

Average case

$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{j=2}^n \frac{t_j}{2} + c_6 \sum_{j=2}^n \left(\frac{t_j}{2} - 1\right) + c_7 \sum_{j=2}^n \left(\frac{t_j}{2} - 1\right) + c_8(n-1)$$

$$S_n = \frac{t_j}{2}$$

$$= S_1 + S_2 + S_3 + S_4 + \dots + S_n$$

$$= \frac{1}{2} + 1 + \frac{3}{2} + 2 + \dots + \frac{n}{2}$$

$$= \frac{n}{2} \left((2 \times \frac{1}{2}) + (n-1) \times \frac{1}{2} \right)$$

$$= \frac{n \left(\frac{1}{2}n + \frac{1}{2} \right)}{2}$$

$$S_n = \frac{t_j}{2} - 1$$

$$= S_1 + S_2 + S_3 + S_4 + \dots + S_n$$

$$= -\frac{1}{2} + 0 + \frac{1}{2} + 1 + \dots + \left(\frac{n}{2} - 1\right)$$

$$= \frac{n \left((2 \times \frac{1}{2}) + (n-1) \times \frac{1}{2} \right)}{2}$$

$$= \frac{n \left(\frac{1}{2}n - \frac{3}{2} \right)}{2}$$

$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{j=2}^n \frac{t_j}{2} + c_6 \sum_{j=2}^n \left(\frac{t_j}{2} - 1\right) + c_7 \sum_{j=2}^n \left(\frac{t_j}{2} - 1\right) + c_8(n-1)$$

$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5 \left(\frac{n \left(\frac{1}{2}n + \frac{1}{2} \right)}{2} - \frac{1}{2} \right) +$$

$$c_6 \left(\frac{n \left(\frac{1}{2}n - \frac{3}{2} \right)}{2} + \frac{1}{2} \right) + c_7 \left(\frac{n \left(\frac{1}{2}n - \frac{3}{2} \right)}{2} + \frac{1}{2} \right) + c_8(n-1)$$

$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5 \left(\frac{\frac{1}{2}n^2 + \frac{1}{2}n}{2} - \frac{1}{2} \right) + \\ c_6 \left(\frac{\frac{1}{2}n^2 - \frac{3}{2}n}{2} + \frac{1}{2} \right) + c_7 \left(\frac{\frac{1}{2}n^2 - \frac{3}{2}n}{2} + \frac{1}{2} \right) + c_8(n-1)$$

$$T(n) = c_1 n + c_2 n - c_2 + c_4 n - c_4 + c_5 n^2 + c_5 n - \frac{1}{2} c_5 + \\ c_6 n^2 - 3c_6 n + \frac{1}{2} c_6 + c_7 n^2 - 3c_7 n + \frac{1}{2} c_7 + c_8 n + c_8$$

$$T(n) = (c_5 + c_6 + c_7)n^2 + (c_1 + c_2 + c_4 + c_5 - 3c_6 - 3c_7 + c_8)n \\ + (-c_2 - c_4 - \frac{1}{2}c_5 + \frac{1}{2}c_6 + \frac{1}{2}c_7 - c_8)$$

$$T(n) = an^2 + bn + c$$

∴ we can say average case is same as worst case.