

HW 4 \Rightarrow ARSHITHA BASAVARAJ

Q) Yes, the above loop will work if the last loop is modified to : $j = 1$ to A.length.

Explanation: (with an example)

i/n array $\Rightarrow A = \{1, 2, 3, 3, 5, 6, 2, 1, 1\}$

Using the algorithm, output array B will be as follows. [Original Algorithm]

B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	B[7]	B[8]	B[9]
1	1	1	2	2	3	3	5	6

Modified algorithm output will be as follows

B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	B[7]	B[8]	B[9]
1	1	1	2	2	3	3	5	6

However, the original algorithm is a stable counting sort, i.e., identical elements are placed in the same order as they occurred in the input array, in the output array.

For instance, 1 ~~has~~ appears twice in the i/h array.

$B[1] \Rightarrow$ the 1 that appears first in the i/h array

$B[2] \Rightarrow$ the 1 " " ^{2nd} in " " "

$B[3] \Rightarrow$ the 1 that " " ^{3rd} " " " "

But modifying the original algorithm, ~~we get~~
we get

$B[3] \Rightarrow$ the 1 that appears 3rd in i/n array

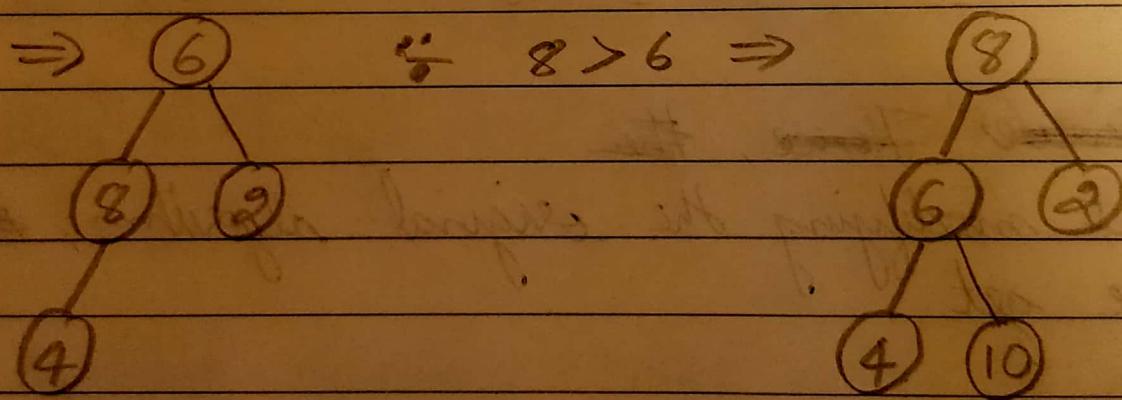
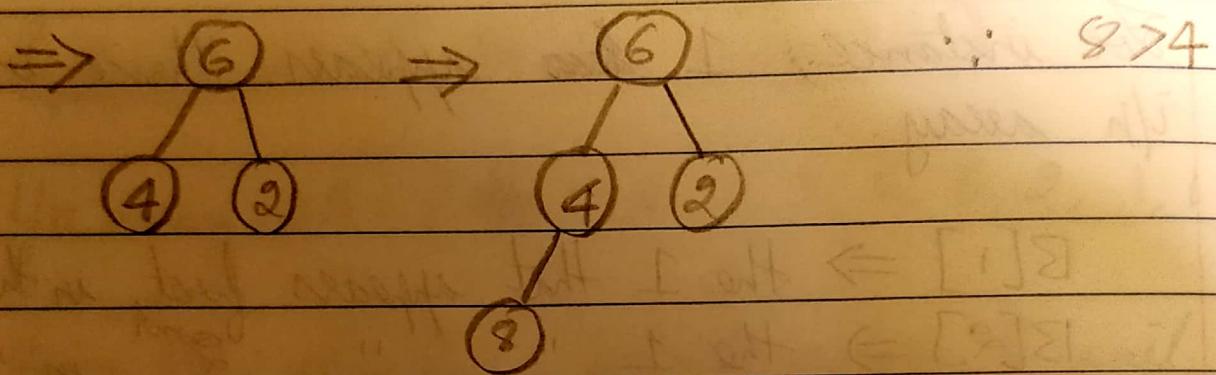
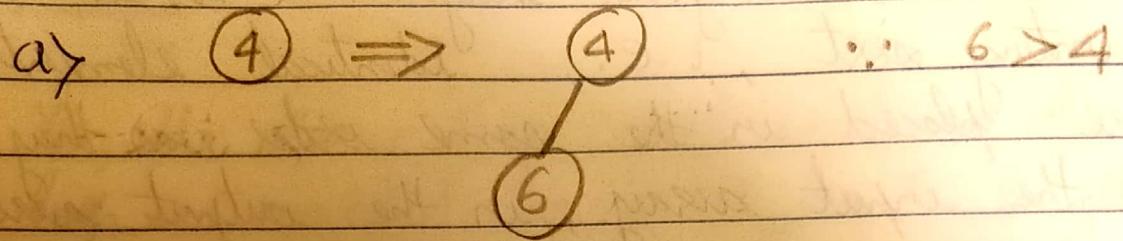
$B[2] \Rightarrow$ the 1 " " 2nd in i/n array

$B[1] \Rightarrow$ " 1 " 1st in i/n array

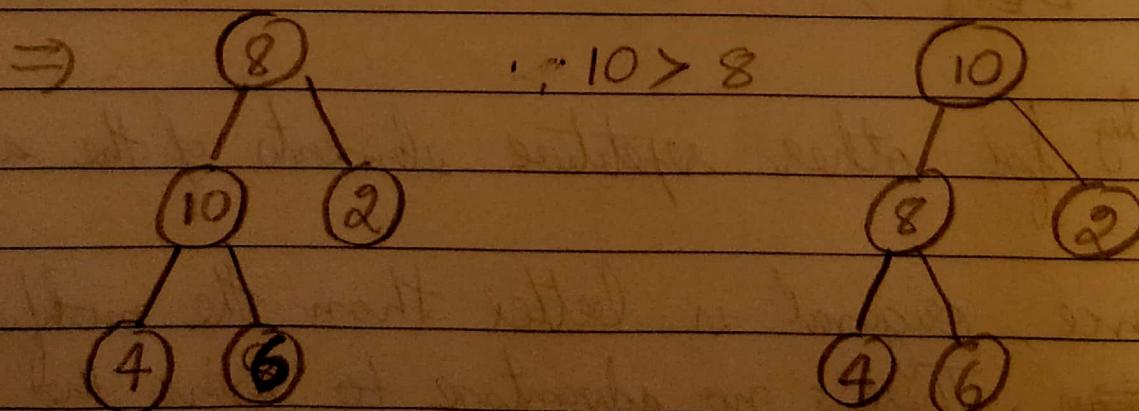
... for other repetitive elements of the array

Hence, original is better than the modified algo. ~~algo~~. There's no advantage to using the modified algorithm.

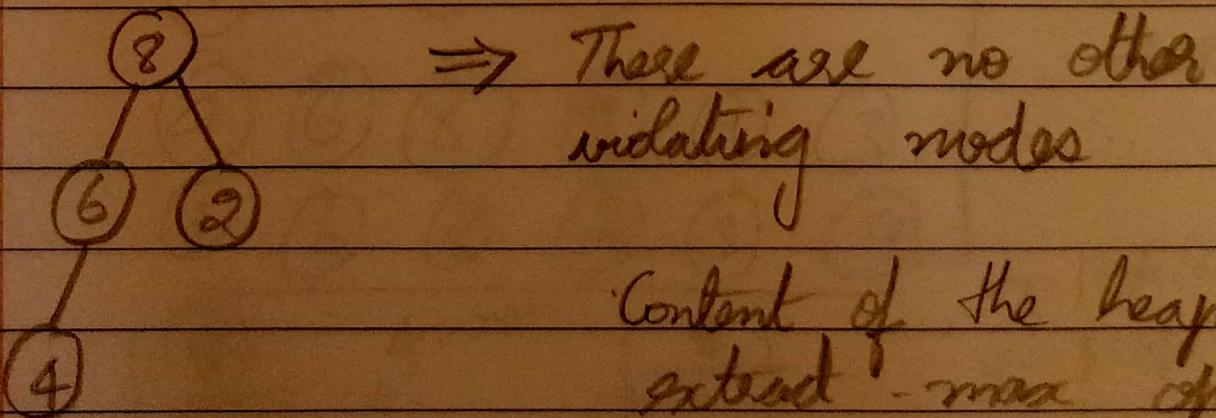
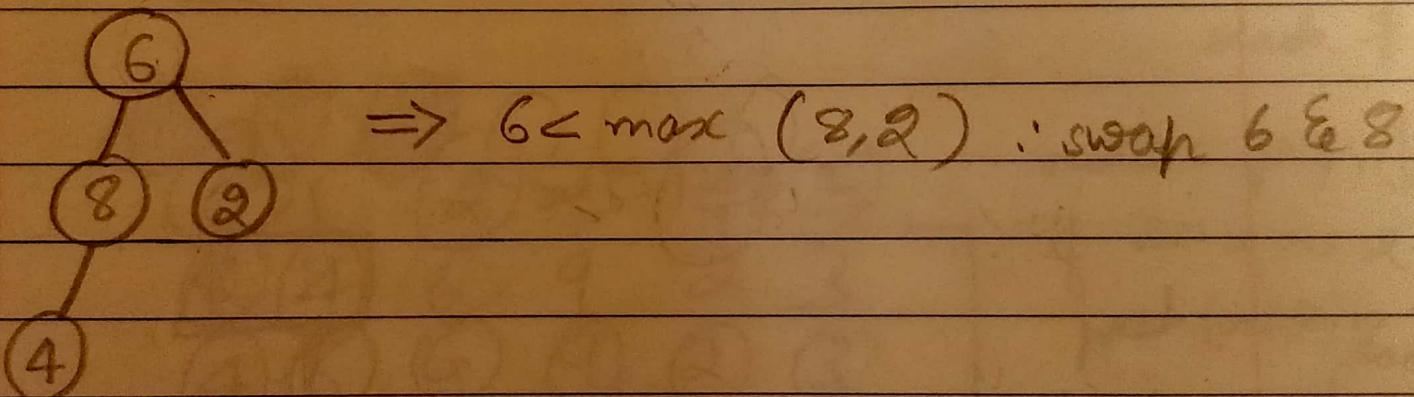
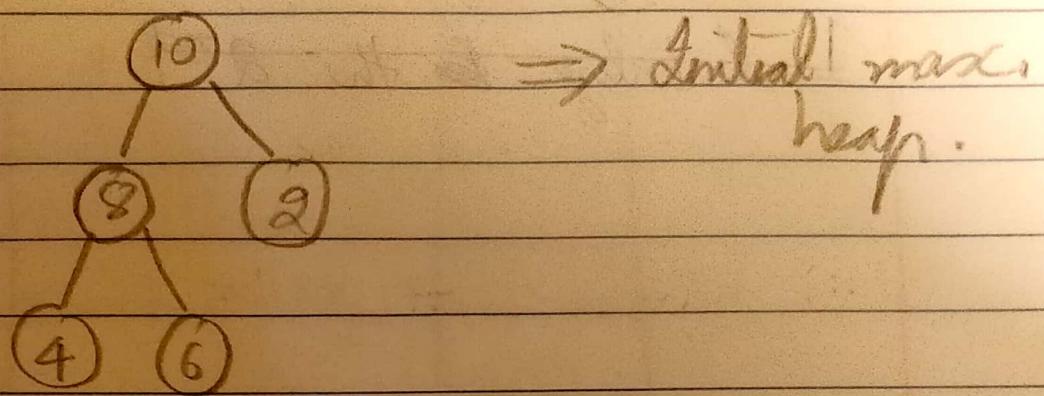
5



$\therefore 10 > 6$



b) Heapify after extracting max & replacing with least element



Content of the heap after extract-max operation

3) In the worst case, Quick Sort has $O(n^2)$ as its runtime.

But if we choose a pivot that ~~the~~ divides the ~~a~~ unsorted array equally, then the time complexity can be reduced to $O(n \log n)$.

This right pivot selection can be achieved by finding the median of medians of the unsorted array.

Median of medians can be found by using the Selection algorithm.

[Pseudo Algorithm description]

Select (k, n, A) // selects the k^{th} smallest
// of n elements in A
// A is the input array with
// n elements.

In our case, we need $\lceil \frac{n}{2} \rceil^{\text{th}}$ element
of the unsorted array.

Procedure

- ① Divide $i|h$ into $\frac{n}{5}$ groups of 5 elements each.
- ② Find the median of each group
- ③ Recursively find the median of medians

Select $(n = \frac{n}{5}, k = \frac{n}{2} \cdot \frac{1}{2})$

④ Use this value as the pivot for quick sort.

E.g.: 3, 8, 4, 1, 2, 6,
5, 9, 8, 7, 5, 5, 3

3 8 ④ 1 2
6 5 9 8 ⑦
5 ⑤ 3

median of } = 5 = pivot
medians

If we use this pivot, after 1st partition of unsorted array we get the following.

3 4 1 2 5 5 3 5 | 8 6 9 8 7

∴ Quick sort with median of medians as the pivot gives $O(n \log n)$ in worst case.

(S1)

1. Let stack 1 be stored with elements of i/n array.

(i) Declare an empty stack 2 (S2)

(ii) Run a loop for 'n' times (n is size of array)

a. Keep pushing elements in S2 till
~~stop~~ $S2.\text{top}() < S1.\text{top}()$

b. If $S1.\text{top}() < S2.\text{top}()$

a. Keep pushing elements in s_2 till
 ~~$s_2.top() < s_1.top()$~~

b. If $s_1.top() < s_2.top()$

then swap [$s_1.top()$, $s_2.top()$]

then push $s_2.top()$ back on
top.

After each iteration, one of the stack is
empty so the process (ii) is repeated on the
other stack.

- (iii) At the end of each iteration, the sorted element is placed in its correct position in the original array
- (iv) The array is then printed out.

[Pseudocode]

Stack S1; //contains elements of the array
Stack S2; //empty initially.

int i;
int temp;
int t;

} // 3 variables

for (i = 0 to n)

{ // After each iteration one stack is empty

if (i % 2 == 0) // & the other is full

while (S1 != empty)

{

while ($s1 \neq \text{empty}$)

{

$t = s1.\text{pop}();$

if ($s2 == \text{empty}$)

$s2.\text{push}(t);$

else

{

if ($s2.\text{top}() > t$) {

// swap

$\text{temp} = s2.\text{pop}();$

$s2.\text{push}(t);$

$s2.\text{push}(\text{temp});$

}

else
}

s2.push(t);

}

}

}

// store back the sorted element in the
original array.

arr[n-1-i] = s2.pop();

}

else

The same operation as above is done. Only this time S1 is empty & S2 is full.

~~Reference~~ Reference : Internet

4a. Result :

0 1 2 3 4 5 6 7 8 9

b. Yes, it sorts all arrays of n integers.

The given algorithm is just another implementation of insertion sort. And therefore, it works.