Ans to the Q. No-5

$$x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-2y=x^2\sin(\ln x) \quad \ldots \quad (i)$$

considering : $x=e^t$

$\Rightarrow xe^{-t}=1$

$\therefore \frac{dy}{dx}=e^{-t}\frac{dy}{dt}$ and $\frac{d^2y}{dx^2}=\left(\frac{d^2y}{dt^2}-\frac{dy}{dt}\right)e^{-2t}$

substituting the values to eqn (i)

$$x^2\left(\frac{d^2y}{dt^2}-\frac{dy}{dt}\right)e^{-2t}+3xe^{-t}\frac{dy}{dt}-2y=e^{t^2}\sin(\ln e^t)$$

$$\Rightarrow\left(\frac{d^2y}{dt^2}-\frac{dy}{dt}\right)+3\frac{dy}{dt}-2y=\sin t$$

$$\Rightarrow \frac{d^2y}{dt^2}+2\frac{dy}{dt}-2y=\sin t \quad \ldots \quad (ii)$$

which is a new 2nd order linear non-homogeneous equation.

from equation ② a probable solution can be

$y = a\cos(t) + b\sin(t)$; [where a and b are constant]

differentiating with respect to $\boxed{t=x}$ we find:

$y' = -a\sin(x)$ ~~+ $b\sin$~~ $+ b\cos x$

$\Rightarrow y'' = -a\cos(x) - b\sin x$

putting the values to equation ② we get:

$$\left(-a\cos(x) - b\sin(x)\right) - 4\left(-a\sin x + b\cos x\right) - 2\left(a\cos t + b\sin t\right) = \sin t$$

$$\Rightarrow -a\cos(x) - b\sin(x) + 4a\sin x - 4b\cos x - 2a\cos t + 2b\sin t = \sin t$$

$$\Rightarrow -a\cos(x) - 4b\cos x - 2a\cos t - b\sin(x) - \cancel{4b}s4a\sin(x) + 2b\sin t = \sin t$$

equating coefficients of $\cos x$ and $\sin x$ we find:

$-a - 4b - 2a = 0$

$\Rightarrow -3a - 4b = 1 \quad \ldots \text{(iii)}$

and,

$-b - 4a + 2b = 0$

$\Rightarrow b - 4a = 1$

$\Rightarrow b = 1 + 4a \quad \ldots \text{(iv)}$

~~repal~~ putting the value of b in equation (iii) we get:

$-3a - 1 - 4a = 1$

$\Rightarrow -7a = 2$

$\Rightarrow a = -\frac{2}{7}$

putting value of a to equation (iv)

$b = 1 - \frac{4 \cdot 2}{7}$

$= \frac{7-8}{7}$

$= -\frac{1}{7}$

therefore the probable solotion becomes:

$y_p = -\frac{2}{7}\cos t - \frac{1}{7}\sin t$.

complementary function

from equation (ii) we can

say

$m^2 + 2m - 2 = 0$

$\Rightarrow m^2 + m + m + 1 - 3 = 0$

$\Rightarrow m(m+1) + (m+1) - 3 = 0$

$\Rightarrow (m+1)^2 = 3$

$\Rightarrow m+1 = \pm\sqrt{3}$

$\Rightarrow m = +\sqrt{3} - 1 \ , \ -\sqrt{3} - 1$

Thus the homogeneous solution (ii) can have

the solution:

$y_c = Ae^{(\sqrt{3}-1)t} + Be^{(-\sqrt{3}-1)t}$

for general solution, of

$y(t) = y_c + y_p$

$= Ae^{(\sqrt{3}-1)t} + Be^{(-\sqrt{3}-1)t} - \frac{2}{7}\cos t$

$- \frac{1}{7}\sin t$

since $x = e^t$

$\Rightarrow t = \ln x$

$\therefore$ we get,

$y = Ae^{(\sqrt{3}-1)\ln x} + Be^{(\sqrt{3}-1)\ln x} - \frac{2}{7}\cos(\ln x)$

$- \frac{1}{7}\sin(\ln x).$

$= Ae^{\ln x^{(\sqrt{3}-1)}} - Be^{\ln x^{\sqrt{3}+1}} - \frac{2}{7}\cos(\ln x)$

$- \frac{1}{7}\sin(\ln x)$

$= A(\sqrt{3}-1) - B(\sqrt{3}+1) - \frac{2}{7}\cos(\ln x) - \frac{1}{7}\sin(\ln x)$

which the general solution.