Theme: Assignment 4:

Date: 6 / 12 / 2020.

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1. $T(n) = T(n-2) + 2^n$, $T(0) = 0$

$= T(n-4) + 2^{n-2} + 2^n$

$= T(n-6) + 2^{n-1} + 2^{n-2} + 2^n$.

$\vdots$

$T(n-2k) + \sum_{i=1}^{k} 2^{(n-2(k-1))}$

$= T(n - 2\cdot\frac{n}{2}) + \sum_{i=1}^{n/2} 2^{(n-2(\frac{n}{2}-1))}$

let;

$n - 2k = 0$

$n = 2k$

$k = n/2$

$= T(0) + \sum_{i=1}^{n/2} 2^{2i}$

$= 0 + \frac{1-4^{n/2}}{1-4}$

$= O(2^n)$

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2. $T(n) = (T(n-2))^2 \qquad T(0) = 2$

$= T((n-4)^2)^2$

$= T((n-6)^2)^4.$

$= T((n-8)^2)^8$

$\vdots$

$T((n-2k)^2)^{2k}.$

Assume;

$n-2k = 0$

$n = 2k$

$k = n/2$

$T((n-2k)^2)^{2k}$

$= (T(0))2^{n/2}$

$= (2)\, 2^{n/2}$

$= 4^{n/2}$

$= 2^n$

$O(2^n)$

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3. $T(n) = T(\sqrt{n}) + \theta(\log_2 n)$ $\quad T(2) = 1.$

$= T(n^{1/2}) + c \cdot \log_2 n$

$= T(n^{1/2})^2 + c \log_2 n^{1/2} + c \cdot \log_2 n$

$= T(n^{1/2})^3 + c \log_2^{1/4} + c \log^{1/2} + c \cdot \log n$

$= T(n^{1/2})^4 + c \log_2 n^{1/8} + c \log^{1/4} + c \log^{1/2} + c \log n^{1/2}.$

$\vdots$

$T(n^{1/2})^k + c \log_2 n \sum_{i=0}^{k-1} \left(\frac{1}{2}\right)^i$

Assume;

$n^{(1/2)^k} = 2$

$\log n^{1/2k} = \log_2 2$

$\frac{1}{2}k \log_2 n = 1$

$\log_2 n = 2^k$

$\log_2 \log_2 n = \log 2^k$

$k = \log_2 \log_2 n$

$T(n) = T(2) + c \log_2 n \sum_{i=0}^{\log \log n - 1} \left(\frac{1}{2}\right)^i$

$= 1 + c \log_2 n \sum_{i=0}^{\lg \lg n - 1} \left(\frac{1}{2}\right)^i$

$= O(\log_2 n)$

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4. $T(n) = T(n/2) + T(\sqrt{n}) + \theta(n)$

$= 2T(n/2) + cn$ // taking dominant term

$= 2(2T(n/4) + cn/2) + cn$

$= 4T(n/4) + 2cn/2) + cn$

$= 8T(n/8 + cn + cn + cn$

$\vdots$

$2^k T(n/2^k) + kcn$

Assume;

$n/2^k = 1$

$n = 2^k$.

$\lg_2 n = \lg_2 2^k$

$\lg_2 n = k$.

$T(n) = 2^{\lg n} - nT(1) + \lg n \cdot cn$

$= O(n \lg_2 n)$.

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5. $T(n) = 2T(n/3) + \theta(n \lg n)$

$= 2T(n/3) + c \cdot n \lg n$

$= 2\left(2T(n/9) + c \cdot \frac{n}{3} \log \frac{n}{3}\right) + c \cdot n \lg n$

$= 4T(n/9) + 2c \cdot n/3 \log n/3 + cn \log n$

$= 4T \cdot \left(2T(n/27) + 2c(n/9) \log(n/9)\right) + Cn \log n$

$= 8T(n/27) + 2cn/9 \log n/9 + c\, cn \log n$

$\vdots$

$= 2^k (n/3k) + cn \sum_{i=0}^{k-1} (2/3)^i \log (n/3)^i$

Assume;

$n/3^k = 1$

$n = 3^k$

$\log_3 n = \log_3 3^k$

$k = \log_3 n$

$T(n) = 2^k (n/3k) + cn \sum_{i=0}^{k-1} (2/3)^i \log (n/3)^i$

$= 2^{\log_3 n} (n/3^{\log_3 n}) + cn \sum_{i=0}^{\log_3 n - 1} \left(\frac{2}{3}\right)^i \log (n/3)^i$

$= O(n \log n)$

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6. $T(n) = T(n/5) + \theta((lg_2 n)^2)$

$= T(n/5) + c(lg\, n)^2$

$= T(n/5^2) + c(\log n/5)^2 + c(\log_2 n)^2$

$= T(n/5^3) + c(\log n/5^2)^2 + c(\log_2 n/5)^2 + c(\log n)^2$

$\vdots$

$T(n/5^k) + c\sum_{i=0}^{k-1} \log(n/5^i)^2$

Assumes

$\frac{n}{5^k} = 1$ $\quad n = 5^k$

$\log_5 n = \log 5^k$

$k = \log_5 n$

$T(n) = (n/5^k) + c\sum_{i=0}^{k-1} \log(n/5^i)^2$

$= T(1) + c\sum_{i=0}^{\log_5 n - 1} \log(n/5^i)^2$

$= 1 +$

$= O(lg_2 n)^2$

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7. $T(n) = 3T(n/5) + \theta((\lg_2 n)^2)$.

$= 3\left(3T(n/5^2) + c.\left(\lg \frac{n}{5}\right)^2\right) + c.\lg n/5$

$= 9T(n/5^2) + 3c(\log n/5)^2 + c\log n/5$

$= 9T\left(3T\left(n/5^3\right) + 3c(\log n/5)^3\right) + c\log n/5 + c\log n/5$

$\vdots$

$= 3^k(n/5^k) + c\sum_{i=0}^{k-1} \log (n/5)^i$

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8. $T(n) = T(n/3) + T(2n/6) + \theta(n)$, $T(1) = 1$.

$T(n/3) + T(n/3) + c \cdot n$.

$2T(n/3) + cn$.

substituting:

$2(2T(n/9) + c \cdot n/3) + c \cdot n$

$4T(n/9) + 2c \cdot n/3 + c \cdot n$

$\cancel{4T(n/9}\; 4(2T(n/27) + c \cdot n/9) + 2c \cdot n/3 + c \cdot n$.

$8T(n/27) + 4cn/9 + 2cn/3 + cn$

⋮

$2^k T(n/3^k) + cn$.

Assume;

$\frac{n}{3^k} = 1$

$n = 3^k$

$\log_3 n = \log 3^k$

$\log_3 n = k$.

$2^{\lg_3 n}(T(1)) + cn$

$n^{\lg^3}(1) + cn$

$O(n)$.

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