Assignment - 5

① $T(n) = T(n-2) + 2^n$, $T(0) = 0$

$= T(n-4) + 2^{n-2} + 2^n$

$= T(n-6) + 2^{n-4} + 2^{n-2} + 2^n$

$\vdots$

$= T(n-2k) + 2^{n-2k+2} + \ldots\ldots + 2^{n-4} + 2^{n-2} + 2^n$

$= T(n-2k) + 2^{n-2(k-1)} + \ldots\ldots + 2^{n-4} + 2^{n-2} + 2^n$

$= T(n-2k) + \sum_{i=1}^{k} 2^{n-2(k-i)}$

Let,

$n - 2k = 0$

$2k = n$

$k = \frac{n}{2}$

now,

$T(n) = T(n - 2\times\frac{n}{2}) + \sum_{i=1}^{n/2} 2^{n-2(\frac{n}{2}-i)}$

$= T(0) + \sum_{i=1}^{n/2} 2^{2i}$

$= 0 + \frac{4(2^{2\times\frac{n}{2}} - 1)}{2^2 - 1}$

$= 0 + \frac{4(2^n - 1)}{3}$

~~$\therefore \theta =$~~ $\therefore \theta(2^n)$ (Ans)

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② $T(n) = (T(n-2))^2, \quad T(0) = 2$

$$= ((T(n-4))^2)^2$$

$$= (T(n-4))^4$$

$$= ((T(n-6))^2)^4$$

$$= (T(n-6))^8$$

$$= ((T(n-8))^2)^8$$

$$= (T(n-8))^{16}$$

$\vdots$

~~= T(n-2~~

$$= (T(n-2k))^{2^k}$$

Let,

$n - 2k = 0$

$2k = n$

$k = \frac{n}{2}$

now,

$$T(n) = (T(n - 2 \times \tfrac{n}{2}))^{2^{n/2}}$$

$$= (T(0))^{2^{n/2}}$$

$$= (2)^{2^{n/2}}$$

~~$\therefore \theta =$~~

$\therefore \theta(2^{2^n})$ (Ans)

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③ $T(n) = T(\sqrt{n}) + \Theta(\log_2 n)$, $T(2) = 1$

$$T(n) = T(n^{\frac{1}{2}}) + c\log_2 n$$

$$= T(n^{\frac{1}{4}}) + c\log_2 n^{\frac{1}{2}}$$

$$= T(n^{\frac{1}{4}}) + \frac{c}{2}\log_2 n + c\log_2 n$$

$$= T(n^{\frac{1}{8}}) + \frac{c}{4}\log_2 n + \frac{c}{2}\log_2 n + c\log_2 n$$

$$= T(n^{\frac{1}{16}}) + \frac{c}{8}\log_2 n + \frac{c}{4}\log_2 n + \frac{c}{2}\log_2 n + c\log_2 n$$

$$\vdots$$

$$= T(n^{\frac{1}{2^k}}) + \sum_{i=0}^{k-1} c\log_2 n \times \left(\frac{1}{2}\right)^i$$

let,

$$n^{\frac{1}{2^k}} = 2$$

$$\log_2 n^{\frac{1}{2^k}} = \log_2 2$$

$$\frac{1}{2^k}\log_2 n = 1$$

$$\log_2 n = 2^k$$

$$\log_2\log_2 n = \log_2 2^k$$

$$\log_2\log_2 n = k$$

now,

$$T(n) = T(2) + \sum_{i=0}^{\log_2\log_2 n - 1} c\left(\frac{1}{2}\right)^i \log_2 n$$

$\therefore \Theta(\log_2 n)$ (Ans)

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(4) $T(n) = T(\frac{n}{2}) + T(\sqrt{n}) + \theta(n)$, $T(1) = 1$

$$T(n) = 2T(n/2) + \theta(n)$$
$$= 2\left(2T(\tfrac{n}{4}) + \tfrac{cn}{2}\right) + cn$$
$$= 4T(\tfrac{n}{4}) + cn + cn$$
$$= 8T(\tfrac{n}{8}) + cn + cn + cn$$
$$= 16T(\tfrac{n}{16}) + cn + cn + cn + cn$$
$$\vdots$$
$$= 2^k T\left(\frac{n}{2^k}\right) + kcn$$

Let,

$$\frac{n}{2^k} = 1$$
$$n = 2^k$$
$$\log_2 n = k$$

now,

$$T(n) = 2^{\log_2 n} T(1) + \log_2 n (cn)$$
$$= 2^{\log_2 n} + cn\log_2 n$$

$\therefore \theta(n\log_2 n)$

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(5) $T(n) = 2T\left(\frac{n}{3}\right) + \theta(n\log_2 n)$, $T(1) = 1$

$$= 2T\left(\frac{n}{3}\right) + cn\log_2 n$$

$$= 2\left(2T\left(\frac{n}{9}\right) + c\frac{n}{3}\log_2\frac{n}{3}\right) + cn\log_2 n$$

$$= 4T\left(\frac{n}{9}\right) + \frac{2}{3}cn\log_2\frac{n}{3} + cn\log_2 n$$

$$= 8T\left(\frac{n}{27}\right) + \frac{4}{9}cn\log_2\frac{n}{9} + \frac{2}{3}cn\log_2\frac{n}{3} + cn\log_2 n$$

$$\vdots$$

$$= 2^k T\left(\frac{n}{3^k}\right) + \sum_{i=0}^{k-1}\left(\frac{2}{3}\right)^i cn\log_2\left(\frac{n}{3^i}\right)$$

let,

$$\frac{n}{3^k} = 1$$

$$n = 3^k$$

$$\log_3 n = k$$

$$T(n) = 2^{\log_3 n}T(1) + \sum_{i=0}^{\log_3 n-1}\left(\frac{2}{3}\right)^i cn\log_2\left(\frac{n}{3^i}\right)$$

$$= 2^{\log_3 n} + \sum_{i=0}^{\log_3 n-1}\left(\frac{2}{3}\right)^i cn\log_2\left(\frac{n}{3^i}\right)$$

$\therefore \theta(n\log_2 n)$ (Ans)

⑥ $T(n) = T\left(\frac{n}{5}\right) + \Theta\left((\log_2 n)^2\right)$, $T(1) = 1$

$$= \cancel{T\left(\frac{n}{25}\right) +}$$

$$= T\left(\frac{n}{5}\right) + c(\log_2 n)^2$$

$$= T\left(\frac{n}{25}\right) + c\left(\log_2 \frac{n}{5}\right)^2 + c(\log_2 n)^2$$

$$= T\left(\frac{n}{125}\right) + c\left(\log_2 \frac{n}{25}\right)^2 + c\left(\log_2 \frac{n}{5}\right)^2 + c(\log_2 n)^2$$

$$\vdots$$

$$= T\left(\frac{n}{5^k}\right) + \sum_{i=0}^{k-1} c\left(\log_2 \frac{n}{5^i}\right)^2$$

let,

$$\frac{n}{5^k} = 1$$

$$n = 5^k$$

$$\log_5 n = k$$

now,

$$T(n) = T(1) + \sum_{i=0}^{\log_5 n - 1} c\left(\log_2 \frac{n}{5^i}\right)^2$$

$$\therefore \Theta\left((\log_2 n)^2\right) \text{ (Ans)}$$

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7 $T(n) = 3T\left(\frac{n}{5}\right) + \theta\left(\left(\log_2 n\right)^2\right)$, $T(1) = 1$

$$= 3T\left(\frac{n}{5}\right) + c\left(\log_2 n\right)^2$$

$$= 3\left(3T\left(\frac{n}{25}\right) + c\left(\log_2 \frac{n}{5}\right)^2\right) + c\left(\log_2 n\right)^2$$

$$= 9T\left(\frac{n}{25}\right) + 3c\left(\log_2 \frac{n}{5}\right)^2 + c\left(\log_2 n\right)^2$$

$$= 9\left(3T\left(\frac{n}{125}\right) + c\left(\log_2 \frac{n}{25}\right)^2\right) + 3c\left(\log_2 \frac{n}{5}\right)^2 + c\left(\log_2 n\right)^2$$

$$T(n) = 3^k T\left(\frac{n}{5^k}\right) + \sum_{i=0}^{k-1} 3^i c\left(\log_2 \frac{n}{5^i}\right)^2$$

Let,

$$\frac{n}{5^k} = 1$$

$$\log_5 n = k$$

$$T(n) = 3^{\log_5 n} T(1) + \sum_{i=0}^{\log_5 n - 1} 3^i c\left(\log_2 \frac{n}{5^i}\right)^2$$

$$= 3^{\log_5 n} + \sum_{i=0}^{\log_5 n - 1} 3^i c\left(\log_2 \frac{n}{5^i}\right)^2$$

$\therefore \theta\left(\left(\log_2 n\right)^2\right)$ (Ans)

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8) $T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{6}\right) + \Theta(n)$, $T(1) = 1$

$$= T\left(\frac{n}{3}\right) + T\left(\frac{2n}{6}\right) + cn$$

$$= 2T\left(\frac{n}{3}\right) + cn$$

$$= 2\left(2T\left(\frac{n}{9}\right) + c\frac{n}{3}\right) + cn$$

$$= 4T\left(\frac{n}{9}\right) + \frac{2}{3}cn + cn$$

$$= 4\left(2T\left(\frac{n}{27}\right) + c\frac{n}{9}\right) + \frac{2}{3}cn + cn$$

$$= 8T\left(\frac{n}{27}\right) + \frac{4}{9}cn + \frac{2}{3}cn + cn$$

$$\vdots$$

$$= 2^k T\left(\frac{n}{3^k}\right) + \sum_{i=0}^{k-1} \left(\frac{2}{3}\right)^i cn$$

let,

$$\frac{n}{3^k} = 1$$

$$n = 3^k$$

$$\log_3 n = k$$

$$T(n) = 2^{\log_3 n} T(1) + \sum_{i=0}^{\log_3 n - 1} \left(\frac{2}{3}\right)^i cn$$

$$= 2^{\log_3 n} + \sum_{i=0}^{\log_3 n - 1} \left(\frac{2}{3}\right)^i cn$$

$\therefore \Theta(n)$ (Ans)

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