Average case

$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{j=2}^{n} \frac{t_j}{2} + c_6 \sum_{j=2}^{n} \left(\frac{t_j}{2} - 1\right) +$$
$$c_7 \sum_{j=2}^{n} \left(\frac{t_j}{2} - 1\right) + c_8(n-1)$$

$$S_n = \frac{t_j}{2}$$
$$= s_1 + s_2 + s_3 + s_4 + \dots + s_n$$
$$= \frac{1}{2} + 1 + \frac{3}{2} + 2 + \dots + \frac{n}{2}$$
$$= \frac{n}{2}\left((2 \times \tfrac{1}{2}) + (n-1) \times \tfrac{1}{2}\right)$$
$$= \frac{n\left(\frac{1}{2}n + \frac{1}{2}\right)}{2}$$

$$S_n = \frac{t_j}{2} - 1$$
$$= s_1 + s_2 + s_3 + s_4 + \dots + s_n$$
$$= -\frac{1}{2} + 0 + \frac{1}{2} + 1 + \dots + \left(\frac{n}{2} - 1\right)$$
$$= \frac{n\left((2 \times -\frac{1}{2}) + (n-1) \times \frac{1}{2}\right)}{2}$$
$$= \frac{n\left(\frac{1}{2}n - \frac{3}{2}\right)}{2}$$

$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{j=2}^{n} \frac{t_j}{2} + c_6 \sum_{j=2}^{n} \left(\frac{t_j}{2} - 1\right) +$$
$$c_7 \sum_{j=2}^{n} \left(\frac{t_j}{2} - 1\right) + c_8(n-1)$$

$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5\left(\frac{n\left(\frac{1}{2}n + \frac{1}{2}\right)}{2} - \frac{1}{2}\right) +$$
$$c_6\left(\frac{n\left(\frac{1}{2}n - \frac{3}{2}\right)}{2} + \frac{1}{2}\right) + c_7\left(\frac{n\left(\frac{1}{2}n - \frac{3}{2}\right)}{2} + \frac{1}{2}\right) + c_8(n-1)$$

$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5\left(\frac{\frac{1}{2}n^2 + \frac{1}{2}n}{2} - \frac{1}{2}\right) +$$
$$c_6\left(\frac{\frac{1}{2}n^2 - \frac{3}{2}n}{2} + \frac{1}{2}\right) + c_7\left(\frac{\frac{1}{2}n^2 - \frac{3}{2}n}{2} + \frac{1}{2}\right) + c_8(n-1)$$

$$T(n) = c_1 n + c_2 n - c_2 + c_4 n - c_4 + \frac{1}{4}c_5 n^2 + \frac{1}{4}c_5 n - \frac{1}{2}c_5$$
$$+ \frac{1}{4}c_6 n^2 - \frac{3}{4}c_6 n + \frac{1}{2}c_6 + \frac{1}{4}c_7 n^2 - \frac{3}{4}c_7 n + \frac{1}{2}c_7 + c_8 n - c_8$$

$$T(n) = \left(\frac{1}{4}c_5 + \frac{1}{4}c_6 + \frac{1}{4}c_7\right)n^2 + \left(c_1 + c_2 + c_4 + \frac{1}{4}c_5 - \frac{3}{4}c_6 - \frac{3}{4}c_7 + c_8\right)n$$
$$+ \left(-c_2 - c_4 - \frac{1}{2}c_5 + \frac{1}{2}c_6 + \frac{1}{2}c_7 - c_8\right)$$

$$T(n) = an^2 + bn + c$$

$\therefore$ We can say average case is same as worst case.

Scanned with CamScanner