Q1)

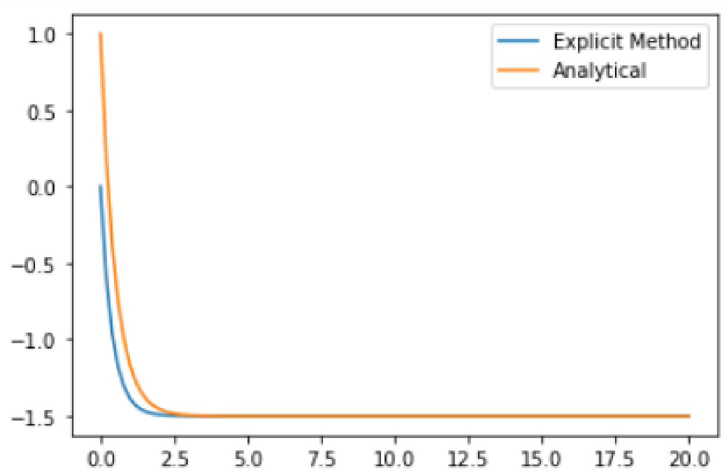
* **FMD 1:** Solve the 1st order ODE $y' + 2y + 3 = 0$ with both implicit and explicit Euler method with I.C. *y(0)=1* and compare with the analytical solution:

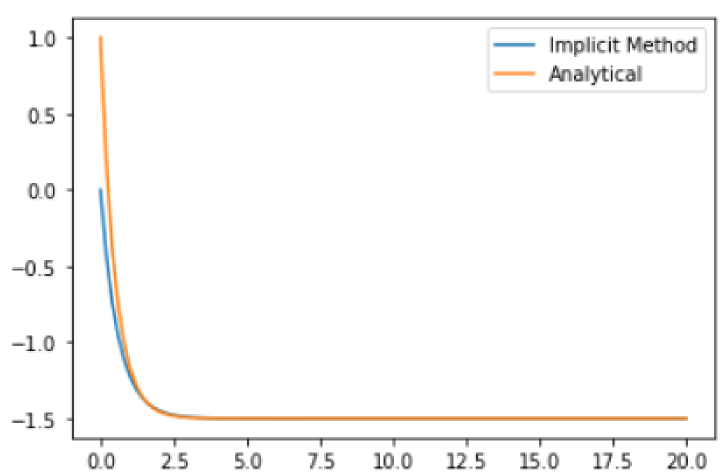
$y(t) = \frac{5}{2}e^{-2t} - \frac{3}{2}$.

By plotting analytical solutions for different values of Δt, show that the explicit method becomes unstable for Δt>0.5 whereas implicit method is unconditionally stable.

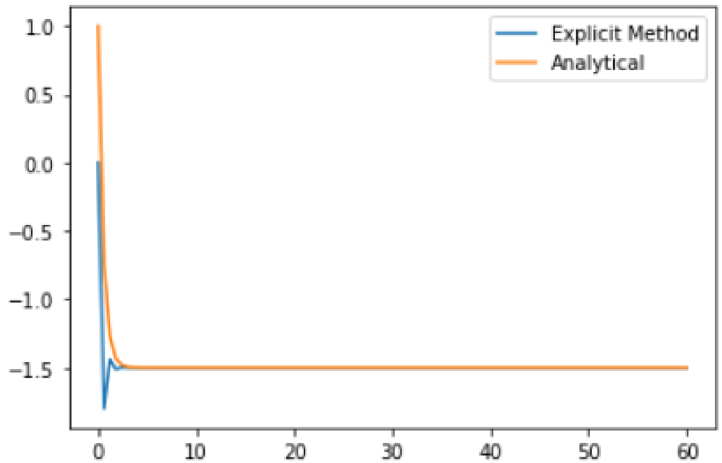
Sol:

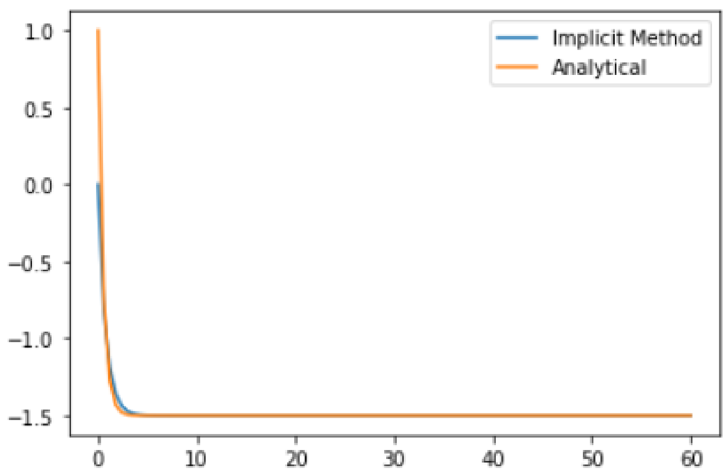
i) h = 0.2



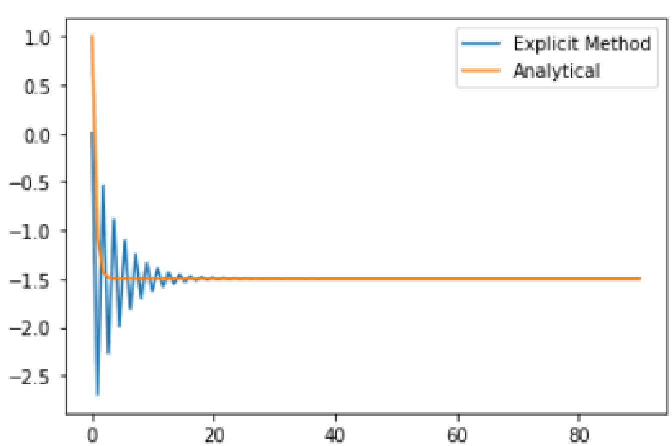


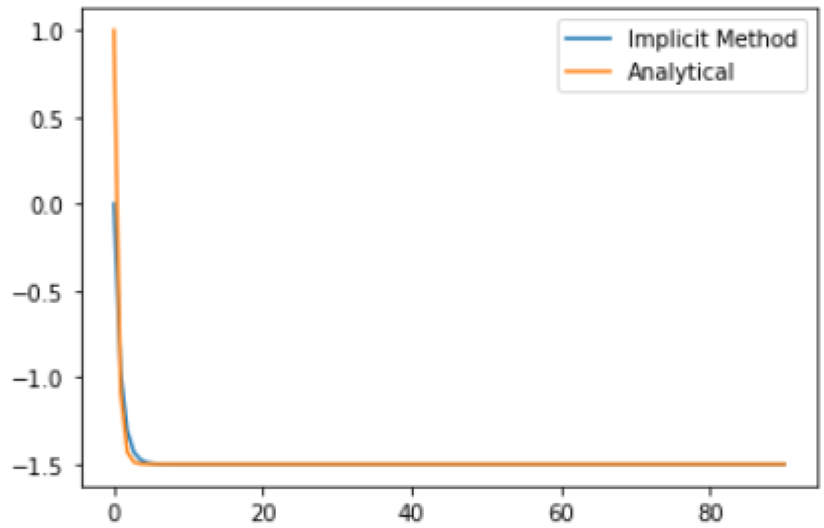
ii) h =0.6





iii) h =0.9





Q2)

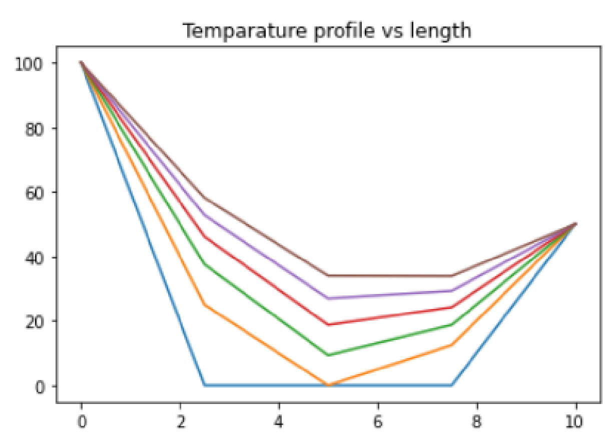
***Heat Equation 1:** Consider the system in which a thin rod of length L=10cm is placed between two heat reservoirs kept at 100 and 50°C, respectively. The initial temperature of the rod is 0°C. Write a code to compute heat evolution for 1st 100sec in the rod using **explicit method** of solving heat equation. Plot the result

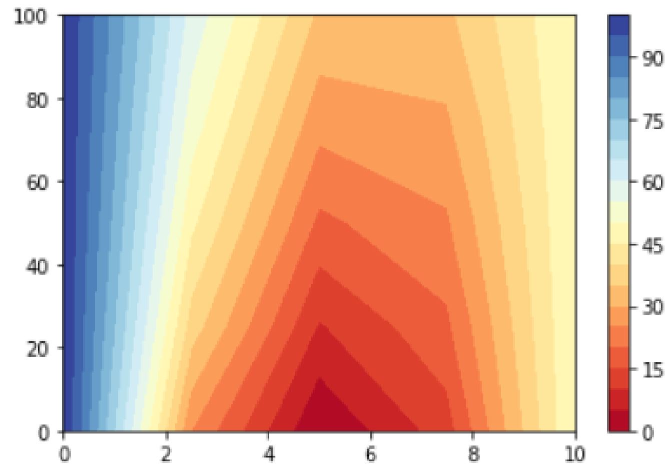
1) In a 2D contour or surface plot
2) And in animated version

for two values of σ (=κΔt/Δx^2) greater and less than 0.5.

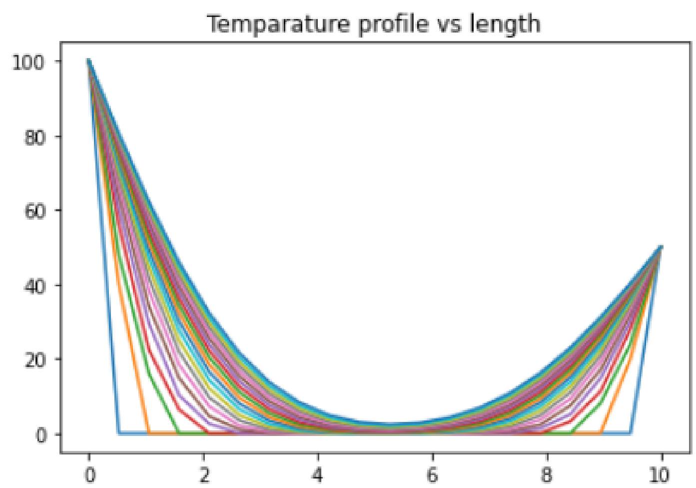
Sol:

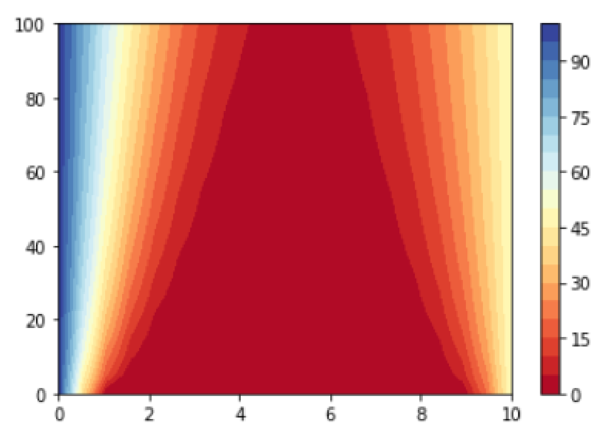
i) sigma = 0.25



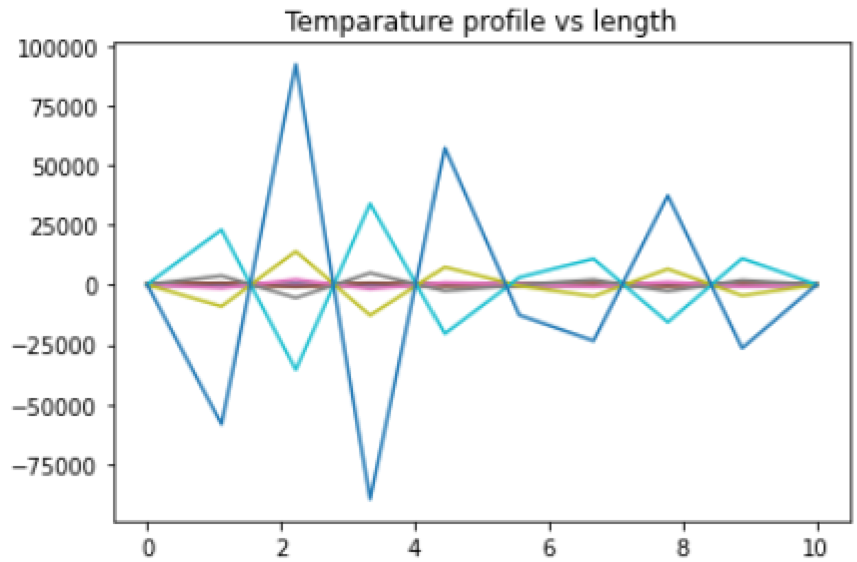


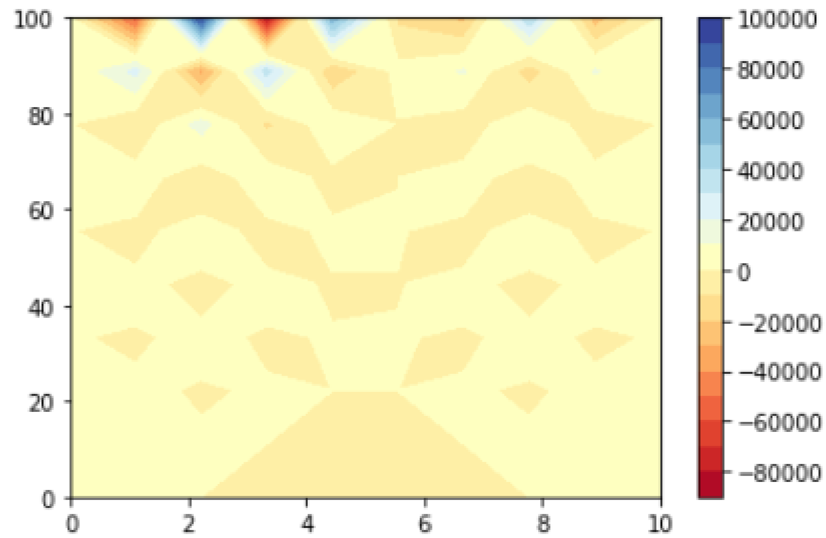
ii) sigma = 0.4





iii) sigma = 1





Q3

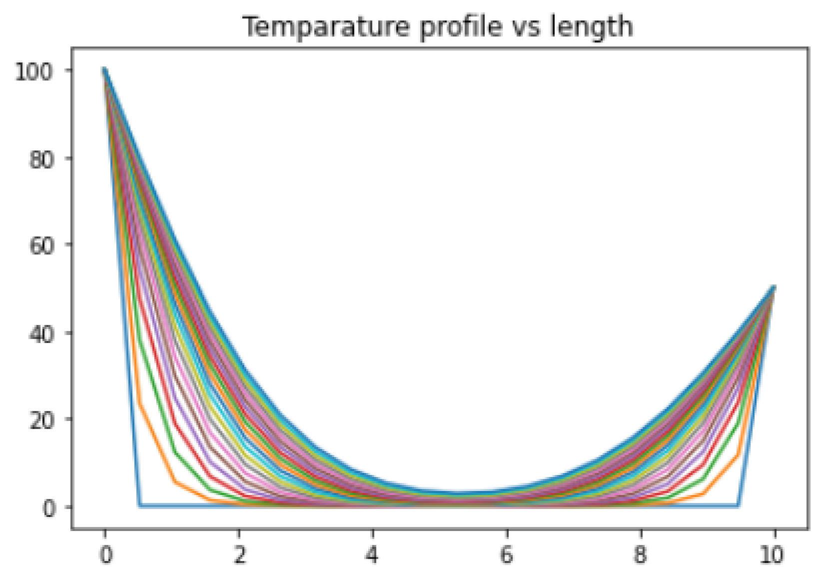
***Heat Equation 2:** Consider the system in which a thin rod of length L=10cm is placed between two heat reservoirs kept at 100 and 50ºC, respectively. The initial temperature of the rod is 0ºC. Write a code to compute heat evolution for 1st 100sec in the rod using **implicit method** of solving heat equation. Plot the result

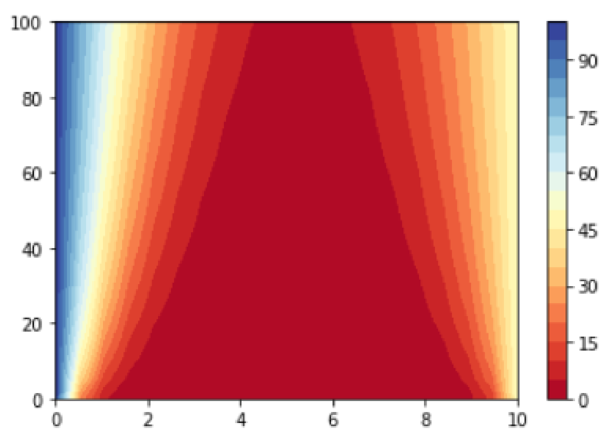
1) In a 2D contour or surface plot
2) And in animated version

for two values of σ (=$\kappa\Delta t/\Delta x$^2) greater and less than 0.5.

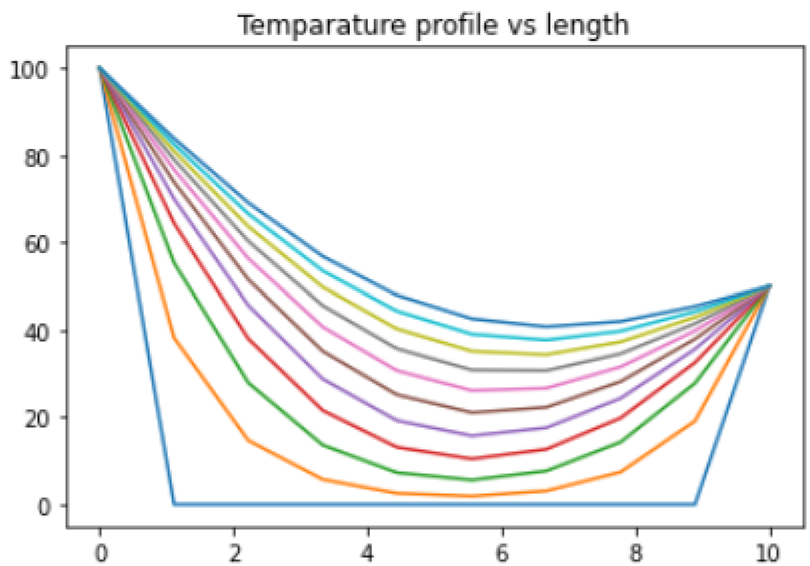
Q3)

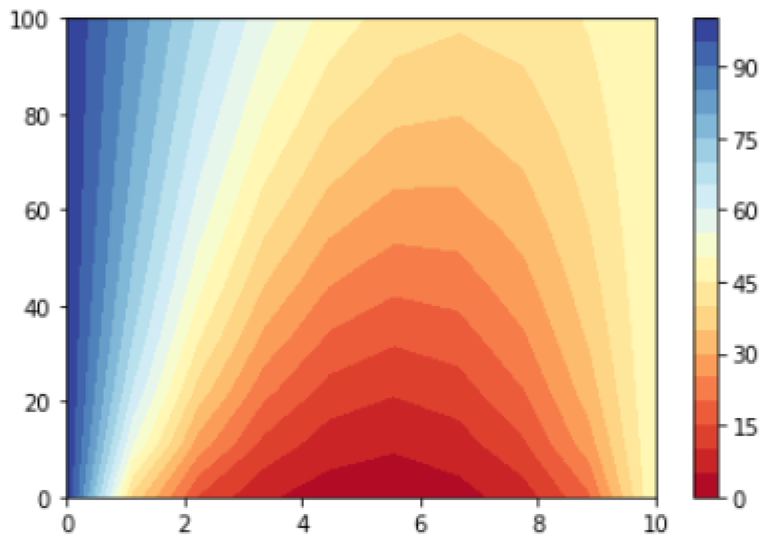
Sol: **Case 1: sigma =0.4**





Sigma = 1


Temparature profile vs length
100
80
60
40
20
0
0
2
4
6
8
10


100
80
60
40
20
0
0
2
4
6
8
10
90
75
60
45
30
15
0

Q4)

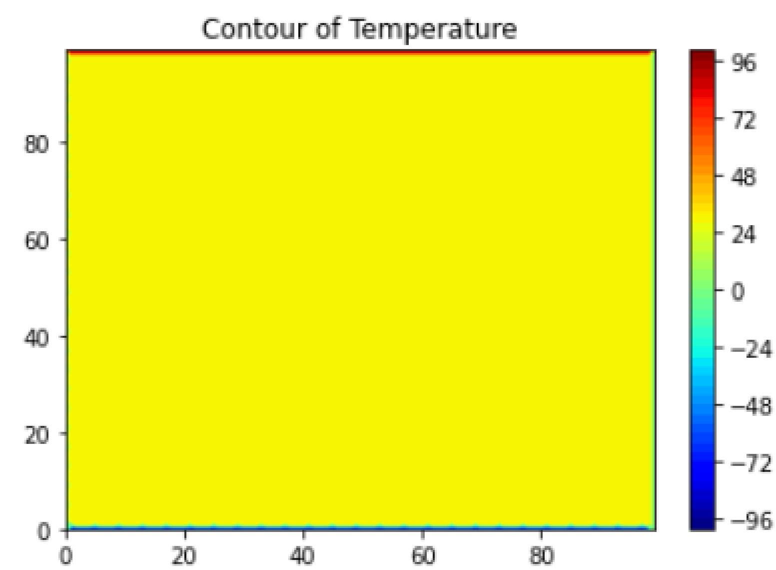
***Laplace equation:** Write a program to solve the two dimensional Laplace equation $T_{xx} + T_{yy} = 0$ describing the steady state temperature distribution on a **square plate of sides L=100 cm**. Use same length for x- and y- increment. Show the temperature distribution T(x, y) using a surface plot for an x-y grid of minimum 20×20 segments. Vary number of greed points to **fill the table below**.

The boundary conditions are:

$T(x = 0) = T(x = L) = 0°$ C, $T(y = 0) = -100°$, $T(y = L) = 100°$ at all time, corner points are assumed to have T equals to the average of adjoining sites.

Sol:

For delta = 2, 4, 5



For delta = 1

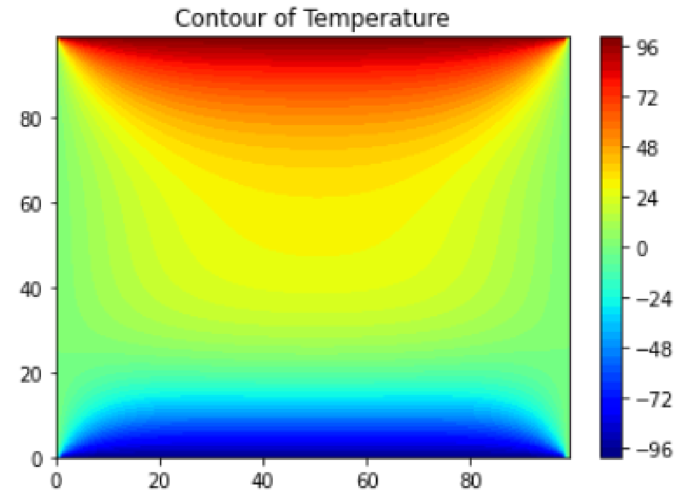


TABLE :

Slno	No. of grid Segments	No of iterations to converger	Total Calculations
1	20 X 20	1	400
2	25 X 25	1	625
3	50 X 50	1	2401
4	100 X 100	349	3351796