Assignment - 3

① $3n^2 + 10n\log_2 n = O(n\log_2 n)$

Here,

~~$f(n) = 3n^2 + 10n\log$~~

$f(n) = 3n^2 + 10n\log_2 n$, $g(n) = n\log_2 n$

In order to be correct,

$$f(n) \leq c \cdot g(n)$$

$$3n^2 + 10n\log_2 n \leq c \cdot n\log_2 n$$

$$3\frac{n^2}{n} + 10\frac{n}{n}\log_2 n \leq c \cdot \frac{n}{n}\log_2 n$$

$$3n + 10\log_2 n \leq c \cdot \log_2 n$$

let, $c = 10$, $n_0 = 1$

$(3\times1) + 10\log_2 1 \leq 10\log_2 1$

$3 \leq 0$

$\therefore$ f(n) is asymptotically faster growing than g(n) and hence it is not upper bounded by g(n). (Disproved)

② $3n^2 + 10n\log_2 n = \Omega(n^2)$

Here,

$f(n) = 3n^2 + 10n\log_2 n$, $g(n) = n^2$

In order to be correct, $f(n) \geq c \cdot g(n)$

$$3n^2 + 10n\log_2 n \geq n^2 \cdot c$$

$$3\frac{n^2}{n} + 10\frac{n}{n}\log_2 n \geq \frac{n^2}{n} \cdot c$$

$$3n + 10\log_2 n \geq n \cdot c$$

Let, $n_0 = 1$, $c = 3$

~~let~~,

$(3\times1) + 10\log_2 1 \geq 3\times1$

$3 \geq 3$

$\therefore$ f(n) is asymptotically faster growing than g(n) and hence it is lower bounded by g(n). (Proved)

③ $3n^2 + 10n\log_2 n = \theta(n^2)$

Here,

$f(n) = 3n^2 + 10n\log_2 n$, $g(n) = n^2$

In order to be correct,

$$C_1 g(n) \leqslant f(n) \leqslant C_2 g(n)$$

$$C_1 n^2 \leqslant 3n^2 + 10n\log_2 n \leqslant C_2 n^2$$

$$C_1 \frac{n^2}{n} \leqslant 3\frac{n^2}{n} + 10\frac{n}{n}\log_2 n \leqslant C_2 \frac{n^2}{n}$$

$$C_1 \cdot n \leqslant 3n + 10\log_2 n \leqslant C_2 \cdot n$$

let, $C_1 = 3$, $C_2 = 10$, $n = 1$

$$(3\times1) \leqslant (3\times1) + 10\log_2 1 \leqslant 10\times1$$

$$3 \leqslant 3 \leqslant 10$$

$n = 2$

$$(3\times2) \leqslant (3\times2) + 10\log_2 2 \leqslant 10\times2$$

$$6 \leqslant 16 \leqslant 20$$

$\therefore$ $f(n)$ is asymptotically faster growing than $C_1 g(n)$ and slower growing than $C_2 g(n)$. Hence ~~it both up~~. $f(n)$ is upper bounded by $C_2 \cdot g(n)$ and lower bounded by $C_1 g(n)$. So we can say $f(n)$ is tight bounded by $\theta(n^2)$. (Proved).

(4) $n\log_2 n + \frac{n}{2} = O(n)$

Here,

$f(n) = n\log_2 n + \frac{n}{2}$, $g(n) = n$

In order to be correct $f(n) \leq c \cdot g(n)$

$$n\log_2 n + \frac{n}{2} \leq c \cdot n$$

$$\frac{n}{n}\log_2 n + \frac{n}{2n} \leq c \cdot \frac{n}{n}$$

$$\log_2 n + \frac{1}{2} \leq c$$

$\therefore$ $f(n)$ is asymptotically faster growing than $g(n)$ and hence $f(n)$ is not upperbounded by $g(n)$. (Disproved)

(5) $10\sqrt{n} + \log_2 n = O(n)$

Here,

$f(n) = 10\sqrt{n} + \log_2 n$, $g(n) = n$

In order to be correct $f(n) \leq c \cdot g(n)$

$$10\sqrt{n} + \log_2 n \leq c \cdot n$$

let, $c = 10$, $n_0 = 1$

$$\cancel{10\sqrt{1} + \log_2(1) \leq}$$

$$10\sqrt{1} + \log_2 1 \leq 10 \times 1$$

$$10 \leq 10$$

$n = 2$

$$10\sqrt{2} + \log_2 2 \leq 10 \times 2$$

$$15 \leq 20$$

$\therefore$ f(n) is asymptotically slower growing than g(n) and hence f(n) is upperbounded by g(n). (Proved)

(6) $\sqrt{n} + \log_2 n = O(\log_2 n)$

Here,

$f(n) = \sqrt{n} + \log_2 n \quad , \quad g(n) = \log_2 n$

In order to be correct $f(n) \leq c \cdot g(n)$

$$\sqrt{n} + \log_2 n \leq c \cdot \log_2 n$$

$$\frac{\sqrt{n}}{\log_2 n} + \frac{\log_2 n}{\log_2 n} \leq c \cdot \frac{\log_2 n}{\log_2 n}$$

$$\frac{\sqrt{n}}{\log_2 n} + 1 \leq c$$

$\therefore$ f(n) is asymptotically faster growing than g(n) and hence f(n) is not ~~too~~ upperbounded by g(n). (Disproved)

(7) $\sqrt{n} + \log_2 n = \Theta(\log_2 n)$

Here,

$f(n) = \sqrt{n} + \log_2 n \quad , \quad g(n) = \log_2 n$

In order to be correct $c_1 g(n) \leq f(8) \leq c_2 g(n)$

$$c_1 \cdot \log_2 n \leq \sqrt{n} + \log_2 n \leq c_2 \cdot \log_2 n$$

$$c_1 \cdot \frac{\log_2 n}{\log_2 n} \leq \frac{\sqrt{n}}{\log_2 n} + \frac{\log_2 n}{\log_2 n} \leq c_2 \cdot \frac{\log_2 n}{\log_2 n}$$

$$c_1 \leq \frac{\sqrt{n}}{\log_2 n} + 1 \leq c_2$$

$\therefore$ f(n) is asymptotically faster growing than both $c_1 \cdot g(n)$ and $c_2 g(n)$. Hence f(n) is lower bounded by $c_1 \cdot g(n)$ but not upperbounded by $c_2 \cdot g(n)$. Therefore we can say f(n) is not tight bounded by g(n). (Disproved)

⑧ $\sqrt{n} + \log_2 n = \theta(n)$

Here,

$f(n) = \sqrt{n} + \log_2 n \quad , g(n) = n$

In order to be correct $c_1 \cdot g(n) \leq f(n) \leq c_2 g(n)$

$$c_1 \cdot n \leq \sqrt{n} + \log_2 n \leq c_2 \cdot n$$

let, $c_1 = 1$, $c_2 = 3$, $n = 6$

$6 \leq 5 \leq 18$

$\therefore$ f(n) is asymptotically slower growing than both $c_1 \cdot g(n)$ and $c_2 \cdot g(n)$. Hence ~~it~~ f(n) is upper bounded by $c_2 g(n)$ but not lower bounded by $c_1 g(n)$. Therefore f(n) is not tight bounded by g(n). (Disproved)

⑨ $2n + \log_2 n = \theta(\sqrt{n})$

Here,

$f(n) = 2n + \log_2 n \quad , g(n) = \sqrt{n}$

In order to be correct $c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$

$$c_1 \cdot \sqrt{n} \leq 2n + \log_2 n \leq c_2 \sqrt{n}$$

$$c_1 \cdot \frac{\sqrt{n}}{\sqrt{n}} \leq \frac{2n}{\sqrt{n}} + \frac{\log_2 n}{\sqrt{n}} \leq c_2 \cdot \frac{\sqrt{n}}{\sqrt{n}}$$

$$c_1 \leq \frac{2n}{\sqrt{n}} + \frac{\log_2 n}{\sqrt{n}} \leq c_2$$

Let, $c_1 = 2$, $c_2 = 8$

$n = 1$

$$2 \leq \frac{2 \times 1}{\sqrt{1}} + \frac{\log_2 1}{\sqrt{1}} \leq 8$$

$$2 \leq 2 \leq 8$$

$n = 3$

$$2 \le \frac{2\times 3}{\sqrt{3}} + \frac{\log_2 3}{\sqrt{3}} \le 8$$

$$2 \le 4 \le 8$$

$n = 20$

$$2 \le \frac{2\times 20}{\sqrt{20}} + \frac{\log_2 20}{\sqrt{20}} \le 8$$

$$2 \le 10 \le 8$$

$\therefore$ $f(n)$ is ~~asymptotid~~ asymptotically faster growing than both $c_1 g(n)$ and $c_2 g(n)$. Hence $f(n)$ is lower bounded by $c_1 g(n)$ but not upper bounded by $c_2 g(n)$. Therefore $f(n)$ is not tight bounded by $g(n)$. (Disproved)

(10) $\frac{1}{2}n^2 - 3n = \Theta(n^2)$

Here,

$f(n) = \frac{1}{2}n^2 - 3n$, $g(n) = n^2$

In order to be correct $c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n)$

$$c_1 n^2 \le \frac{1}{2}n^2 - 3n \le c_2 n^2$$

$$c_1 \cdot \frac{n^2}{n} \le \frac{1}{2}\frac{n^2}{n} - 3\frac{n}{n} \le c_2 \cdot \frac{n^2}{n}$$

$$c_1 \cdot n \le \frac{1}{2}n - 3 \le c_2 \cdot n$$

$c_1 = 1$, $c_2 = 5$

$n = 1$

$$(1\times 1) \le \left(\tfrac{1}{2}\times 1\right) - 3 \le 5\times 1$$

$$1 \le -2.5 \le 5$$

$n = 6$

$$(1\times 6) \le \left(\tfrac{1}{2}\times 6\right) - 3 \le 5\times 6$$

$$6 \le 0 \le 30$$

$\therefore$ $f(n)$ is asymptotically slower growing than both $c_1 \cdot g(n)$ and $c_2 \cdot g(n)$. Hence $f(n)$ is upper bounded by $c_2 \cdot g(n)$ but ~~$c_1 g(n)$~~ not lower bounded by $c_1 g(n)$. Therefore we can say $f(n)$ is not ~~lightly~~ tight bounded by $g(n)$. (Disproved)

(11) $6n^3 = \theta(n^2)$

Here,

$f(n) = 6n^3 \quad , \quad g(n) = n^2$

In order to be correct $c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$

$$c_1 \cdot n^2 \leq 6n^3 \leq c_2 \cdot n^2$$

$$c_1 \cdot \frac{n^2}{n^2} \leq 6\frac{n^3}{n^2} \leq c_2 \cdot \frac{n^2}{n^2}$$

$$c_1 \leq 6n \leq c_2$$

Let, $c_1 = 2 \quad , \quad c_2 = 6$

$n = 1$

$$2 \leq 6 \times 1 \leq 6$$

$$2 \leq 6 \leq 6$$

$n = 3$

$$2 \leq 6 \times 3 \leq 6$$

$$2 \leq 18 \leq 6$$

$\therefore$ $f(n)$ is asymptotically faster growing than both $c_1 \cdot g(n)$ and $c_2 \cdot g(n)$. Hence $f(n)$ is lower bounded by $c_1 \cdot g(n)$ but not upper bounded by $c_2 \cdot g(n)$. Therefore $f(n)$ is not tight bounded by $g(n)$. (Disproved)

(12) $\sqrt{n} + \log_2 n = \Omega(1)$

Here,

$f(n) = \sqrt{n} + \log_2 n$, $g(n) = 1$

In order to be correct, $f(n) \geq c \cdot g(n)$

$$\sqrt{n} + \log_2 n \geq c \cdot 1$$

$$\sqrt{n} + \log_2 n \geq c$$

let, $c = 2$, $n = 2$

$$\sqrt{2} + \log_2 2 \geq 2$$

$$2.4 \geq 2$$

$n = 4$

$$\sqrt{4} + \log_2 4 \geq 2$$

$$4 \geq 2$$

$\therefore f(n)$ is asymptotically faster growing than $g(n)$. Hence $f(n)$ is lower bounded by $g(n)$. (Proved)

(13) $\sqrt{n} + \log_2 n = \Omega(\log_2 n)$

Here,

$f(n) = \sqrt{n} + \log_2 n$, $g(n) = \log_2 n$

In order to be correct, $f(n) \geq c \cdot g(n)$

$$\sqrt{n} + \log_2 n \geq c \cdot \log_2 n$$

Let,

$c = 1$, $n = 1$

$$\sqrt{1} + \log_2 1 \geq 1 \cdot \log_2 1$$

$$1 \geq 0$$

Scanned with CamScanner

$n = 4$

$$\sqrt{4} + \log_2 4 \geq 1 \cdot \log_2 4$$
$$4 \geq 2$$

$n = 8$

$$\sqrt{8} + \log_2 8 \geq 1 \cdot \log_2 8$$
$$5.8 \geq 3$$

$\therefore$ f(n) is asymptotically faster growing than g(n). Hence f(n) is lower bounded by g(n). (Proved)

14 $\sqrt{n} + \log_2 n = \Omega(n)$

Here,

$f(n) = \sqrt{n} + \log_2 n$, $g(n) = n$

In order to be correct, $f(n) \geq c \cdot g(n)$

$$\sqrt{n} + \log_2 n \geq c \cdot n$$

Let, $c = 5$

$n = 1$ $\quad \sqrt{1} + \log_2 1 \geq 5 \times 1$
$$1 \geq 5$$

$n = 5$ $\quad \sqrt{5} + \log_2 5 \geq 5 \times 5$
$$5 \geq 25$$

$\therefore$ f(n) is asymptotically slower growing than g(n). Hence f(n) is not lower bounded by g(n). (Disproved)