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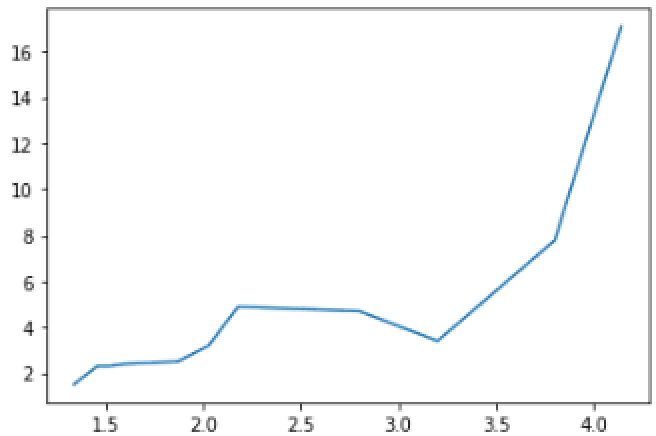
Q1)Plot and integrate the tabulated data given below by
a suitable method. Take the end points of the table as integration limits.

x	1.34	1.46	1.52	1.6	1.87	2.03	2.18	2.8	3.2	3.8	4.15
$f(x)$	1.5	2.3	2.3	2.4	2.5	3.2	4.9	4.7	3.4	7.8	17.1

Plot:



This is x vs F(x) plot
14.467999999999996 is the integration value for the given data.

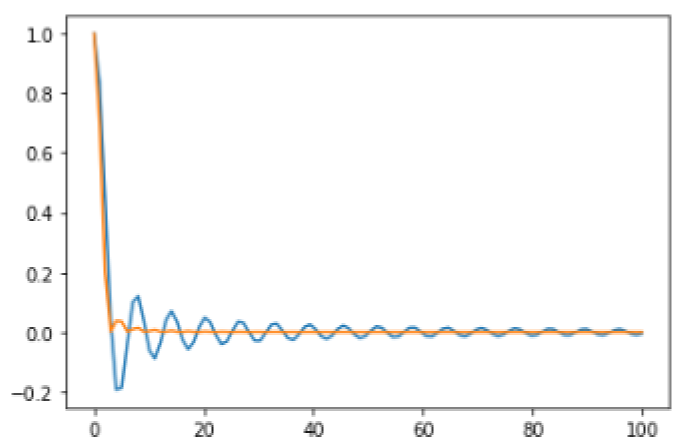
Q2)Write a function in python trap (f, a, b, N) which
will evaluate the integral ∫ f(x)dx using the trapezoidal rule with N steps.
Also develop a similar function simp (f, a, b, N) for the Simpson's rule. Use
these functions to evaluate the numerical integrals in the subsequent
problems,
wherever applicable.
Use these functions to evaluate the following integrals in limits a to b for [a,b]
interval divided into N (=2n, n=1, 2, 3, ...10) intervals with:
1) trapezoidal rules
2) Simpson's rule

Sol:
Function 1:
f(x) = x^2
The Integral value from -1 to 1 from the trapezoidal method is 0.6666679407195909.
The integral value from -1 to 1 from the Simpson method is 0.6666666666666675.

Function 2:
f(x)=sin(x)
The Integral value from 0 to pi from the trapezoidal method is 1.9999984281999592.
The integral value from 0 to pi from the Simpson method is 2.000000000000613.

Function 3:
f(x) = (sin x / x)^2
The Integral value from 0 to infinity from the trapezoidal method is 1.5658182076234524.
The integral value from 0 to infinity from the Simpson method is 1.5658182776254017.

The plot of this function is

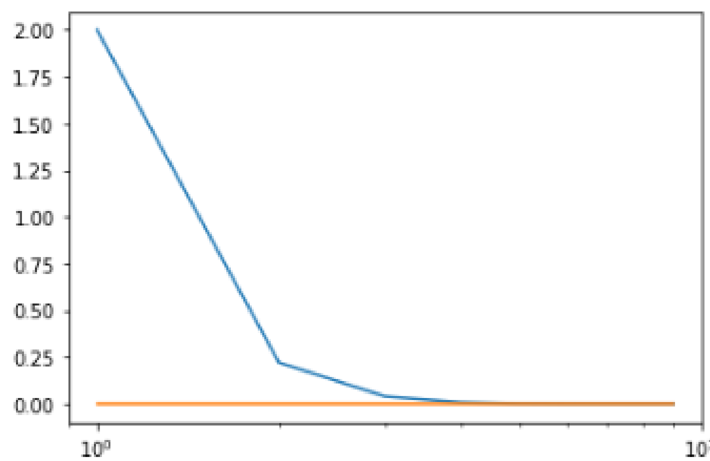


Blue is sinx / x
Orange is (sinx / x)^2

Show plot of log e(N) with varying log N for all the integrals. Are these results keeping with how you expect the error to scale with N?

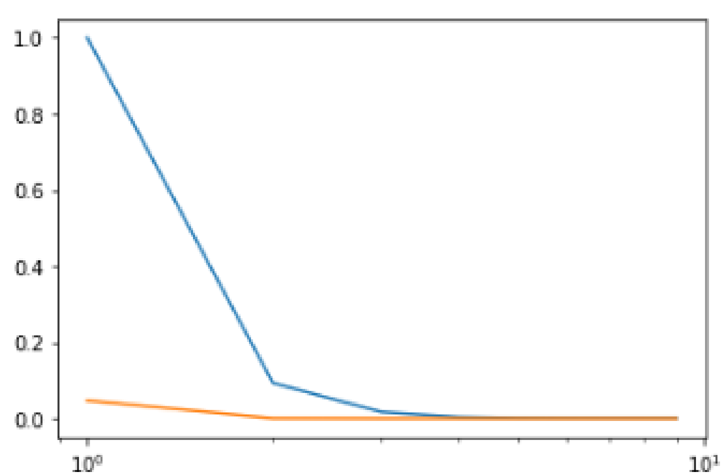
Function 1:

Log N vs Error plot for Trapezoidal(Blue) and Simpson(Orange) method:

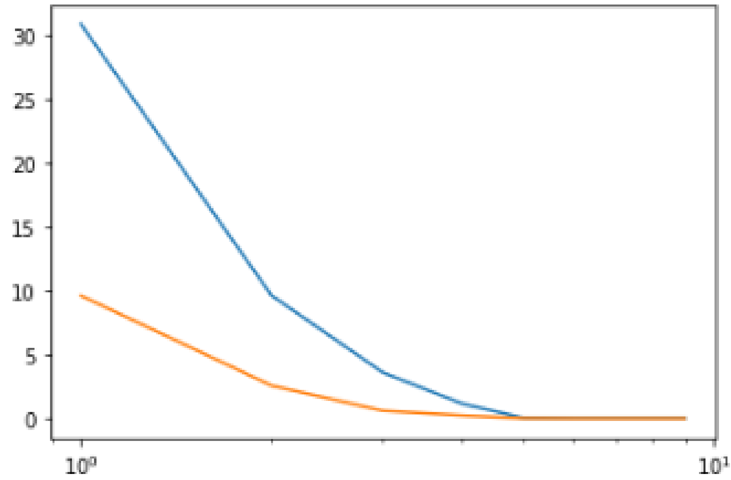


Function 2:

Log N vs Error plot for Trapezoidal(Blue) and Simpson(Orange) method:



Function 3:



Q3)Consider a simple harmonic oscillator with potential for a particle with m = 1Kg and 0= 2 sec-1. Express this integral in terms of dimensionles variables.

a) Numerically calculate the time period of oscillation by integrating the equation with Trapezoidal method and check this against the expected value.

Note that the integrand will diverge at the limits. So the limits has to be redefined i.e. b(1-e) in place of b. Numerically obtained values of T will also diverge for very low values of . Make a log-log plot of e vs. T. Then choose a suitable value of that will provide a reasonably accurate value of T. Verify that T does not depend on the amplitude of oscillation.

Sol:

The Time period for Amplitude =2 is given as 0.9936339879121445 s

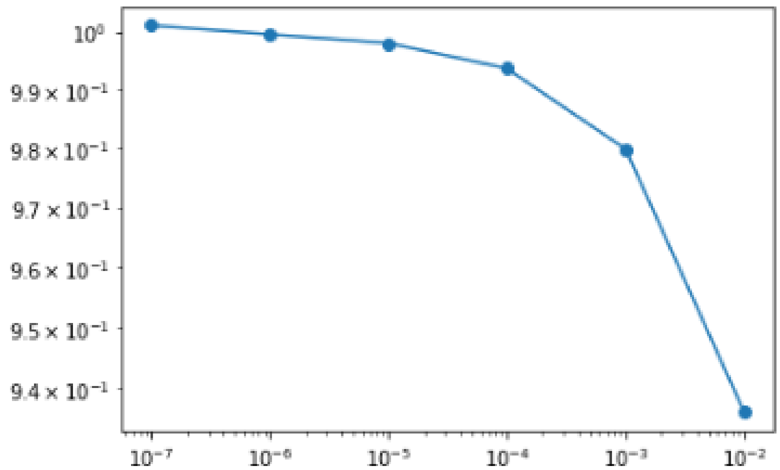
The Time period for Amplitude =4 is given as 0.995499008979651 s

The Time period for Amplitude =8 is given as 0.9968185927401884 s

From the above values we can say that Time period dosent depend upon the amplitude of the oscillator.

We observe that the time period we get from Numerical calculation is close to what we get from the analytical calculations i.e.; 1s

Plot:
log -log plot of epsilon vs T

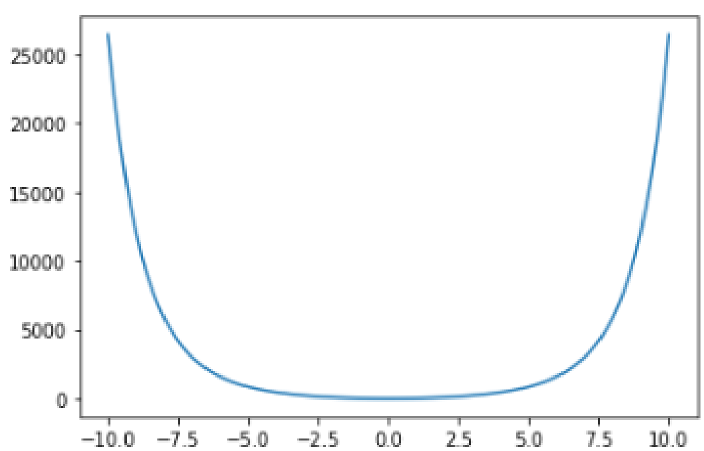


The x axis represent epsilon and the y axis represent T.

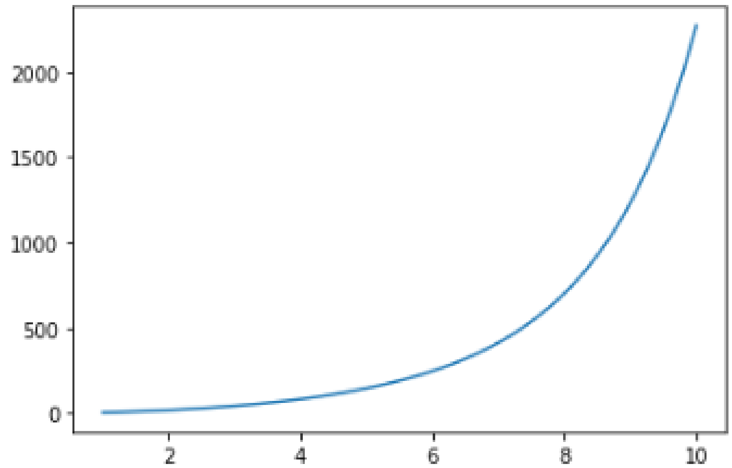
Part B:
Plot time period as a function of the amplitude of bound motion in the range [-10 to 10] meters with L = 5m. Observe the variation of T for small amplitude.
Solve for the Potential V(x)

$$V(x) = \frac{m{\omega_0}^2 L^2}{2}\left[\exp\left(\frac{x^2}{L^2}\right) - 1\right].$$

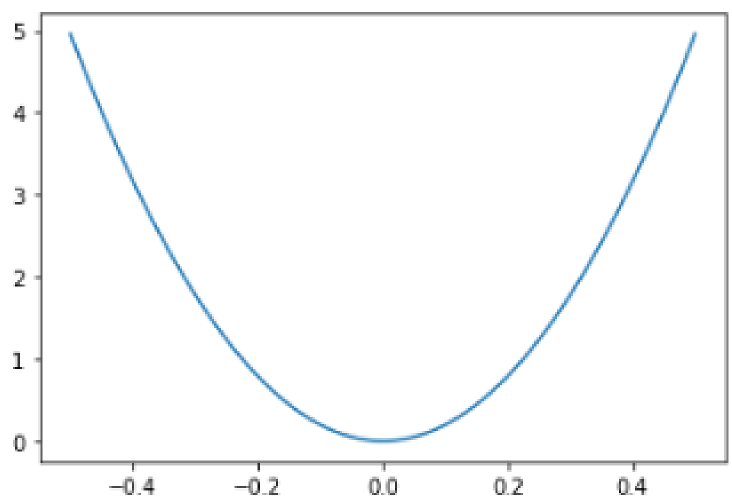
Sol:
The Plot of the V(x)
X vs V(x)



Plot (Amplitude vs Time period)



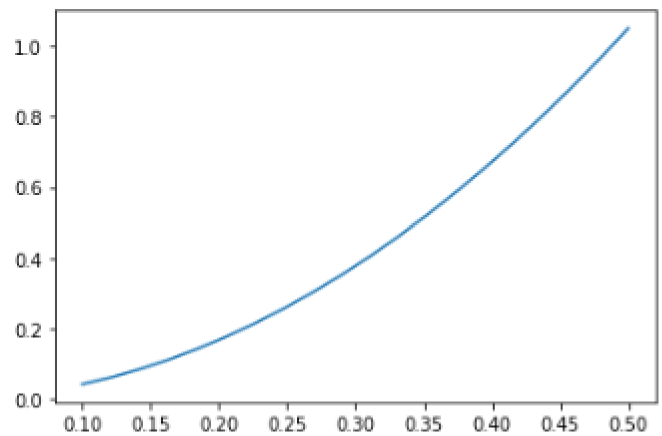
X-axis: Amplitude
Y-axis: Time period
Plot V(x) for small x:



This resembles the Simple Harmonic Oscillator at small amplitudes.

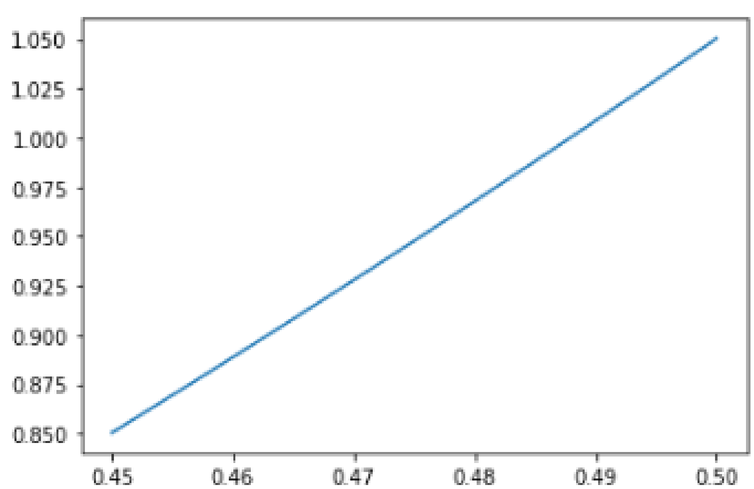
Plot Variation of T with small amplitudes:

Amplitude vs T:



X-axis : Amplitude
Y-axis : Time Period
We observe that as amplitude increases the Time period increases.

Zooming at 0.45 to 0.5:



We observe that at amplitude around 0.5, the time period of the Harmonic Oscillator is 1s and same as the previous case.