CSC446/546 —Operations Research II: Simulations

Assignment 1
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Useful Tip: For a single channel queuing system, if λ is the average arrival rate and μ is the average departure (service) rate, then the "utilization" of the system is defined as $\rho = \lambda/\mu$. Please note that $1/\lambda$ is the average inter-arrival (time between arrivals) time and $1/\mu$ is the average service time.

CSc 446 students can receive bonus marks by answering questions marked "For CSc 546 students only"

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1. A courier service receives shipment packages from both business and home customers with *equal probability*. Let us simulate the package arrivals by tossing a coin due to this equal probability. Assume that a home customer is represented by the outcome "Head" and a business customer by the outcome "Tail". The distribution of random time between package arrivals is specified as:

Time between arrivals (in minutes)	*Toss*
6	Head *(H)*
2	Tail *(T)*

Table 1: Distribution of time between Arrivals

Packages are processed by a single server in first-come-first-serve order. The processing time for each package is assumed to be *constant* at 2 minutes for each home customer and 4 minutes for each business customer. Construct a simulation table (similar to the one on Slide 26, Chapter 2 slides of class notes) for up to 20 customers. Assume the sequence for the 20 customers is: *THTTTHTHHTHHHTHTTTHT*. Start with simulation clock at t=*0* and assume that system starts empty (i.e, the first customer T arrives to the system after 2 mins as specified by the toss). Create the event list in the *chronological* order of events in a table format. Include event number, customer number, whether the event is an arrival or a departure, simulation clock when that event will be processed.

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2. For the Problem 1 above, compute the following from your simulation table:

a. Average service time as computed from the simulation table

b. Average inter-arrival time as computed from the simulation table

c. Server utilization as computed from the simulation table

d. Theoretical Server utilization (calculate from theoretical inter-arrival times and service times). Is there any discrepancy between this and the simulated results? If so why?

e. Average time customer spends in the system

f. Assume that the packages received from home and business customers are not of equal probability, but say, with a probability p a package is received from the home customers and with a probability $1 - p$ it is from the business customers.

   1. What will be the average service time?
   2. What are the bounds of p so that the system is stable?

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3. Let us re-visit the Able-Baker call center problem (Slides 31 to 39 of Chapter 2 class notes). Change the policy to "random" on who gets the call when the servers are idle and when a new call arrives; i.e., either Able or Baker is chosen with equal probability (e.g., by tossing a coin). Modify the simulation steps 1, 2 and 3 (Slides 33 to 35) to take care of this new policy.

a. Modify the posted spreadsheet of Call center problem to include this policy.

b. Modify the spreadsheet to include other performance metrics (see Chapter 2 slides 27 to 28 as an example). Specifically compute probability of idle server for both Able and Baker.

c. Run one trail with 100 customers and report the performance metrics that you obtained through this simulation.

d. Run an experiment with 200 trails and note the average caller delay with the modified policy. Compare this by repeating this experiment with the original policy. Which policy is better and why? Justify your analysis.

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4. This problem is to study the Java example code of the single server queue provided on the course website. Please note that you may need to compile the java files in this file set on your platform by using an appropriate compiler. The simulation is run by: "java Sim xxxx", where xxxx is the initial seed number. Indicate what seed numbers you used. **Note that the mean service times and arrival times are just printed off from the input in the code provided. You need to modify the code so that you actually measure the actual inter-arrival and service times observed and then print the final averages for this exercise**.

a. What is the mean and type of distribution used for the inter-arrival time?

b. What is the mean and type of distribution used for the service time?

c. What is the theoretical server Utilization?

d. Run the simulation for 1000 customers with two different seeds (two runs) and note the outputs: mean inter-arrival time, mean service time, server utilization and mean response time

e. Run the simulation for 5000 customers with two different seeds (two runs) and note the outputs: mean inter-arrival time, mean service time, server utilization and mean response time.

f. What do you infer from the above runs?

g. Change the service distribution to *exponential* with the same mean and note the outputs: mean inter-arrival time, mean service time, server utilization and mean response time for 1000 customers with two different seeds

h. Run the above simulation with 5 different seeds (five runs) for 5000 customers and note the mean inter-arrival time, mean service time, server utilization and mean response time in the program output. Calculate the mean and variance of each of these observed variables across the runs. What do you infer from the results? Are they any good?

i. **(For CSc 546 students only)**This Java simulation code does not include any warm up period. Modify the code to start collecting statistics after a specified "warm up period". As discussed in the class, warm up period is used to establish a steady state behavior of the system. Instead of time, you can use a specified number of customers for the warm up period. Repeat the simulation listed just above (five runs of 500 customers each) with two different sets of warm up periods: 1) warm up period ends after 100 customers; 2) warm up period ends after 1000 customers. What do you infer from these results?

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5. This experiment is to see the outcomes of service times (Example2.2 Service Times). This spreadsheet can run one experiment to produce service times for 25 callers and computes the frequency table for these service times. Check whether this frequency table matches with the programmed distribution or not. One experiment is not enough. So you have to repeat the experiment, note the frequencies in each iteration and compute the cumulative counts. Conduct 1, 5 and 10 trails and compute the frequency tables and provide a summary on how well it compares with the intended distribution. Provide justification to your conclusions.

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6. This problem looks at the effect of variance ***(what if scenarios)*** in the air-supply expedition:

a. Set $\sigma_x = 600$ meters and σ_y =300 meters in the spread sheet of target hitting example. Conduct a simulation of 200 trials. What was the average number of hits?

b. Repeat above with a simulation of 400 trials. What was the average number of hits?

c. Set $\sigma_x = 50$ meters and σ_y =300 meters in the spread sheet of target hitting example. Conduct a simulation of 200 trials. What was the average number of hits?

d. Set $\sigma_x = 50$ meters and σ_y =500 meters in the spread sheet of target hitting example. Conduct a simulation of 200 trials. What was the average number of hits?

e. Set $\sigma_x = 2\sigma_y$. What is the value of σ_x if the average number of hits is to be about 0.6 based on on experiment of 400 trials.

f. What do you infer from these simulations?

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7. This problem is to use simulation for parameter estimation. Assume that you are given a weighted coin. When you toss the coin, the outcome is heads with probability p and tails with probability $1 - p$. Write a simple program (in any language or tool of your choice) to estimate the parameter p. Run the simulation until it simulates 10000 times of tossing.

a. When $p = 0.3$, what is its estimated value, $\hat{p}$? What is the normalized estimation error, calculated as $|p - \hat{p}|/p$?

b. When $p = 0.1$, what is its estimated value, $\hat{p}$? What is the normalized estimation error, calculated as $|p - \hat{p}|/p$?

c. when $p = 0.001$, what is its estimated value, $\hat{p}$? What is the normalized estimation error, calculated as $|p - \hat{p}|/p$?

d. when $p = 0.0001$, what is its estimated value, $\hat{p}$? What is the normalized estimation error, calculated as $|p - \hat{p}|/p$?

e. What can you conclude from this experiment? What is your suggestion to reduce the normalized estimation error? Please try your solution to see if it works well.

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8. **(For CSc 546 students only)** The single-channel Java simulation code that is provided to you uses FIFO (First-In-First-Out) queuing structure to en-queue and de-queue customers. The purpose of this problem is to study performance of the system if the queue is changed to a LIFO (Last-In-First-Out). Modify the code provided to achieve the LIFO order to service customers. Compare the performance of LIFO order of service with the FIFO order by running *appropriate* simulations.