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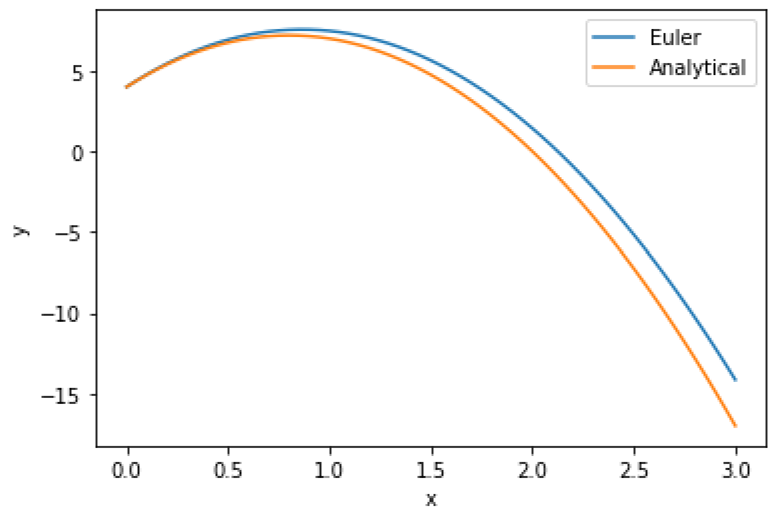
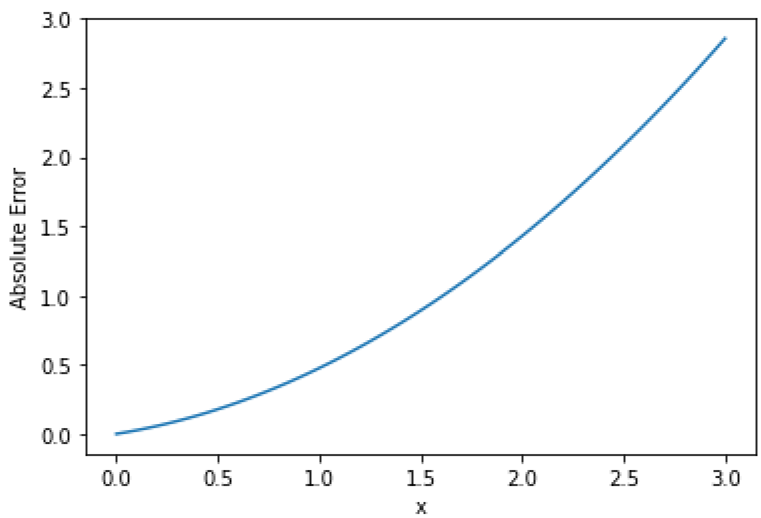
1) Define a function Eul (f(x,y), x0, y0, h, N) that will return the the value of y after N steps of size h. Use this to solve the two problems below:
ODE1: a) Solve the ODE (1+x)dy/ dx −2 y+18 x=0with y (0)=4 and interval h=0.05 in the interval (0, 3) (i.e. 60 steps) by using Euler's method. Plot the analytical solution (y=−5x2+8 x+4) and the numerical solution in the same figure to visually inspect the outcome.
b) Plot the absolute error at each point for Euler's method as a function of x.

Sol:



Plot of Analytical and solution obtained from Euler Method.

Plot of error in Euler Method:



Q2)*Coupled ODE 1: While solving a set of M coupled ODEs the function 'Eul' may be extended to return an array Y= Eul (F(x,y), x0, y0, h, N,M), where Y is the M-dimensional array of solutions and F is a M-dimensional array of RHS functions for coupled ODEs. Using this modified function find the phase space trajectory of a particle of unit mass in a potential V(x). You need to solve the Hamilton's equations of the system, given by:

$dp/dt =-\partial H/\partial x$

$dx/dt =\partial H/\partial p$

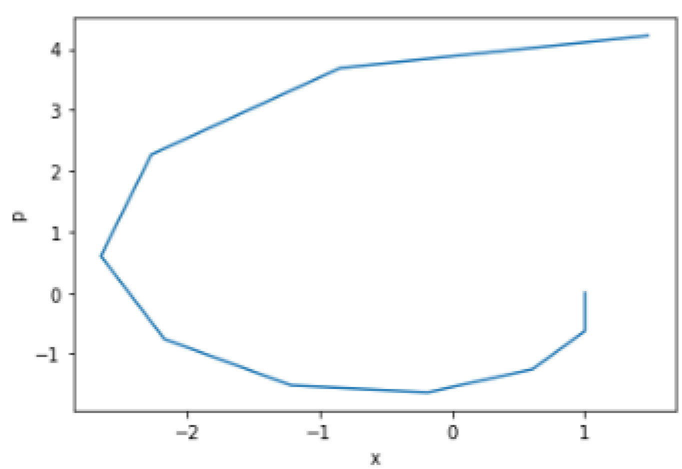
(a.) Start by taking simple harmonic oscillator potential $V (x)=1/2k x2$, k=1 N/m.

What is the period T of this oscillator? Use h=T/S with S=10, 100 and 1000 for the step size and follow the trajectory over a time 10 T considering the initial condition (x, px) = (1.0, 0.0). Fro all the cases, compare the following

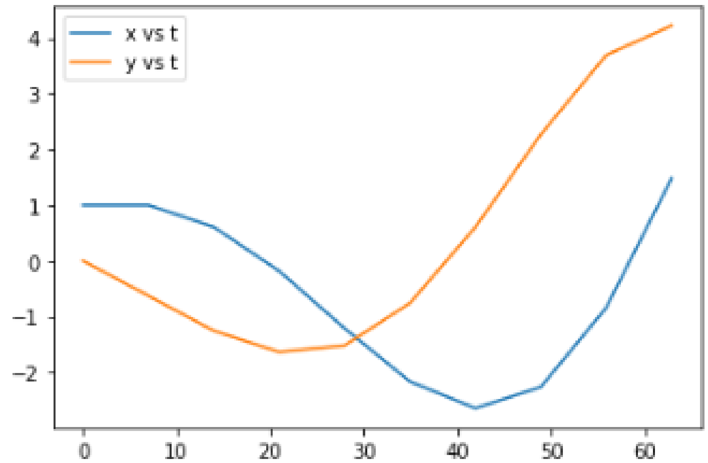
(i) Show the trajectory i.e. x and p as functions of time and also the phase space . Compare these with the analytical solutions.

(ii) The energy E is expected to be conserved. Plot the relative change in energy dE/E=(E_n-E)/E as a function of time, here E_n and E are respectively the numerically calculated energy and the actual energy respectively.

a)For the case S = 10

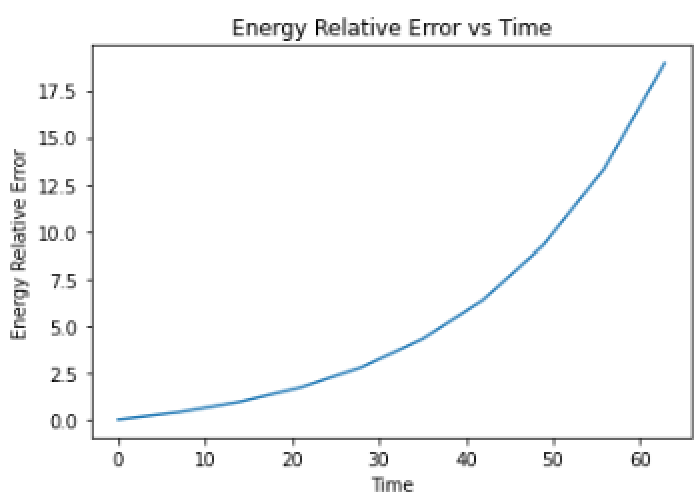


Phase Space Plot



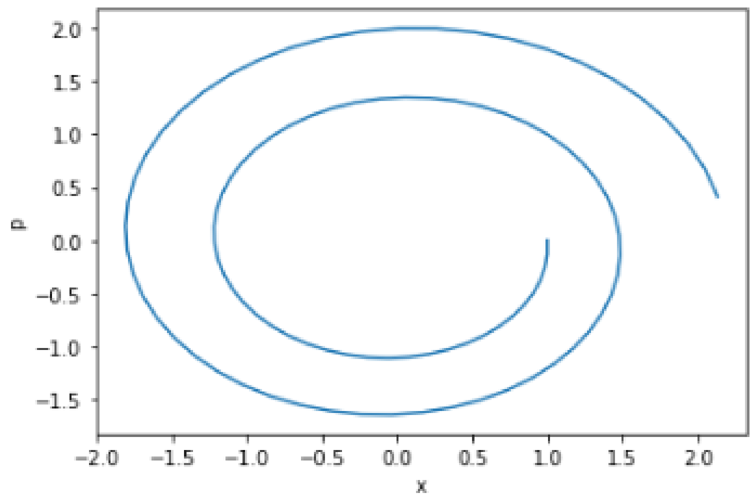
Here x is the position i.e; t vs x(t)
And y is the momentum p i.e; t vs p(t)

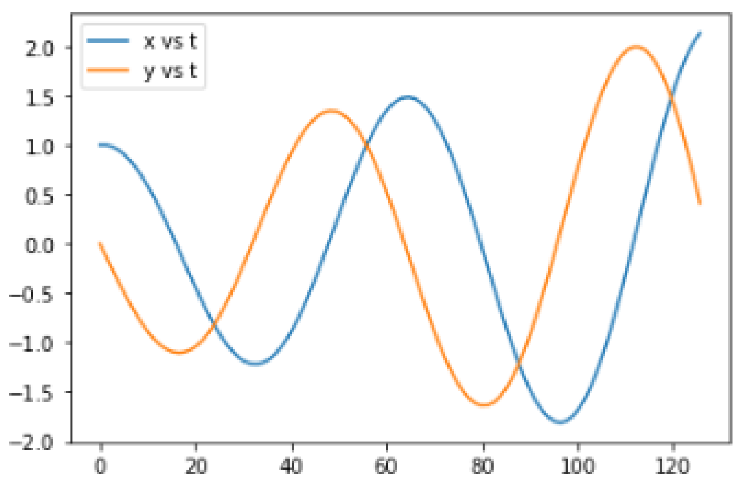
Error Plot:



We observe that as the number of the steps in the interval is just 10 which is small, the error in the Energy rapidly increases and the contour shows a spiral moving inwards in phase space.

b) For case S=100

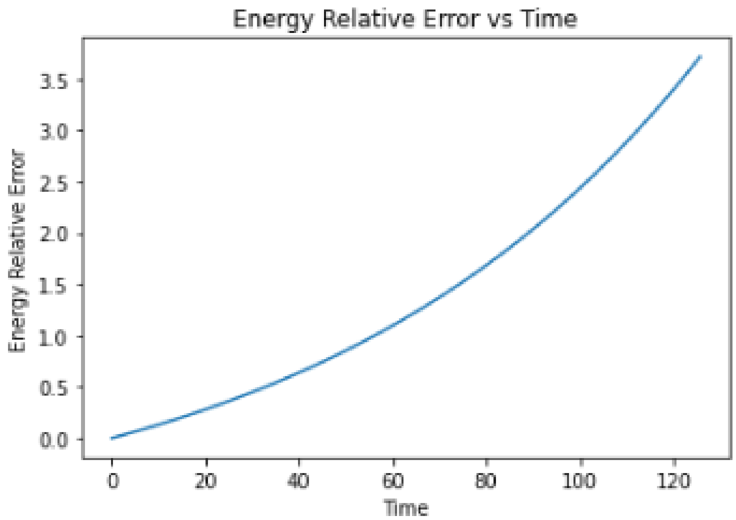




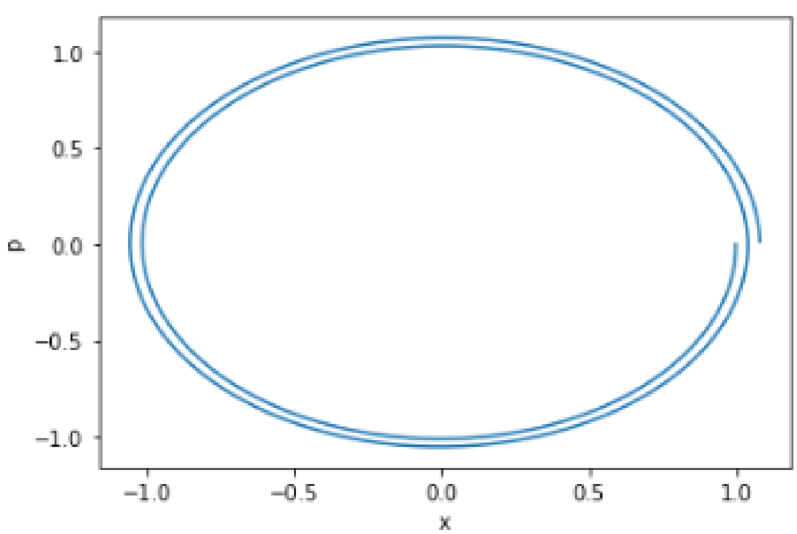
t vs x(t)   and   t vs p(t)

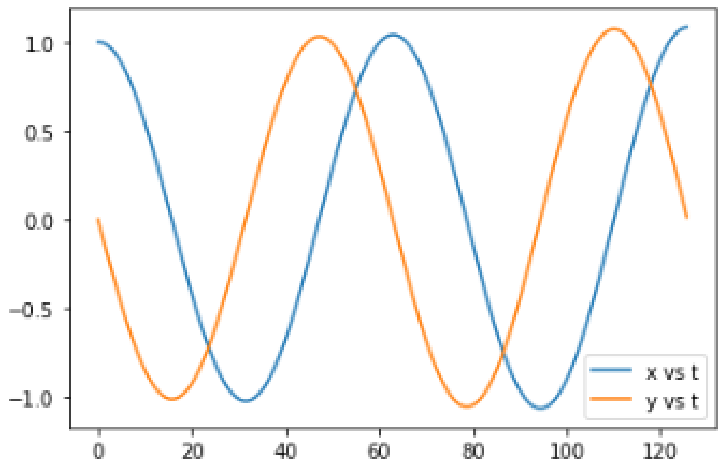
We can observe from the x(t) and y(t) plots , their amplitudes get increased. And the contour in this case is still a spiral but has an elliptical form which is different from the previous case.

Energy Relative Error vs Time:

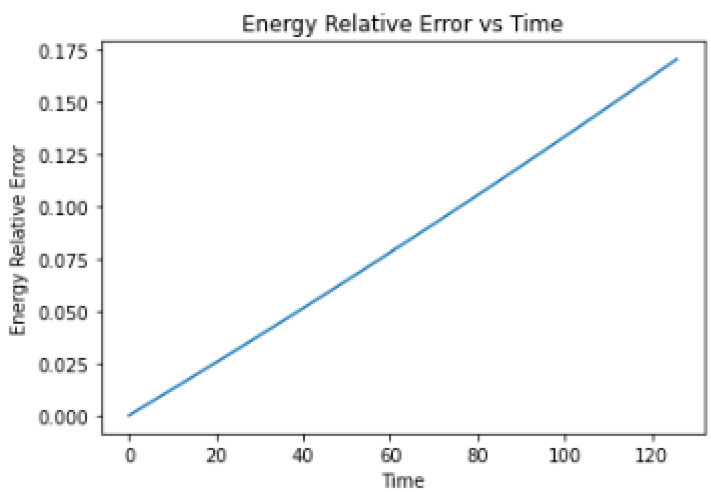


c) S= 1000



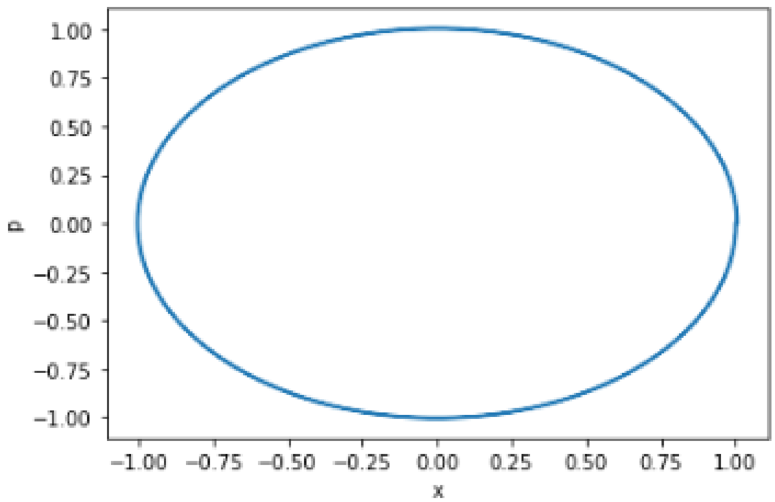


Energy Relative error vs time :

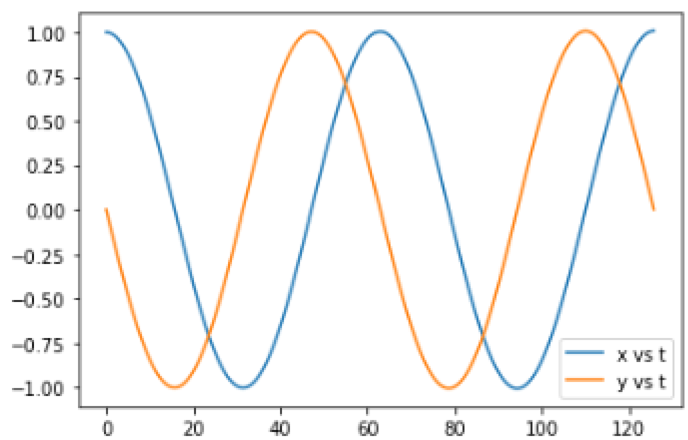


The increase in error is Linear and very less when compared to the previous cases.

S= 10000



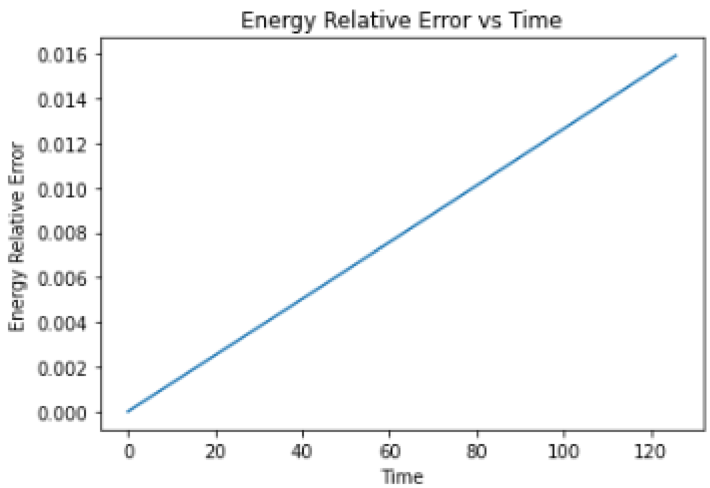
The Contour is almost always remained Elleptical .



The x(t) and p(t) values are oscillating values which is due to the fact of conservation of Energy in the system.

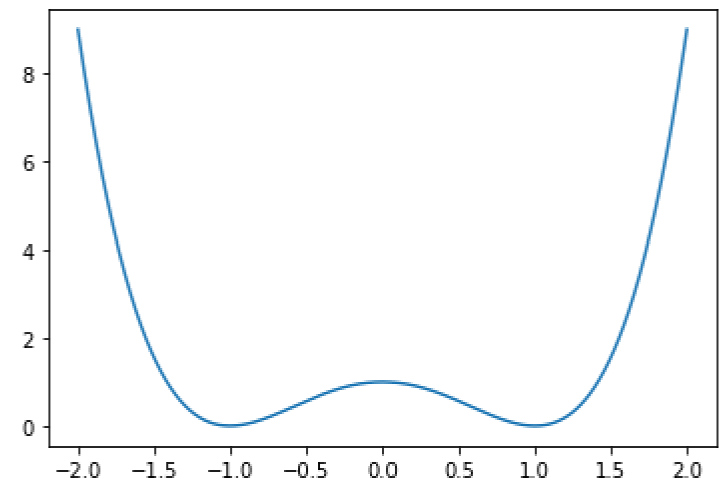
Energy Error Plot vs Time:

The error range is much small when compared to the previous cases.

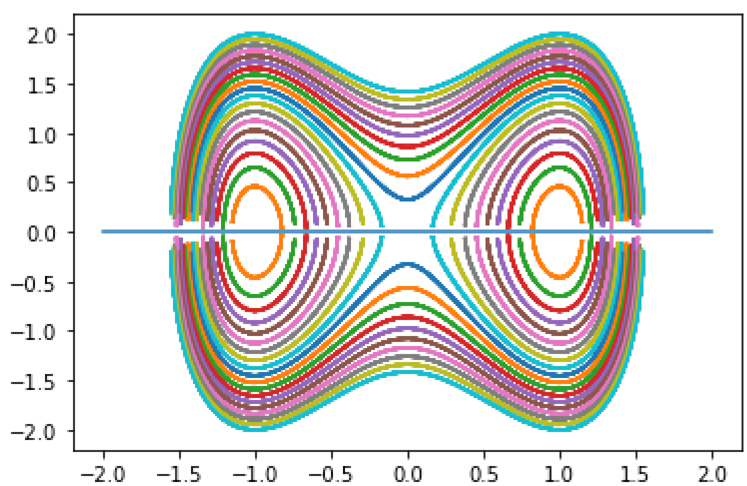


b) Now consider a double well potential V (x)=(x2−1)2. Start by plotting this potential for x = [-2, 2]. Considering phase space, plot the contours corresponding to different values of energy E. Analyse these to discuss the various kinds of trajectories possible in this potential.
For this new potential, repeat the same exercises as in (a) for the initial conditions (x, px) = (1.0, 0.1), (-1.0, 0.1) and (1.0, 10.0).

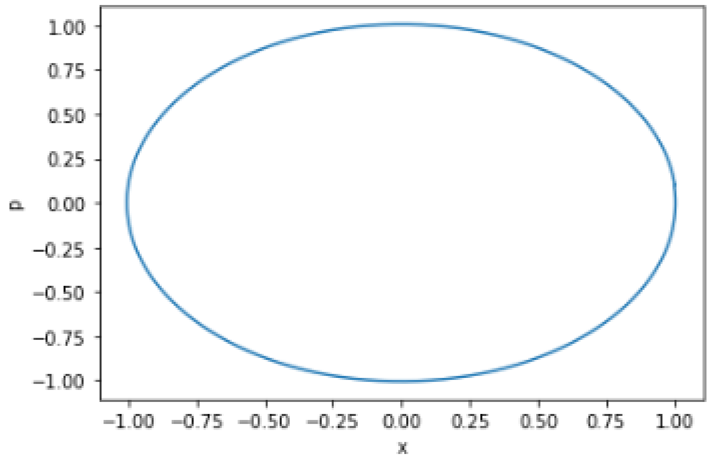
Sol: PLot x vs V(x)

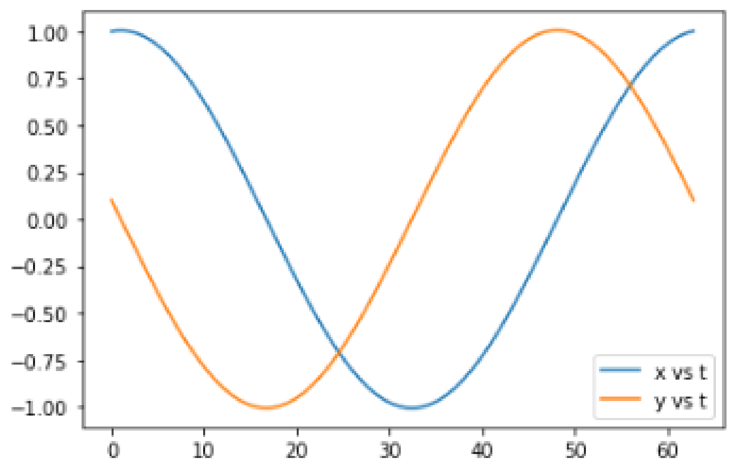


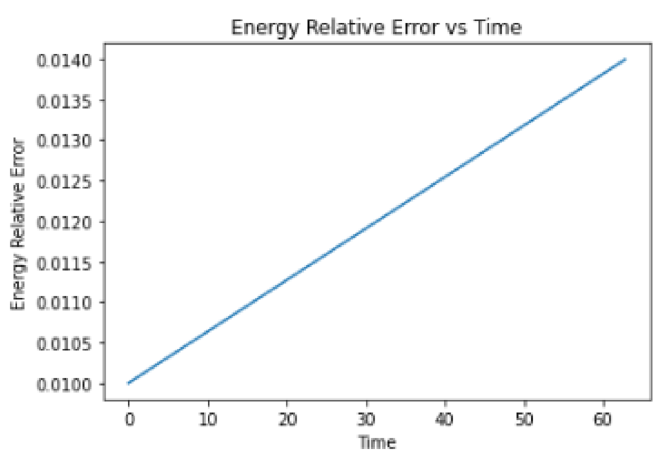
Contours in Phase space for different Energies:



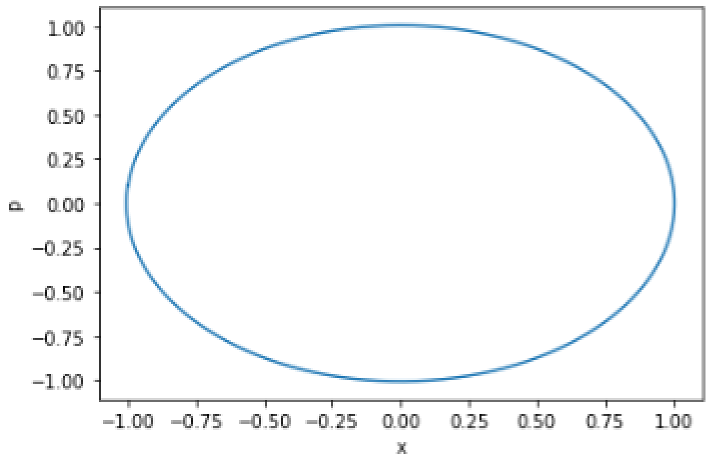
Case 1 :
X, p = 1, 0.1


1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
p
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x

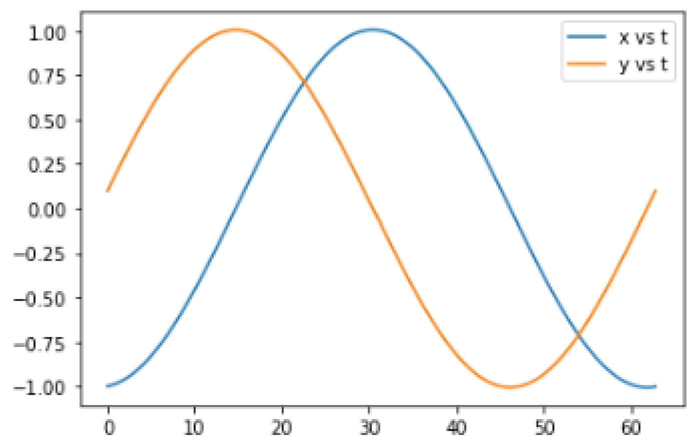

1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
10
20
30
40
50
60
x vs t
y vs t


Energy Relative Error vs Time
0.0140
0.0135
0.0130
0.0125
0.0120
0.0115
0.0110
0.0105
0.0100
Energy Relative Error
0
10
20
30
40
50
60
Time

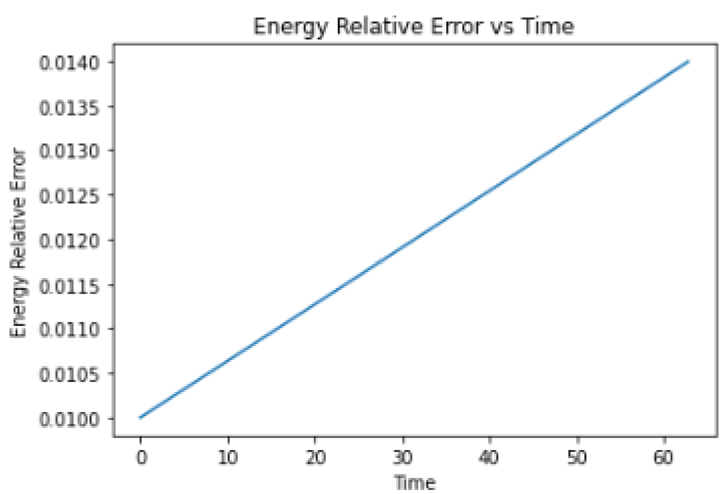
Case 2 : X, p = -1, 0.1



1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
p
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
x

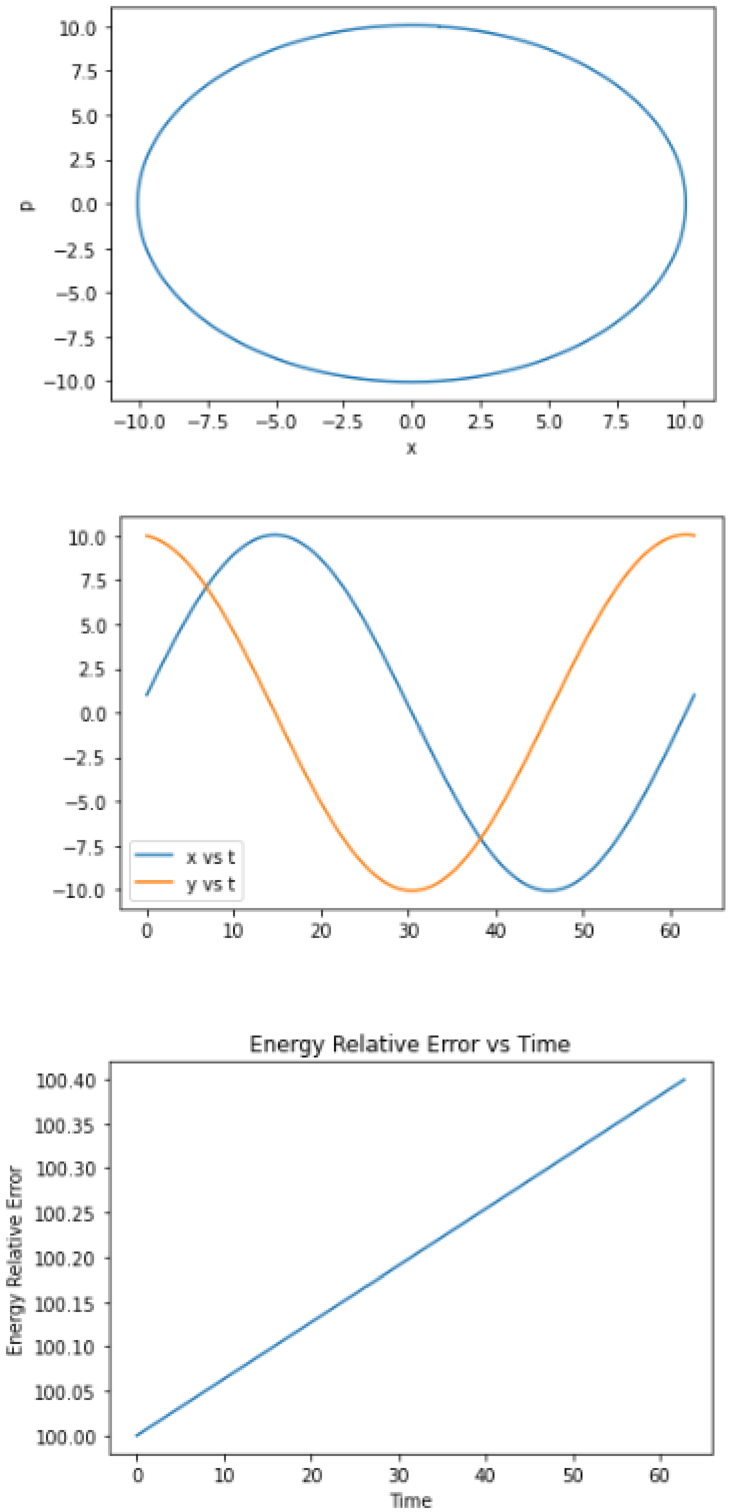


x vs t
y vs t
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0 10 20 30 40 50 60



Energy Relative Error vs Time
Energy Relative Error
0.0140
0.0135
0.0130
0.0125
0.0120
0.0115
0.0110
0.0105
0.0100
0 10 20 30 40 50 60
Time

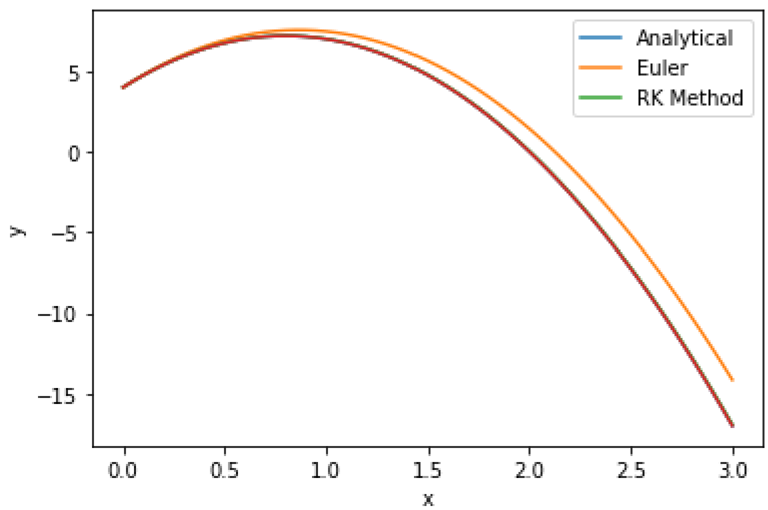
Case 3 : X, P = 1, 10


p
x
x vs t
y vs t
Energy Relative Error vs Time
Energy Relative Error
Time

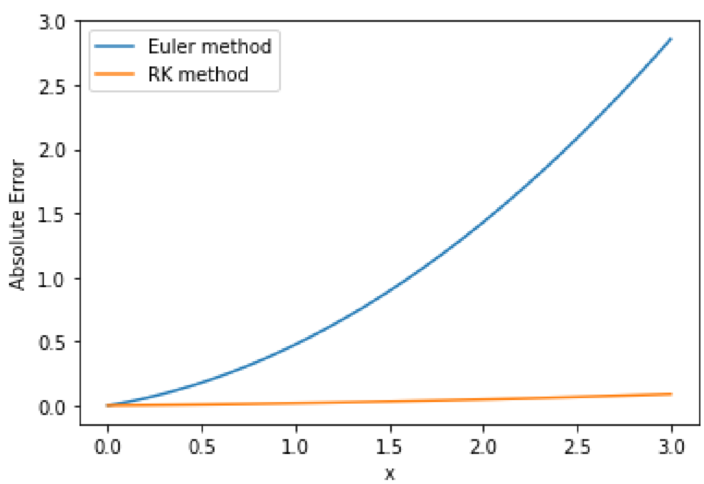
3)*R-K 2nd order: Define a function 'RK2 =(f(x,y), x0, y0, h, N)' which will return the solution of a ODE of the form dy/dx =f (x , y) for N steps of size h using 2nd order Runge-Kutta (R-K) method.
Solve the ODE (1+x)dy/dx −2 y+18 x=0with y (0)=4 and increment h=0.05 in the interval (0, 3) by using function RK2. Compare this with the analytical solution and that from Euler's method. Compare the absolute error in Energy with the results from the Euler's method.

Sol:

The following is the comparison of Analytical , Euler , Range Kutta Method. Solution of RK Method is Same as Analytical Solution.



Absolute errors in Both Methods:



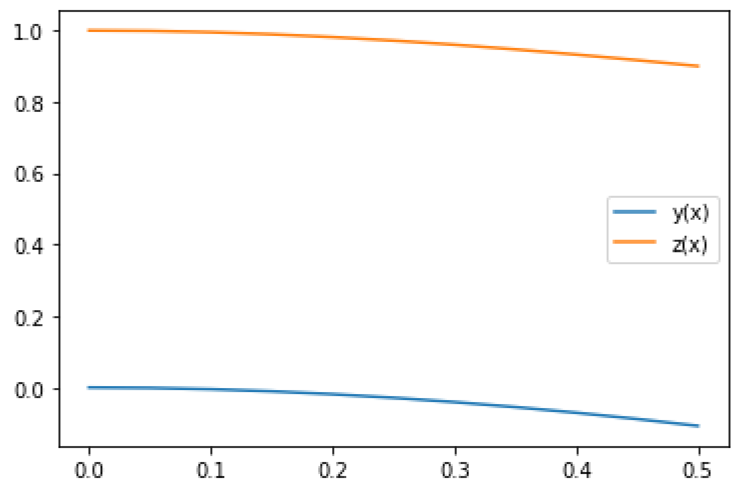
4) *Coupled ODE 2: Extend the function ‘RK2’ to generalize it for M coupled ODEs as Y= RK2 (F(x,y), x0, y0, h, N,M), where Y is the array of solutions and F is a M-dimensional array of RHS functions for coupled ODEs. Solve the following coupled ODE by R-K 2nd order method for x = [0.0 0.5] with h=0.05. I.C. are y(0)=0, z(0)=1.

dy/dx =−x−yz

dz/dx =−y−xz

Plot the solutions y(x) and z(x).

Sol:



5) *Coupled ODE 4: Write a program to follow the motion of an electron (e) in an electric field E(x, t) and a magnetic field B(x, t). Numerically determine the trajectory of an electron for 1 micro second with 1 nano second of time resolution by solving Lorentz force equation:

$$m d\vec{v}/dt = q(\vec{E}+\vec{v}\vec{B}).$$

Assume that the particle starts at the origin with velocity v = (1.0, 1.0, 1.0) m/sec for the following field configurations:

(i) Uniform magnetic field 10-4 Tesla along the z-axis.

(ii) Uniform magnetic field 10-4 Tesla along the z-axis and a uniform electric field 1V/m along the y-axis.

Visualize these trajectories by plotting different two dimensional sections (x-y) , (x,z) etc.

How will you ensure that your step size is small enough? You can possibly check conservation of energy i.e. Kinetic + Potential. Show the accuracy to which your trajectory conserves energy as the particle evolves.

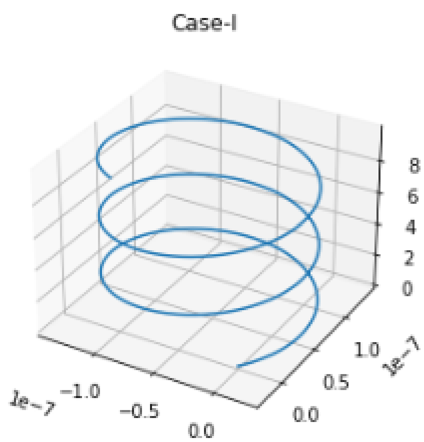
[Use parameters q=-1.610-19C, me=9.1110-31 Kg]

Sol: Case 1 : For t = 1 micro Second

B=[0, 0, 10**(-4)]

E=[0, 0, 0]

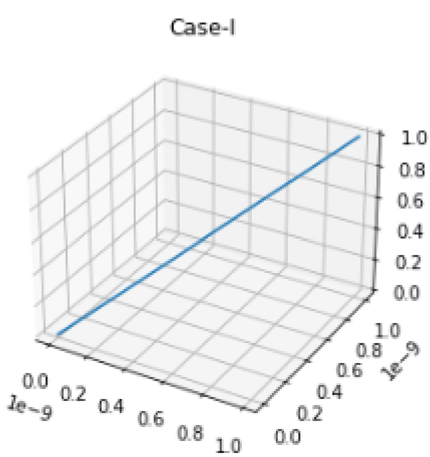
Path is given by :



Its a Helix path

For t = 1 nano second

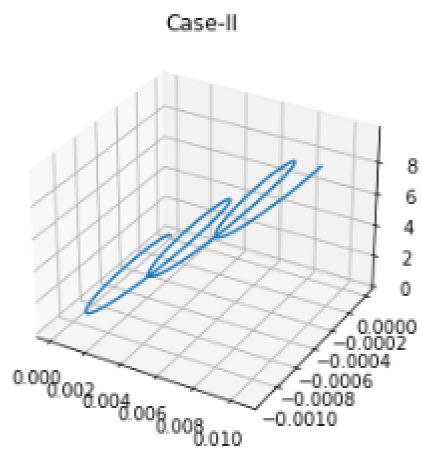
Path is given by :



Case 2 :

T = 1 micro second
B=[0, 0, 10**(-4)]
E=[0, 1, 0]

Path is given by:



Its a Cycloid Path

T = 1 nano second :

The path is given by :