CSC446/546 —OR II: Simulations
Assignment 3
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Due date: November 17, 2015

- This assignment walks you through techniques and testing of random number generation and variates.
- Please use the chat-room on conneX course page to ask any questions related to this assignment.
- CSc 446 students can receive bonus marks by answering questions marked *"For CSc 546 students only"*
- Please mention what section (CSC 446 or 546) you belong to on your submssion.

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1. A random number generator uses the following algorithm to produce random numbers $X_i, i = 0, 1, 2, \ldots$. Write a program to generate 1000 numbers using this algorithm and test the generator for uniformity (using Chi-square test) for 0.05 level of significance (α). The test uses $n = 10$ intervals. Discuss any logical problems you might have encountered using this algorithm. The steps in the algorithm are:

Initialization:
    Set X_0=a real number between 0 and 1. Use at least 8 decimal places;
    Set X_1= another real number between 0 and 1. Use at least 8 decimal places;
while ($i \leq value$){
    $X_i = X_{i-1} + X_{i-2}$;
    if $X_i \geq 1, \{X_i = X_i - 1.0\}$;
    i++;}

Submit your code on conneX.

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2. This problem is to test the random number generator used in either java or C programs. Use standard libraries in either of these languages and produce ten random numbers. Then use Kolmogorv-Smirnov method with a 0.05 level of significance (α) to test whether the random number generator passes the uniformity test or not. What can you conclude from this? Generate 1000 numbers and perform a uniformilty test by conducting a Chi-Square test.

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3. Consider the multiplicative congruential generator under the following circumstances:

a. $a = 11, m = 16, X_0 = 7$

b. $a = 11, m = 16, X_0 = 8$

c. $a = 7, m = 16, X_0 = 7$

d. $a = 7, m = 16, X_0 = 8$

Generate enough values to complete a cycle and analyze each case. What inferences can be drawn? Is the maximum period achieved?

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4. Develop a random-variate generator for a random variable X with the pdf

$$f(x) = \begin{cases} e^{2x}, & -\infty < x \le 0 \\ e^{-2x}, & 0 < x < \infty \end{cases}$$

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5. Begining with the first number, test for the auto correlation between every third number for the following sequence of numbers (use $\alpha = 0.05$): 0.594, 0.928, 0.515, 0.055, 0.507, 0.351, 0.262, 0.797, 0.788, 0.442, 0.097, 0.798, 0.227, 0.127, 0.474, 0.825, 0.007, 0.182, 0.929, 0.852.

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6. Write a program to generate exponential random variate with $\lambda = 1$ using standard *rand()* function in 'C'. Note that the *rand()* function produces integer random numbers in the range [0,*RANDMAX*]. So, you need to first find the value of *RANDMAX* and use this to normalize (or scale) the generated number. Submit your code on conneX.

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7. Generate $N = 1000$ samples for the exponential random variate (X) that you developed in the problem above (Problem 6). Our goal is to see the behavior of the density function for this random variate. For this purpose, you will produce a histogram by observing the number of occurences for X in an interval. Use 11 bins in increments of 0.5 from 0 up to a max value of 5.5(i.e., [0,0.5),[0,5,1.0),...,[5.0,5.5)) to keep these counts. Count all the $X \ge 5$ into the last bin [5.0, 5,5). Normalize the counts to the number of samples (N). This gives the relative frequency and would be used as an approximation to the actual exponential density function $f(x) = exp(-x) = e^{-x}$ as $\lambda = 1$. Compare the developed histogram with actual density at the bin midpoints. Build the cumulative distributon from the histogram data and compare it with exponential CDF.

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8. ***(CSC 546 Only)*** Develop an acceptance-rejection technique for generating a geometric random variable X with parameter p on the range $0, 1, 2 \ldots$. We already discussed one method in the class. (*Hint:* X can be thought of as the number of trials before the first success occurs in a sequence of independent Bernoulli trials.)