Tasks for 10/10/2022 and 17/10/2022:

(Problems marked with '*' needs to be attended during the class and later submitted as assignments. Other problems are practice problems.)

*Define a function *Eul* ($f(x,y)$, x_0, y_0, h, N) that will return the the value of y after N steps of size h. Use this to solve the two problems below:

***ODE1: a)** Solve the ODE $(1+x)\frac{dy}{dx}-2y+18x=0 \, with \, y(0)=4$ and interval h=0.05 in the interval (0, 3) (*i.e.* 60 steps) by using Euler's method. Plot the analytical solution ($y=-5x^2+8x+4$) and the numerical solution in the same figure to visually inspect the outcome.

b) Plot the absolute error at each point for Euler's method as a function of x.

ODE2: a) Solve the one-dimensional trajectory of a projectile fired from cannon located at the origin using the Euler method. Assume the initial projected speed is 700 m/s, and using different firing angles starting from 20 degrees to 60 degrees with an interval of 5 degrees. Neglect the effects of the air resistance. Plot the trajectories of the projectile for different firing angles. Also plot the range of the projectile against the firing angles and show numerically that the maximum range of the projectile corresponds to a firing angle of 45 degrees.

b) Plot the numerical results with the analytical solutions for the range and the duration of the projectile.

***Coupled ODE 1:** While solving a set of M coupled ODEs the function '*Eul*' may be extended to return an array $Y= Eul\,(F(x,y), x_0, y_0, h, N,M\,)$, where Y is the M-dimensional array of solutions and F is a M-dimensional array of RHS functions for coupled ODEs. Using this modified function find the phase space trajectory of a particle of *unit mass* in a potential $V(x)$. You need to solve the Hamilton's equations of the system, given by:

$$\frac{dp}{dt}=\frac{-\partial H}{\partial x}$$

$$\frac{dx}{dt}=\frac{\partial H}{\partial p}$$

(a.) Start by taking simple harmonic oscillator potential $V(x)=\frac{1}{2}k\,x^2$, k=1 N/m.

What is the period T of this oscillator? Use h=T/S with S=10, 100 and 1000 for the step size and follow the trajectory over a time 10 T considering the initial condition (x, p_x) = (1.0, 0.0). Fro all the cases, compare the following

(i) Show the trajectory i.e. x and p as functions of time and also the phase space . Compare these with the analytical solutions.

(ii) The energy E is expected to be conserved. Plot the relative change in energy dE/E=(E_n-E)/E as a function of time, here E_n and E are respectively the numerically calculated energy and the actual energy respectively.

b) Now consider a double well potential $V(x)=(x^2-1)^2$. Start by plotting this potential for x = [-2, 2]. Considering phase space, plot the contours corresponding to different values of energy E. Analyse these to discuss the various kinds of trajectories possible in this potential.

For this new potential, repeat the same exercises as in **(a)** for the initial conditions (x, p_x) = (1.0, 0.1), (-1.0, 0.1) and (1.0, 10.0).

***R-K 2nd order:** Define a function 'RK2 =($f(x,y)$, x_0, y_0, h, N)' which will return the solution of a ODE of the form $\frac{dy}{dx}=f(x,y)$ for N steps of size h using 2nd order Runge-Kutta (R-K) method.

Solve the ODE $(1+x)\frac{dy}{dx}-2y+18x=0 \, with \, y(0)=4$ and increment h=0.05 in the interval (0, 3) by using function RK2. Compare this with the analytical solution and that from Euler's method. Compare the absolute error in Energy with the results from the Euler's method.

***Coupled ODE 2:** Extend the function 'RK2' to generalize it for M coupled ODEs as Y= *RK2* ($F(x,y)$, x_0, y_0, h, N,M), where Y is the array of solutions and F is a M-dimensional array of RHS functions for coupled ODEs. Solve the following coupled ODE by R-K 2nd order method for x = [0.0 0.5] with h=0.05. I.C. are $y(0)=0$, $z(0)=1$.

$$\frac{dy}{dx}=-x-yz$$

$$\frac{dz}{dx}=-y-xz$$

Plot the solutions $y(x)$ and $z(x)$.

Coupled ODE 2: Solve the following 2nd order ODE

$$\frac{d^2y}{dx^2}+0.5\frac{dy}{dx}+4y=5$$

Initial condition: $y(0)=y'(0)=0$;

by applying of R-K 2nd order method for x-range of $0 \le x \le 5$ and h=0.1. Plot y(x) alongside the analytical solution.

***Coupled ODE 4:** Write a program to follow the motion of an electron (*e*) in an electric field *E(x, t)* and a magnetic field *B(x, t)*. Numerically determine the trajectory of an electron for 1 micro second with 1 nano second of time resolution by solving Lorentz force equation:

$$m\frac{d\vec{v}}{dt}=q(\vec{E}+\vec{v}\vec{B}).$$

Assume that the particle starts at the origin with velocity v = (1.0, 1.0, 1.0) m/sec for the following field configurations:

(i) Uniform magnetic field 10^{-4} Tesla along the z-axis.

(ii) Uniform magnetic field 10^{-4} Tesla along the z-axis and a uniform electric field 1V/m along the y-axis.

Visualize these trajectories by plotting different two dimensional sections (x-y) , (x,z) etc.

How will you ensure that your step size is small enough? You can possibly check conservation of energy i.e. Kinetic + Potential. Show the accuracy to which your trajectory conserves energy as the particle evolves.

[Use parameters q=-1.6$\times 10^{-19}$C, m_e=9.11$\times 10^{-31}$ Kg]

[Hint: Write down the Lorentz force equation in component form. That will give three coupled equations in velocity components. Position equations are directly solvable from obtained velocity values.]