

Theme: Assignment 2.

Date: 24 / 11 / 2020

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1.  $3n^2 + 10n \log_2 n = O(n \log_2 n)$

$$= 3n^2 + 10n \log_2 n \leq c \cdot (n \log_2 n).$$

$$= \frac{3n^2}{n \log_2 n} + \frac{10n \log_2 n}{n \log_2 n} \leq c$$

Disapproved, since  $n^2$  is asymptotically faster growing than  $n \log n$  and hence not upperbounded by it.

2.  $3n^2 + 10n \log_2 n = \Omega(n^2)$

$$\frac{3n^2}{n^2} + \frac{10n \log_2 n}{n^2} \geq c$$

$$3 + \frac{10 \log_2 n}{n} \geq c$$

$$n=2 \quad c=7.$$

$$3 + \frac{10 \log_2(2)}{(2)} \geq 7$$

$$8 \geq 7$$

Rough:

$$n=3 \quad c=13$$

$$3 + \frac{10 \log_2(3)}{3} \geq 13$$

$$8.2 \geq 13.$$

Approved, since  $n^2$  asymptotically grows like  $n^2$  and hence  $\Omega(n^2)$  it is  $\Omega(n^2)$

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3.  $3n^2 + 10n \log_2 n = \theta(n^2)$

$$C_1 n_2 \leq 3n^2 + 10n \log_2 n \leq C_2 n_2$$

For  $C_1$ ;

$$C_1 n_2 \leq 3n^2 + 10n \log_2 n$$

$$C_1 \leq \frac{3n^2}{n^2} + \frac{10n \log_2 n}{n^2}$$

$$C_1 \leq 3 + \frac{10 \log_2 n}{n}$$

For  $n = 3$ ,  $C = 13$ ;

$$13 \leq 3 + \frac{10 \log_2(3)}{3}$$

$$7 \leq 8.2.$$

For  $C_2$ ;

$$\frac{3n^2}{n^2} + \frac{10n \log_2 n}{n^2} \leq C_2$$

$$3 + \frac{10 \log_2(3)}{(3)} \leq 9$$

$$8.2 \leq 9.$$

Rough:

$$n=3 \quad C=13$$

$$13 \leq 3 + \frac{10(\log_2(3))}{(2)} \quad // \text{maximum}$$

$$13 \leq 8.2$$

Approved, since  ~~$\theta(n^2)$~~   $\theta(n^2)$  faster grower than  $n \log n$  and hence not upperbounded by it.

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4.  $n \log_2 n + \frac{n}{2} = O(n)$

$$= n \log_2 n + \frac{n}{2} \leq C \cdot n$$

$$= \frac{\cancel{n} \log_2 n}{\cancel{n}} + \frac{n^2}{2} \leq C.$$

$$= \log_2 n + \frac{n^2}{2} \leq C$$

Disapproved, since  $n \log n$  is not asymptotically upperbounded by  $C$ .  
 $n^2$  grows exponentially.

5.  $10\sqrt{n} + \log_2 n = O(n)$

$$= 10\sqrt{n} + \log_2 n \leq C \cdot n$$

$$= \frac{10\sqrt{n}}{n} + \frac{\log_2 n}{n} \leq C.$$

$$10n^{\frac{1}{2}-1} + \frac{\log_2(n)}{(n)} \leq C$$

for  $n=2$   $C=5$

$$10(2)^{\frac{1}{2}-1} + \frac{\log(2)}{(2)} \leq 5$$

$$3.53 \leq 5$$

for  $n=3$   $C=5$

$$10(3)^{\frac{1}{2}-1} + \frac{\log(3)}{(3)} \leq 5$$

$$1 \leq 5$$

Approved, since  $10\sqrt{n}$  is asymptotically upperbounded by  $n$ .

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6.  $\sqrt{n} + \log_2 n = O(\log_2 n)$

$$\sqrt{n} + \log_2 n \leq C \cdot \log_2 n. \quad n=3 \quad C=5$$

$$\frac{\sqrt{n}}{\log_2 n} + \frac{\log_2 n}{\log_2 n} \leq C$$

$$\frac{3^{1/2}}{\log_2 3} + 1 \leq 5$$

$$2.09 \leq 5$$

$$\frac{n^{1/2}}{\log n} + 1 \leq C$$

Disapproved, since  $\sqrt{n}$  is not asymptotically upper bounded by  $C$ .

7.  $\sqrt{n} + \log_2 n = \Theta(\log_2 n)$

$$= C_1 \log_2 n \leq \sqrt{n} + \log_2(n) \leq C_2 \cdot \log_2 n$$

= For  $C_1$ ;

$$C_1 \leq \frac{\sqrt{n} + \log_2(n)}{\log_2 n} = \frac{\sqrt{n}}{\log_2 n} + \frac{\log_2 n}{\log_2 n}$$

$$C_1 \leq \frac{\sqrt{n}}{\log_2 n} + 1$$

For  $n=2 \quad C_1=2$

$$C_1 \leq \frac{\sqrt{2}}{\log(2)} + 1$$

$$1 \leq 2.41$$

For  $C_2$ ;

$$\frac{\sqrt{n}}{\log_2 n} + 1 \leq C_2$$

Disapproved, can't be bounded.

$\sqrt{n}$  is asymptotically fast growing.

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8.  $\sqrt{n} + \log_2 n = \Theta(n)$

$$C_1 \cdot n \leq \sqrt{n} + \log_2 n \leq C_2 \cdot n$$

$$C_1 \cdot n \leq \sqrt{n} + \log_2 n$$

$$C_1 \leq \frac{\sqrt{n}}{n} + \frac{\log_2 n}{n}$$

$$\frac{\sqrt{n}}{n} + \frac{\log_2 n}{n} \leq C_2$$

$$n=5$$

When  $n=1$   $C_1 \leq 1$

$n=2$   $C_1 \leq 1.2$

$n=3$   $C_1 \leq 1.1$

~~$n=5$   $C_1 \leq$~~

Disapproved,  $\sqrt{n}$  is asymptotically faster slower growing than  $n$ .

9.  $2n + \log_2 n = \Theta(\sqrt{n})$

$$C_1 \cdot \sqrt{n} \leq 2n + \log_2 n \leq C_2 \cdot \sqrt{n}$$

$$C_1 \cdot \sqrt{n} \leq 2n + \log_2 n$$

$$C_1 \leq \frac{2n}{\sqrt{n}} + \frac{\log_2 n}{\sqrt{n}}$$

$n=1$   $C_1 \leq 2$

$n=2$   $C_1 \leq 2.1$

$n=3$   $C_1 \leq 2.06$

$n=4$   $C_1 \leq 2$

$n=6$   $C_1 \leq 1.6$

$n=7$   $C_1 \leq 1.81$

$n=10$   $C_1 \leq 1.6$

$n=100$   $C_1 \leq 0.8$

can't be lower.

for  $C_2$ ,

$$\frac{2n}{\sqrt{n}} + \frac{\log_2 n}{\sqrt{n}} \leq C_2$$

$n=3$   $C=3$

$$2.06 \leq 3$$

Disapproved, this can't be lower bounded.



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10.  $\frac{1}{2}n^2 - 3n = \theta(n^2)$

$$C_1 \cdot n_2 \leq \frac{1}{2}n^2 - 3n \leq C_2 \cdot n_2$$

$$C_1 \cdot n_2 \leq \frac{1}{2}n^2 - 3n$$

$$C_1 \leq \frac{\frac{1}{2}(n)^2}{n_2} - \frac{3n}{n_2}$$

$$n=1 \quad C_1 \leq -2.5 \quad -2.5 \leq 1 \quad \text{"true"}$$

$$n=2 \quad C_1 \leq -1/2$$

$$n=3 \quad C_1 \leq 0.5$$

$$n=10 \quad C_1 \leq 4.7.$$

$$\text{For } C_2; \quad \frac{\frac{1}{2}(n^2)}{n_2} - \frac{3}{n} \leq C_2$$

$$n=1000 \quad 49.9.$$

Disapproved, ~~since~~ for arbitrary large  $n$ ,  
since  $C$  is constant.

11.  $6n^3 = \theta(n^2)$

$$C_1 \cdot n_2 \leq 6n^3 \leq C_2 \cdot n_2$$

$$C_1 \leq \frac{6n^3}{n_2}$$

$$C_1 \leq 6n$$

$$n=2 \quad C=2$$

$$2 \leq 2$$

$$\text{for } C_2; \quad 6n \leq C_2$$

Disapproved,  $n^3$  can't be  
upper bounded.

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12.  $\sqrt{n} + \log_2 n = \Omega(1)$

$$\sqrt{n} + \log_2 n \geq c \cdot 1$$

$$n=2; 2.41 \text{ ————— } 2.41 \geq 1$$

$$n=1 \quad c=1$$

$$\sqrt{1} + \log_2(1) \geq 1$$

$$1 \geq 1$$

Approved, since  $\sqrt{n}$  is asymptotically faster growing than 1.

13.  $\sqrt{n} + \log_2(n) = \Omega(\log_2 n)$

$$\sqrt{n} + \log_2 n \geq c \cdot \log_2 n$$

$$\frac{\sqrt{n}}{\log_2 n} + \frac{\log_2 n}{\log_2 n} \geq c$$

$$\frac{\sqrt{n}}{\log_2 n} + 1 \geq c$$

$$c=1 \quad n=2$$

$$\frac{\sqrt{2}}{\log(2)} + 1 \geq 1$$

$$2.41 \geq 1$$

Approved,  $\sqrt{n}$  is asymptotically faster growing than  $\log n$ .

$$n=2 \quad 2.41$$

$$n=3 \quad 2.09$$

$$n=7 \quad 1.94$$

$$n=9 \quad 1.94$$

$$n=12 \quad 1.96$$

$$n=16 \quad 2$$

$$n=100 \quad 2.58$$

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14.

$$\sqrt{n} + \log_2 n = \Omega(n)$$

$$\sqrt{n} + \log_2 n \geq c \cdot (n)$$

$$\frac{\sqrt{n}}{n} + \frac{\log_2 n}{n} \geq c$$

$$n=1 - 1$$

$$n=2 - 1.20$$

$$n=4 - 1$$

$$n=5 - 0.9$$

$$n=10 - 0.6$$

Disapproved, this can't be bounded by  $c$ .  
 $\sqrt{n}$  asymptotically slower growing than  $n$ .

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