Q3) Given,

$\beta_n = \alpha / \bar{O}_n$ , $\bar{O}_n = \bar{O}_{n-1} + \alpha(1 - \bar{O}_{n-1})$

$\bar{O}_0 = 0$

$Q_{n+1} = Q_n + \beta_n (R_n - Q_n)$ [Considering for a particular action].

$Q \rightarrow$ Estimated value function

$\beta_n \rightarrow$ Step size

$R_n \rightarrow$ Reward

$n \rightarrow$ time step.

at $n = 1$

$Q_2 = Q_1 + \beta_1 (R_1 - Q_1)$

$\beta_1 = \dfrac{\alpha}{\bar{O}_1}$ $\bar{O}_1 = \bar{O}_0 + \alpha(1 - \bar{O}_0)$

$= 0 + \alpha$

$\beta_1 = 1$

$Q_2 = Q_1 + R_1 - Q_1$

$\boxed{Q_2 = R_1}$

This makes it independent of $Q_1$

Considering the general form also,

$$Q_{n+1} = Q_n + \beta_n (R_n - Q_n)$$

$$Q_{n+1} = \beta_n R_n + Q_n (1 - \beta_n)$$

$$Q_{n+1} = \beta_n R_n + (1-\beta_n)[\beta_{n-1} R_{n-1} + (1-\beta_{n-1}) Q_{n-1}]$$

$$Q_{n+1} = \beta_n R_n + (1-\beta_n)\beta_{n-1} R_{n-1} + (1-\beta_n)(1-\beta_{n-1}) Q_{n-1}$$

$$Q_{n+1} = \beta_n R_n + (1-\beta_n)\beta_{n-1} R_{n-1} + \dots\dots +$$
$$(1-\beta_n)(1-\beta_{n-1})\dots(1-\beta_2)\beta_1 R_1 +$$
$$(1-\beta_n)(1-\beta_{n-1})\dots(1-\beta_2)(1-\beta_1) Q_1$$

$$\beta_1 = \alpha / \bar{o}_1 = 1$$

$1 - \beta_1 = 0$, hence the coefficient of $Q_1$ is zero making $Q_{n+1}$ independent of $Q_1$

$$Q_{n+1} = \sum_{i=1}^{n} R_i \beta_i \prod_{j=i+1}^{n} (1-\beta_j)$$