CSC446/546 —OR II: Simulations

Assignment 2

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- This assignment walks you through various system modeling scenarios to conduct performance studies.
- Please use the chat-room on conneX course page to ask any questions related to this assignment.
- CSc 446 students can receive bonus marks by answering questions marked *"For CSc 546 students only"*

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1. Suppose a random variable X has the density function as shown in Figure 1.

a. Write the density equation $f(x)$ for this random variable.

b. Write the Cumulative Distribution Function (CDF) $F(x)$ of this random variable.

c. What is the value of c?

d. What is the probability $P(0.5 \leq X \leq 1.5)$?

e. What is $E[X]$? What is the variance of X?

f. What will be the density of Y if we used a transformation $Y = 2X$?

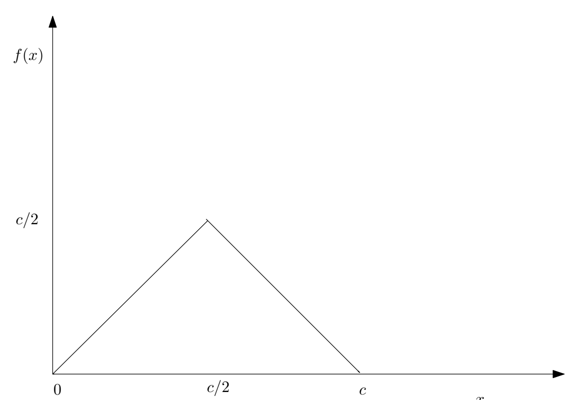

Figure 1: Density function of random variable X

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2. You are given a sequence of independent and identically distributed (IID) random variables $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$. Find the mean (expectation) and variance of $\mathbf{Y} = (\mathbf{X}_1 + \mathbf{X}_2 + \ldots + \mathbf{X}_n) = \sum_{i=1}^{n} \mathbf{X}_i$ given that mean of $\mathbf{X}_i = \eta$ and variance of $\mathbf{X}_i = \sigma^2_{\mathbf{X}_i} = \sigma^2$. Note that for two independent random variables X and Y, the expectation $E[XY] = E[X].E[Y]$ and $E[X+Y] = E[X] + E[Y]$. (**PS**:We need this result to solve Problem 8).

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3. A terminal at an airport has one queue to check-in for Terminal 1 passengers. After the check-in, Terminal1 passengers join a security check queue along with Terminal 2 passengers as shown in Figure 2. The arrivals are Poisson distributed with rates λ_1 and λ_2 for Terminal 1 and 2 passengers respectively. The service times are exponentially distributed with service rates μ_1, μ_S for Terminal 1 check-in and Security-Check respectively. The arrival rates are 10 customers/hour for Terminal

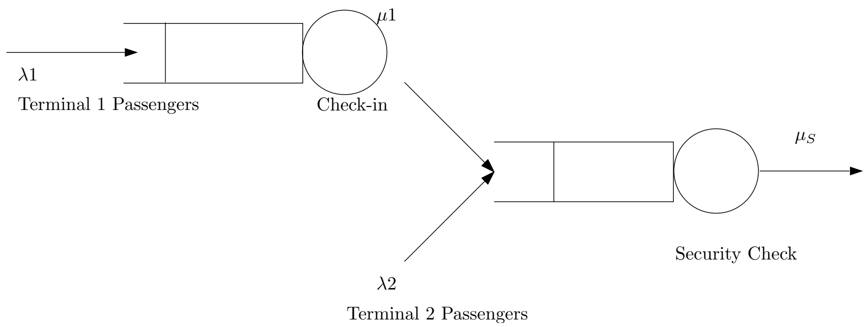

Figure 2: Queuing at an airport

1 passengers and 20 customers/hour for Terminal 2 passengers. The average service times at the Terminal 1 check-in is 2 minutes per customer. The Security-Check queue maintains an utilization of 80% all the time.

- What is the average service time at the Security-Check?
- What are the performance metrics (ρ, L, L_Q, w, w_Q) for Terminal 1 Check-in?
- What are the performance metrics (ρ, L, L_Q, w, w_Q) for Security-Check?
- What is the total average time spent by Terminal 1 passengers in the system?
- What is the total average number of Terminal 1 passengers in the system?
- What is the average time spent by Terminal 2 passengers in the system?

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4. Simulate the Security-Check queue in Problem 3, by re-using the sample code of single-queue and single-server queuing system (FIFO) provided in Assignment 1. Run the simulation 5 times with different seeds, with each run processing up to 50000 customers. Compute the average values across simulations for the performance metrics. Use a table to compare the simulation results with the theoretical results obtained from Problem 3. Please note that you have to collect the stats for the performance metrics (ρ, w, w_Q) in your code. You can use Little's law to obtain L and L_Q. Please submit this code along with your assignment.

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5. Arrivals to an airport are all directed to the same runway. At certain time of the day, these arrivals form a Poisson process with rate 30 per hour. The time to land an aircraft is a normal distribution with a mean and standard deviation of 90 seconds. Determine L_Q, w_Q, L and w for this airport. If a delayed aircraft burns $5000 worth of fuel per hour on the average, determine the average cost per aircraft of delay in waiting to land.

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6. Study the effect of *pooling servers* (having multiple servers draw customers from a single queue, rather than each having its own queue) by comparing the performance measures (ρ, L, w, w_Q, L_Q) for two $M/M/1$ queues, each with arrival rate λ and service rate μ, to an $M/M/2$ queue with arrival rate 2λ and service rate μ for each server. Which one is better?

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7. A self-service car wash has 4 washing stalls. When in a stall, a customer may choose from among three options: rinse only; wash and rinse; and wash, rinse, and wax. Each option has a fixed time to complete: rinse only, 3 minutes; wash and rinse, 7 minutes; wash, rinse, and wax, 12 minutes. The owners have observed that 20% of customers rinse only; 70% wash and rinse; and 10% wash, rinse, and wax. There are no scheduled appointments, and customers arrive at a rate of about 34 cars per hour. There is room for only 3 cars to wait in the parking lot, so, currently, many customers are lost. The owners wants to know how much more business they will do if they add another stall. Adding a stall will take away one space in the parking lot. Develop a queuing model of the system. Estimate the rate at which customers will be lost in the current and proposed system. Carefully state any assumptions or approximations you make.

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8. Patients arrive for a physical examination according to a Poisson process at the rate 1 per hour. The physical examination requires three stages, each one independently exponentially distributed with a service time of 15 minutes. A patient must go through all the three stages before the next patient is admitted to the treatment facility.

- What is the mean and variance of total examination time?
- Is this an exponential distribution?
- Compute the average number of delayed patients, L_Q for this system.

- Compute the total mean time a customer spends in the system.

(*Hint:* See problem 2).

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9. A repair and inspection facility consists of two stations (see Figure 3): a repair station with two technicians, and an inspection station with 1 inspector. Each repair technician works at the rate 3 items per hour; the inspector can inspect 8 items per hour. Approximately 10% of all items fail inspection and are sent back to the repair station. This percentage holds even for items that have been repaired two or more times. If items arrive at the rate of 5 per hour,

a. What is the long-run expected delay that items experience at each of the two stations, assuming a Poisson arrival process and exponentially distributed service times?

b. What is the maximum arrival rate that the system can handle without adding personnel?

(*Hint:* For any stable system, the "the law of conservation" must hold. That is, the rate of external arrivals to the system must be the same as the rate of departures from the system. Customers neither can be created nor destroyed. First, compute x using this principle).

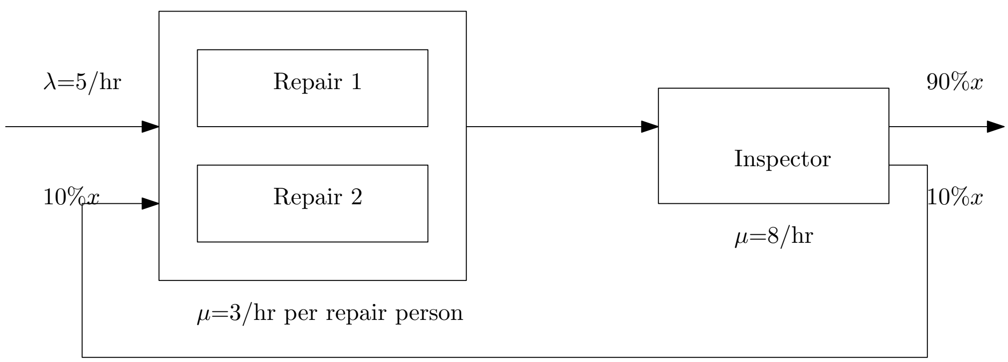

Figure 3: Repair and Inspection Facility

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10. ***(CSC 546 students only)*** Compute the average number in the system (L) from the state probabilities $P(L = k) = P_k = (1-\rho)\rho^k$ of an $M/M/1$ queuing system. (*Hint:* Write $\sum_{k=0}^{\infty} k\rho^k$ as $\rho\frac{\partial}{\partial\rho}\sum_{k=0}^{\infty}\rho^k$. Note that $\sum_{k=0}^{\infty}\rho^k$ is a sum of geometric series).