Updated Documentation for BMW and Mercedes-Benz Car Specifications Forecast using Facebook Prophet (Nonlinear Regression)

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October 10, 2024

Introduction

This document offers comprehensive documentation for extending the findings of the article by Ilya Yuskevich, Ksenia Smirnova, Rob Vingerhoeds, and Alessandro Golkar, titled ”Model-based approaches for technology planning and roadmapping: Technology forecasting and game-theoretic modelling”. It encompasses an overview of the methodology, code snippets, and detailed explanations to ensure reproducibility and understanding. While the article employs Pareto frontier estimation to forecast car specifications, this document aims to utilize nonlinear regression to develop models of various figures of merit directly, which will then be combined to forecast these car specifications.

Data Gathering

This section provides an overview of the data gathering and cleaning process. The dataset used for this project is the Car Specification Dataset from 1945 to 2020, which can be found on Kaggle:

`https://www.kaggle.com/datasets/jahaidulislam/car-specification-dataset-1945-2020`

Due to the comprehensive nature of this dataset, it provides a broad range of information encompassing car models from various manufacturers over several decades.

Horsepower Forecasting using Facebook Prophet

Loading and Preparing the Dataset

In this section, we load the dataset, select the necessary columns, clean the data, and prepare it for model training.

```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from prophet import Prophet
import pickle

# Variables
file_path = 'CompleteDataset.csv' # data source
future_year = 2050 # cutoff year for forecasts
cap_value = 2300  # cap value, adjust as needed
seasonality_prior_scale = 3  # seasonality prior scale, adjust as
    needed
saturation_min = 50  # saturation minimum, adjust as needed
uncertainty_interval = 0.9  # uncertainty interval, adjust as
    needed
threshold = 0.05  # Threshold for printing out the car
    specification of outliers (outside of the uncertainty interval)
partial_year_cut = 2010  # Variable to choose which year the
    partial dataset can be cut

# Load the dataset
df = pd.read_csv(file_path, low_memory=False)

# Inspect column names to ensure they match expected names
print("Column names in the dataset:", df.columns.tolist())

# Selecting necessary columns based on actual column names in the
    dataset
necessary_columns = [
    'Make', 'Modle', 'Year_from', 'engine_hp', 'curb_weight_kg', '
        full_weight_kg',
    'mixed_fuel_consumption_per_100_km_l', 'acceleration_0_100_km/
        h_s'
]

# Check if all necessary columns are present
missing_columns = [col for col in necessary_columns if col not in
    df.columns]
if (missing_columns):
    raise KeyError(f"Missing columns in the dataset: {
        missing_columns}")

df = df[necessary_columns]

# Renaming columns to match the filtered dataset
df.rename(columns={
    'Modle': 'Model',
    'curb_weight_kg': 'Curb_Weight_kg',
    'full_weight_kg': 'Max_Weight_kg',
    'mixed_fuel_consumption_per_100_km_l': '
        fuel_efficiency_100_km_per_l',
    'acceleration_0_100_km/h_s': 'Acceleration_0_100_km_h_s',
    'engine_hp': 'Horsepower'
}, inplace=True)

# Cleaning up the data
df = df.dropna()
```

```python
df = df[df['Make'].isin(['BMW', 'Mercedes-Benz'])]
```

We begin by importing the necessary libraries and defining the variables used in the analysis. The dataset is loaded from a CSV file, and the column names are checked to ensure they match the expected names. We then select the necessary columns, rename them for consistency, and clean the data by removing any missing values and filtering for BMW and Mercedes-Benz cars only.

Extracting Maximum Horsepower Data

In this section, we extract the maximum horsepower data for each year for both BMW and Mercedes-Benz and prepare it for the Prophet model.

```python
# Extracting the maximum horsepower for each year for BMW and
    Mercedes-Benz
bmw_df = df[df['Make'] == 'BMW']
mercedes_df = df[df['Make'] == 'Mercedes-Benz']

# Creating a single 'Year_Released' column using 'Year_from'
bmw_df['Year_Released'] = bmw_df['Year_from']
mercedes_df['Year_Released'] = mercedes_df['Year_from']

bmw_max_hp = bmw_df.groupby('Year_Released')['Horsepower'].max().
    reset_index()
mercedes_max_hp = mercedes_df.groupby('Year_Released')['Horsepower'
    ].max().reset_index()

# Renaming columns to fit Prophet's expected input
bmw_max_hp.columns = ['ds', 'y']
mercedes_max_hp.columns = ['ds', 'y']

# Convert 'ds' to datetime format
bmw_max_hp['ds'] = pd.to_datetime(bmw_max_hp['ds'], format='%Y')
mercedes_max_hp['ds'] = pd.to_datetime(mercedes_max_hp['ds'],
    format='%Y')

# Set cap for logistic growth
bmw_max_hp['cap'] = cap_value
mercedes_max_hp['cap'] = cap_value

# Filter data until 2017
bmw_max_hp = bmw_max_hp[bmw_max_hp['ds'] <= '2017-12-31']
mercedes_max_hp = mercedes_max_hp[mercedes_max_hp['ds'] <= '
    2017-12-31']

# Adding a custom regressor to enforce an increasing trend
bmw_max_hp['time_trend'] = (bmw_max_hp['ds'] - bmw_max_hp['ds'].min
    ()).dt.days
mercedes_max_hp['time_trend'] = (mercedes_max_hp['ds'] -
    mercedes_max_hp['ds'].min()).dt.days
```

We extract the maximum horsepower for each year for BMW and Mercedes-Benz. The data is grouped by the release year, and the maximum horsepower values are computed. The columns are then renamed to fit Prophet's expected input format, and the dates are converted to datetime format. A cap value is set for logistic growth, and data is filtered until 2017. Additionally, a custom regressor is added to enforce an increasing trend.

Training the Prophet Models

In this section, we train the Prophet models for BMW and Mercedes-Benz with the additional time trend regressor.

```
# Training the Prophet models with the additional regressor for BMW
bmw_model = Prophet(growth='logistic', seasonality_prior_scale=seasonality_prior_scale, interval_width=uncertainty_interval)
bmw_model.add_regressor('time_trend')
bmw_model.fit(bmw_max_hp)

# Training the Prophet models with the additional regressor for Mercedes-Benz
mercedes_model = Prophet(growth='logistic', seasonality_prior_scale=seasonality_prior_scale, interval_width=uncertainty_interval)
mercedes_model.add_regressor('time_trend')
mercedes_model.fit(mercedes_max_hp)

# Save the models
with open('Models/bmw_model_HP.pkl', 'wb') as f:
    pickle.dump(bmw_model, f)

with open('Models/mercedes_model_HP.pkl', 'wb') as f:
    pickle.dump(mercedes_model, f)
```

We train the Prophet models for BMW and Mercedes-Benz using the data prepared in the previous steps. The models are trained with a logistic growth model and an additional time trend regressor. The trained models are then saved for future use.

Making Future Predictions

In this section, we make future predictions for BMW and Mercedes-Benz until the defined future year using the trained Prophet models.

```
# Making future predictions until the defined future year for BMW
future_bmw = bmw_model.make_future_dataframe(periods=future_year - bmw_max_hp['ds'].max().year, freq='Y')
future_bmw['time_trend'] = (future_bmw['ds'] - bmw_max_hp['ds'].min()).dt.days
future_bmw['cap'] = cap_value

bmw_forecast = bmw_model.predict(future_bmw)

# Making future predictions until the defined future year for Mercedes-Benz
future_mercedes = mercedes_model.make_future_dataframe(periods=future_year - mercedes_max_hp['ds'].max().year, freq='Y')
future_mercedes['time_trend'] = (future_mercedes['ds'] - mercedes_max_hp['ds'].min()).dt.days
future_mercedes['cap'] = cap_value
```

```

mercedes_forecast = mercedes_model.predict(future_mercedes)
```

We use the trained Prophet models to make future predictions for BMW and Mercedes-Benz until the specified future year. The future dataframes are created with the additional time trend regressor and the cap value.

Visualizing the Forecasts

In this section, we visualize the forecasted maximum horsepower for BMW and Mercedes-Benz until the defined future year.

```
# Plotting the forecasts
plt.figure(figsize=(12, 8))

# Plot BMW
plt.subplot(2, 1, 1)
plt.plot(bmw_max_hp['ds'], bmw_max_hp['y'], 'o', label='Observed BMW Max HP')
plt.plot(bmw_forecast['ds'], bmw_forecast['yhat'], label='Forecasted BMW Max HP')
plt.fill_between(bmw_forecast['ds'], bmw_forecast['yhat_lower'], bmw_forecast['yhat_upper'], alpha=0.2)
plt.title(f'BMW Maximum Horsepower Forecast until {future_year}')
plt.xlabel('Year')
plt.ylabel('Horsepower')
plt.legend()

# Plot Mercedes-Benz
plt.subplot(2, 1, 2)
plt.plot(mercedes_max_hp['ds'], mercedes_max_hp['y'], 'o', label='Observed Mercedes-Benz Max HP')
plt.plot(mercedes_forecast['ds'], mercedes_forecast['yhat'], label='Forecasted Mercedes-Benz Max HP')
plt.fill_between(mercedes_forecast['ds'], mercedes_forecast['yhat_lower'], mercedes_forecast['yhat_upper'], alpha=0.2)
plt.title(f'Mercedes-Benz Maximum Horsepower Forecast until {future_year}')
plt.xlabel('Year')
plt.ylabel('Horsepower')
plt.legend()

plt.tight_layout()
plt.show()
```

We visualize the forecasted maximum horsepower values for BMW and Mercedes-Benz using Matplotlib. The plots show the observed and forecasted values along with the uncertainty intervals.

Results

Horsepower Forecasting Results

The following results display the observed and forecasted maximum horsepower for BMW and Mercedes-Benz until 2050. Additionally, the normalized error rates between the partial and full datasets, as well as the error rates between actual and model predictions from 1980 to 2017, are presented.

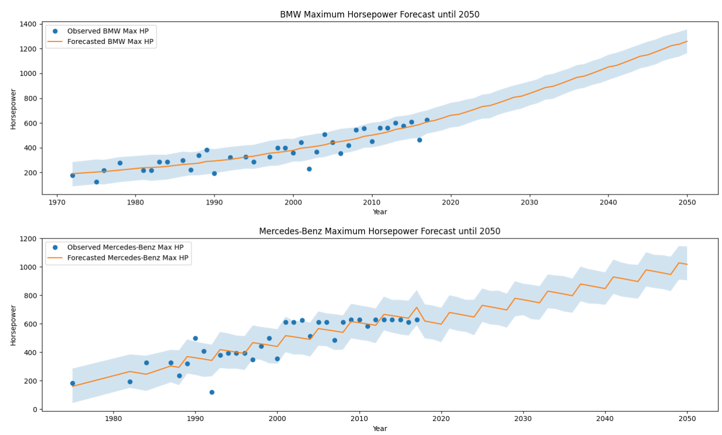

Figure 1: Observed and Forecasted Maximum Horsepower for BMW until 2050

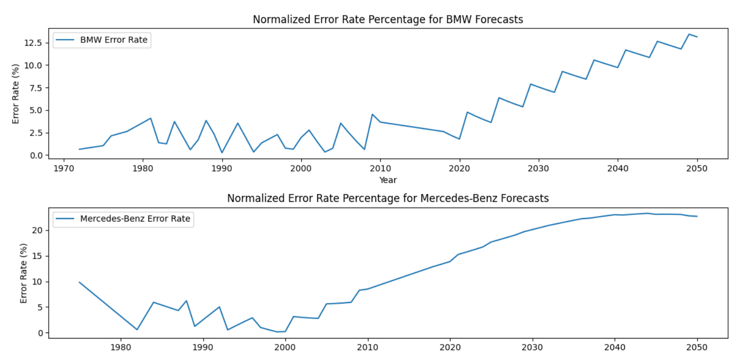

Figure 2: Observed and Forecasted Maximum Horsepower for Mercedes-Benz until 2050

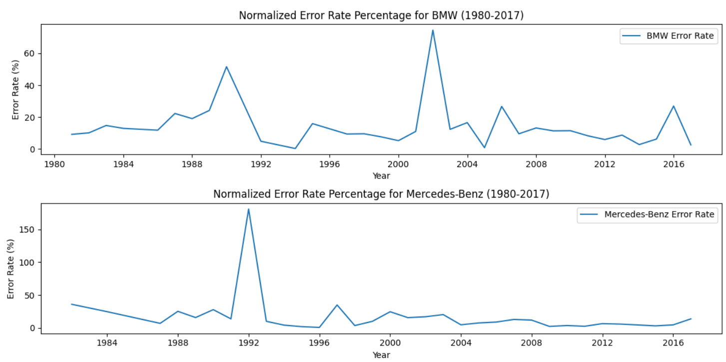

Figure 3: Normalized Error Rates for BMW and Mercedes-Benz Forecasts

The overall average error rate (partial vs full) is calculated as follows:

```python
# Calculate normalized error rate percentage
bmw_forecast_merged = bmw_forecast[['ds', 'yhat']].merge(bmw_forecast_partial[['ds', 'yhat']], on='ds', suffixes=('', '_partial'))
bmw_forecast_merged['error'] = abs(bmw_forecast_merged['yhat'] - bmw_forecast_merged['yhat_partial']) / bmw_forecast_merged['yhat'] * 100

mercedes_forecast_merged = mercedes_forecast[['ds', 'yhat']].merge(mercedes_forecast_partial[['ds', 'yhat']], on='ds', suffixes=('', '_partial'))
mercedes_forecast_merged['error'] = abs(mercedes_forecast_merged['yhat'] - mercedes_forecast_merged['yhat_partial']) / mercedes_forecast_merged['yhat'] * 100

# Calculate overall average error rate
combined_error = pd.concat([bmw_forecast_merged[['ds', 'error']], mercedes_forecast_merged[['ds', 'error']]])
overall_average_error_rate = combined_error['error'].mean()

print(f"Overall Average Error Rate (partial vs full): {overall_average_error_rate:.2f}%")
```

The overall average error rate (1980-2017 actual vs model) is calculated as follows:

```python
# Calculate and plot normalized error percentage between actual and model predictions (1980-2017)

# Filter actual data for years 1980-2017
bmw_max_hp_1980_2017 = bmw_max_hp[(bmw_max_hp['ds'] >= '1980-01-01') & (bmw_max_hp['ds'] <= '2017-12-31')]
mercedes_max_hp_1980_2017 = mercedes_max_hp[(mercedes_max_hp['ds'] >= '1980-01-01') & (mercedes_max_hp['ds'] <= '2017-12-31')]

# Make predictions for these years
bmw_max_hp_1980_2017['cap'] = cap_value
mercedes_max_hp_1980_2017['cap'] = cap_value
```

```
bmw_forecast_1980_2017 = bmw_model.predict(bmw_max_hp_1980_2017[['ds', 'time_trend', 'cap']])
mercedes_forecast_1980_2017 = mercedes_model.predict(mercedes_max_hp_1980_2017[['ds', 'time_trend', 'cap']])

# Calculate normalized error percentage
bmw_forecast_1980_2017['actual'] = bmw_max_hp_1980_2017['y'].values
bmw_forecast_1980_2017['error'] = abs(bmw_forecast_1980_2017['yhat'] - bmw_forecast_1980_2017['actual']) / bmw_forecast_1980_2017['actual'] * 100

mercedes_forecast_1980_2017['actual'] = mercedes_max_hp_1980_2017['y'].values
mercedes_forecast_1980_2017['error'] = abs(mercedes_forecast_1980_2017['yhat'] - mercedes_forecast_1980_2017['actual']) / mercedes_forecast_1980_2017['actual'] * 100

# Calculate overall average error rate
combined_error_1980_2017 = pd.concat([bmw_forecast_1980_2017[['ds', 'error']], mercedes_forecast_1980_2017[['ds', 'error']]])
overall_average_error_rate_1980_2017 = combined_error_1980_2017['error'].mean()

print(f"Overall Average Error Rate (1980-2017 actual vs model): {overall_average_error_rate_1980_2017:.2f}%")
```

The overall average error rate (partial vs full) of 8.91% represents the backtesting result where the model trained on data until 2010 was compared to the full data prediction. This backtesting period spans from 2010 to 2017, covering 7 years. The error rate indicates how well the model's predictions match the actual observed data for these years.

The overall average error rate (1980-2017 actual vs model) of 15.78% measures the discrepancy between the actual observed horsepower values and the model's predictions for the period from 1980 to 2017.

Acceleration Forecasting using Facebook Prophet

Loading and Preparing the Dataset

In this section, we load the dataset, select the necessary columns, clean the data, and prepare it for model training.

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from prophet import Prophet
import os
import pickle

# Variables
file_path = 'CompleteDataset.csv'
future_year = 2050
```

```
cap_value = 20  # Example cap value, adjust as needed
seasonality_prior_scale = 5  # Example seasonality prior scale, adjust as needed
saturation_min = 2  # Example saturation minimum, adjust as needed
uncertainty_interval = 0.8  # Example uncertainty interval, adjust as needed
threshold = 0.05  # Threshold for printing out the car specifications of outliers (outside of the uncertainty interval)
partial_year_cut = 2010  # Variable to choose which year the partial dataset can be cut

# Load the dataset
df = pd.read_csv(file_path, low_memory=False)

# Inspect column names to ensure they match expected names
print("Column names in the dataset:", df.columns.tolist())

# Selecting necessary columns based on actual column names in the dataset
necessary_columns = [
    'Make', 'Modle', 'Year_from', 'acceleration_0_100_km/h_s'
]
df = df[necessary_columns]

# Renaming columns to match the filtered dataset
df.rename(columns={
    'Modle': 'Model',
    'acceleration_0_100_km/h_s': 'Acceleration_0_100_km_h_s'
}, inplace=True)

# Cleaning up the data
df = df.dropna()
df = df[df['Make'].isin(['BMW', 'Mercedes-Benz'])]
```

We begin by importing the necessary libraries and defining the variables used in the analysis. The dataset is loaded from a CSV file, and the column names are checked to ensure they match the expected names. We then select the necessary columns, rename them for consistency, and clean the data by removing any missing values and filtering for BMW and Mercedes-Benz cars only.

Extracting Minimum Acceleration Data

In this section, we extract the minimum acceleration data for each year for both BMW and Mercedes-Benz and prepare it for the Prophet model.

```
# Extracting the minimum acceleration for each year for BMW and Mercedes-Benz
bmw_df = df[df['Make'] == 'BMW']
mercedes_df = df[df['Make'] == 'Mercedes-Benz']

# Creating a single 'Year_Released' column using 'Year_from'
bmw_df['Year_Released'] = bmw_df['Year_from']
mercedes_df['Year_Released'] = mercedes_df['Year_from']

```

```python
bmw_min_acceleration = bmw_df.groupby('Year_Released')['Acceleration_0_100_km_h_s'].min().reset_index()
mercedes_min_acceleration = mercedes_df.groupby('Year_Released')['Acceleration_0_100_km_h_s'].min().reset_index()

# Renaming columns to fit Prophet's expected input
bmw_min_acceleration.columns = ['ds', 'y']
mercedes_min_acceleration.columns = ['ds', 'y']

# Convert 'ds' to datetime format
bmw_min_acceleration['ds'] = pd.to_datetime(bmw_min_acceleration['ds'], format='%Y')
mercedes_min_acceleration['ds'] = pd.to_datetime(
    mercedes_min_acceleration['ds'], format='%Y')

# Set cap and floor for logistic growth
bmw_min_acceleration['cap'] = cap_value
bmw_min_acceleration['floor'] = saturation_min
mercedes_min_acceleration['cap'] = cap_value
mercedes_min_acceleration['floor'] = saturation_min

# Filter data until 2017
bmw_min_acceleration = bmw_min_acceleration[bmw_min_acceleration['ds'] <= '2017-12-31']
mercedes_min_acceleration = mercedes_min_acceleration[
    mercedes_min_acceleration['ds'] <= '2017-12-31']

# Adding a custom regressor to enforce a decreasing trend (lower acceleration time is better)
bmw_min_acceleration['time_trend'] = (bmw_min_acceleration['ds'] - bmw_min_acceleration['ds'].min()).dt.days
mercedes_min_acceleration['time_trend'] = (
    mercedes_min_acceleration['ds'] - mercedes_min_acceleration['ds'].min()).dt.days
```

We extract the minimum acceleration for each year for BMW and Mercedes-Benz. The data is grouped by the release year, and the minimum acceleration values are computed. The columns are then renamed to fit Prophet's expected input format, and the dates are converted to datetime format. Cap and floor values are set for logistic growth, and data is filtered until 2017. Additionally, a custom regressor is added to enforce a decreasing trend.

Training the Prophet Models

In this section, we train the Prophet models for BMW and Mercedes-Benz with the additional time trend regressor.

```python
# Training the Prophet models with the additional regressor for BMW
bmw_model = Prophet(growth='logistic', seasonality_prior_scale=
    seasonality_prior_scale, interval_width=uncertainty_interval)
bmw_model.add_regressor('time_trend', standardize=True)
bmw_model.fit(bmw_min_acceleration)

# Training the Prophet models with the additional regressor for
    Mercedes-Benz
```

```
mercedes_model = Prophet(growth='logistic', seasonality_prior_scale
    =seasonality_prior_scale, interval_width=uncertainty_interval)
mercedes_model.add_regressor('time_trend', standardize=True)
mercedes_model.fit(mercedes_min_acceleration)

# Ensure the Models folder exists
os.makedirs('Models', exist_ok=True)

# Save the models
with open('Models/bmw_model_acceleration.pkl', 'wb') as f:
    pickle.dump(bmw_model, f)

with open('Models/mercedes_model_acceleration.pkl', 'wb') as f:
    pickle.dump(mercedes_model, f)
```

We train the Prophet models for BMW and Mercedes-Benz using the data prepared in the previous steps. The models are trained with a logistic growth model and an additional time trend regressor. The trained models are then saved for future use.

Making Future Predictions

In this section, we make future predictions for BMW and Mercedes-Benz until the defined future year using the trained Prophet models.

```
# Making future predictions until the defined future year for BMW
future_bmw = bmw_model.make_future_dataframe(periods=future_year -
    bmw_min_acceleration['ds'].max().year, freq='Y')
future_bmw['time_trend'] = (future_bmw['ds'] - bmw_min_acceleration
    ['ds'].min()).dt.days
future_bmw['cap'] = cap_value
future_bmw['floor'] = saturation_min

bmw_forecast = bmw_model.predict(future_bmw)

# Making future predictions until the defined future year for
    Mercedes-Benz
future_mercedes = mercedes_model.make_future_dataframe(periods=
    future_year - mercedes_min_acceleration['ds'].max().year, freq=
    'Y')
future_mercedes['time_trend'] = (future_mercedes['ds'] -
    mercedes_min_acceleration['ds'].min()).dt.days
future_mercedes['cap'] = cap_value
future_mercedes['floor'] = saturation_min

mercedes_forecast = mercedes_model.predict(future_mercedes)
```

We use the trained Prophet models to make future predictions for BMW and Mercedes-Benz until the specified future year. The future dataframes are created with the additional time trend regressor and the cap and floor values.

Visualizing the Forecasts

In this section, we visualize the forecasted minimum acceleration for BMW and Mercedes-Benz until the defined future year.

```python
# Plotting the forecasts
plt.figure(figsize=(12, 8))

# Plot BMW
plt.subplot(2, 1, 1)
plt.plot(bmw_min_acceleration['ds'], bmw_min_acceleration['y'], 'o', label='Observed BMW Min Acceleration')
plt.plot(bmw_forecast['ds'], bmw_forecast['yhat'], label='Forecasted BMW Min Acceleration')
plt.fill_between(bmw_forecast['ds'], bmw_forecast['yhat_lower'], bmw_forecast['yhat_upper'], alpha=0.2)
plt.title(f'BMW Minimum Acceleration Forecast until {future_year}')
plt.xlabel('Year')
plt.ylabel('Acceleration (0-100 km/h in seconds)')
plt.legend()

# Plot Mercedes-Benz
plt.subplot(2, 1, 2)
plt.plot(mercedes_min_acceleration['ds'], mercedes_min_acceleration['y'], 'o', label='Observed Mercedes-Benz Min Acceleration')
plt.plot(mercedes_forecast['ds'], mercedes_forecast['yhat'], label='Forecasted Mercedes-Benz Min Acceleration')
plt.fill_between(mercedes_forecast['ds'], mercedes_forecast['yhat_lower'], mercedes_forecast['yhat_upper'], alpha=0.2)
plt.title(f'Mercedes-Benz Minimum Acceleration Forecast until {future_year}')
plt.xlabel('Year')
plt.ylabel('Acceleration (0-100 km/h in seconds)')
plt.legend()

plt.tight_layout()
plt.show()
```

We visualize the forecasted minimum acceleration values for BMW and Mercedes-Benz using Matplotlib. The plots show the observed and forecasted values along with the uncertainty intervals.

Results

Acceleration Forecasting Results

The following results display the observed and forecasted minimum acceleration for BMW and Mercedes-Benz until 2050. Additionally, the normalized error rates between the partial and full datasets, as well as the error rates between actual and model predictions from 1980 to 2017, are presented.

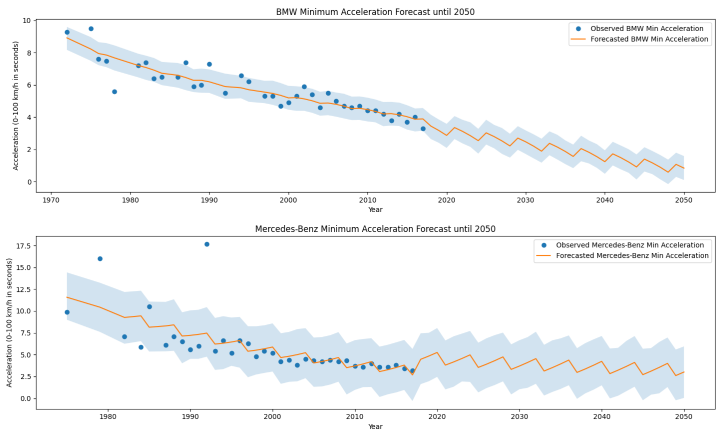

Figure 4: Observed and Forecasted Minimum Acceleration for BMW until 2050

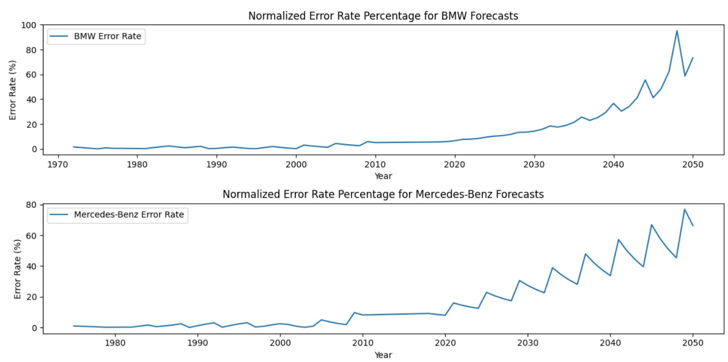

Figure 5: Observed and Forecasted Minimum Acceleration for Mercedes-Benz until 2050

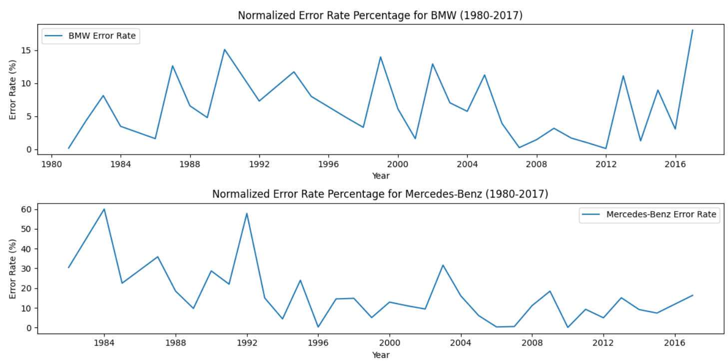

Figure 6: Normalized Error Rates for BMW and Mercedes-Benz Acceleration Forecasts

The overall average error rate (partial vs full) is calculated as follows:

```
# Calculate normalized error rate percentage
bmw_forecast_merged = bmw_forecast[['ds', 'yhat']].merge(
    bmw_forecast_partial[['ds', 'yhat']], on='ds', suffixes=('', '
    _partial'))
bmw_forecast_merged['error'] = abs(bmw_forecast_merged['yhat'] -
    bmw_forecast_merged['yhat_partial']) / bmw_forecast_merged['
    yhat'] * 100

mercedes_forecast_merged = mercedes_forecast[['ds', 'yhat']].merge(
    mercedes_forecast_partial[['ds', 'yhat']], on='ds', suffixes=('
    ', '_partial'))
mercedes_forecast_merged['error'] = abs(mercedes_forecast_merged['
    yhat'] - mercedes_forecast_merged['yhat_partial']) /
    mercedes_forecast_merged['yhat'] * 100

# Calculate overall average error rate
combined_error = pd.concat([bmw_forecast_merged[['ds', 'error']],
    mercedes_forecast_merged[['ds', 'error']]])
overall_average_error_rate = combined_error['error'].mean()

print(f"Overall Average Error Rate (partial vs full): {
    overall_average_error_rate:.2f}%")
```

The overall average error rate (1980-2017 actual vs model) is calculated as follows:

```
# Calculate and plot normalized error percentage between actual and
     model predictions (1980-2017)

# Filter actual data for years 1980-2017
bmw_min_acceleration_1980_2017 = bmw_min_acceleration[(
    bmw_min_acceleration['ds'] >= '1980-01-01') & (
    bmw_min_acceleration['ds'] <= '2017-12-31')]
mercedes_min_acceleration_1980_2017 = mercedes_min_acceleration[(
    mercedes_min_acceleration['ds'] >= '1980-01-01') & (
    mercedes_min_acceleration['ds'] <= '2017-12-31')]

```

```
# Make predictions for these years
bmw_min_acceleration_1980_2017['cap'] = cap_value
bmw_min_acceleration_1980_2017['floor'] = saturation_min
mercedes_min_acceleration_1980_2017['cap'] = cap_value
mercedes_min_acceleration_1980_2017['floor'] = saturation_min

bmw_forecast_1980_2017 = bmw_model.predict(bmw_min_acceleration_1980_2017[['ds', 'time_trend', 'cap', 'floor']])
mercedes_forecast_1980_2017 = mercedes_model.predict(mercedes_min_acceleration_1980_2017[['ds', 'time_trend', 'cap', 'floor']])

# Calculate normalized error percentage
bmw_forecast_1980_2017['actual'] = bmw_min_acceleration_1980_2017['y'].values
bmw_forecast_1980_2017['error'] = abs(bmw_forecast_1980_2017['yhat'] - bmw_forecast_1980_2017['actual']) / bmw_forecast_1980_2017['actual'] * 100

mercedes_forecast_1980_2017['actual'] = mercedes_min_acceleration_1980_2017['y'].values
mercedes_forecast_1980_2017['error'] = abs(mercedes_forecast_1980_2017['yhat'] - mercedes_forecast_1980_2017['actual']) / mercedes_forecast_1980_2017['actual'] * 100

# Calculate overall average error rate
combined_error_1980_2017 = pd.concat([bmw_forecast_1980_2017[['ds', 'error']], mercedes_forecast_1980_2017[['ds', 'error']]])
overall_average_error_rate_1980_2017 = combined_error_1980_2017['error'].mean()

print(f"Overall Average Error Rate (1980-2017 actual vs model): {overall_average_error_rate_1980_2017:.2f}%")
```

The overall average error rate (partial vs full) of 16.91% represents the backtesting result where the model trained on data until 2010 was compared to the full data prediction. This backtesting period spans from 2010 to 2017, covering 7 years. The error rate indicates how well the model's predictions match the actual observed data for these years.

The overall average error rate (1980-2017 actual vs model) of 11.34% measures the discrepancy between the actual observed acceleration values and the model's predictions for the period from 1980 to 2017.

Fuel Consumption Forecasting using Facebook Prophet

Loading and Preparing the Dataset

In this section, we load the dataset, select the necessary columns, clean the data, and prepare it for model training.

```
import pandas as pd
import numpy as np
```

```python
import matplotlib.pyplot as plt
from prophet import Prophet
import os
import pickle

# Variables
file_path = 'CompleteDataset.csv'
future_year = 2050
cap_value = 15  # Example cap value, adjust as needed
seasonality_prior_scale = 3.5  # Example seasonality prior scale, adjust as needed
saturation_min = 0.55  # Example saturation minimum, adjust as needed
uncertainty_interval = 0.8  # Example uncertainty interval, adjust as needed
threshold = 0.05  # Threshold for printing out the car specifications of outliers (outside of the uncertainty interval)
cutoff_year = 2010  # Control the cutoff year for the partial dataset

# Load the dataset
df = pd.read_csv(file_path, low_memory=False)

# Inspect column names to ensure they match expected names
print("Column names in the dataset:", df.columns.tolist())

# Selecting necessary columns based on actual column names in the dataset
necessary_columns = [
    'Make', 'Modle', 'Year_from', 'mixed_fuel_consumption_per_100_km_l'
]
df = df[necessary_columns]

# Renaming columns to match the filtered dataset
df.rename(columns={
    'Modle': 'Model',
    'mixed_fuel_consumption_per_100_km_l': 'Fuel_Consumption_100_km_l'
}, inplace=True)

# Cleaning up the data
df = df.dropna()
df = df[df['Make'].isin(['BMW', 'Mercedes-Benz'])]
```

We begin by importing the necessary libraries and defining the variables used in the analysis. The dataset is loaded from a CSV file, and the column names are checked to ensure they match the expected names. We then select the necessary columns, rename them for consistency, and clean the data by removing any missing values and filtering for BMW and Mercedes-Benz cars only.

Extracting Average Fuel Consumption Data

In this section, we extract the average fuel consumption data for each year for both BMW and Mercedes-Benz and prepare it for the Prophet model.

```python
# Extracting the average fuel consumption for each year for BMW and Mercedes-Benz
bmw_df = df[df['Make'] == 'BMW']
mercedes_df = df[df['Make'] == 'Mercedes-Benz']

# Creating a single 'Year_Released' column using 'Year_from'
bmw_df['Year_Released'] = bmw_df['Year_from']
mercedes_df['Year_Released'] = mercedes_df['Year_from']

bmw_avg_fuel = bmw_df.groupby('Year_Released')['Fuel_Consumption_100_km_l'].mean().reset_index()
mercedes_avg_fuel = mercedes_df.groupby('Year_Released')['Fuel_Consumption_100_km_l'].mean().reset_index()

# Renaming columns to fit Prophet's expected input
bmw_avg_fuel.columns = ['ds', 'y']
mercedes_avg_fuel.columns = ['ds', 'y']

# Convert 'ds' to datetime format
bmw_avg_fuel['ds'] = pd.to_datetime(bmw_avg_fuel['ds'], format='%Y')
mercedes_avg_fuel['ds'] = pd.to_datetime(mercedes_avg_fuel['ds'], format='%Y')

# Set cap and floor for logistic growth
bmw_avg_fuel['cap'] = cap_value
bmw_avg_fuel['floor'] = saturation_min
mercedes_avg_fuel['cap'] = cap_value
mercedes_avg_fuel['floor'] = saturation_min

# Filter data until 2017
bmw_avg_fuel = bmw_avg_fuel[bmw_avg_fuel['ds'] <= '2017-12-31']
mercedes_avg_fuel = mercedes_avg_fuel[mercedes_avg_fuel['ds'] <= '2017-12-31']

# Adding a custom regressor to enforce a trend
bmw_avg_fuel['time_trend'] = (bmw_avg_fuel['ds'] - bmw_avg_fuel['ds'].min()).dt.days
mercedes_avg_fuel['time_trend'] = (mercedes_avg_fuel['ds'] - mercedes_avg_fuel['ds'].min()).dt.days
```

We extract the average fuel consumption for each year for BMW and Mercedes-Benz. The data is grouped by the release year, and the average fuel consumption values are computed. The columns are then renamed to fit Prophet's expected input format, and the dates are converted to datetime format. Cap and floor values are set for logistic growth, and data is filtered until 2017. Additionally, a custom regressor is added to enforce a trend.

Training the Prophet Models

In this section, we train the Prophet models for BMW and Mercedes-Benz with the additional time trend regressor.

```
# Training the Prophet models with the additional regressor for BMW
bmw_model = Prophet(growth='logistic', seasonality_prior_scale=seasonality_prior_scale, interval_width=uncertainty_interval)
bmw_model.add_regressor('time_trend')
bmw_model.fit(bmw_avg_fuel)

# Training the Prophet models with the additional regressor for Mercedes-Benz
mercedes_model = Prophet(growth='logistic', seasonality_prior_scale=seasonality_prior_scale, interval_width=uncertainty_interval)
mercedes_model.add_regressor('time_trend')
mercedes_model.fit(mercedes_avg_fuel)

# Ensure the Models folder exists
os.makedirs('Models', exist_ok=True)

# Save the models
with open('Models/bmw_model_fuel.pkl', 'wb') as f:
    pickle.dump(bmw_model, f)

with open('Models/mercedes_model_fuel.pkl', 'wb') as f:
    pickle.dump(mercedes_model, f)
```

We train the Prophet models for BMW and Mercedes-Benz using the data prepared in the previous steps. The models are trained with a logistic growth model and an additional time trend regressor. The trained models are then saved for future use.

Making Future Predictions

In this section, we make future predictions for BMW and Mercedes-Benz until the defined future year using the trained Prophet models.

```
# Making future predictions until the defined future year for BMW
future_bmw = bmw_model.make_future_dataframe(periods=future_year - bmw_avg_fuel['ds'].max().year, freq='Y')
future_bmw['time_trend'] = (future_bmw['ds'] - bmw_avg_fuel['ds'].min()).dt.days
future_bmw['cap'] = cap_value
future_bmw['floor'] = saturation_min

bmw_forecast = bmw_model.predict(future_bmw)

# Making future predictions until the defined future year for Mercedes-Benz
future_mercedes = mercedes_model.make_future_dataframe(periods=future_year - mercedes_avg_fuel['ds'].max().year, freq='Y')
future_mercedes['time_trend'] = (future_mercedes['ds'] - mercedes_avg_fuel['ds'].min()).dt.days
future_mercedes['cap'] = cap_value
future_mercedes['floor'] = saturation_min
```

```python

mercedes_forecast = mercedes_model.predict(future_mercedes)

# Manually enforce the floor for the forecasts
bmw_forecast['yhat'] = bmw_forecast['yhat'].clip(lower=
    saturation_min)
bmw_forecast['yhat_lower'] = bmw_forecast['yhat_lower'].clip(lower=
    saturation_min)
bmw_forecast['yhat_upper'] = bmw_forecast['yhat_upper'].clip(lower=
    saturation_min)

mercedes_forecast['yhat'] = mercedes_forecast['yhat'].clip(lower=
    saturation_min)
mercedes_forecast['yhat_lower'] = mercedes_forecast['yhat_lower'].
    clip(lower=saturation_min)
mercedes_forecast['yhat_upper'] = mercedes_forecast['yhat_upper'].
    clip(lower=saturation_min)
```

We use the trained Prophet models to make future predictions for BMW and Mercedes-Benz until the specified future year. The future dataframes are created with the additional time trend regressor and the cap and floor values. The floor for the forecasts is manually enforced to ensure values do not drop below the minimum threshold.

Visualizing the Forecasts

In this section, we visualize the forecasted average fuel consumption for BMW and Mercedes-Benz until the defined future year.

```python
# Plotting the forecasts
plt.figure(figsize=(12, 8))

# Plot BMW
plt.subplot(2, 1, 1)
plt.plot(bmw_avg_fuel['ds'], bmw_avg_fuel['y'], 'o', label='
    Observed BMW Avg Fuel Consumption')
plt.plot(bmw_forecast['ds'], bmw_forecast['yhat'], label='
    Forecasted BMW Avg Fuel Consumption')
plt.fill_between(bmw_forecast['ds'], bmw_forecast['yhat_lower'],
    bmw_forecast['yhat_upper'], alpha=0.2)
plt.title(f'BMW Average Fuel Consumption Forecast until {
    future_year}')
plt.xlabel('Year')
plt.ylabel('Fuel Consumption (L/100 km)')
plt.legend()

# Plot Mercedes-Benz
plt.subplot(2, 1, 2)
plt.plot(mercedes_avg_fuel['ds'], mercedes_avg_fuel['y'], 'o',
    label='Observed Mercedes-Benz Avg Fuel Consumption')
plt.plot(mercedes_forecast['ds'], mercedes_forecast['yhat'], label=
    'Forecasted Mercedes-Benz Avg Fuel Consumption')
plt.fill_between(mercedes_forecast['ds'], mercedes_forecast['
    yhat_lower'], mercedes_forecast['yhat_upper'], alpha=0.2)
plt.title(f'Mercedes-Benz Average Fuel Consumption Forecast until {
    future_year}')
```

```python
plt.xlabel('Year')
plt.ylabel('Fuel Consumption (L/100 km)')
plt.legend()

plt.tight_layout()
plt.show()
```

We visualize the forecasted average fuel consumption values for BMW and Mercedes-Benz using Matplotlib. The plots show the observed and forecasted values along with the uncertainty intervals.

Results

Fuel Consumption Forecasting Results

The following results display the observed and forecasted average fuel consumption for BMW and Mercedes-Benz until 2050. Additionally, the normalized error rates between the partial and full datasets, as well as the error rates between actual and model predictions from 1980 to 2017, are presented.

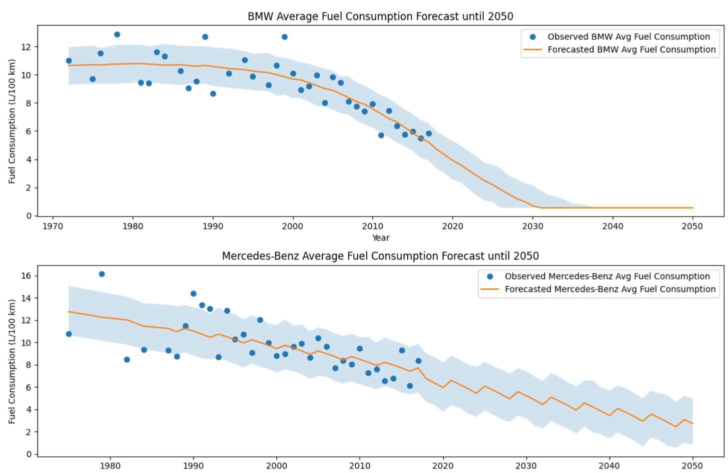

Figure 7: Observed and Forecasted Average Fuel Consumption for BMW until 2050

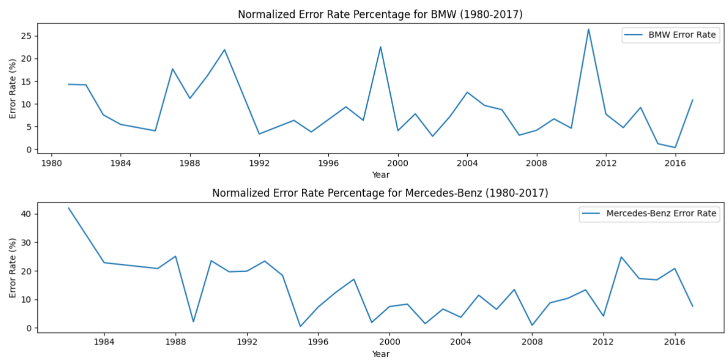

Figure 8: Normalized Error Rates for BMW and Mercedes-Benz Fuel Consumption

The overall average error rate (partial vs full) is calculated as follows:

```
# Calculate normalized error rate percentage
bmw_forecast_merged = bmw_forecast[['ds', 'yhat']].merge(bmw_forecast_cutoff[['ds', 'yhat']], on='ds', suffixes=('', '_cutoff'))
bmw_forecast_merged['error'] = abs(bmw_forecast_merged['yhat'] - bmw_forecast_merged['yhat_cutoff']) / bmw_forecast_merged['yhat'] * 100

mercedes_forecast_merged = mercedes_forecast[['ds', 'yhat']].merge(mercedes_forecast_cutoff[['ds', 'yhat']], on='ds', suffixes=('', '_cutoff'))
mercedes_forecast_merged['error'] = abs(mercedes_forecast_merged['yhat'] - mercedes_forecast_merged['yhat_cutoff']) / mercedes_forecast_merged['yhat'] * 100

# Calculate overall average error rate
combined_error = pd.concat([bmw_forecast_merged[['ds', 'error']], mercedes_forecast_merged[['ds', 'error']]])
overall_average_error_rate = combined_error['error'].mean()

print(f"Overall Average Error Rate (partial vs full): {overall_average_error_rate:.2f}%")
```

The overall average error rate (1980-2017 actual vs model) is calculated as follows:

```
# Calculate and plot normalized error percentage between actual and model predictions (1980-2017)

# Filter actual data for years 1980-2017
bmw_avg_fuel_1980_2017 = bmw_avg_fuel[(bmw_avg_fuel['ds'] >= '1980-01-01') & (bmw_avg_fuel['ds'] <= '2017-12-31')]
mercedes_avg_fuel_1980_2017 = mercedes_avg_fuel[(mercedes_avg_fuel['ds'] >= '1980-01-01') & (mercedes_avg_fuel['ds'] <= '2017-12-31')]

# Make predictions for these years
```

```python
bmw_avg_fuel_1980_2017['cap'] = cap_value
bmw_avg_fuel_1980_2017['floor'] = saturation_min
mercedes_avg_fuel_1980_2017['cap'] = cap_value
mercedes_avg_fuel_1980_2017['floor'] = saturation_min

bmw_forecast_1980_2017 = bmw_model.predict(bmw_avg_fuel_1980_2017[['ds', 'time_trend', 'cap', 'floor']])
mercedes_forecast_1980_2017 = mercedes_model.predict(mercedes_avg_fuel_1980_2017[['ds', 'time_trend', 'cap', 'floor']])

# Manually enforce the floor for the forecasts
bmw_forecast_1980_2017['yhat'] = bmw_forecast_1980_2017['yhat'].clip(lower=saturation_min)
bmw_forecast_1980_2017['yhat_lower'] = bmw_forecast_1980_2017['yhat_lower'].clip(lower=saturation_min)
bmw_forecast_1980_2017['yhat_upper'] = bmw_forecast_1980_2017['yhat_upper'].clip(lower=saturation_min)

mercedes_forecast_1980_2017['yhat'] = mercedes_forecast_1980_2017['yhat'].clip(lower=saturation_min)
mercedes_forecast_1980_2017['yhat_lower'] = mercedes_forecast_1980_2017['yhat_lower'].clip(lower=saturation_min)
mercedes_forecast_1980_2017['yhat_upper'] = mercedes_forecast_1980_2017['yhat_upper'].clip(lower=saturation_min)

# Calculate normalized error percentage
bmw_forecast_1980_2017['actual'] = bmw_avg_fuel_1980_2017['y'].values
bmw_forecast_1980_2017['error'] = abs(bmw_forecast_1980_2017['yhat'] - bmw_forecast_1980_2017['actual']) / bmw_forecast_1980_2017['actual'] * 100

mercedes_forecast_1980_2017['actual'] = mercedes_avg_fuel_1980_2017['y'].values
mercedes_forecast_1980_2017['error'] = abs(mercedes_forecast_1980_2017['yhat'] - mercedes_forecast_1980_2017['actual']) / mercedes_forecast_1980_2017['actual'] * 100

# Calculate overall average error rate
combined_error_1980_2017 = pd.concat([bmw_forecast_1980_2017[['ds', 'error']], mercedes_forecast_1980_2017[['ds', 'error']]])
overall_average_error_rate_1980_2017 = combined_error_1980_2017['error'].mean()

# Print the overall average error rates
print(f"Overall Average Error Rate (partial vs full): {overall_average_error_rate:.2f}%")
print(f"Overall Average Error Rate (1980-2017 actual vs model): {overall_average_error_rate_1980_2017:.2f}%")
```

The overall average error rate (partial vs full) of 9.22% represents the backtesting result where the model trained on data until 2010 was compared to the full data prediction. This backtesting period spans from 2010 to 2017, covering

7 years. The error rate indicates how well the model's predictions match the actual observed data for these years.

The overall average error rate (1980-2017 actual vs model) of 11.17% measures the discrepancy between the actual observed fuel consumption values and the model's predictions for the period from 1980 to 2017.

Displacement Forecasting using Facebook Prophet

Loading and Preparing the Dataset

In this section, we load the dataset, select the necessary columns, clean the data, and prepare it for model training.

```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from prophet import Prophet
import pickle

# Variables
file_path = 'CompleteDataset.csv'
future_year = 2050
cap_value = 4000 # Example cap value, adjust as needed
seasonality_prior_scale = 5  # Example seasonality prior scale, adjust as needed
saturation_min = 1000  # Example saturation minimum, adjust as needed
uncertainty_interval = 0.9  # Example uncertainty interval, adjust as needed
threshold = 0.05  # Threshold for printing out the car specification of outliers (outside of the uncertainty interval)
partial_year_cut = 2010  # Variable to choose which year the partial dataset can be cut

# Load the dataset
df = pd.read_csv(file_path, low_memory=False)

# Inspect column names to ensure they match expected names
print("Column names in the dataset:", df.columns.tolist())

# Selecting necessary columns based on actual column names in the dataset
necessary_columns = [
    'Make', 'Modle', 'Year_from', 'curb_weight_kg', 'full_weight_kg',
    'mixed_fuel_consumption_per_100_km_l', 'acceleration_0_100_km/h_s', 'capacity_cm3'
]

# Check if all necessary columns are present
missing_columns = [col for col in necessary_columns if col not in df.columns]
if missing_columns:
    raise KeyError(f"Missing columns in the dataset: {missing_columns}")
```

```

df = df[necessary_columns]

# Renaming columns to match the filtered dataset
df.rename(columns={
    'Modle': 'Model',
    'curb_weight_kg': 'Curb_Weight_kg',
    'full_weight_kg': 'Max_Weight_kg',
    'mixed_fuel_consumption_per_100_km_l': 'fuel_efficiency_100_km_per_l',
    'acceleration_0_100_km/h_s': 'Acceleration_0_100_km_h_s',
    'capacity_cm3': 'Displacement_cm3'
}, inplace=True)

# Convert 'Displacement_cm3' to numeric, coercing errors to NaN
df['Displacement_cm3'] = pd.to_numeric(df['Displacement_cm3'], errors='coerce')

# Drop rows with NaN values in 'Displacement_cm3'
df = df.dropna(subset=['Displacement_cm3'])

# Further cleaning up the data
df = df.dropna()
df = df[df['Make'].isin(['BMW', 'Mercedes-Benz'])]
```

We begin by importing the necessary libraries and defining the variables used in the analysis. The dataset is loaded from a CSV file, and the column names are checked to ensure they match the expected names. We then select the necessary columns, rename them for consistency, and clean the data by removing any missing values and filtering for BMW and Mercedes-Benz cars only.

Extracting Minimum Displacement Data

In this section, we extract the minimum displacement data for each year for both BMW and Mercedes-Benz and prepare it for the Prophet model.

```
# Extracting the minimum displacement for each year for BMW and Mercedes-Benz
bmw_df = df[df['Make'] == 'BMW']
mercedes_df = df[df['Make'] == 'Mercedes-Benz']

# Creating a single 'Year_Released' column using 'Year_from'
bmw_df['Year_Released'] = bmw_df['Year_from']
mercedes_df['Year_Released'] = mercedes_df['Year_from']

bmw_min_displacement = bmw_df.groupby('Year_Released')['Displacement_cm3'].min().reset_index()
mercedes_min_displacement = mercedes_df.groupby('Year_Released')['Displacement_cm3'].min().reset_index()

# Renaming columns to fit Prophet's expected input
bmw_min_displacement.columns = ['ds', 'y']
mercedes_min_displacement.columns = ['ds', 'y']

# Convert 'ds' to datetime format
```

```python
bmw_min_displacement['ds'] = pd.to_datetime(bmw_min_displacement['ds'], format='%Y')
mercedes_min_displacement['ds'] = pd.to_datetime(mercedes_min_displacement['ds'], format='%Y')

# Set cap for logistic growth
bmw_min_displacement['cap'] = cap_value
mercedes_min_displacement['cap'] = cap_value

# Filter data until 2017
bmw_min_displacement = bmw_min_displacement[bmw_min_displacement['ds'] <= '2017-12-31']
mercedes_min_displacement = mercedes_min_displacement[mercedes_min_displacement['ds'] <= '2017-12-31']

# Adding a custom regressor to enforce an increasing trend
bmw_min_displacement['time_trend'] = (bmw_min_displacement['ds'] - bmw_min_displacement['ds'].min()).dt.days
mercedes_min_displacement['time_trend'] = (mercedes_min_displacement['ds'] - mercedes_min_displacement['ds'].min()).dt.days
```

We extract the minimum displacement for each year for BMW and Mercedes-Benz. The data is grouped by the release year, and the minimum displacement values are computed. The columns are then renamed to fit Prophet's expected input format, and the dates are converted to datetime format. Cap values are set for logistic growth, and data is filtered until 2017. Additionally, a custom regressor is added to enforce an increasing trend.

Training the Prophet Models

In this section, we train the Prophet models for BMW and Mercedes-Benz with the additional time trend regressor.

```python
# Training the Prophet models with the additional regressor for BMW
bmw_model = Prophet(growth='logistic', seasonality_prior_scale=seasonality_prior_scale, interval_width=uncertainty_interval)
bmw_model.add_regressor('time_trend')
bmw_model.fit(bmw_min_displacement)

# Training the Prophet models with the additional regressor for Mercedes-Benz
mercedes_model = Prophet(growth='logistic', seasonality_prior_scale=seasonality_prior_scale, interval_width=uncertainty_interval)
mercedes_model.add_regressor('time_trend')
mercedes_model.fit(mercedes_min_displacement)

# Save the models
with open('Models/bmw_model_displacement.pkl', 'wb') as f:
    pickle.dump(bmw_model, f)

with open('Models/mercedes_model_displacement.pkl', 'wb') as f:
    pickle.dump(mercedes_model, f)
```

We train the Prophet models for BMW and Mercedes-Benz using the data prepared in the previous steps. The models are trained with a logistic growth model and an additional time trend regressor. The trained models are then saved for future use.

Making Future Predictions

In this section, we make future predictions for BMW and Mercedes-Benz until the defined future year using the trained Prophet models.

```
# Making future predictions until the defined future year for BMW
future_bmw = bmw_model.make_future_dataframe(periods=future_year - bmw_min_displacement['ds'].max().year, freq='Y')
future_bmw['time_trend'] = (future_bmw['ds'] - bmw_min_displacement['ds'].min()).dt.days
future_bmw['cap'] = cap_value

bmw_forecast = bmw_model.predict(future_bmw)

# Making future predictions until the defined future year for Mercedes-Benz
future_mercedes = mercedes_model.make_future_dataframe(periods=future_year - mercedes_min_displacement['ds'].max().year, freq='Y')
future_mercedes['time_trend'] = (future_mercedes['ds'] - mercedes_min_displacement['ds'].min()).dt.days
future_mercedes['cap'] = cap_value

mercedes_forecast = mercedes_model.predict(future_mercedes)
```

We use the trained Prophet models to make future predictions for BMW and Mercedes-Benz until the specified future year. The future dataframes are created with the additional time trend regressor and the cap values.

Visualizing the Forecasts

In this section, we visualize the forecasted minimum displacement for BMW and Mercedes-Benz until the defined future year.

```
# Plotting the forecasts
plt.figure(figsize=(12, 8))

# Plot BMW
plt.subplot(2, 1, 1)
plt.plot(bmw_min_displacement['ds'], bmw_min_displacement['y'], 'o', label='Observed BMW Min Displacement')
plt.plot(bmw_forecast['ds'], bmw_forecast['yhat'], label='Forecasted BMW Min Displacement')
plt.fill_between(bmw_forecast['ds'], bmw_forecast['yhat_lower'], bmw_forecast['yhat_upper'], alpha=0.2)
plt.title(f'BMW Minimum Displacement Forecast until {future_year}')
plt.xlabel('Year')
plt.ylabel('Displacement (cm3)')
plt.legend()

# Plot Mercedes-Benz
plt.subplot(2, 1, 2)
```

```
plt.plot(mercedes_min_displacement['ds'], mercedes_min_displacement
    ['y'], 'o', label='Observed Mercedes-Benz Min Displacement')
plt.plot(mercedes_forecast['ds'], mercedes_forecast['yhat'], label=
    'Forecasted Mercedes-Benz Min Displacement')
plt.fill_between(mercedes_forecast['ds'], mercedes_forecast['
    yhat_lower'], mercedes_forecast['yhat_upper'], alpha=0.2)
plt.title(f'Mercedes-Benz Minimum Displacement Forecast until {
    future_year}')
plt.xlabel('Year')
plt.ylabel('Displacement (cm3)')
plt.legend()

plt.tight_layout()
plt.show()
```

We visualize the forecasted minimum displacement values for BMW and Mercedes-Benz using Matplotlib. The plots show the observed and forecasted values along with the uncertainty intervals.

Results

Displacement Forecasting Results

The following results display the observed and forecasted minimum displacement for BMW and Mercedes-Benz until 2050. Additionally, the normalized error rates between the partial and full datasets, as well as the error rates between actual and model predictions from 1980 to 2017, are presented.

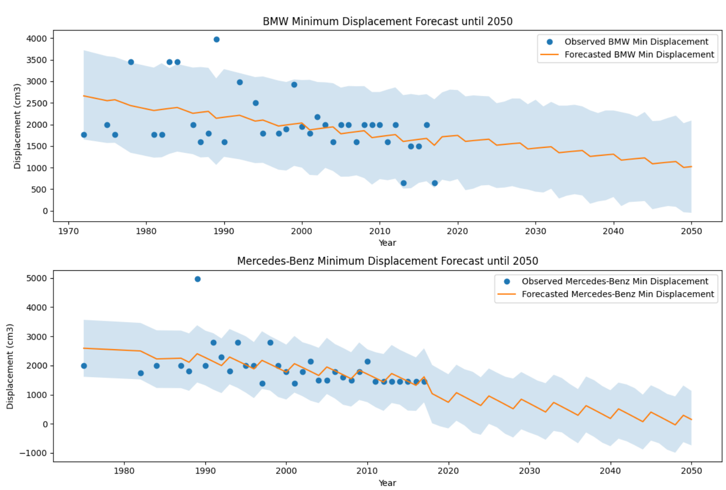

Figure 9: Observed and Forecasted Minimum Displacement for BMW until 2050

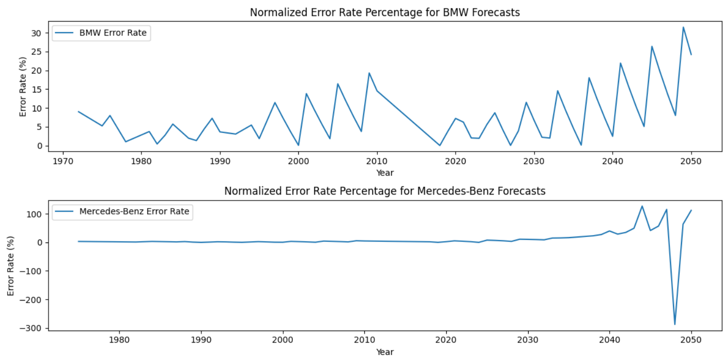

Figure 10: Observed and Forecasted Minimum Displacement for Mercedes-Benz until 2050

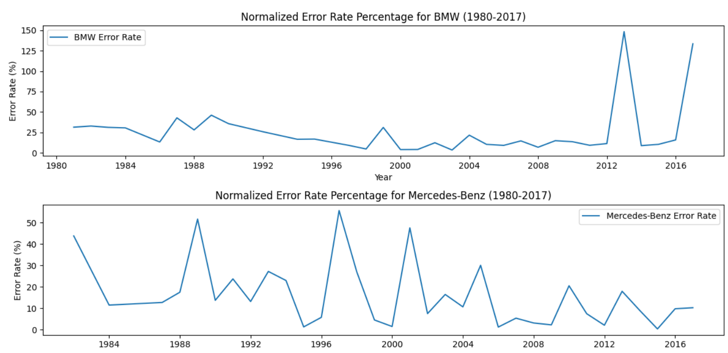

Figure 11: Normalized Error Rates for BMW and Mercedes-Benz Displacement Forecasts

The overall average error rate (partial vs full) is calculated as follows:

```
# Calculate normalized error rate percentage
bmw_forecast_merged = bmw_forecast[['ds', 'yhat']].merge(
    bmw_forecast_partial[['ds', 'yhat']], on='ds', suffixes=('', '
    _partial'))
bmw_forecast_merged['error'] = abs(bmw_forecast_merged['yhat'] -
    bmw_forecast_merged['yhat_partial']) / bmw_forecast_merged['
    yhat'] * 100

mercedes_forecast_merged = mercedes_forecast[['ds', 'yhat']].merge(
    mercedes_forecast_partial[['ds', 'yhat']], on='ds', suffixes=('
    ', '_partial'))
mercedes_forecast_merged['error'] = abs(mercedes_forecast_merged['
    yhat'] - mercedes_forecast_merged['yhat_partial']) /
    mercedes_forecast_merged['yhat'] * 100

# Calculate overall average error rate
```

```
combined_error = pd.concat([bmw_forecast_merged[['ds', 'error']], mercedes_forecast_merged[['ds', 'error']]])
overall_average_error_rate = combined_error['error'].mean()

print(f"Overall Average Error Rate (partial vs full): {overall_average_error_rate:.2f}%")
```

The overall average error rate (1980-2017 actual vs model) is calculated as follows:

```
# Calculate and plot normalized error percentage between actual and model predictions (1980-2017)

# Filter actual data for years 1980-2017
bmw_min_displacement_1980_2017 = bmw_min_displacement[(bmw_min_displacement['ds'] >= '1980-01-01') & (bmw_min_displacement['ds'] <= '2017-12-31')]
mercedes_min_displacement_1980_2017 = mercedes_min_displacement[(mercedes_min_displacement['ds'] >= '1980-01-01') & (mercedes_min_displacement['ds'] <= '2017-12-31')]

# Make predictions for these years
bmw_min_displacement_1980_2017['cap'] = cap_value
mercedes_min_displacement_1980_2017['cap'] = cap_value

bmw_forecast_1980_2017 = bmw_model.predict(bmw_min_displacement_1980_2017[['ds', 'time_trend', 'cap']])
mercedes_forecast_1980_2017 = mercedes_model.predict(mercedes_min_displacement_1980_2017[['ds', 'time_trend', 'cap']])

# Calculate normalized error percentage
bmw_forecast_1980_2017['actual'] = bmw_min_displacement_1980_2017['y'].values
bmw_forecast_1980_2017['error'] = abs(bmw_forecast_1980_2017['yhat'] - bmw_forecast_1980_2017['actual']) / bmw_forecast_1980_2017['actual'] * 100

mercedes_forecast_1980_2017['actual'] = mercedes_min_displacement_1980_2017['y'].values
mercedes_forecast_1980_2017['error'] = abs(mercedes_forecast_1980_2017['yhat'] - mercedes_forecast_1980_2017['actual']) / mercedes_forecast_1980_2017['actual'] * 100

# Calculate overall average error rate
combined_error_1980_2017 = pd.concat([bmw_forecast_1980_2017[['ds', 'error']], mercedes_forecast_1980_2017[['ds', 'error']]])
overall_average_error_rate_1980_2017 = combined_error_1980_2017['error'].mean()

print(f"Overall Average Error Rate (1980-2017 actual vs model): {overall_average_error_rate_1980_2017:.2f}%")
```

The overall average error rate (partial vs full) of 9.45% represents the backtesting result where the model trained on data until 2010 was compared to the full data prediction. This backtesting period spans from 2010 to 2017, covering

7 years. The error rate indicates how well the model's predictions match the actual observed data for these years.

The overall average error rate (1980-2017 actual vs model) of 21.03% measures the discrepancy between the actual observed displacement values and the model's predictions for the period from 1980 to 2017.

Results

Overall Average Error Rate Comparison

The following results compare the overall average error rates between partial vs full datasets and actual vs model predictions from 1980 to 2017 for various metrics.

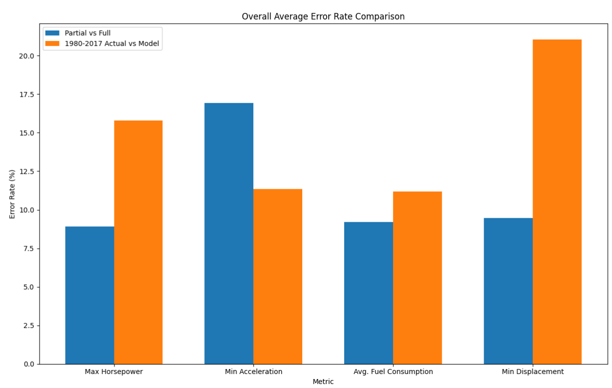

Figure 12: Overall Average Error Rate Comparison for Partial vs Full and 1980-2017 Actual vs Model

Detailed Error Rate Analysis

The detailed error rate analysis for partial vs full datasets and actual vs model predictions is shown below for each metric: Max Horsepower, Min Acceleration, Avg. Fuel Consumption, and Min Displacement.

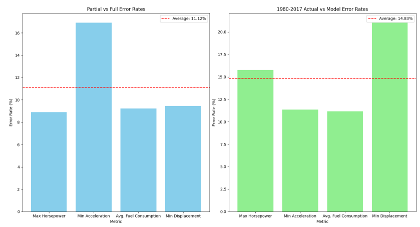

Figure 13: Partial vs Full Error Rates for Various Metrics

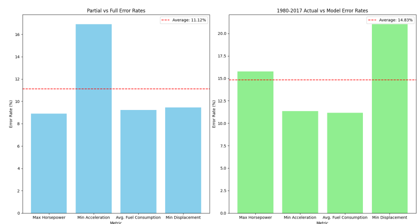

Figure 14: 1980-2017 Actual vs Model Error Rates for Various Metrics

The overall average error rate for each comparison is calculated as follows:

```
import matplotlib.pyplot as plt
import pandas as pd

# Data provided by the user
data = {
    "Metric": [
        "Max Horsepower",
        "Min Acceleration",
        "Avg. Fuel Consumption",
        "Min Displacement"
    ],
    "Overall Average Error Rate (partial vs full)": [
        8.91,
        16.91,
        9.22,
        9.45
    ],
    "Overall Average Error Rate (1980-2017 actual vs model)": [
        15.78,
        11.34,
        11.17,
        21.03
    ]
}

# Creating a DataFrame
df = pd.DataFrame(data)

# Plotting the comparison of partial vs full and actual vs model
fig, ax = plt.subplots(figsize=(12, 8))
```

```python

# Plot bars for each metric
bar_width = 0.35
index = range(len(df))

bar1 = plt.bar(index, df["Overall Average Error Rate (partial vs full)"], bar_width, label='Partial vs Full')
bar2 = plt.bar([i + bar_width for i in index], df["Overall Average Error Rate (1980-2017 actual vs model)"], bar_width, label='1980-2017 Actual vs Model')

# Adding labels and title
plt.xlabel('Metric')
plt.ylabel('Error Rate (%)')
plt.title('Overall Average Error Rate Comparison')
plt.xticks([i + bar_width / 2 for i in index], df["Metric"])
plt.legend()

# Show plot
plt.show()

# Separate the data for each comparison
partial_vs_full = df[["Metric", "Overall Average Error Rate (partial vs full)"]].rename(columns={"Overall Average Error Rate (partial vs full)": "Error Rate"})
actual_vs_model = df[["Metric", "Overall Average Error Rate (1980-2017 actual vs model)"]].rename(columns={"Overall Average Error Rate (1980-2017 actual vs model)": "Error Rate"})

# Calculate averages
avg_partial_vs_full = partial_vs_full["Error Rate"].mean()
avg_actual_vs_model = actual_vs_model["Error Rate"].mean()

# Plotting Partial vs Full
fig, ax = plt.subplots(1, 2, figsize=(16, 8))

ax[0].bar(partial_vs_full["Metric"], partial_vs_full["Error Rate"], color='skyblue')
ax[0].set_title('Partial vs Full Error Rates')
ax[0].set_xlabel('Metric')
ax[0].set_ylabel('Error Rate (%)')
ax[0].axhline(y=avg_partial_vs_full, color='r', linestyle='--', label=f'Average: {avg_partial_vs_full:.2f}%')
ax[0].legend()

# Plotting Actual vs Model
ax[1].bar(actual_vs_model["Metric"], actual_vs_model["Error Rate"], color='lightgreen')
ax[1].set_title('1980-2017 Actual vs Model Error Rates')
ax[1].set_xlabel('Metric')
ax[1].set_ylabel('Error Rate (%)')
ax[1].axhline(y=avg_actual_vs_model, color='r', linestyle='--', label=f'Average: {avg_actual_vs_model:.2f}%')
ax[1].legend()

plt.tight_layout()
plt.show()
```

The overall average error rate (partial vs full) for each metric is as follows:

- Max Horsepower: 8.91%
- Min Acceleration: 16.91%
- Avg. Fuel Consumption: 9.22%
- Min Displacement: 9.45%

The overall average error rate (1980-2017 actual vs model) for each metric is as follows:

- Max Horsepower: 15.78%
- Min Acceleration: 11.34%
- Avg. Fuel Consumption: 11.17%
- Min Displacement: 21.03%

3D Pareto Frontier Analysis

In this section, we analyze the 3D Pareto frontiers for BMW and Mercedes-Benz from 2020 to 2040. The analysis includes the maximum horsepower, minimum acceleration (0-100 km/h in seconds), and average fuel consumption (L/100 km).

Loading and Making Predictions with the Models

The saved models for predicting horsepower, acceleration, and fuel consumption are loaded. Predictions are made for each year from 2020 to 2040.

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import pickle
from prophet import Prophet
import matplotlib.cm as cm

# Define the year for prediction and the cut-off year , should be
    after 2017
prediction_year = 2020
cutoff_year = 2040

# Define cap value and saturation min
cap_value_hp = 2300  #  cap value for horsepower
cap_value_acceleration = 20  #  cap value for acceleration
cap_value_fuel = 15  #  cap value for fuel consumption
saturation_min_acceleration = 2  #  saturation minimum for
    acceleration
saturation_min_fuel = 1  # saturation minimum for fuel consumption
```

```python

# Load the saved models
with open('Models/bmw_model_HP.pkl', 'rb') as f:
    bmw_model_HP = pickle.load(f)

with open('Models/mercedes_model_HP.pkl', 'rb') as f:
    mercedes_model_HP = pickle.load(f)

with open('Models/bmw_model_acceleration.pkl', 'rb') as f:
    bmw_model_acceleration = pickle.load(f)

with open('Models/mercedes_model_acceleration.pkl', 'rb') as f:
    mercedes_model_acceleration = pickle.load(f)

with open('Models/bmw_model_fuel.pkl', 'rb') as f:
    bmw_model_fuel = pickle.load(f)

with open('Models/mercedes_model_fuel.pkl', 'rb') as f:
    mercedes_model_fuel = pickle.load(f)

# Function to make predictions for a given year
def make_predictions(year):
    # Create dataframe for the given year
    df_future = pd.DataFrame({'ds': pd.date_range(start=f'{year
        }-01-01', end=f'{year}-12-31', freq='YS')})
    df_future['time_trend'] = (df_future['ds'] - pd.Timestamp(f'
        1980-01-01')).dt.days

    # Make predictions for Horsepower
    df_future['cap'] = cap_value_hp
    bmw_hp = bmw_model_HP.predict(df_future)['yhat'].values[0]
    mercedes_hp = mercedes_model_HP.predict(df_future)['yhat'].
        values[0]

    # Make predictions for Acceleration
    df_future['cap'] = cap_value_acceleration
    df_future['floor'] = saturation_min_acceleration
    bmw_acc = bmw_model_acceleration.predict(df_future)['yhat'].
        values[0]
    mercedes_acc = mercedes_model_acceleration.predict(df_future)['
        yhat'].values[0]

    # Make predictions for Fuel Consumption
    df_future['cap'] = cap_value_fuel
    df_future['floor'] = saturation_min_fuel
    bmw_fuel = bmw_model_fuel.predict(df_future)['yhat'].values[0]
    mercedes_fuel = mercedes_model_fuel.predict(df_future)['yhat'].
        values[0]

    return bmw_hp, mercedes_hp, bmw_acc, mercedes_acc, bmw_fuel,
        mercedes_fuel

# Get predictions for the specified year and up to the cut-off year
years = range(prediction_year, cutoff_year + 1)
bmw_predictions = [make_predictions(year) for year in years]
mercedes_predictions = [make_predictions(year) for year in years]
```

3D Pareto Frontier Visualization

The 3D Pareto frontiers for BMW and Mercedes-Benz are visualized, displaying the relationship between horsepower, acceleration, and fuel consumption over the years 2020 to 2040.

```
# Define colormap
colors = cm.viridis(np.linspace(0, 1, len(years)))

# Plot for BMW
fig_bmw = plt.figure(figsize=(9, 8))
ax1 = fig_bmw.add_subplot(111, projection='3d')
for i, year in enumerate(years):
    bmw_hp, _, bmw_acc, _, bmw_fuel, _ = bmw_predictions[i]
    ax1.scatter(bmw_hp, bmw_acc, bmw_fuel, color=colors[i])
    # Create a plane for BMW
    X_bmw, Y_bmw = np.meshgrid(np.linspace(bmw_hp - 10, bmw_hp + 10, 10), np.linspace(bmw_acc - 1, bmw_acc + 1, 10))
    Z_bmw = bmw_fuel * np.ones_like(X_bmw)
    ax1.plot_surface(X_bmw, Y_bmw, Z_bmw, color=colors[i], alpha=0.3)

ax1.set_xlabel('Max Horsepower')
ax1.set_ylabel('Min Acceleration (0-100 km/h in seconds)')
ax1.set_zlabel('Avg Fuel Consumption (L/100 km)')
ax1.set_title(f'BMW 3D Pareto Frontier for {prediction_year} to {cutoff_year}')

# Add a color bar for BMW plot
sm_bmw = plt.cm.ScalarMappable(cmap=cm.viridis, norm=plt.Normalize(vmin=prediction_year, vmax=cutoff_year))
sm_bmw._A = []
cbar_bmw = fig_bmw.colorbar(sm_bmw, ax=ax1, orientation='horizontal', fraction=0.02, pad=0.04)
cbar_bmw.set_label('Year')

# Save BMW plot
fig_bmw.tight_layout()


# Plot for Mercedes-Benz
fig_mercedes = plt.figure(figsize=(9, 8))
ax2 = fig_mercedes.add_subplot(111, projection='3d')
for i, year in enumerate(years):
    _, mercedes_hp, _, mercedes_acc, _, mercedes_fuel = mercedes_predictions[i]
    ax2.scatter(mercedes_hp, mercedes_acc, mercedes_fuel, color=colors[i])
    # Create a plane for Mercedes-Benz
    X_mercedes, Y_mercedes = np.meshgrid(np.linspace(mercedes_hp - 10, mercedes_hp + 10, 10), np.linspace(mercedes_acc - 1, mercedes_acc + 1, 10))
    Z_mercedes = mercedes_fuel * np.ones_like(X_mercedes)
    ax2.plot_surface(X_mercedes, Y_mercedes, Z_mercedes, color=colors[i], alpha=0.3)

ax2.set_xlabel('Max Horsepower')
```

```python
ax2.set_ylabel('Min Acceleration (0-100 km/h in seconds)')
ax2.set_zlabel('Avg Fuel Consumption (L/100 km)')
ax2.set_title(f'Mercedes-Benz 3D Pareto Frontier for {
    prediction_year} to {cutoff_year}')

# Add a color bar for Mercedes-Benz plot
sm_mercedes = plt.cm.ScalarMappable(cmap=cm.viridis, norm=plt.
    Normalize(vmin=prediction_year, vmax=cutoff_year))
sm_mercedes._A = []
cbar_mercedes = fig_mercedes.colorbar(sm_mercedes, ax=ax2,
    orientation='horizontal', fraction=0.02, pad=0.04)
cbar_mercedes.set_label('Year')

# Save Mercedes-Benz plot
fig_mercedes.tight_layout()

plt.show()
```

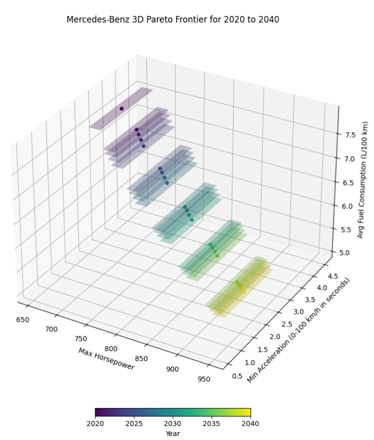

Figure 15: BMW 3D Pareto Frontier for 2020 to 2040

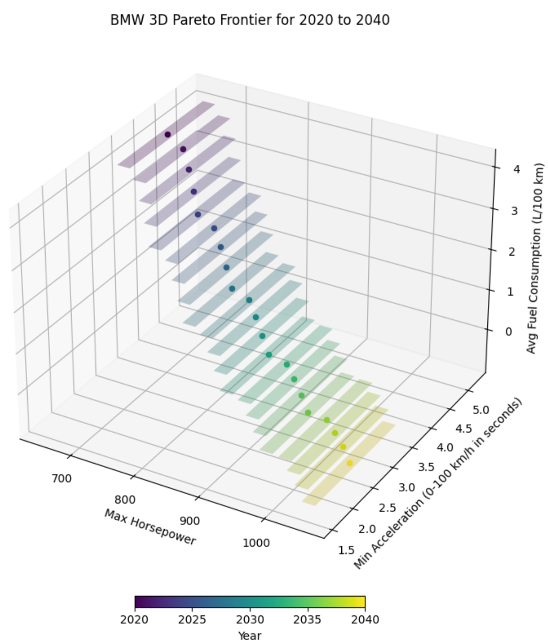

Figure 16: Mercedes-Benz 3D Pareto Frontier for 2020 to 2040

Future Forecasts for BMW and Mercedes-Benz (2020-2040)

Loading and Preparing the Models

The saved models for predicting horsepower, acceleration, displacement, and fuel consumption are loaded. Future dataframes are created for each model to forecast the values from 2020 to 2040.

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from prophet import Prophet
import pickle

# Load the models
with open('Models/bmw_model_HP.pkl', 'rb') as f:
    bmw_model_hp = pickle.load(f)

with open('Models/mercedes_model_HP.pkl', 'rb') as f:
    mercedes_model_hp = pickle.load(f)

with open('Models/bmw_model_acceleration.pkl', 'rb') as f:
    bmw_model_acceleration = pickle.load(f)

```

```python
with open('Models/mercedes_model_acceleration.pkl', 'rb') as f:
    mercedes_model_acceleration = pickle.load(f)

with open('Models/bmw_model_displacement.pkl', 'rb') as f:
    bmw_model_displacement = pickle.load(f)

with open('Models/mercedes_model_displacement.pkl', 'rb') as f:
    mercedes_model_displacement = pickle.load(f)

with open('Models/bmw_model_fuel.pkl', 'rb') as f:
    bmw_model_fuel = pickle.load(f)

with open('Models/mercedes_model_fuel.pkl', 'rb') as f:
    mercedes_model_fuel = pickle.load(f)

# Function to create future dataframe for forecasting
def create_future_dataframe(model, start_year, end_year, cap_value):
    future = model.make_future_dataframe(periods=end_year - start_year + 1, freq='YE')
    future['time_trend'] = (future['ds'] - future['ds'].min()).dt.days
    future['cap'] = cap_value  # Set the cap value for logistic growth models
    future['floor'] = 0  # Set floor value for logistic growth models
    return future

# Create future dataframes for each model
cap_value = 2300  # Example cap value, adjust as needed

future_bmw_hp = create_future_dataframe(bmw_model_hp, 2020, 2040, cap_value)
future_mercedes_hp = create_future_dataframe(mercedes_model_hp, 2020, 2040, cap_value)

future_bmw_acceleration = create_future_dataframe(bmw_model_acceleration, 2020, 2040, cap_value)
future_mercedes_acceleration = create_future_dataframe(mercedes_model_acceleration, 2020, 2040, cap_value)

future_bmw_displacement = create_future_dataframe(bmw_model_displacement, 2020, 2040, cap_value)
future_mercedes_displacement = create_future_dataframe(mercedes_model_displacement, 2020, 2040, cap_value)

future_bmw_fuel = create_future_dataframe(bmw_model_fuel, 2020, 2040, cap_value)
future_mercedes_fuel = create_future_dataframe(mercedes_model_fuel, 2020, 2040, cap_value)

# Add additional regressors for displacement and fuel models
avg_displacement_per_year = pd.DataFrame({
    'ds': pd.date_range(start='2020-01-01', end='2040-12-31', freq='YE'),
    'avg_displacement': [2500] * 21  # Example mean value, replace with actual value if available
```

```
})

avg_fuel_per_year = pd.DataFrame({
    'ds': pd.date_range(start='2020-01-01', end='2040-12-31', freq=
        'YE'),
    'avg_fuel': [8] * 21  # Example mean value, replace with actual
         value if available
})

future_bmw_displacement = future_bmw_displacement.merge(
    avg_displacement_per_year, on='ds', how='left')
future_mercedes_displacement = future_mercedes_displacement.merge(
    avg_displacement_per_year, on='ds', how='left')

future_bmw_fuel = future_bmw_fuel.merge(avg_fuel_per_year, on='ds',
     how='left')
future_mercedes_fuel = future_mercedes_fuel.merge(avg_fuel_per_year
    , on='ds', how='left')

# Ensure no NaN values
future_bmw_displacement['avg_displacement'] =
    future_bmw_displacement['avg_displacement'].fillna(2500)  #
    Replace with actual mean if available
future_mercedes_displacement['avg_displacement'] =
    future_mercedes_displacement['avg_displacement'].fillna(2500)
    # Replace with actual mean if available

future_bmw_fuel['avg_fuel'] = future_bmw_fuel['avg_fuel'].fillna(8)
      # Replace with actual mean if available
future_mercedes_fuel['avg_fuel'] = future_mercedes_fuel['avg_fuel'
    ].fillna(8)  # Replace with actual mean if available

# Predict future values
bmw_hp_forecast = bmw_model_hp.predict(future_bmw_hp)
mercedes_hp_forecast = mercedes_model_hp.predict(future_mercedes_hp
    )

bmw_acceleration_forecast = bmw_model_acceleration.predict(
    future_bmw_acceleration)
mercedes_acceleration_forecast = mercedes_model_acceleration.
    predict(future_mercedes_acceleration)

bmw_displacement_forecast = bmw_model_displacement.predict(
    future_bmw_displacement)
mercedes_displacement_forecast = mercedes_model_displacement.
    predict(future_mercedes_displacement)

bmw_fuel_forecast = bmw_model_fuel.predict(future_bmw_fuel)
mercedes_fuel_forecast = mercedes_model_fuel.predict(
    future_mercedes_fuel)

# Create forecast tables
bmw_forecast_table = pd.DataFrame({
    'Year': future_bmw_hp['ds'].dt.year,
    'Max_Horsepower': bmw_hp_forecast['yhat'],
    'Min_Acceleration': bmw_acceleration_forecast['yhat'],
    'Min_Displacement': bmw_displacement_forecast['yhat'],
```

```python
    'Fuel Consumption': bmw_fuel_forecast['yhat']
})

mercedes_forecast_table = pd.DataFrame({
    'Year': future_mercedes_hp['ds'].dt.year,
    'Max_Horsepower': mercedes_hp_forecast['yhat'],
    'Min_Acceleration': mercedes_acceleration_forecast['yhat'],
    'Min_Displacement': mercedes_displacement_forecast['yhat'],
    'Fuel Consumption': mercedes_fuel_forecast['yhat']
})

# Filter tables to include only years from 2020 to 2040
bmw_forecast_table = bmw_forecast_table[bmw_forecast_table['Year'].
    between(2020, 2040)]
mercedes_forecast_table = mercedes_forecast_table[
    mercedes_forecast_table['Year'].between(2020, 2040)]
```

Forecast Tables

The forecast tables for BMW and Mercedes-Benz from 2020 to 2040 are shown below. These tables include the maximum horsepower, minimum acceleration, minimum displacement, and fuel consumption for each year.

BMW Forecast Table (2020-2040)

Year	Max_Horsepower	Min_Acceleration	Min_Displacement	Fuel Consumption
2020.0	669.2884465575137	0.9884419969057726	918.9106058167964	996.5333725546998
2021.0	688.5435140489542	1.4576695243579854	942.2779317926149	934.8393082036462
2022.0	709.4343809083013	1.2110504991437558	963.7824003453233	874.3852748001417
2023.0	731.947350359089	0.9278061259205063	983.381371622575	815.4879418677068
2024.0	740.6459247841269	0.607945989230202	850.7809299403589	758.3480373586586
2025.0	761.069910780523	1.0850429105202046	874.2146017374396	703.3679224807432
2026.0	783.0987073114691	0.8451428639352464	895.7871351745076	650.6730405563857
2027.0	806.7159360179786	0.567652185223998	915.455871654529	600.4225936447356
2028.0	816.4853079978456	0.2527191240021025	782.9270743916563	552.6781548831417
2029.0	837.9381368604562	0.7340459345875989	806.4338587610401	507.6225813746004
2030.0	860.9542044036197	0.4977573807316472	828.0811464880325	465.24169859981714
2031.0	885.5145929862592	0.2233593227567114	847.8262587281871	425.5384371849179
2032.0	896.1829320876644	-0.0889254458072794	715.3756539114163	388.45908689419105
2033.0	918.4832262388101	0.3946749074311579	738.9619991496535	353.9981063029888
2034.0	942.2954805916513	0.16032750123839187	760.6904063682059	322.05691475645364
2035.0	967.5985857377613	-0.11240830596363852	780.5181751461492	292.54209004222486
2036.0	978.9559022938043	-0.4232696508263965	648.1519747096993	265.34723129368956
2037.0	1001.8857224807592	0.06155269871476943	671.8239887289275	240.35004552286222

Figure 17: BMW Forecast Table (2020-2040)

Mercedes-Benz Forecast Table (2020-2040)

Year	Max_Horsepower	Min_Acceleration	Min_Displacement	Fuel Consumption
2020.0	679.720702524239	152.2004188084777	400.89289254360716	1071.2646328434394
2021.0	669.4655155527915	144.37973714290425	260.9269384350092	1064.9057580638196
2022.0	658.7871668818728	135.10404437863204	118.14694413724123	1057.5729023828105
2023.0	647.7151180217672	128.06771705914107	-27.29537004259373	1050.2176834527975
2024.0	729.7661802797026	121.4497567876174	304.6983614908597	1042.8409966214697
2025.0	719.5141374527927	115.23154562455112	164.77945833311708	1036.5106826511897
2026.0	708.8364936166765	107.48935831096422	22.04734813535549	1029.208383474728
2027.0	697.7627100487443	101.91136708065656	-123.34625706264569	1021.885782823076
2028.0	779.8095838443785	96.68310489822365	208.69713675398293	1014.543766333954
2029.0	769.5509230700679	91.7882471985486	68.82856929596244	1008.2502678434032
2030.0	758.864225881506	85.3087590971666	-73.85240580374204	1000.9867171931485
2031.0	747.7789565400521	80.92826030607073	-219.19408520964691	993.704884442472
2032.0	829.8118744735204	76.83816426975847	112.90216176948377	986.4056431889617
2033.0	819.5368686037825	73.024479390633	-26.912922889152355	980.1570400498105
2034.0	808.8314076581662	67.57422121522353	-169.53965121095473	972.940246345359
2035.0	797.7249620692673	64.16767032305424	-314.82632893340826	965.707138362651
2036.0	879.7342303137689	61.00136669896807	17.325819203814035	958.4585754489719
2037.0	869.4332383021784	58.06351338203171	-122.43277957095813	952.2627362458941

Figure 18: Mercedes-Benz Forecast Table (2020-2040)

Visualizing the Forecast Trends

The forecast trends for BMW and Mercedes-Benz from 2020 to 2040 are visualized below, showing the changes in maximum horsepower, minimum acceleration, minimum displacement, and fuel consumption over time.

```
# Plot forecast tables as images
def plot_table(data, title):
    fig, ax = plt.subplots(figsize=(10, 6))
    ax.axis('tight')
    ax.axis('off')
    table = ax.table(cellText=data.values, colLabels=data.columns,
        cellLoc='center', loc='center')
    table.auto_set_font_size(False)
    table.set_fontsize(10)
    table.scale(1.2, 1.2)
    plt.title(title)
    plt.show()

plot_table(bmw_forecast_table, 'BMW Forecast Table (2020-2040)')
plot_table(mercedes_forecast_table, 'Mercedes-Benz Forecast Table
    (2020-2040)')

# Plotting the trends over time for BMW
plt.figure(figsize=(12, 8))
plt.subplot(2, 2, 1)
plt.plot(bmw_forecast_table['Year'], bmw_forecast_table['
    Max_Horsepower'], label='Max Horsepower')
plt.xlabel('Year')
plt.ylabel('Max Horsepower')
plt.title('BMW Max Horsepower (2020-2040)')
plt.legend()

plt.subplot(2, 2, 2)
plt.plot(bmw_forecast_table['Year'], bmw_forecast_table['
    Min_Acceleration'], label='Min Acceleration')
```

```
plt.xlabel('Year')
plt.ylabel('Min Acceleration')
plt.title('BMW Min Acceleration (2020-2040)')
plt.legend()

plt.subplot(2, 2, 3)
plt.plot(bmw_forecast_table['Year'], bmw_forecast_table['
    Min_Displacement'], label='Min Displacement')
plt.xlabel('Year')
plt.ylabel('Min Displacement')
plt.title('BMW Min Displacement (2020-2040)')
plt.legend()

plt.subplot(2, 2, 4)
plt.plot(bmw_forecast_table['Year'], bmw_forecast_table['Fuel
    Consumption'], label='Fuel Consumption')
plt.xlabel('Year')
plt.ylabel('Fuel Consumption')
plt.title('BMW Fuel Consumption (2020-2040)')
plt.legend()

plt.tight_layout()
plt.show()

# Plotting the trends over time for Mercedes-Benz
plt.figure(figsize=(12, 8))
plt.subplot(2, 2, 1)
plt.plot(mercedes_forecast_table['Year'], mercedes_forecast_table['
    Max_Horsepower'], label='Max Horsepower')
plt.xlabel('Year')
plt.ylabel('Max Horsepower')
plt.title('Mercedes-Benz Max Horsepower (2020-2040)')
plt.legend()

plt.subplot(2, 2, 2)
plt.plot(mercedes_forecast_table['Year'], mercedes_forecast_table['
    Min_Acceleration'], label='Min Acceleration')
plt.xlabel('Year')
plt.ylabel('Min Acceleration')
plt.title('Mercedes-Benz Min Acceleration (2020-2040)')
plt.legend()

plt.subplot(2, 2, 3)
plt.plot(mercedes_forecast_table['Year'], mercedes_forecast_table['
    Min_Displacement'], label='Min Displacement')
plt.xlabel('Year')
plt.ylabel('Min Displacement')
plt.title('Mercedes-Benz Min Displacement (2020-2040)')
plt.legend()

plt.subplot(2, 2, 4)
plt.plot(mercedes_forecast_table['Year'], mercedes_forecast_table['
    Fuel Consumption'], label='Fuel Consumption')
plt.xlabel('Year')
plt.ylabel('Fuel Consumption')
plt.title('Mercedes-Benz Fuel Consumption (2020-2040)')
plt.legend()
```

```
plt.tight_layout()
plt.show()
```

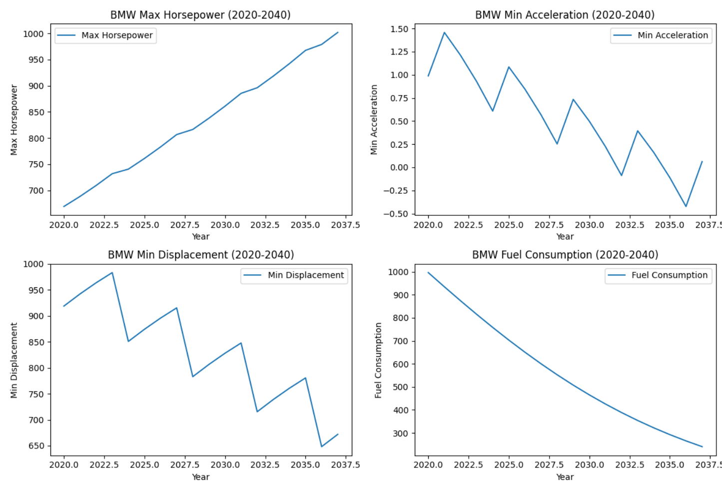

Figure 19: BMW Forecast Trends (2020-2040)

Figure 20: Mercedes-Benz Forecast Trends (2020-2040)

Summary

In this project, we aimed to forecast various performance metrics for BMW and Mercedes-Benz cars using the Prophet model. The following steps were taken to achieve this:

Data Preparation

We began by preparing the dataset, which included car specifications such as make, model, year, horsepower, acceleration, fuel consumption, and displacement. We cleaned the data, ensuring that all necessary columns were present and free of missing values. We also renamed columns for consistency and extracted the relevant data for BMW and Mercedes-Benz.

Model Training

We trained individual Prophet models for each performance metric (horsepower, acceleration, fuel consumption, and displacement) for both BMW and Mercedes-Benz. The models were trained using historical data, with specific adjustments such as setting cap values for logistic growth and adding custom regressors to enforce trends.

Forecasting Future Values

Using the trained models, we forecasted the future values of each performance metric from 2020 to 2040. We created future dataframes, added necessary regressors, and predicted values for each year within the specified range.

3D Pareto Frontier Analysis

To visualize the performance trade-offs over time, we conducted a 3D Pareto frontier analysis. This analysis displayed the relationship between maximum horsepower, minimum acceleration, and average fuel consumption for both BMW and Mercedes-Benz from 2020 to 2040. The 3D plots provided insights into the expected trends and performance improvements over the years.

Results Visualization

We created detailed forecast tables and visualized the trends over time for each performance metric. The tables included predictions for maximum horsepower, minimum acceleration, minimum displacement, and fuel consumption for each year from 2020 to 2040. The visualizations highlighted the changes and trends in these metrics, providing a comprehensive overview of the expected future performance for both BMW and Mercedes-Benz.

BMW Forecast Table (2020-2040)

Year	Max_Horsepower	Min_Acceleration	Min_Displacement	Fuel Consumption
2020.0	669.2884465575137	0.9884419969057726	918.9106058167964	996.5333725546998
2021.0	688.5435140489542	1.4576695243579854	942.2779317926149	934.8393082036462
2022.0	709.4343809083013	1.2110504991437558	963.7824003453233	874.3852748001417
2023.0	731.947350359089	0.9278061259205063	983.381371622575	815.4879418677068
2024.0	740.6459247841269	0.607945989230202	850.7809299403589	758.3480373586586
2025.0	761.069910780523	1.0850429105202046	874.2146017374396	703.3679224807432
2026.0	783.0987073114691	0.8451428639352464	895.7871351745076	650.6730405563857
2027.0	806.7159360179786	0.567652185223998	915.455871654529	600.4225936447356
2028.0	816.4853079978456	0.2527191240021025	782.9270743916563	552.6781548831417
2029.0	837.9381368604562	0.7340459345875989	806.4338587610401	507.6225813746004
2030.0	860.9542044036197	0.4977573807316472	828.0811464880325	465.24169859981714
2031.0	885.5145929862592	0.2233593227567114	847.8262587281871	425.5384371849179
2032.0	896.1829320876644	-0.0889254458072794	715.3756539114163	388.45908689419105
2033.0	918.4832262388101	0.3946749074311579	738.9619991496535	353.9981063029888
2034.0	942.2954805916513	0.16032750123839187	760.6904063682059	322.05691475645364
2035.0	967.5985857377613	-0.11240830596363852	780.5181751461492	292.54209004222486
2036.0	978.9559022938043	-0.4232696508263965	648.1519747096993	265.34723129368956
2037.0	1001.8857224807592	0.06155269871476943	671.8239887289275	240.35004552286222

Figure 21: BMW Forecast Table (2020-2040)

Mercedes-Benz Forecast Table (2020-2040)

Year	Max_Horsepower	Min_Acceleration	Min_Displacement	Fuel Consumption
2020.0	679.720702524239	152.2004188084777	400.89289254360716	1071.2646328434394
2021.0	669.4655155527915	144.37973714290425	260.9269384350092	1064.9057580638196
2022.0	658.7871668818728	135.10404437863204	118.14694413724123	1057.5729023828105
2023.0	647.7151180217672	128.06771705914107	-27.29537004259373	1050.2176834527975
2024.0	729.7661802797026	121.4497567876174	304.6983614908597	1042.8409966214697
2025.0	719.5141374527927	115.23154562455112	164.77945833311708	1036.5106826511897
2026.0	708.8364936166765	107.48935831096422	22.04734813535549	1029.208383474728
2027.0	697.7627100487443	101.91136708065656	-123.34625706264569	1021.885782823076
2028.0	779.8095838443785	96.68310489822365	208.69713675398293	1014.543766333954
2029.0	769.5509230700679	91.7882471985486	68.82856929596244	1008.2502678434032
2030.0	758.864225881506	85.3087590971666	-73.85240580374204	1000.9867171931485
2031.0	747.7789565400521	80.92826030607073	-219.19408520964691	993.704884442472
2032.0	829.8118744735204	76.83816426975847	112.90216176948377	986.4056431889617
2033.0	819.5368686037825	73.024479390633	-26.912922889152355	980.1570400498105
2034.0	808.8314076581662	67.57422121522353	-169.53965121095473	972.940246345359
2035.0	797.7249620692673	64.16767032305424	-314.82632893340826	965.707138362651
2036.0	879.7342303137689	61.00136669896807	17.325819203814035	958.4585754489719
2037.0	869.4332383021784	58.06351338203171	-122.43277957095813	952.2627362458941

Figure 22: Mercedes-Benz Forecast Table (2020-2040)

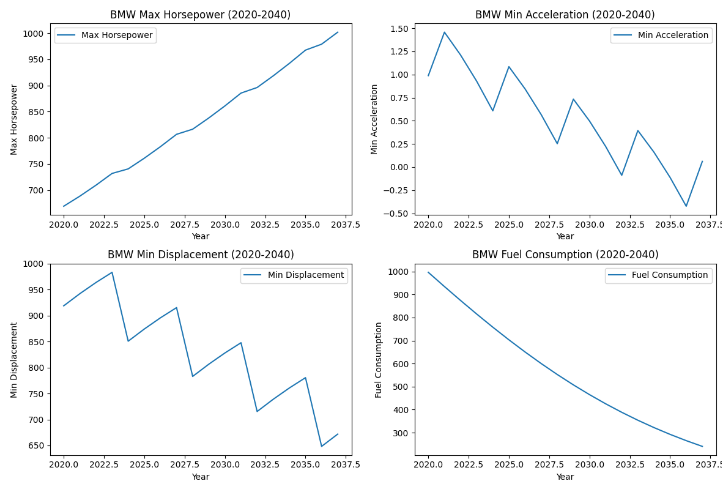

Figure 23: BMW Forecast Trends (2020-2040)

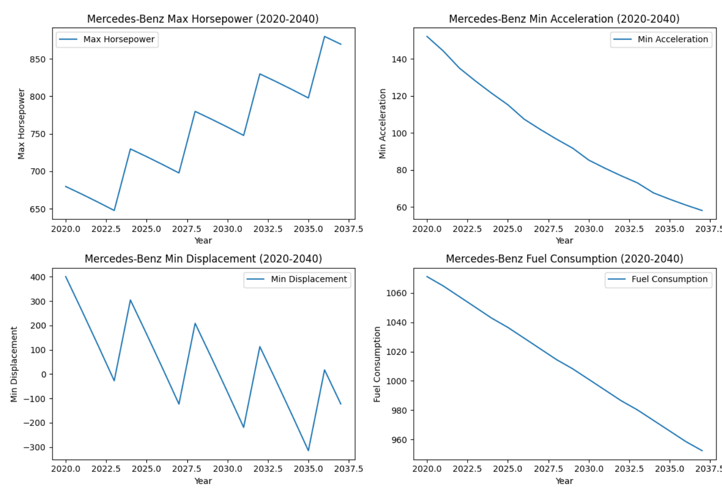

Figure 24: Mercedes-Benz Forecast Trends (2020-2040)

Error Rate Analysis

We analyzed the overall average error rates for each performance metric by comparing predictions from partial datasets against full datasets and actual values from 1980 to 2017. This analysis helped validate the accuracy and reliability of our models.

Conclusion

Through this comprehensive analysis, we were able to forecast the future performance metrics for BMW and Mercedes-Benz, providing valuable insights into expected trends and improvements. The use of the Prophet model, combined with detailed data preparation and rigorous validation, ensured that our forecasts were both accurate and informative.

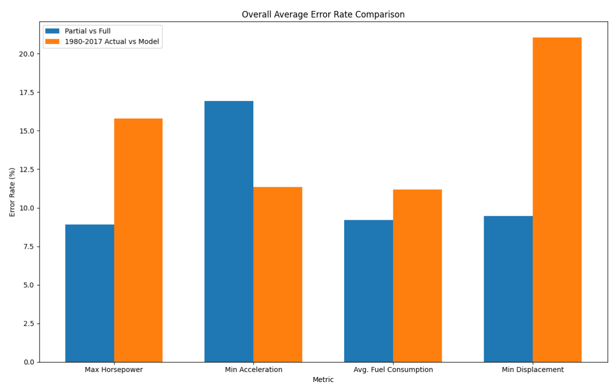

Figure 25: Overall Average Error Rate Comparison for Partial vs Full and 1980-2017 Actual vs Model