i) $(x^2+2xy-y^2)dx+(y^2+2xy-x^2)dy=0$

non-linear, homogeneous equation.

$$M = x^2+2xy-y^2$$

$$M_y = 0+2x-2y$$

$$M_y = 2x-2y$$

$$N = y^2+2xy-x^2$$

$$N_x = 0+2y-2x$$

$$N_x = -2x+2y$$

as $M_y \neq N_x$ so it is not exact.

$$Mx = x(x^2+2xy-y^2)$$

$$= x^3+2x^2y-xy^2$$

$$Ny = y(y^2+2xy-x^2)$$

$$= y^3+2xy^2-x^2y$$

Intregrating Factor $= \dfrac{1}{Mx+Ny}$

$$= \frac{1}{x^3+2x^2y-xy^2+y^3+2xy^2-x^2y}$$

$$= \frac{1}{x^3+x^2y+xy^2+y^3}$$

$$= \frac{1}{x^2(x+y)+y^2(x+y)}$$

$$= \frac{1}{(x^2+y^2)(x+y)}$$

$$\frac{1}{(x^2+y^2)(x+y)}(x^2+2xy-y^2)\,dx + \frac{1}{(x^2+y^2)(x+y)}(y^2+2xy-x^2)\,dy = 0$$

$$\left(\frac{x^2+xy+xy-y^2}{(x^2+y^2)(x+y)}\right)dx + \left(\frac{y^2+xy+xy-x^2}{(x^2+y^2)(x+y)}\right)dy = 0$$

$$\cancel{\left(\frac{x(x+y)-y(x+y)}{(x^2+y^2)(x+y)}\right)dx + y(y+x)-x(}$$

$$\cancel{\left(\frac{x(x+y)+y(x-y)}{(x+y)^2(x+y)}\right)dx + \frac{y(y+x)+x(y-x)}{(x+y)^2(x}}$$

$$\left(\frac{x(x+y)+y(x-y)}{(x^2+y^2)(x+y)}\right)dx + \left(\frac{y(y+x)+x(y-x)}{(x^2+y^2)(x+y)}\right)dy = 0$$

$$\frac{(x+y)(x-y)}{(x^2+y^2)(x+y)}\,dx + \frac{(x+y)(y-x)}{(x^2+y^2)(x+y)}\,dy = 0$$

$$\frac{x-y}{x^2+y^2}\,dx + \frac{y-x}{x^2+y^2}\,dy = 0$$

$$M = \frac{x-y}{x^2+y^2}$$

$$M_y = \frac{(x^2+y^2)(-1)-(x-y)(2y)}{(x^2+y^2)^2}$$

$$\cancel{M_y = -x^2-y^2-x+y}$$

$$M_y = \frac{-x^2-y^2-(2xy-2y^2)}{(x^2+y^2)^2}$$

$$= \frac{-x^2-y^2-2xy+2y^2}{(x^2+y^2)^2}$$

$$M_x = \frac{-x^2 - 2xy + y^2}{(x^2+y^2)^2}$$

$$N = \frac{y-x}{x^2+y^2}$$

$$N_x = \frac{x^2+y^2(-1) - (y-x)(2x)}{(x^2+y^2)^2}$$

$$= \frac{-x^2 - y^2 - 2xy + 2x^2}{(x^2+y^2)^2}$$

$$= \frac{x^2 - 2xy - y^2}{(x^2+y^2)^2}$$

3) $\frac{di}{dt} + \frac{1}{t} i = f(t)$; $i(1) = 0$, $f(t) = \begin{cases} \sin t & 1 \leq t < \pi \\ 0 & t \geq \pi \end{cases}$

$$\mu(t) = e^{\int \frac{1}{t} dx}$$

$$= e^{\ln t} \Rightarrow t$$

when $f(t) = 0$,

$$i \cdot \mu(t) = \int 0 \cdot \mu(t)\, dt + c$$

$$i \times t = \int 0 \times t \; dt + c$$

$$i \times t = c$$

$$0 \times 1 = c$$

$$0 = c$$

$$i \times t = 0$$

when $f(t) = \sin t$

$$i \times t = \int \sin t \times t \; dt + c$$

~~$i \times t = \ldots$~~

~~$i \times t = [\ldots$~~

$$i \times t = [-t \cos(t) + \sin(t)] + c$$

$$0 \times 1 = -1 \cos(1) + \sin(1) + c$$

$$0 = 0.3 + c$$

$$c = -0.3$$

$$it = -t\cos(t) + \sin(t) - 0.3$$

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When. $f(t) = 0$

$it = \int 0 \times t \, dt + c$

$it = c$

$i = \frac{1}{t} c$.