ECE 653 - ASSIGNMENT 2

ARSHAD MOMIN

Student ID- 20986176

Email- a5momin@uwaterloo.ca

Question 1 :

(a) How many execution paths does Prog1 have? List all the paths as a sequence of line numbers taken on the path.

Answer:

(a) Path 1: Considering all conditional statements to be true.
Execution path:
1→ 2→ 3→ 4→ 8→9→ 10 →11→12→13→17

(b) Path 2: Considering the conditional statement "if 2 * (x + y) > 21 then" to be FALSE and all other conditional statements to be TRUE.
Execution path:
1→ 2→ 3→ 4→ 8→9→ 10 →11→14→15→16→17

(c) Path 3: Considering the conditional statement "if x + y > 15 then" to be FALSE and all other conditional statements to be TRUE.
Execution path:
1→ 2→ 5→ 6→ 7→8→9→ 10 →11→12→13→17

(d) Path 4: Considering all the conditional statements to be FALSE.
Execution path:
1→ 2→ 5→ 6→ 7→8→9→ 10 →11→14→15→16→17

(b) Symbolically execute each path and provide the resulting path condition. Show the steps of symbolic execution as a table. An example of executing the first line is given below:

Answer:

Symbolic Execution for path 1:

Edge	Symbolic State	Path condition
1→2	$x \to X_0$, $y \to Y_0$	TRUE
2→3	$x \to X_0$, $y \to Y_0$	$X_0 + Y_0 > 15$
3→4	$x \to X_0 + 7$, $y \to Y_0$	$X_0 + Y_0 > 15$
4→8	$x \to X_0 + 7$, $y \to Y_0 - 12$	$X_0 + Y_0 > 15$
8→9	$x \to X_0 + 7$, $y \to Y_0 - 12$	$X_0 + Y_0 > 15$
9→10	$x \to X_0 + 9$, $y \to Y_0 - 12$	$X_0 + Y_0 > 15$
10→11	$x \to X_0 + 9$, $y \to Y_0 - 12$	$X_0 + Y_0 > 15$
11→12	$x \to X_0 + 9$, $y \to Y_0 - 12$	$X_0 + Y_0 > 15 \land 2 * (X_0 + Y_0) > 21$
12→13	$x \to 3(X_0 + 9)$, $y \to Y_0 - 12$	$X_0 + Y_0 > 15 \land 2 * (X_0 + Y_0) > 21$
13→17	$x \to 3(X_0 + 9)$, $y \to 2(Y_0 - 12)$	$X_0 + Y_0 > 15 \land 2 * (X_0 + Y_0) > 21$

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Symbolic Execution for path 2:

Edge	Symbolic State	Path condition
1→2	$x \rightarrow X_0$, $y \rightarrow Y_0$	TRUE
2→3	$x \rightarrow X_0$, $y \rightarrow Y_0$	$X_0 + Y_0 > 15$
3→4	$x \rightarrow X_0 + 7$, $y \rightarrow Y_0$	$X_0 + Y_0 > 15$
4→8	$x \rightarrow X_0 + 7$, $y \rightarrow Y_0 - 12$	$X_0 + Y_0 > 15$
8→9	$x \rightarrow X_0 + 7$, $y \rightarrow Y_0 - 12$	$X_0 + Y_0 > 15$
9→10	$x \rightarrow X_0 + 9$, $y \rightarrow Y_0 - 12$	$X_0 + Y_0 > 15$
10→11	$x \rightarrow X_0 + 9$, $y \rightarrow Y_0 - 12$	$X_0 + Y_0 > 15$
11→14	$x \rightarrow X_0 + 9$, $y \rightarrow Y_0 - 12$	$X_0 + Y_0 > 15 \wedge 2* (X_0 + Y_0) \leq 21$
14→15	$x \rightarrow 4(X_0 + 9)$, $y \rightarrow Y_0 - 12$	$X_0 + Y_0 > 15 \wedge 2* (X_0 + Y_0) \leq 21$
15→16	$x \rightarrow 4(X_0 + 9)$, $y \rightarrow Y_0 - 12$	$X_0 + Y_0 > 15 \wedge 2* (X_0 + Y_0) \leq 21$
16→17	$x \rightarrow 4(X_0 + 9)$, $y \rightarrow 3Y_0 + 4X_0$	$X_0 + Y_0 > 15 \wedge 2* (X_0 + Y_0) \leq 21$

Symbolic Execution for path 3:

Edge	Symbolic State	Path condition
1→2	$x \rightarrow X_0$, $y \rightarrow Y_0$	TRUE
2→ 5	$x \rightarrow X_0$, $y \rightarrow Y_0$	$X_0 + Y_0 \leq 15$
5→6	$x \rightarrow X_0$, $y \rightarrow Y_0$	$X_0 + Y_0 \leq 15$
6→ 7	$x \rightarrow X_0$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15$
7→8	$x \rightarrow X_0 - 2$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15$
8→ 9	$x \rightarrow X_0 - 2$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15$
9→10	$x \rightarrow X_0$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15$
10→11	$x \rightarrow X_0$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15$
11→12	$x \rightarrow X_0$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15 \wedge 2* (X_0 + Y_0) > 21$
12→13	$x \rightarrow 3 * X_0$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15 \wedge 2* (X_0 + Y_0) > 21$
13→17	$x \rightarrow 3 * X_0$, $y \rightarrow 2(Y_0 + 10)$	$X_0 + Y_0 \leq 15 \wedge 2* (X_0 + Y_0) > 21$

Symbolic Execution for path 4:

Edge	Symbolic State	Path condition
1→2	$x \rightarrow X_0$, $y \rightarrow Y_0$	TRUE
2→ 5	$x \rightarrow X_0$, $y \rightarrow Y_0$	$X_0 + Y_0 \leq 15$
5→6	$x \rightarrow X_0$, $y \rightarrow Y_0$	$X_0 + Y_0 \leq 15$
6→ 7	$x \rightarrow X_0$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15$
7→8	$x \rightarrow X_0 - 2$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15$
8→ 9	$x \rightarrow X_0 - 2$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15$
9→10	$x \rightarrow X_0$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15$
10→11	$x \rightarrow X_0$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15$
11→14	$x \rightarrow X_0$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15 \wedge 2* (X_0 + Y_0) \leq 21$
14→15	$x \rightarrow X_0$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15 \wedge 2* (X_0 + Y_0) \leq 21$
15→16	$x \rightarrow 4X_0$, $y \rightarrow Y_0 + 10$	$X_0 + Y_0 \leq 15 \wedge 2* (X_0 + Y_0) \leq 21$
16→17	$x \rightarrow 4X_0$, $y \rightarrow 3Y_0 + 4X_0 + 30$	$X_0 + Y_0 \leq 15 \wedge 2* (X_0 + Y_0) \leq 21$

(c) For each path in part (b), indicate whether it is feasible or not. For each feasible path, give values for Xo and Yo that satisfy the path condition.

Answer:

1. Path 1 is feasible with values of Xo=7 and Yo=10
2. Path 2 is not feasible due to conditional statement "Xo+Yo > 15 AND 2* (Xo+Yo) ≤ 21"
3. Path 3 is feasible with values of Xo=5 and Yo=8
4. Path 4 is feasible with values of Xo= 4 and Yo=5

Question 2:

(a) The constraint at-most-one (a1, . . . , an) is satisfied if at most one of the Boolean variables a1, . . . , an is true. For example, at-most-one ($\top,\bot,\bot$) is true, and at-most-one($\top,\bot,\top$) is false. Encode the constraint :
at-most-one (a1, a2, a3, a4) into an equivalent set of clauses (i.e., in CNF).

Answer: at -most- one (a1, a2, a3, a4) is satisfied if at most one of a1, a2, a3, a4 is true. Thus, the following must be satisfied with the below method:

$$\neg((a1 \land a2) \lor (a1 \land a3) \lor (a1 \land a4) \lor (a2 \land a3) \lor (a3 \land a4))$$

The above constraint becomes equivalent to as follows:

$$(\neg a1 \lor \neg a2) \land (\neg a1 \lor \neg a3) \land (\neg a1 \lor \neg a4) \land (\neg a2 \lor \neg a3) \land (\neg a2 \lor \neg a4) \land (\neg a3 \lor \neg a4)$$

(b) Show whether the following First Order Logic (FOL) sentence is valid or not. Either give a proof of validity or show a model in which the sentence is false.

$$(\forall x \cdot \exists y \cdot P(x) \lor Q(y)) \Longleftrightarrow (\forall x \cdot P(x)) \lor (\exists y \cdot Q(y))$$

Answer:

The above FOL (First order logic) sentence is valid by Algebraic formulations as below:

$$\begin{aligned}
\forall x \cdot \exists y \cdot (P(x) \lor Q(y) &= \forall x \cdot ((\exists y \cdot P(x)) \lor (\exists y \cdot Q(y)) \\
&= \forall x \cdot (P(x) \lor (\exists y \cdot Q(y))) && \text{Since: } \{\exists y \cdot P(x) = P(x)\} \\
&= (\forall x \cdot (P(x)) \lor (\forall x \cdot \exists y \cdot Q(y)) \\
&= (\forall x \cdot (P(x)) \lor (\exists y \cdot Q(y)) && \text{Since: } \{\forall x \cdot \exists y = \exists y\}
\end{aligned}$$

(d) Consider the following FOL formula Φ: $\exists x \exists y \exists z\ (P(x, y) \wedge P(z, y) \wedge P(x, z) \wedge \neg P(z, x))$ For each of the following FOL models, explain whether they satisfy or violate the formula Φ.

b) $M2 = \langle S2, P2 \rangle$, where $S2 = N$ and $P2 = \{(x, x + 1) \mid x \in N\}$. Does $M2 \models \Phi$?

Answer:

The above FOL (First order logic) model does not satisfy the formula (violates) Φ

With respect to the contradiction and assuming $x,y,z \in N$ (elements of set N) satisfies:

$$\mathbf{(P(x, y) \wedge P(z, y) \wedge P(x, z) \wedge \neg P(z, x))}$$

Which can be formulated as below:

$$y = x+1 \wedge y = z+1 \wedge z = x+1 \wedge x \neq z+1$$

since

1. y=z+1
2. z=x+1
3. Therefore z=y-1 from "y=z+1"

Substituting value of z=y-1 in equation "z=x+1"

y-1=x+1

and y=x+2

The below equations lead to a contradiction:

$$\mathbf{y = x+1 \wedge y = x+2}$$

Therefore, the above FOL (First order logic) model does not satisfy the formula (violates) Φ

c) $M3 = \langle S3, P3 \rangle$, where $S3 = P(N)$, the powerset of natural numbers, and $P3 = \{(A,B) \mid A,B \in P(N) \wedge A \subseteq B\}$. Does $M3 \models \Phi$?

Answer:

Considering the below equation:

$$x \subseteq y \ \wedge z \subseteq y \ \wedge x \subseteq z \ \wedge z \neg\subseteq x$$

Since the above equation implies x being subset of y, z being subset of y and x being subset of z, we can say that the statement satisfies the formula assuming x=1, z=2 and y=(0,1,2).

Meaning which it satisfies the equation:

$$(\mathbf{P(x, y) \land P(z, y) \land P(x, z) \land \neg P(z, x)})$$

(e) Extend your encoding from part (a) to n variables and use at most O(n) clauses and variables. If your solution is based on external resources, make sure to properly reference them.

Answer:

Let A = {a1, a2, . . . , an}.

The standard encoding of the At Most One constraint can be formulated as below equation for n variables:

at-most-one(X) ≡ {ai ∨ aj | ai, aj ∈ A, i < j}

$$(\neg a1 \lor \neg a2) \land (\neg a1 \lor \neg a3) \land (\neg a1 \lor \neg a4) \land (\neg a2 \lor \neg a3) \land (\neg a2 \lor \neg a4) \land (\neg a3 \lor \neg a4)$$

The encoding part as above from part (a) can be extended as:

$$(\neg a_m \lor \neg a_{m+1}) \land (\neg a_m \lor \neg a_{m+2}) \land (\neg a_m \lor \neg a_{m+3}) \land \left(\neg a_m \lor \neg a_{m+(n-1)}\right)$$

In this above case,

1. n= Total number of variables
2. m= Variable number