<u>Tasks for 12/09/2022 and 19/09/2022:</u>

(Problems marked with '*' needs to be attended during the class and later submitted as assignments. Other problems are practice problems.)

Polynomial interpolation 1: Given the three data points $(x, y) = (1.0, 8.0), (2.1, 20.6)$ and $(5.0, 13.7)$, write a program to return the value of y for any arbitrary x in the range [1.0, 5.0] using **second order polynomial**. Use Lagrange method of interpolation to construct the polynomial. **Plot the polynomial** along with the data points.

***Numerical integration 1:** Plot and integrate the tabulated data given below by a suitable method. Take the end points of the table as integration limits.

x	1.34	1.46	1.52	1.6	1.87	2.03	2.18	2.8	3.2	3.8	4.15
$f(x)$	1.5	2.3	2.3	2.4	2.5	3.2	4.9	4.7	3.4	7.8	17.1

***Numerical integration 2:** Write a function in python *trap* (f, a, b, N) which will evaluate the integral $\int_a^b f(x)dx$ using the trapezoidal rule with N steps. Also develop a similar function *simp* (f, a, b, N) for the Simpson's rule. Use these functions to evaluate the numerical integrals in the subsequent problems, wherever applicable.

Use these functions to evaluate the following integrals in limits a to b for $[a,b]$ interval divided into N $(=2^n, n=1, 2, 3, ...10)$ **intervals** with:

1) trapezoidal rules
2) Simpson's rule

$f(x)$	x^2	$sin(x)$	$\left(\frac{Sinx}{x}\right)^2$
a	-1	0	0
b	1	π	∞
Analytical value	2/3	2	$\pi/2$

Let y_N be the numerical result for N number of intervals used. Let y_{AN} be the analytic result. We define the relative error as

$$e(N) = |\frac{y_{N} - y_{AN}}{y_{AN}}|$$

where the error depends on *N*. Show plot of *log e(N)* with varying *log N* for all the integrals. Are these results keeping with how you expect the error to scale with *N*?

Numerical integration 3:

a) Evaluate the following integrals

f(x)	$x^6 - 7x^3 + 5$	$x^2 e^{x-1}$	$\frac{sinx}{\sqrt{x}}$
A	-1	0	0
B	1	5	1

by using 1) Trapezoidal rule with 100 points

2) Simpson's rule for with 51 point

3) Gauss quadrature method for 6 points (Please see the table below)

And compare the values in a table.

Table for Gauss Quadrature method

n = 2

i	**weight - w_i**	**abscissa - x_i**
1	1.0000000000000000	-0.5773502691896257
2	1.0000000000000000	0.5773502691896257

n = 3

i	**weight - w_i**	**abscissa - x_i**
1	0.8888888888888888	0.0000000000000000
2	0.5555555555555556	-0.7745966692414834
3	0.5555555555555556	0.7745966692414834

n = 4

i	**weight - w_i**	**abscissa - x_i**
1	0.6521451548625461	-0.3399810435848563
2	0.6521451548625461	0.3399810435848563
3	0.3478548451374538	-0.8611363115940526

i	weight - w_i	abscissa - x_i
4	0.3478548451374538	0.8611363115940526

n = 5

i	weight - w_i	abscissa - x_i
1	0.5688888888888889	0.0000000000000000
2	0.4786286704993665	-0.5384693101056831
3	0.4786286704993665	0.5384693101056831
4	0.2369268850561891	-0.9061798459386640
5	0.2369268850561891	0.9061798459386640

n = 6

i	weight - w_i	abscissa - x_i
1	0.3607615730481386	0.6612093864662645
2	0.3607615730481386	-0.6612093864662645
3	0.4679139345726910	-0.2386191860831969
4	0.4679139345726910	0.2386191860831969
5	0.1713244923791704	-0.9324695142031521
6	0.1713244923791704	0.9324695142031521

***Numerical integration 4:** We consider the bound 1-D motion of a particle of mass m in a time independent potential $V(x)$. The fact that the energy E will be conserved allows us to integrate the equation of motion and obtain a solution in closed form. The time period of the oscillation T is given by:

$$T = \int_a^b \frac{\sqrt{2m}}{\sqrt{E - V(x)}} dx$$

Where the limits a and b are obtained by solving $V(x)=E$, $a<x<b$.

a) Consider a simple harmonic oscillator with potential $\frac{1}{2} m {\omega_0}^2 x^2$ for a particle with m = 1Kg and ω_0= 2π sec^{-1}. Express this integral in terms of dimensionless variables.

a) Numerically calculate the time period of oscillation by integrating the equation with Trapezoidal method and check this against the expected value.

Note that the integrand will diverge at the limits. So the limits has to be redefined *i.e.* $b(1-\varepsilon)$ in place of b. Numerically obtained values of T will also diverge for very low values of ε. Make a log-log plot of ε vs. T. Then choose a suitable value of ε that will provide a reasonably accurate value of T. Verify that T does not depend on the amplitude of oscillation.

b) Solve for the time period for the potential $V(x) = \frac{m{\omega_0}^2L^2}{2}\left[\exp\left(\frac{x^2}{L^2}\right) - 1\right]$.
Plot time period as a function of the amplitude of bound motion in the range [-10 to 10] meters with L = 5m. Observe the variation of T for small amplitude.

2D integral 1: In order to compute $\iint_{0,0}^{1,1} f(x,y)dxdy$ with h =k=0.25,

1) Formulate 2 dimensional Trapezoidal and Simpson coefficient matrices. You need to use the numpy function 'meshgrid' for this.
2) Evaluate the integral for:

a) $f(x,y) = (5x^3y + 2x)$

b) $f(x,y) = (5xy - y^4)$