# Homework 2

1.) Asymptotics: Place following from smallest to largest

$3n^7+2,\ \log_n(\sqrt{n}),\ 70{,}000,\ \sum_{k=1}^{n}\frac{n}{2^k},\ \frac{7n}{10},\ \log(n^2),\ (5n+2)^n,\ 3^n,$
$\sqrt[5]{n},\ (\log(n))^2$

simplifying expressions:

- $\log_n(\sqrt{n}) = \log_n(n^{1/2}) = 1/2$ by definition of log $\log_a(b)=c$ is equiv. to $b=a^c$ $= \Theta(1)$

- $\sum_{k=1}^{n}\left(\frac{n}{2^k}\right) = n\cdot\sum_{k=1}^{n}\frac{1}{2^k} = n\underline{\sum_{k=1}^{n}}\left(\frac{1}{2}\right)^k$

$n\left(\sum_{k=0}^{n}\left(\left(\frac{1}{2}\right)^k\right) - \left(\frac{1}{2}\right)^0\right) = n\left(\sum_{k=0}^{n}\left(\left(\frac{1}{2}\right)^k\right) - 1\right) \Rightarrow$ Geometric series p. 1147

$= n\left(\frac{\left(\frac{1}{2}\right)^{n+1}-1}{\frac{1}{2}-1} - 1\right) = n\left(\frac{\left(\frac{1}{2}\right)^{n+1}-1}{-\frac{1}{2}} - 1\right)$

$= n\left(-2\left(\frac{1}{2}\right)^{n+1} + 2 - 1\right) = n\left(-2\left(\frac{1}{2}\right)^{n+1} + 1\right)$

$= n - 2n\left(\frac{1}{2}\right)^{n+1} = n - 2n(2)^{-(n+1)} = \boxed{n - 2^{-n}\cdot n} = \Theta(n)$

Ordering expressions:

$$\boxed{70{,}000 = \log_n(\sqrt{n})\ ;\ \log(n^2)\ ;\ \log(n)^2\ ;\ \sqrt[5]{n}\ ;\ \frac{7n}{10} = \sum_{k=1}^{n}\frac{n}{2^k}\ ;\ 3n^7+2\ ;\ 3^n\ ;\ (5n+2)^n}$$

2.) Asymptotic Comparison

specify with "yes" or "no" whether the comparisons are true.

| A | B | O | o | Ω | ω | Θ |
|---|---|---|---|---|---|---|
| n! | $\bar{e}^{n}$ | no | no | yes | yes | no |
| $10n^2+6n-2$ | $\binom{n}{2}$ | yes | no | yes | no | yes |
| $\sum_{k=2}^{n}\frac{1}{\ln k}$ | $\log(\log n)^n$ | yes | yes | no | no | no |
| $\log(n^7)$ | $e^{\ln(\ln n)}$ | yes | no | yes | no | yes |
| $\sum_{k=1}^{n}\left(\frac{1}{k+1}-\frac{1}{k}\right)$ | $\sqrt{n}$ | yes | yes | no | no | no |

Simplifying expressions:

- $\binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{n\cdot(n-1)}{2} = \frac{n^2-n}{2}$

- $e^{\ln(\ln(n))} = \ln(n)$ by $b^{\log_b(k)} = k$

- $\sum_{k=1}^{n}\left(\frac{1}{k+1} - \frac{1}{k}\right) \Rightarrow$ telescoping series

  $= \frac{1}{n+1} - 1$

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  $\left[\sum_{k=1}^{n}(a_k - a_{k-1}) = a_n - a_0\right]$

  in this case we have $a_{k+1} - a_k$, but same principle applies.

- $n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ by Stirling's Approx

- $\sum_{k=2}^{n}\frac{1}{\ln(k)}$ is $O\left(\sum_{k=1}^{n} 1\right) \Rightarrow \sum_{k=1}^{n} 1$ is $\Theta(n)$

- $\log(\log n)^n \Rightarrow$ log rules $\Rightarrow n\cdot\log(\log n)$

- $\log(n^7) \Rightarrow$ log rules $\Rightarrow 7\log(n)$