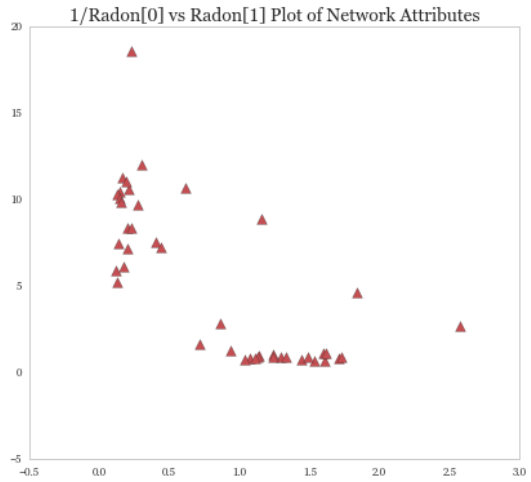
```
In [133]: freq,k = fk_plot(stat_all.values,'Network Attributes')
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:533: ComplexWa
  return array(a, dtype, copy=False, order=order, subok=True)
```

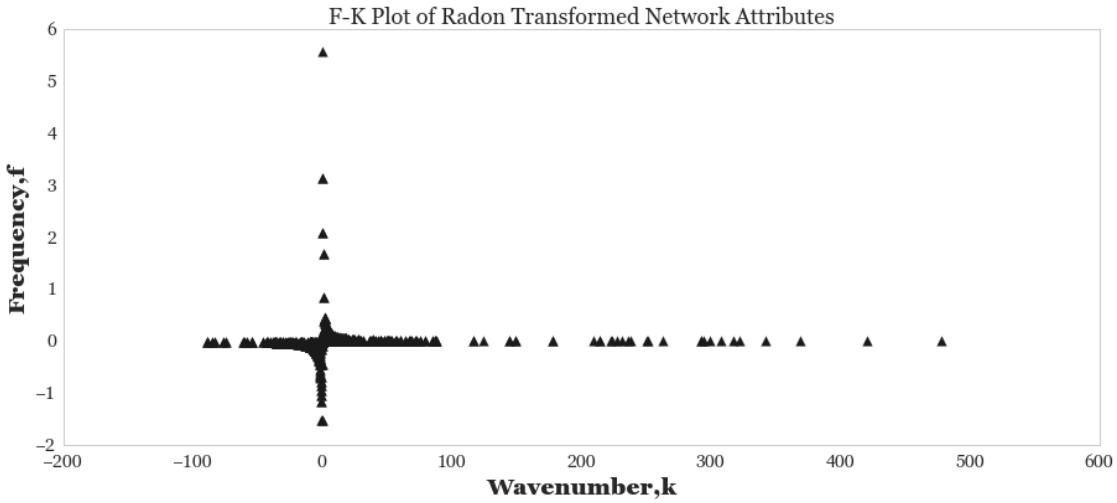


```
In [134]: radon_att = plot_radon(stat_all.values,'Network Attributes')

C:\Users\arsha_000\Anaconda3\lib\site-packages\skimage\transform\radon_transform.py
  warn('Radon transform: image must be zero outside the '
```






```
In [135]: freq,k = fk_plot(radon_att,'Radon Transformed Network Attributes')

C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:533: ComplexWa
  return array(a, dtype, copy=False, order=order, subok=True)
```



8 Towards Global Measures

8.1 Subgraph Stationarity, ζ

This is computed from the adjacency matrix of graphs and is done in two steps. Essentially this is comparing the common subraphs between two networks and deriving a correlation score. This is done in two steps.

Step 1: Calculate Correlation, Ct between the graphs at time, t and time, t+1.

$$C(t) = \frac{A(t) \cap A(t+1}{A(t) \cup A(t+1)}$$

Step 2: Calculate the Subgraph Stationarity, ζ

$$\zeta = \frac{\sum_{t=0}^{tmax-1} C(t, t+1)}{tmax - t0 - 1}$$

From the Subgraph Stationariy ζ we can calculate the amount of members that change at each time step as

$$1 - \zeta$$

```
In [136]: Ct_y = []
          for i in range(0,len(all_year_G)-1):
              x = int(i)
              y = x +1
              Ct_y.append(subgraph_stat(all_year_G[x],all_year_G[y]))
```

```
In [137]: Ct_m = []
          for i in range(0,len(all_month_G)-1):
              x = int(i)
              y = x +1
              Ct_m.append(subgraph_stat(all_month_G[x],all_month_G[y]))
```

```
In [138]: Ct_y
```

```
Out[138]: [0.30310524825452068,
           0.37838536898931635,
           0.55196098792073822,
           0.66442754991336916]
```

```
In [139]: tmp3 = np.zeros(len(Ct_y)+1)
          tmp3[1:]=Ct_y
          tmp3
```

```
Out[139]: array([ 0.        ,  0.30310525,  0.37838537,  0.55196099,  0.66442755])
```

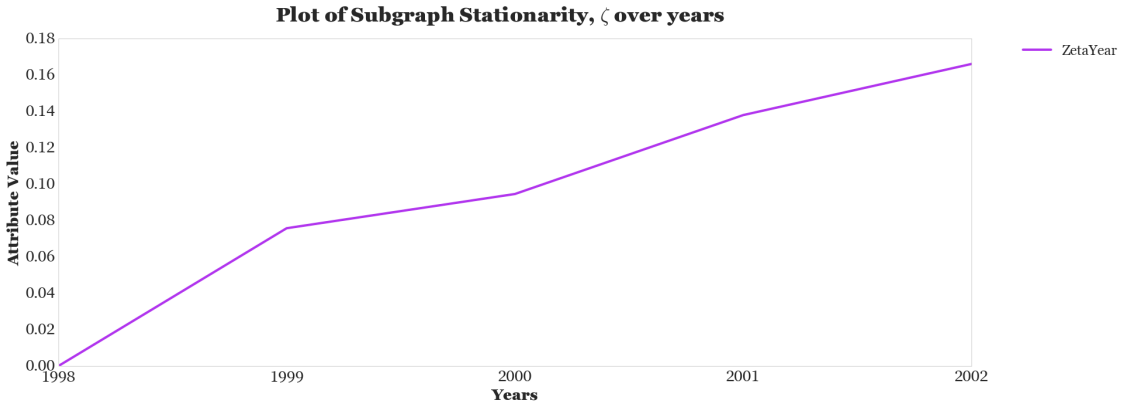
```
In [140]: Ct_m[:5]
```

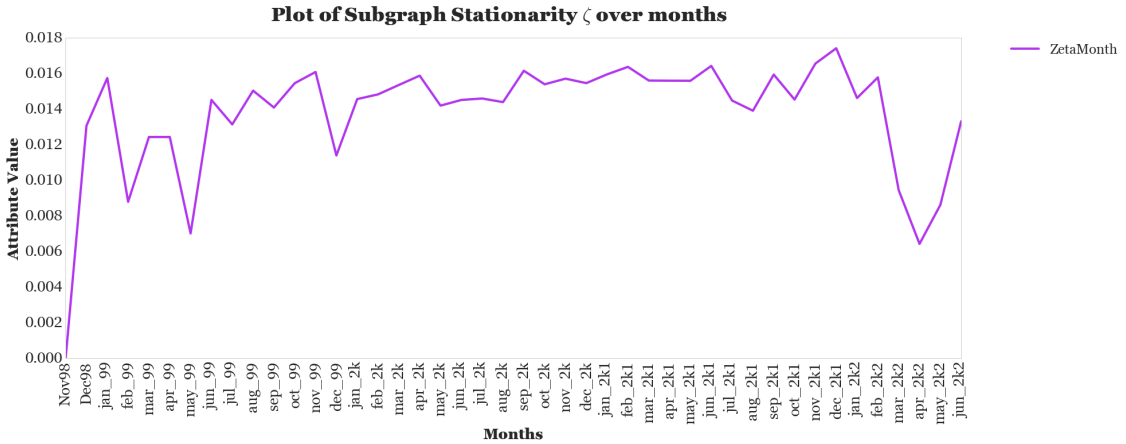
```
Out[140]: [0.56195148694901642,
           0.67700320038633011,
           0.3779644730092272,
           0.53452248382484879,
           0.53452248382484879]
```

```
In [141]: tmp4 = np.zeros(len(Ct_m)+1)
          tmp4[1:]=Ct_m
          tmp4

Out[141]: array([ 0.        ,  0.56195149,  0.6770032 ,  0.37796447,  0.53452248,
                  0.53452248,  0.30151134,  0.62418778,  0.56493268,  0.64655455,
                  0.60572805,  0.66449864,  0.69156407,  0.48989795,  0.62609903,
                  0.63748937,  0.66022529,  0.68268832,  0.61033232,  0.62400363,
                  0.62754405,  0.61876404,  0.69455555,  0.66183399,  0.67545887,
                  0.66466807,  0.6858916 ,  0.70393524,  0.67087945,  0.67054686,
                  0.67039676,  0.70622784,  0.62254302,  0.59803739,  0.68533074,
                  0.62489999,  0.7118685 ,  0.748884  ,  0.62864935,  0.67852807,
                  0.40610041,  0.2763854 ,  0.37047929,  0.57259833])

In [142]: np.cumsum(tmp3)/tmp3.shape[0]

Out[142]: array([ 0.        ,  0.06062105,  0.13629812,  0.24669032,  0.37957583])

In [143]: zeta_y= pd.DataFrame(tmp3)
          zeta_y = zeta_y.cumsum(axis=1)/(zeta_y.shape[0]-1)
          zeta_y =  zeta_y.T
          zeta_y.columns=years[1:]
          zeta_y =  pd.DataFrame(zeta_y).T
          zeta_y.columns=['ZetaYear']
          zeta_y.head()

Out[143]:       ZetaYear
          1998  0.000000
          1999  0.075776
          2000  0.094596
          2001  0.137990
          2002  0.166107

In [144]: zeta_m = pd.DataFrame(tmp4)
          zeta_m = zeta_m.cumsum(axis=1)/(zeta_m.shape[0]-1)
          zeta_m =  zeta_m.T
          zeta_m.columns=months
          zeta_m =  pd.DataFrame(zeta_m).T
          zeta_m.columns=['ZetaMonth']
          zeta_m.head()

Out[144]:         ZetaMonth
          Nov98    0.000000
          Dec98    0.013069
          jan_99   0.015744
          feb_99   0.008790
          mar_99   0.012431

In [145]: zeta_y.plot(fontsize=22, figsize=(28,10))
          plt.suptitle("Plot of Subgraph Stationarity, $\zeta$ over years", fontsiz
```

```
plt.xlabel("Years", fontsize=25)
plt.ylabel("Attribute Value", fontsize=25)
plt.legend(fontsize=24, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
plt.yticks(fontsize=26)

zeta_m.plot(fontsize=22, rot=90, figsize=(28,10))
plt.suptitle("Plot of Subgraph Stationarity $\zeta$ over months", fontsiz
plt.xlabel("Months", fontsize=25)
plt.ylabel("Attribute Value", fontsize=25)
plt.legend(fontsize=24, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xticks(np.arange(len(months)), months,fontsize=26)
plt.yticks(fontsize=26)
```

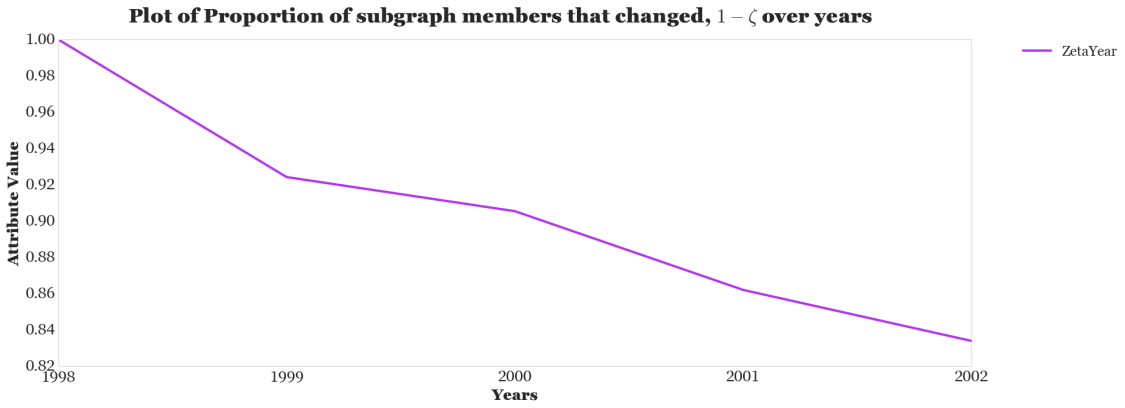
```
Out[145]: (array([ 0.   ,  0.002,  0.004,  0.006,  0.008,  0.01 ,  0.012,  0.014,
                   0.016,  0.018]), <a list of 10 Text yticklabel objects>)
```

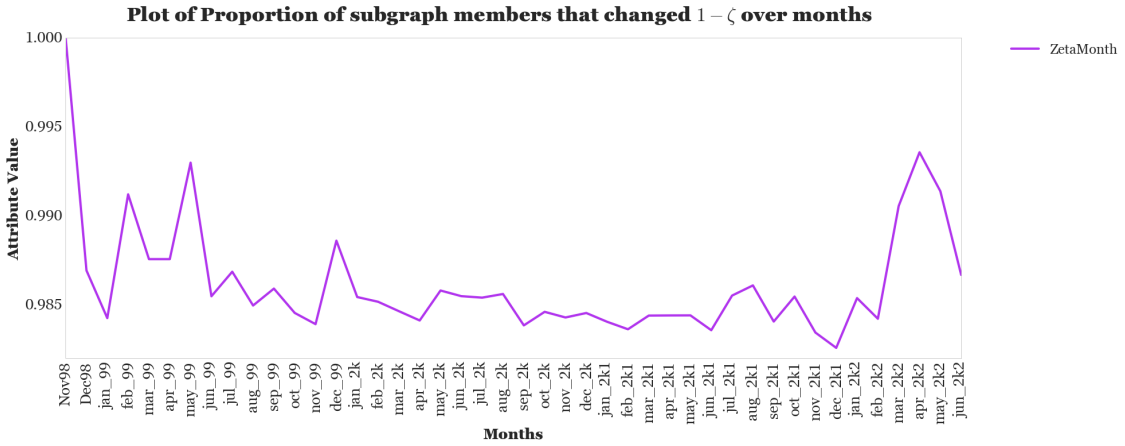




In [146]:
```
(1-zeta_y).plot(fontsize=22, figsize=(28,10))
plt.suptitle("Plot of Proportion of subgraph members that changed, $1-\ze
plt.xlabel("Years", fontsize=25)
plt.ylabel("Attribute Value", fontsize=25)
plt.legend(fontsize=24, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
plt.yticks(fontsize=26)

(1-zeta_m).plot(fontsize=22, rot=90, figsize=(28,10))
plt.suptitle("Plot of Proportion of subgraph members that changed $1-\zet
plt.xlabel("Months", fontsize=25)
plt.ylabel("Attribute Value", fontsize=25)
plt.legend(fontsize=24, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
plt.xticks(np.arange(len(months)), months,fontsize=26)
plt.yticks(fontsize=26)
```

Out[146]:
```
(array([ 0.98 ,  0.985,  0.99 ,  0.995,  1.   ,  1.005]),
 <a list of 6 Text yticklabel objects>)
```





```
In [147]: y_avgst_all['SubgraphStat']=zeta_y
          stat_all['SubgraphStat']=zeta_m
```

8.2 Persistence & Emergence

Persistence and Emergence are discussed in the paper below.

> At the very beginning timestep where t = 1oratany timestep when both Pi,t and Pi,t−1 are zero, Emergence is equal to 0, which indicates a neutral change.When t is larger than 1 and max{Pi,t, Pi,t−1} ?= 0, it is defined to be the normalized change of Persistence at the most recent timesteps. Depending on whether the Persistence contains negative values, normalization terms are defined differently.

Ref:

[1] W. Wei and K. M. Carley, "Measuring Temporal Patterns in Dynamic Social Networks," ACM Trans. Knowl. Discov. Data, vol. 10, no. 1, pp. 1–27, 2015.

```
In [148]: stat_all.head()
```

Out[148]:

	AvgDeg	AvgCloseness	AvgBet	AvgEig	AvgKatz	AvgLoad	\
Nov98	0.142857	0.192308	0.125000	0.222019	0.185435	0.125000	
Dec98	0.145455	0.242737	0.086869	0.233760	0.175111	0.086869	
jan_99	0.083333	0.117057	0.076923	0.132902	0.057692	0.076923	
feb_99	0.500000	0.555556	0.333333	0.569036	0.390223	0.333333	
mar_99	0.200000	0.259259	0.166667	0.328670	0.277684	0.166667	

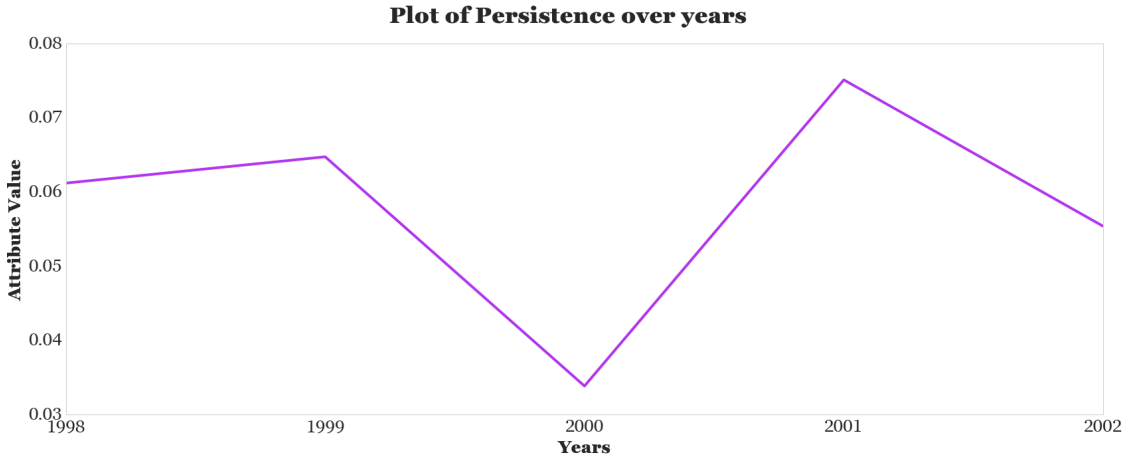
	Density	AvgClustCoef	AlgebraicConnect	MeanKurv	...
Nov98	0.250000	0.000000	1.0	-0.196247	...
Dec98	0.218182	0.246465	1.0	-0.199212	...
jan_99	0.153846	0.000000	1.0	-0.206795	...
feb_99	0.666667	0.000000	1.0	-0.058750	...
mar_99	0.333333	0.000000	1.0	-0.157405	...

	InstAccel	cosInstPhase	IAwIF	IAwIP	MeanResistanceDi
Nov98	-0.048538	0.369891	-2.164452e-01	0.345301	1.6038
Dec98	-0.012273	0.210647	1.753912e+00	1.255140	1.8040
jan_99	-0.144891	0.149447	9.562196e-01	2.265634	1.7512
feb_99	-0.451282	0.667721	-1.233581e-17	2.714095	0.9904
mar_99	-0.279494	0.550674	1.049833e+00	1.703481	1.4787

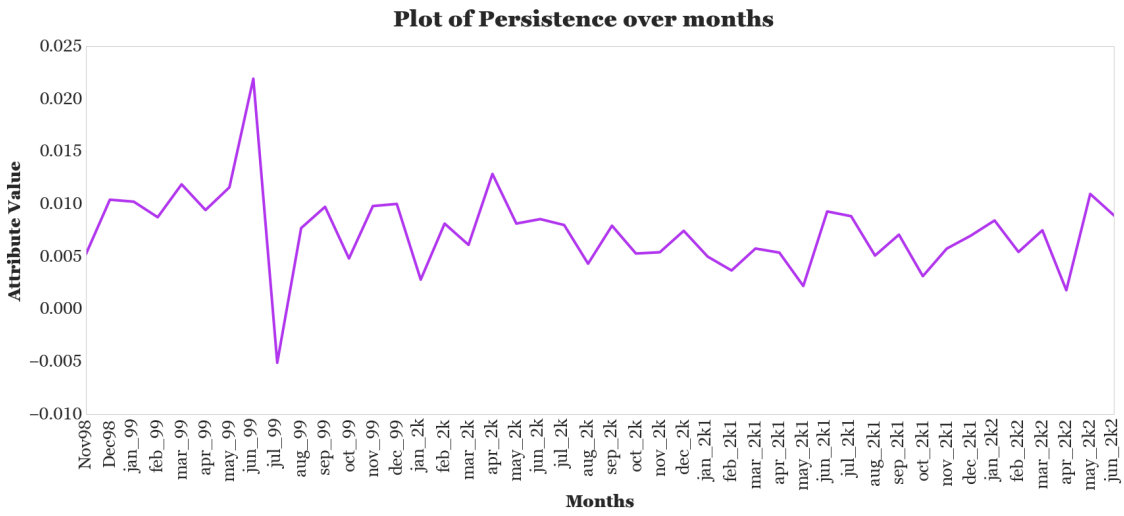
	StatRat	LogKPCARatio	LogICARatio	NormNMFRatio	SubgraphStat
Nov98	0.894427	0.000000	0.000000	0.000000	0.000000
Dec98	0.908295	-0.310794	1.048870	1.478061	0.013069
jan_99	0.930949	0.538398	1.345886	0.895689	0.015744
feb_99	0.774597	-1.051911	-0.853645	0.487906	0.008790
mar_99	0.866025	1.697420	0.614797	1.318329	0.012431

[5 rows x 25 columns]

```
In [149]: persistence_y = y_avgst_all.mean(axis=1)/y_avgst_all.shape[0]
          persistence_m = stat_all.mean(axis=1)/stat_all.shape[0]

In [150]: persistence_y.plot(fontsize=22, figsize=(28,10))
          plt.suptitle("Plot of Persistence over years", fontsize=33)
          plt.xlabel("Years", fontsize=25)
          plt.ylabel("Attribute Value", fontsize=25)
          #plt.legend(fontsize=24, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0
          plt.xticks(years[1:],[i for i in years[1:]],fontsize=26)
          plt.yticks(fontsize=26)

Out[150]: (array([ 0.02,  0.03,  0.04,  0.05,  0.06,  0.07,  0.08,  0.09]),
           <a list of 8 Text yticklabel objects>)
```




```
In [151]: persistence_m.plot(fontsize=22, rot=90, figsize=(28,10))
          plt.suptitle("Plot of Persistence over months", fontsize=33)
          plt.xlabel("Months", fontsize=25)
          plt.ylabel("Attribute Value", fontsize=25)
          #plt.legend(fontsize=24, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0
          plt.xticks(np.arange(len(months)), months,fontsize=26)
          plt.yticks(fontsize=26)

Out[151]: (array([-0.01 , -0.005,  0.   ,  0.005,  0.01 ,  0.015,  0.02 ,  0.025,
                  0.03 ]), <a list of 9 Text yticklabel objects>)
```




```
In [152]: def emergence(per):
              tmp = np.asarray(per)
              emerg = []
              for i in range(len(tmp)-1):
                  x= int(i)
                  y = x +1
                  #print(tmp[y], tmp[x])
                  if tmp[y]==tmp[x]:
                      emerg.append(0)
                  elif tmp[y] < (0) or tmp[x] < 0:
                      res = (tmp[y]-tmp[x])/(abs(tmp[y])+abs(tmp[x]))
                      emerg.append(res)
                  else:
                      res = (tmp[y]-tmp[x])/max([tmp[y],tmp[x]])
                      emerg.append(res)
              tmp2 = np.zeros(len(emerg)+1)
              tmp2[1:] = emerg

              return tmp2
```

```
In [153]: emerg_y = emergence(persistence_y)
          emerg_m = emergence(persistence_m)
```

```
In [154]: emerg_y_df = pd.DataFrame(emerg_y).T
          emerg_y_df.columns=years[1:]
          emerg_y_df = emerg_y_df.T
          emerg_y_df.columns = ['EmergenceYear']
          emerg_y_df.head()
```

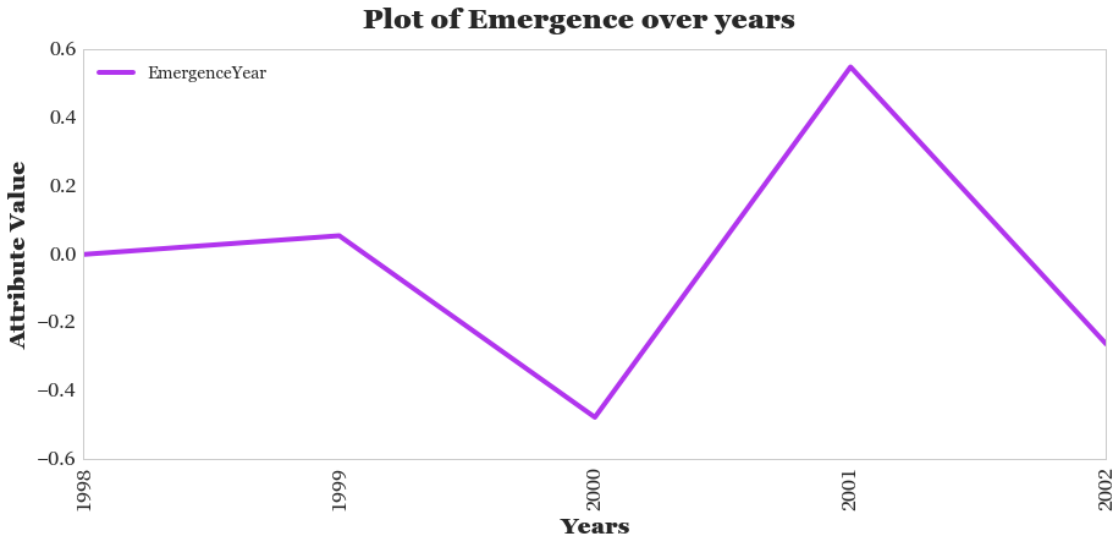
```
Out[154]:       EmergenceYear
          1998       0.000000
          1999       0.054781
          2000      -0.477204
          2001       0.549371
          2002      -0.262637
```

```
In [155]: emerg_m_df = pd.DataFrame(emerg_m).T
          emerg_m_df.columns=months
          emerg_m_df = emerg_m_df.T
          emerg_m_df.columns = ['EmergenceMonth']
          emerg_m_df.head()
```

```
Out[155]:         EmergenceMonth
          Nov98         0.000000
          Dec98         0.497251
          jan_99       -0.017980
          feb_99       -0.144898
          mar_99        0.263661
```

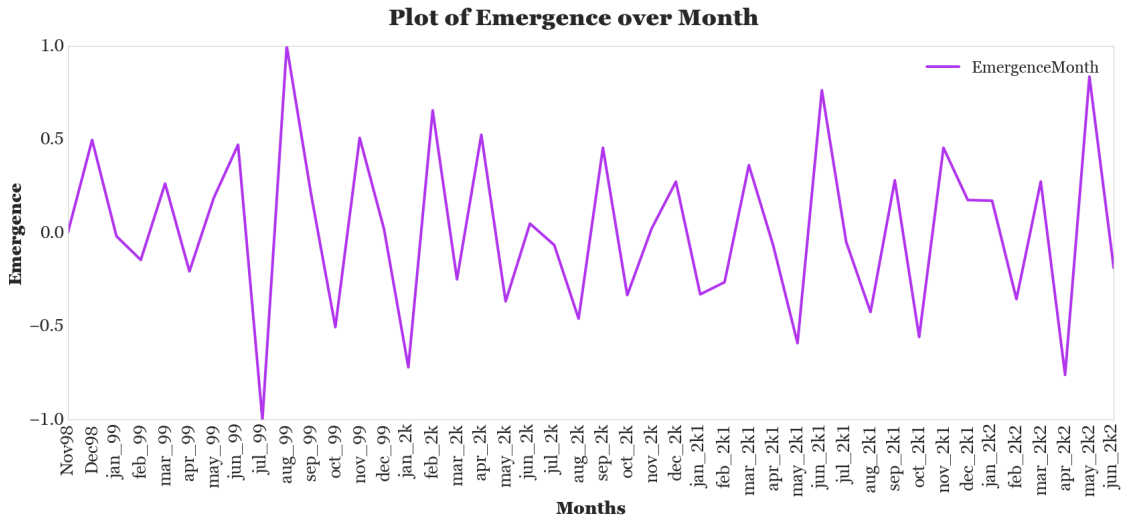
```
In [156]: emerg_y_df.plot(fontsize=22, rot=90)
          plt.suptitle("Plot of Emergence over years", fontsize=22)
          plt.xlabel("Years", fontsize=18)
          plt.ylabel("Attribute Value", fontsize=18)
          plt.legend(fontsize=14, loc=2)
          plt.xticks(years[1:],[i for i in years[1:]],fontsize=16)
          plt.yticks(fontsize=16)
```

```
Out[156]: (array([-0.8, -0.6, -0.4, -0.2,  0. ,  0.2,  0.4,  0.6,  0.8]),
           <a list of 9 Text yticklabel objects>)
```



```
In [157]: emerg_m_df.plot(fontsize=22, figsize=(28,10), rot=90)
          plt.suptitle("Plot of Emergence over Month", fontsize=33)
          plt.xlabel("Months", fontsize=25)
          plt.ylabel("Emergence", fontsize=25)
          plt.legend(fontsize=24, loc=1)
          plt.xticks(np.arange(len(months)), months,fontsize=26)
          plt.yticks(fontsize=26)
```

```
Out[157]: (array([-1. , -0.5,  0. ,  0.5,  1. ]), <a list of 5 Text yticklabel obje
```



8.3 NRMS of Emergence

The NRMS is a repeatability metric used in 4D seismic to quantify the likeness of two traces.

It is defined as:

$$NRMS = \frac{200RMS(a-b)}{RMS(a)+RMS(b)}$$

where the RMS operator is

$$RMS = \sqrt{\frac{\sum_(x)^2}{N}}$$

```
In [158]: nrms_emerg = []
          for i in range(len(emerg_m)-1):
              x = int(i)
              y = x + 1
              nrms_emerg.append(nrms(emerg_m[x],emerg_m[y]))
```

```
In [159]: nrms_emerg[:2]
```

```
Out[159]: [200.0, 200.0]
```

8.4 NRMS of Network Attributes

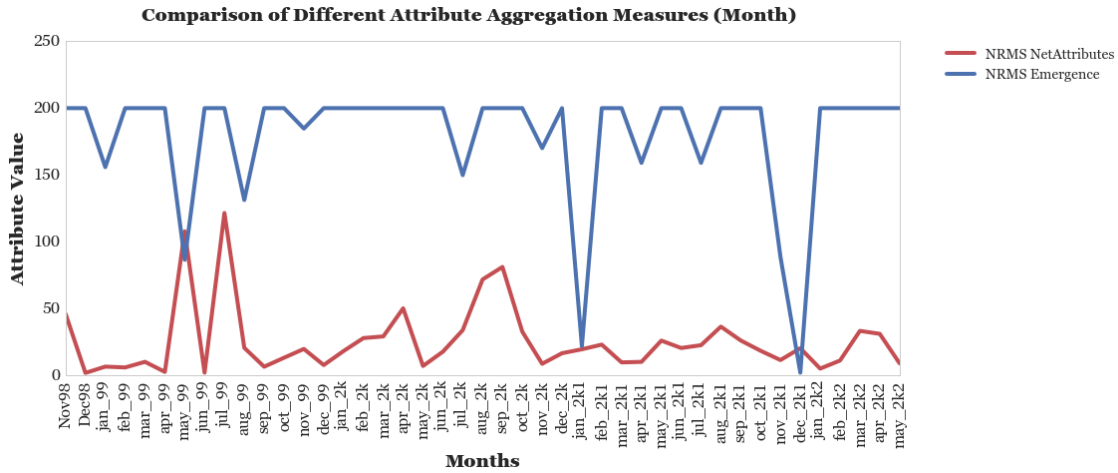
```
In [160]: rms_netatt = stat_all.apply(rms,axis=1).values
```

```
In [161]: nrms_netatt = []
          for i in range(rms_netatt.shape[0]-1):
              x = int(i)
              y = x + 1
              nrms_netatt.append(nrms(rms_netatt[x],rms_netatt[y]))
```

```
In [162]: nrms_netatt[:5]
```

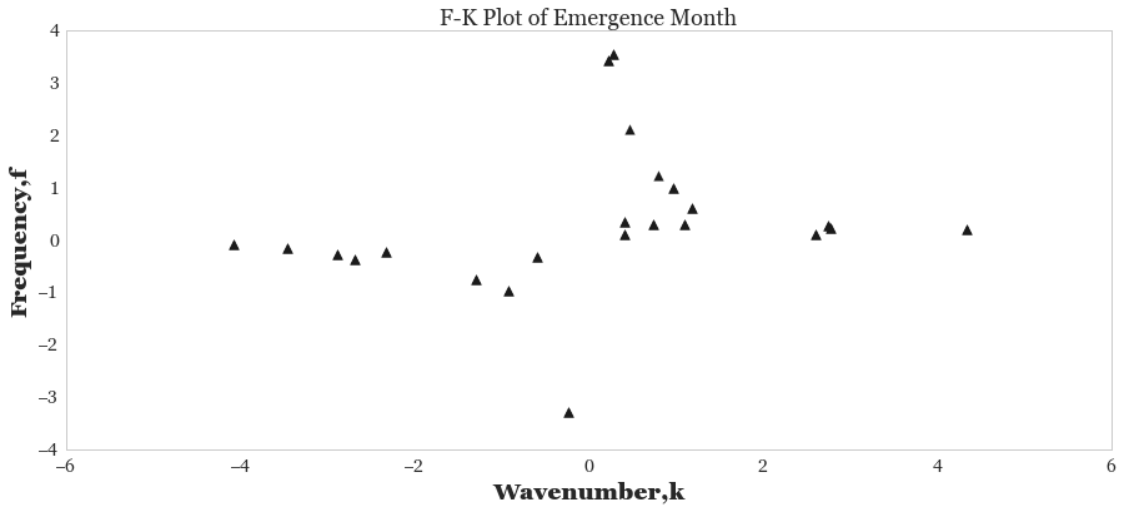
```
Out[162]: [46.0841600460964,
           1.8625099143923407,
           6.5776745948450746,
           5.9532927855931357,
           10.165893850595467]
```

```
In [163]: plt.suptitle("Comparison of Different Attribute Aggregation Measures (Mon
          #plt.plot(emerg_m, 'k',label='EmergenceMonth')
          plt.plot(nrms_netatt, 'r',label='NRMS NetAttributes')
          plt.plot(nrms_emerg, 'b',label='NRMS Emergence')
          plt.xlabel("Months", fontsize=18)
          plt.ylabel("Attribute Value", fontsize=18)
          plt.legend(fontsize=14, bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.
          plt.xticks(np.arange(len(months)), months,fontsize=16, rotation=90)
          plt.yticks(fontsize=16)
          plt.autoscale()
```




```
In [164]: f,k = fk_plot(emerg_m,'Emergence Month')
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:533: ComplexWa
  return array(a, dtype, copy=False, order=order, subok=True)
```




```
In [165]: print("Min/Max Freq Index:",np.argmax(f), np.argmin(f))
          print("Min/Max wavnumber Index:",np.argmax(k), np.argmin(k))
```

```
Min/Max Freq Index: 30 16
Min/Max wavnumber Index: 41 2
```

```
In [166]: print(emerg_m_df.iloc[np.argmax(f)])
          print(emerg_m_df.iloc[np.argmin(f)])
          print(emerg_m_df.iloc[np.argmax(k)])
          print(emerg_m_df.iloc[np.argmin(k)])
```

```
EmergenceMonth   -0.590481
Name: may_2k1, dtype: float64
EmergenceMonth   -0.248978
Name: mar_2k, dtype: float64
EmergenceMonth   -0.76042
Name: apr_2k2, dtype: float64
EmergenceMonth   -0.01798
Name: jan_99, dtype: float64
```

8.5 Sort by freq and wavenumber indices

```
In [167]: emerg_m_df.iloc[np.argsort(f)].head(n=10)
```

```
Out[167]:          EmergenceMonth
          mar_2k        -0.248978
          mar_2k1        0.362056
          feb_2k         0.655270
          apr_2k1       -0.067431
```

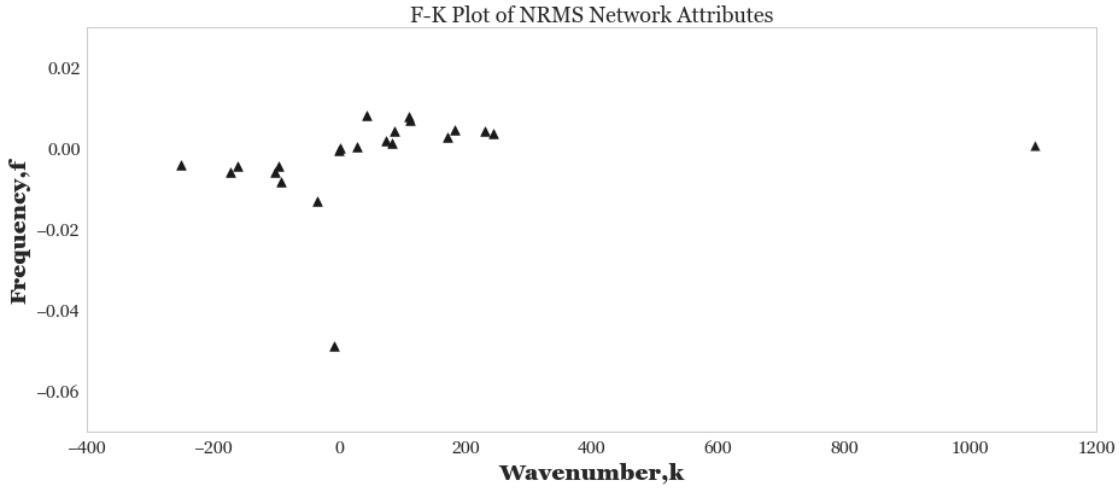
```
apr_2k          0.524839
feb_2k1        -0.264164
jul_2k1        -0.048353
nov_99          0.508278
jun_2k          0.049512
dec_2k          0.273808
```

```
In [168]: emerg_m_df.iloc[np.argsort(k)].head(n=10)
```

```
Out[168]:          EmergenceMonth
jan_99          -0.017980
may_2k2          0.836037
aug_99           1.000000
oct_2k1         -0.557419
dec_99           0.020220
jun_2k1          0.763038
nov_99           0.508278
jul_2k1         -0.048353
sep_99           0.207610
sep_2k1          0.281002
```

```
In [169]: f0,k0 = fk_plot(nrms_netatt,'NRMS Network Attributes')
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:533: ComplexWa
  return array(a, dtype, copy=False, order=order, subok=True)
```

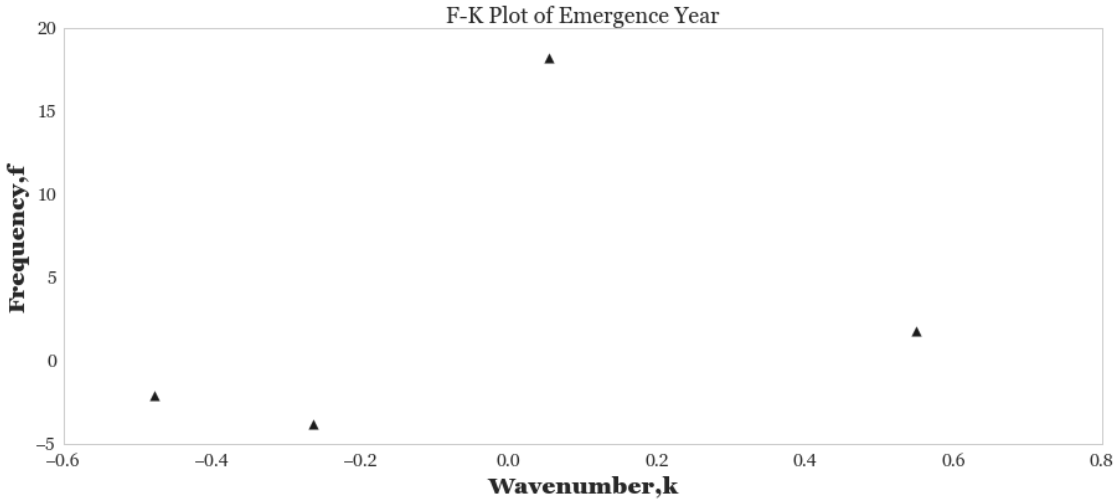



```
In [170]: nrms_netatt_df = pd.DataFrame(nrms_netatt).T
          nrms_netatt_df.columns=months[1:]
          nrms_netatt_df = nrms_netatt_df.T
          nrms_netatt_df.columns = ['EmergenceMonth']
          nrms_netatt_df.head()
```

```
Out[170]:          EmergenceMonth
          Dec98         46.084160
          jan_99         1.862510
          feb_99         6.577675
          mar_99         5.953293
          apr_99        10.165894

In [171]: nrms_netatt_df.iloc[np.argsort(f0)].head(n=10)

Out[171]:           EmergenceMonth
          mar_99          5.953293
          apr_2k2        33.311145
          oct_2k1        26.140238
          sep_99         20.652872
          dec_2k          8.691456
          jul_2k         17.731219
          jun_2k          7.046076
          jan_2k1        16.619401
          jan_99          1.862510
          jun_2k2         9.020304

In [172]: nrms_netatt_df.iloc[np.argsort(k0)].head(n=10)

Out[172]:           EmergenceMonth
          jan_2k          7.751234
          jun_2k1        26.035354
          aug_2k1        22.613574
          nov_99         13.219984
          dec_2k1        11.497456
          jul_99          2.074721
          jan_2k1        16.619401
          jun_2k          7.046076
          sep_99         20.652872
          oct_2k1        26.140238

In [173]: print(nrms_netatt_df.iloc[np.argmax(f0)])
          print(nrms_netatt_df.iloc[np.argmin(f0)])
          print(nrms_netatt_df.iloc[np.argmax(k0)])
          print(nrms_netatt_df.iloc[np.argmin(k0)])

EmergenceMonth    46.08416
Name: Dec98, dtype: float64
EmergenceMonth    5.953293
Name: mar_99, dtype: float64
EmergenceMonth    6.577675
Name: feb_99, dtype: float64
EmergenceMonth    7.751234
Name: jan_2k, dtype: float64
```

```
In [174]: f1,k1 = fk_plot(emerg_y_df.values,'Emergence Year')
```

```
C:\Users\arsha_000\Anaconda3\lib\site-packages\numpy\core\numeric.py:533: ComplexWa
  return array(a, dtype, copy=False, order=order, subok=True)
```




```
In [175]: print(emerg_y_df.iloc[np.argmax(f1)])
          print(emerg_y_df.iloc[np.argmin(f1)])
          print(emerg_y_df.iloc[np.argmax(k1)])
          print(emerg_y_df.iloc[np.argmin(k1)])
```

```
EmergenceYear    0.549371
Name: 2001, dtype: float64
EmergenceYear   -0.477204
Name: 2000, dtype: float64
EmergenceYear    0.0
Name: 1998, dtype: float64
EmergenceYear    0.0
Name: 1998, dtype: float64
```

9 Attribute Correlations

```
In [176]: stat_all['Emergence']=emerg_m_df
```

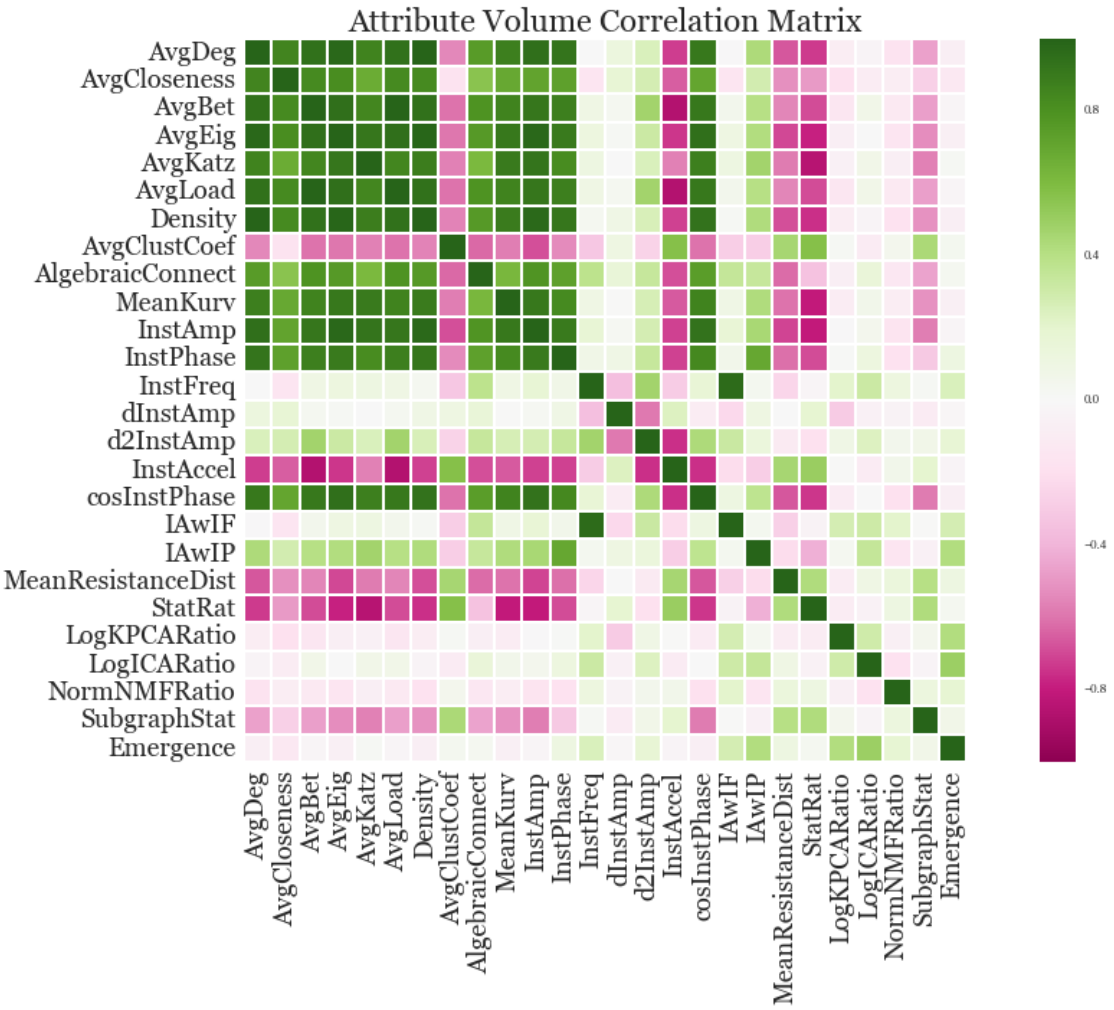
```
In [177]: corr_m = stat_all.corr()
```

9.1 Correlation Matrix

```
In [178]: plt.figure(figsize=(18,9))
          sns.heatmap(corr_m, cmap='PiYG',robust=True, fmt='d', linewidths=1, squar
```

```
        plt.title("Attribute Volume Correlation Matrix", fontsize=22)
        plt.xticks(fontsize=18)
        plt.yticks(fontsize=18)
```

```
Out[178]: (array([  0.5,   1.5,   2.5,   3.5,   4.5,   5.5,   6.5,   7.5,   8.5,
                    9.5,  10.5,  11.5,  12.5,  13.5,  14.5,  15.5,  16.5,  17.5,
                   18.5,  19.5,  20.5,  21.5,  22.5,  23.5,  24.5,  25.5]),
           <a list of 26 Text yticklabel objects>)
```



9.2 Correlation Dendogram

```
In [ ]: sns.dendrogram(corr_m)
        plt.suptitle("Dendogram of Network Attributes", fontsize=22)
        plt.xlabel("Attributes", fontsize=20)
        plt.xticks(fontsize=18)
        plt.yticks(fontsize=18)
        plt.close()
```

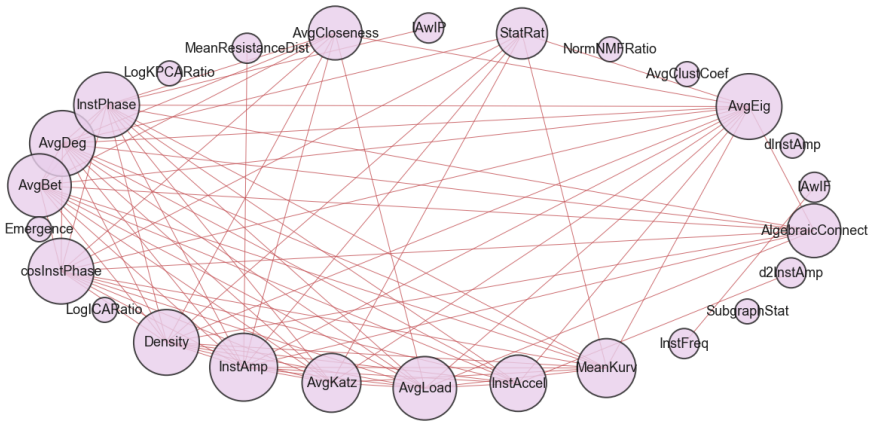
```
In [ ]: g = sns.dendrogram(corr_m, method='average', metric='chebyshev')
        plt.suptitle("Dendogram of Correlation Matrix Network Attributes", fontsize
        plt.xlabel("Attributes", fontsize=20)
        plt.xticks(fontsize=18)
        plt.yticks(fontsize=18)

In [181]: g.reordered_ind[:5]

Out[181]: [18, 8, 1, 11, 16]
```

9.3 Correlation Matrix > 0.7

```
In [182]: threshold = 0.7
          corr_m.values[np.where(abs(corr_m.values) < threshold)] = 0

In [183]: plt.figure(figsize=(18,9))
          sns.heatmap(corr_m.iloc[g.reordered_ind, g.reordered_ind], cmap='PiYG',ro
          plt.title("Attribute Volume Correlation Matrix Reordered showing Correlat
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)

Out[183]: (array([  0.5,   1.5,   2.5,   3.5,   4.5,   5.5,   6.5,   7.5,   8.5,
                    9.5,  10.5,  11.5,  12.5,  13.5,  14.5,  15.5,  16.5,  17.5,
                   18.5,  19.5,  20.5,  21.5,  22.5,  23.5,  24.5,  25.5]),
           <a list of 26 Text yticklabel objects>)
```

9.4 Correlation Network of Attributes

```
In [184]: names = corr_m.index.values

In [185]: G_corr = nx.Graph(corr_m.values)
          pos=nx.fruchterman_reingold_layout(G_corr, iterations=1000, k=200)

In [186]: #ref: https://github.com/traims/correlation-based-networks/blob/master/cr
          components = nx.connected_components(G_corr)
          plt.figure(figsize=(26,12))
          plt.suptitle("Correlation Network of Attributes", fontsize=33)
          plt.axis('off')
          for i in components:
              component = G_corr.subgraph(i)
              nx.draw_networkx(component, with_labels = True,node_size = [component
                  labels = dict([(x, names[x]) for x in component.nodes()]), pos=po
                              edge_color='r', node_color='#E9CFEC', linewidths=2, f
```




```
In [187]: nx.write_gexf(G_corr,'corr_mat_att.gexf')
```

10 Feature Ranking with Gradient Boositng Regressor

10.1 Feature Ranking for predicting Emergence

Here I use gradient boosting regression to predict Emergence based on all the other network attributes. Since Emergence is a gloabal measure that indicates changes in activity over time. Since the attribute volume is small I use a 50/50 test/train split for this analysis. Also this should prevent overfitting from this sample size.

```
In [188]: stat_all.drop(['Emergence'],axis=1).apply(sc.stats.zscore, axis=1).head()
```

```
Out[188]:          AvgDeg  AvgCloseness    AvgBet    AvgEig   AvgKatz   AvgLoad  \
          Nov98  -0.214871     -0.093339 -0.258757 -0.020320 -0.110231 -0.258757
          Dec98  -0.529175     -0.364494 -0.628349 -0.379690 -0.478973 -0.628349
          jan_99 -0.596366     -0.541489 -0.606797 -0.515705 -0.638090 -0.606797
          feb_99  0.160964      0.238483 -0.071593  0.257293  0.007788 -0.071593
          mar_99 -0.574917     -0.469233 -0.634363 -0.345447 -0.436374 -0.634363

                   Density  AvgClustCoef  AlgebraicConnect  MeanKurv      ...
          Nov98   0.048448     -0.565962          1.891678 -1.048266      ...
          Dec98  -0.406061     -0.358183          0.917410 -1.112631      ...
          jan_99 -0.481624     -0.731970          0.895282 -1.068478      ...
          feb_99  0.393521     -0.536707          0.858635 -0.618683      ...
          mar_99 -0.337129     -0.931597          0.851807 -1.212315      ...

                  InstAccel  cosInstPhase     IAwIF     IAwIP  MeanResistanceDist
```

```
Nov98   -0.685250      0.343098 -1.097907  0.282664           3.375699
Dec98   -0.796177     -0.418817  2.193641  1.349315           2.278515
jan_99  -0.967745     -0.488781  0.824041  2.954789           2.117783
feb_99  -1.166399      0.394993 -0.536707  3.250384           0.845335
mar_99  -1.430049      0.050478  0.940679  2.106398           1.705665

         StatRat  LogKPCARatio  LogICARatio  NormNMFRatio  SubgraphStat
Nov98   1.632218     -0.565962    -0.565962     -0.565962     -0.565962
Dec98   0.762171     -1.301518     1.000138      1.726677     -0.753279
jan_99  0.782920      0.144139     1.458126      0.725541     -0.706350
feb_99  0.544121     -2.004482    -1.727834      0.144089     -0.524442
mar_99  0.612876      2.095589     0.164834      1.419516     -0.909428

[5 rows x 25 columns]
```

In [189]:
```
X= stat_all.drop(['Emergence'],axis=1).apply(lambda x: sc.stats.zscore(x)
y = stat_all.Emergence
```

In [190]:
```
X = X.astype(np.float32)
offset = int(X.shape[0] * 0.5)
X_train, y_train = X[:offset], y[:offset]
X_test, y_test = X[offset:], y[offset:]
```

In [191]:
```
from sklearn.metrics import mean_squared_error
from sklearn import ensemble
```

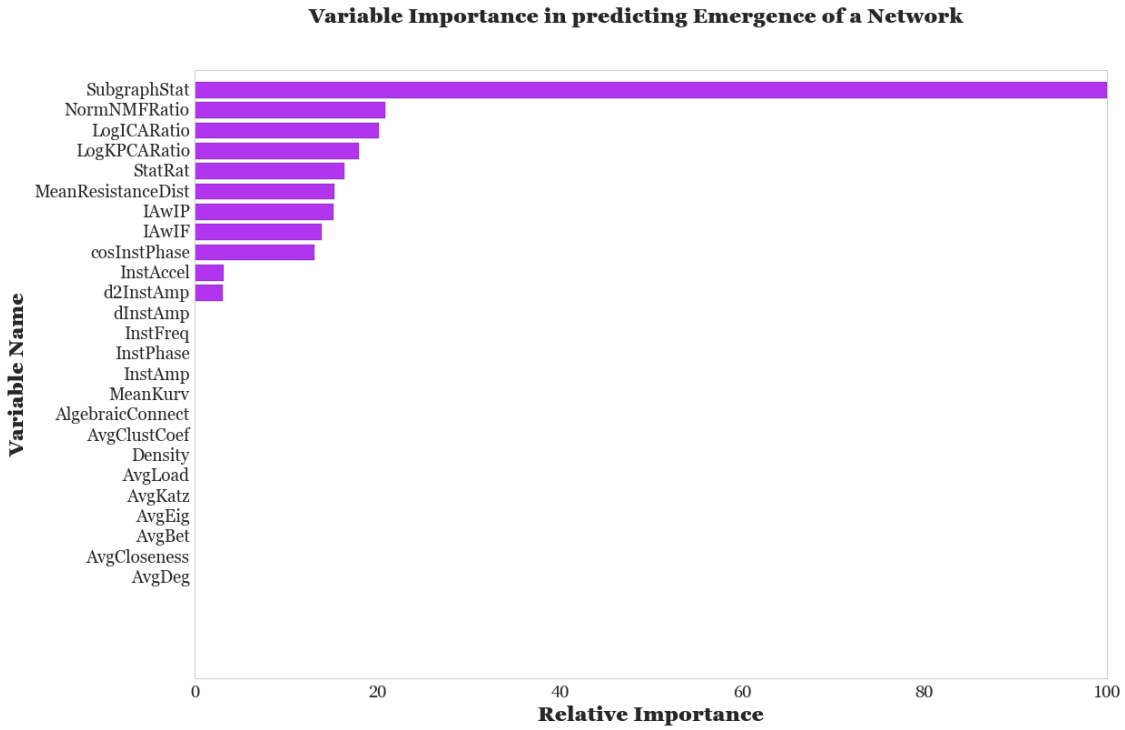
In [192]:
```
params = {'n_estimators': 10, 'max_depth': 10, 'min_samples_split': 10,
          'learning_rate': 0.1, 'loss': 'ls'}
clf = ensemble.GradientBoostingRegressor(**params)
clf.fit(X_train, y_train)
```

Out[192]:
```
GradientBoostingRegressor(alpha=0.9, init=None, learning_rate=0.1, loss='
              max_depth=10, max_features=None, max_leaf_nodes=None,
              min_samples_leaf=1, min_samples_split=10,
              min_weight_fraction_leaf=0.0, n_estimators=10, presort='auto
              random_state=None, subsample=1.0, verbose=0, warm_start=Fals
```

In [193]:
```
feature_importance = clf.feature_importances_

# make importances relative to max importance
plt.figure(figsize=(18,12))
feature_importance = 100.0 * (feature_importance / feature_importance.max
sorted_idx = np.argsort(feature_importance)
feature_names = list(X)
pos = np.arange(sorted_idx.shape[0])
plt.barh(pos, feature_importance[sorted_idx], align='center')
plt.yticks(pos, feature_names, fontsize=19)
plt.ylabel("Variable Name", fontsize=22)
plt.xlabel('Relative Importance', fontsize=22)
```

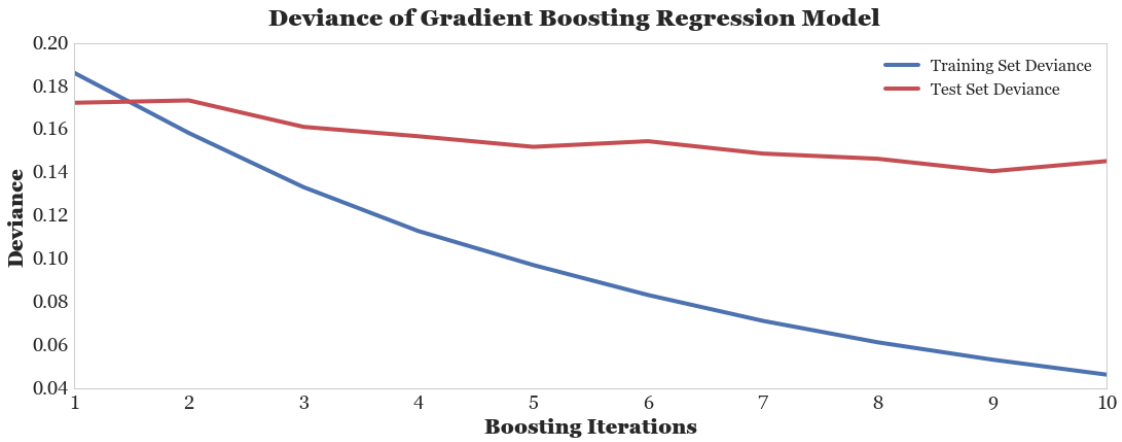
```
        plt.suptitle('Variable Importance in predicting Emergence of a Network',
        plt.xticks(fontsize=18)
```

```
Out[193]: (array([   0.,   20.,   40.,   60.,   80.,  100.]),
           <a list of 6 Text xticklabel objects>)
```




```
In [194]: test_score = np.zeros((params['n_estimators'],), dtype=np.float64)

          for i, y_pred in enumerate(clf.staged_predict(X_test)):
              test_score[i] = clf.loss_(y_test, y_pred)

          plt.figure(figsize=(18, 6))
          plt.suptitle('Deviance of Gradient Boosting Regression Model',fontsize=22
          plt.plot(np.arange(params['n_estimators']) + 1, clf.train_score_, 'b-',
                   label='Training Set Deviance')
          plt.plot(np.arange(params['n_estimators']) + 1, test_score, 'r-',
                   label='Test Set Deviance')
          plt.legend(loc=1, fontsize=16)
          plt.xlabel('Boosting Iterations')
          plt.ylabel('Deviance')
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)
```

```
Out[194]: (array([ 0.04,  0.06,  0.08,  0.1 ,  0.12,  0.14,  0.16,  0.18,  0.2 ,  0
           <a list of 10 Text yticklabel objects>)
```



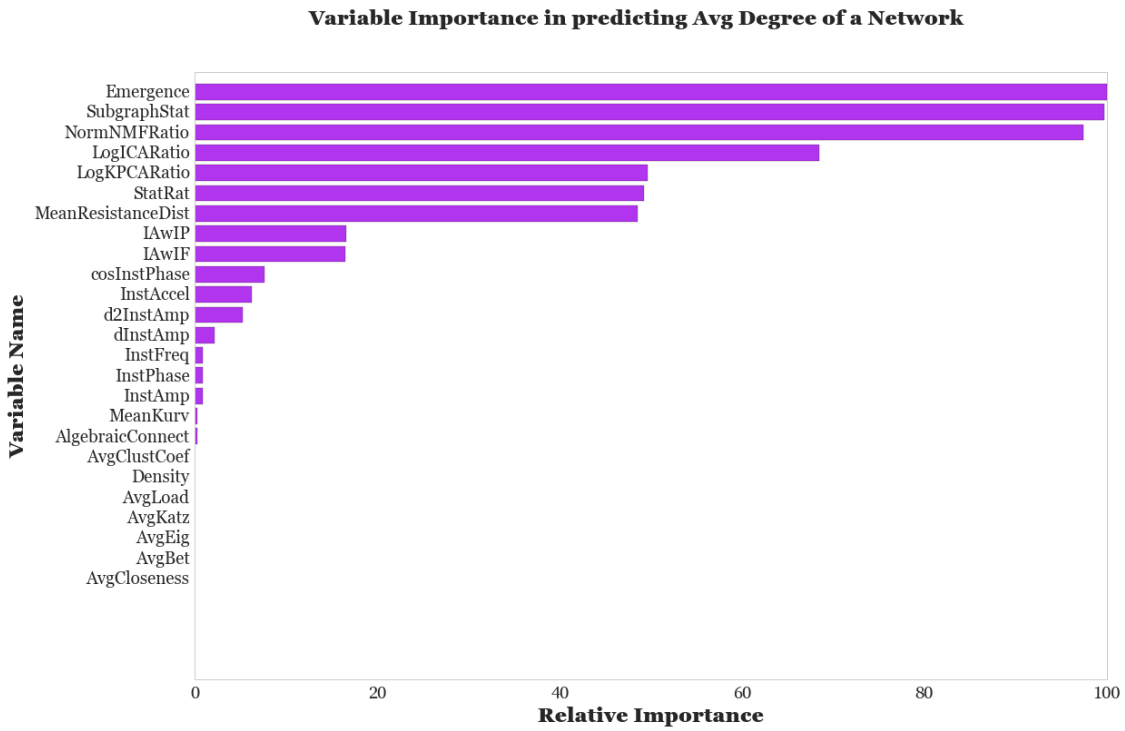

```
In [195]: mse = mean_squared_error(y_test, clf.predict(X_test))
          print("Gradient Boosting MSE: " ,mse)

Gradient Boosting MSE:  0.145520611556
```

10.2 Feature Ranking for predicting Average Degree

```
In [196]: X= stat_all.drop(['AvgDeg'],axis=1).apply(lambda x: sc.stats.zscore(x), a
          y = stat_all.AvgDeg

In [198]: X = X.astype(np.float32)
          offset = int(X.shape[0] * 0.5)
          X_train, y_train = X[:offset], y[:offset]
          X_test, y_test = X[offset:], y[offset:]

In [200]: params = {'n_estimators': 10, 'max_depth': 10, 'min_samples_split': 10,
                    'learning_rate': 0.1, 'loss': 'ls'}
          clf = ensemble.GradientBoostingRegressor(**params)
          clf.fit(X_train, y_train)

Out[200]: GradientBoostingRegressor(alpha=0.9, init=None, learning_rate=0.1, loss='
                       max_depth=10, max_features=None, max_leaf_nodes=None,
                       min_samples_leaf=1, min_samples_split=10,
                       min_weight_fraction_leaf=0.0, n_estimators=10, presort='auto
                       random_state=None, subsample=1.0, verbose=0, warm_start=Fals

In [201]: feature_importance = clf.feature_importances_

          # make importances relative to max importance
          plt.figure(figsize=(18,12))
          feature_importance = 100.0 * (feature_importance / feature_importance.max
          sorted_idx = np.argsort(feature_importance)
```

```
feature_names = list(X)
pos = np.arange(sorted_idx.shape[0])
plt.barh(pos, feature_importance[sorted_idx], align='center')
plt.yticks(pos, feature_names, fontsize=19)
plt.ylabel("Variable Name", fontsize=22)
plt.xlabel('Relative Importance', fontsize=22)
plt.suptitle('Variable Importance in predicting Avg Degree of a Network',
plt.xticks(fontsize=18)
```

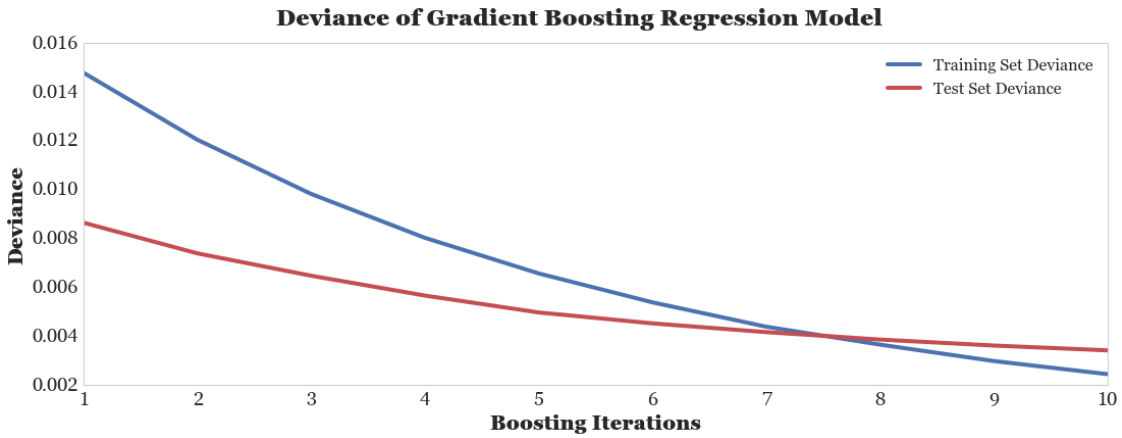
```
Out[201]: (array([   0.,   20.,   40.,   60.,   80.,  100.]),
           <a list of 6 Text xticklabel objects>)
```




```
In [202]: test_score = np.zeros((params['n_estimators'],), dtype=np.float64)

          for i, y_pred in enumerate(clf.staged_predict(X_test)):
              test_score[i] = clf.loss_(y_test, y_pred)

          plt.figure(figsize=(18, 6))
          plt.suptitle('Deviance of Gradient Boosting Regression Model',fontsize=22
          plt.plot(np.arange(params['n_estimators']) + 1, clf.train_score_, 'b-',
                   label='Training Set Deviance')
          plt.plot(np.arange(params['n_estimators']) + 1, test_score, 'r-',
                   label='Test Set Deviance')
          plt.legend(loc=1, fontsize=16)
```

```
plt.xlabel('Boosting Iterations')
plt.ylabel('Deviance')
plt.xticks(fontsize=18)
plt.yticks(fontsize=18)
```

```
Out[202]: (array([ 0.002,  0.004,  0.006,  0.008,  0.01 ,  0.012,  0.014,  0.016]),
           <a list of 8 Text yticklabel objects>)
```




```
In [203]: mse = mean_squared_error(y_test, clf.predict(X_test))
          print("Gradient Boosting MSE: " ,mse)
```

```
Gradient Boosting MSE:  0.0034215367522
```

11 Signal to Noise ratio

Here I calculate the signal to noise ratio of the attribute volume. This is done for each variable by:

$$SNR = \frac{Mean_{att}}{StdDev_{att}}$$

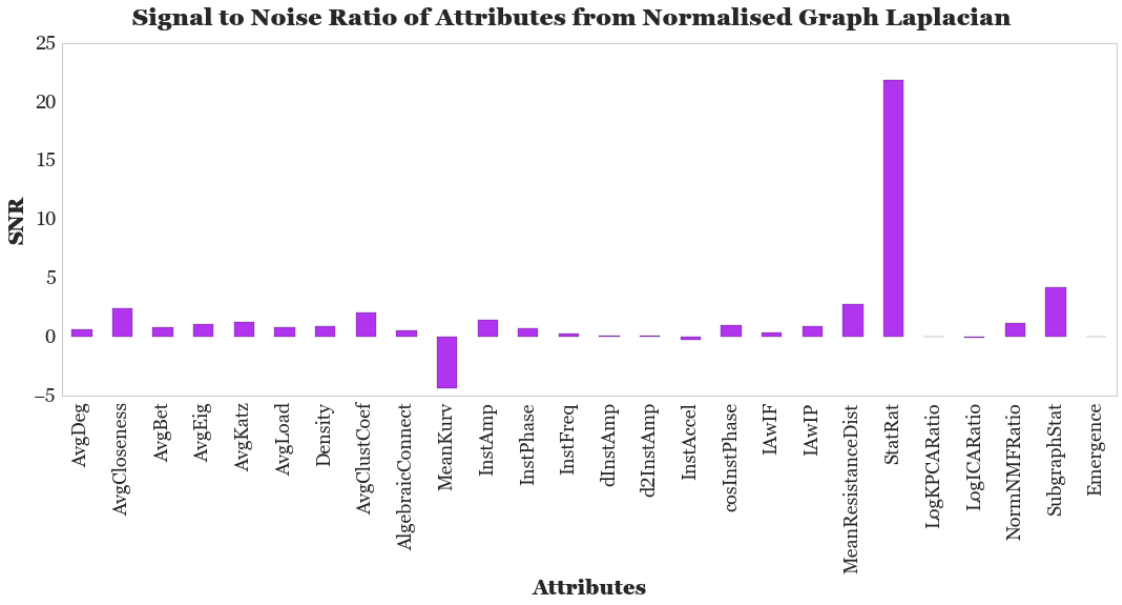
A ratio higher than 1:1 indicates more signal than noise.

```
In [204]: snr_att = stat_all.mean()/stat_all.std()
          snr_att[snr_att>1].count()
```

```
Out[204]: 10
```

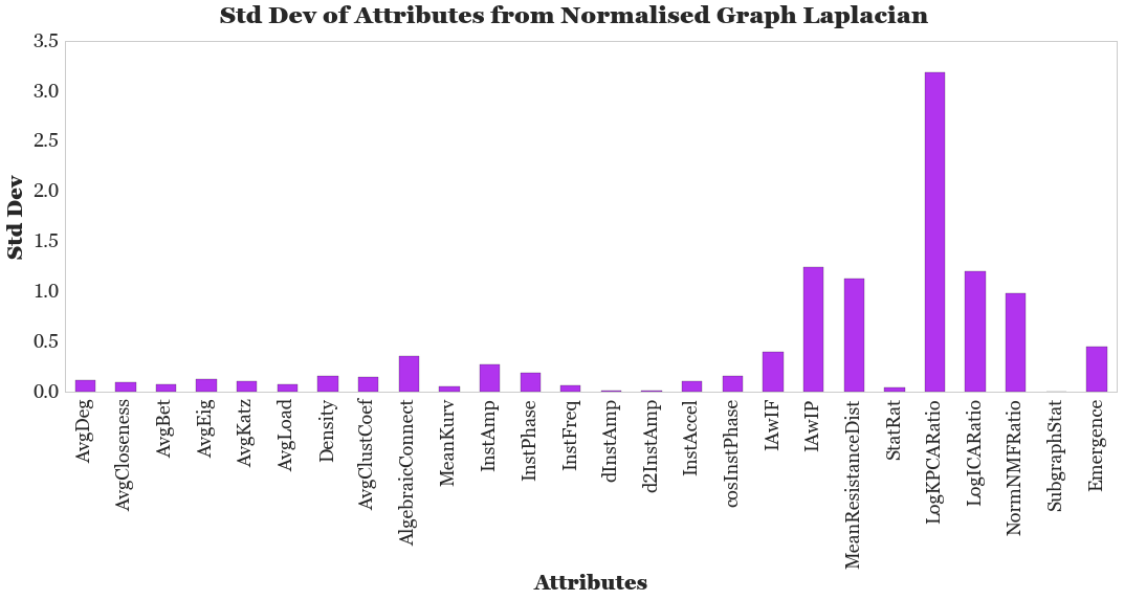
```
In [205]: (stat_all.mean()/stat_all.std()).plot(kind='bar', fontsize=18,figsize=(18
          plt.suptitle('Signal to Noise Ratio of Attributes from Normalised Graph I
          plt.xlabel('Attributes')
          plt.ylabel('SNR')
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)
```

```
Out[205]: (array([ -5.,   0.,   5.,  10.,  15.,  20.,  25.]),
           <a list of 7 Text yticklabel objects>)
```

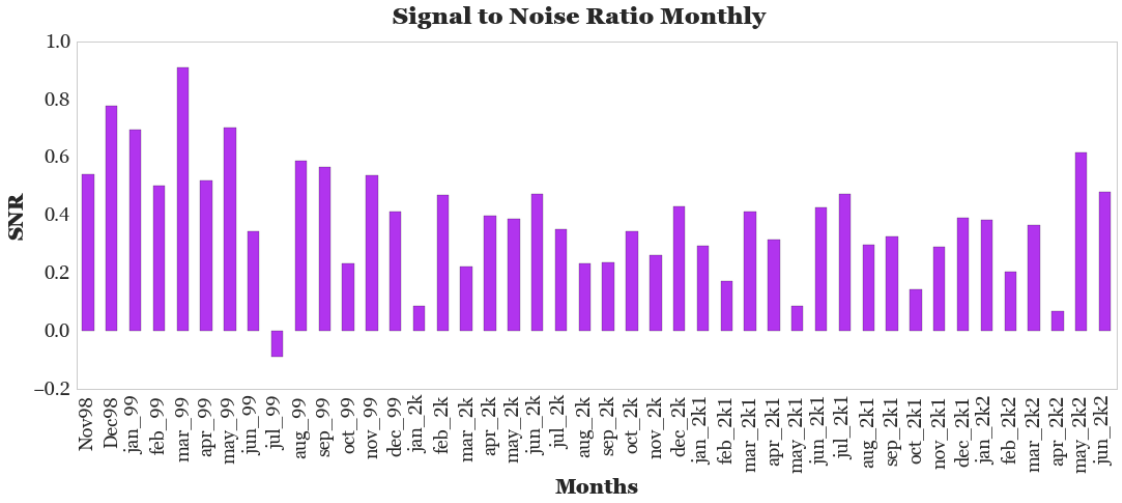



```
In [209]: stat_all.std().plot(kind='bar', fontsize=18,figsize=(18,6))
          plt.suptitle('Std Dev of Attributes from Normalised Graph Laplacian', for
          plt.xlabel('Attributes')
          plt.ylabel('Std Dev')
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)
```

```
Out[209]: (array([ 0. ,  0.5,  1. ,  1.5,  2. ,  2.5,  3. ,  3.5]),
           <a list of 8 Text yticklabel objects>)
```



```
In [206]: (stat_all.mean(axis=1)/stat_all.std(axis=1)).plot(kind='bar', fontsize=18
          plt.suptitle('Signal to Noise Ratio Monthly', fontsize=22)
          plt.xlabel('Months')
          plt.ylabel('SNR')
          plt.xticks(fontsize=18)
          plt.yticks(fontsize=18)

Out[206]: (array([-0.2,  0. ,  0.2,  0.4,  0.6,  0.8,  1. ,  1.2]),
           <a list of 8 Text yticklabel objects>)
```




```
In [207]: stat_all.to_excel('enron_dynet_attvol.xlsx')
```

12 Thoughts and Observations

In this study I have explored the analysis of dynamic networks. The key challenge was to find measures that would allow easy comparison among the networks at each time steps. This is complicated by the fact that the networks are changing over time so attempting to take some direct correlation of attributes will not work because they are of different lenghts. To remedy this situation temporal aggregation was necessary.

Since we are dealing with a large number of networks over time it is not feasible to conduct visual analysis through the inspection of node link diagrams or of derived graph matrices such as the Laplacian, Modularity and Adjacency Matrices. Since these matrices are of different sizes at each timestep instead of considering the whole mateix which may consist of commonalities between timesteps it is better to focus on the differences of these matrices and the difference of the attributes derived from these matrices to characterise change.

To this end I explored a range of measures such as traditional centrality and network statistical measures in addition to novel measures derived from the complex trace and from the decomposition of the Normalised Graph Laplacian. It is surprising to note that the complex trace attributes

have such high correlation to centrality measures. But it encouraging that the trends highlighted by these traditional measures are reflected by these new measures and in some cases they highlight additional areas of interest which are not as clear from the traditional measures.

The interesting finding is that the decomposition based attributes are not particularly strongly correlated to any of the old or new measures. This is useful because it suggests that we are able to get additional information from these measures while the correlated measures confirms what we have already seen. This could be as a result of scale but as the regression based feature ranking shows using the zscores of all attributes the decomposition based measures are particularly useful in trying to predict network statistics. The reason for doing this is to test given a sequence of dynamic networks can the properties of the network at a time, t+1 in the future be reasonably predicted. The assumption is that the wide range of measures used characterise the dynamics of the network to a sufficient extent to allow for predictive analytics.

It is clear from the study that both tradtional and the new measures suggested are very useful in the meaningful characterisation and analysis of dynamic networks. With regards to global measures such as Emergence having a reasonable large set of measures makes it stable to variations in particular attributes.

Also I have shown that it also possible to measure graph evolution directly by comparing the subgraphs of the networks at different time steps. The Subgraph Stationarity measure relies only on the adjacency matrices of the networks in question and gives a good control on the trend weshould expect to see from other measures. This is a very useful independent measure but its inclusion in the Emergence calculation clearly has benefits.

In [ ]: