<u>Tasks for 22/08/2022:</u>

(Problems marked with '*' needs to be attended during the class and later submitted as assignments. Other problems are practice problems.)

Evaluation of *Sin(x)* within a given error limit by adding up the series: Use power series expansion in order to evaluate *Sin(x)* for a given *x*. Truncate the series when the value reaches within the accuracy of allowed error *e* (User defined error limit) set by you.

Computer arithmetic 2: It is desired to calculate all integral powers of the number $x = (\sqrt{5} - 1)/2$.
It turns out that the integral powers of *x* satisfy a recursive relation:

$$x^{n+1} = x^{n-1} - x^n$$

Show that the above recurrence relation is unstable by calculating x^{14}, x^{30}, x^{40} and x^{50} from the recurrence relation and comparing with the actual values obtained by using inbuilt function (e.g., 'pow' in python).

***Computer arithmetic 3:** Consider the logistic map: $x_{n+1} = Ax_n(1 - x_n)$, where, x_n is the n^{th} iteration of *x* for a starting value of $0 \leq x \leq 1$. Here A is a constant.

(a) Write a code to generate the logistic map. Start by varying the value of *A* to observe the following:

- With *A* between 0 and 1, the value of x_n will eventually go to zero, independent of the initial value of *x*.
- With *A* between 1 and 2, the population will quickly approach the value $\frac{A-1}{A}$, independent of the initial *x* value.
- With *A* between 2 and 3, x_n will also eventually approach the value $\frac{A-1}{A}$, but first will fluctuate around that value for some time.
- With *A* between 3 and $1+\sqrt{6}=3.449$, from almost all initial conditions x_n will approach permanent oscillations between two values.

Plot x_n *vs.* *n* for all this condition for a value of $n>50$.

(Work out upto this point in the class. Rest can be taken as homework)

(b) For an initial value $x=0.3$, vary the value of A from 0.5 to 3.99 in total 250 steps. For each value of A, note the values of x_n for $n=150$, then make a plot of A *vs.* x_n and see the bifurcation and chaos. Now change the initial value of x. Do you see any change in the plot?

(c) For $A = 3.0$, choose two points x and x' close to 1 where, $x' = x + 0.01$ and iterate. Plot $log(|x_n - x'_n| / 0.01)$ as a function of n. See if it is approaching a straight line for high n. Check for other values around 3, and you will see error dropping to a platue and eventually to 0 soon enough. This happens because at A=3, near the bifurcation memory, the systems approaches equilibrium in a dramatically slow manner.

***Root finding 1:** How many real roots does the polynomial $f(x) = 2x^3 - 5$ has? Find the roots using the method of bisection.