1a. [ARSHITHA BASAVARAJ]

$$\sum_{i=12}^{N} 5^i = 5^{12} + 5^{12}.5 + 5^{12}.5^2 + \ldots + 5^{12}.5^{(N-12)} \rightarrow (1)$$

(1) is a geometric series where r is positive & greater than zero.

$$\Rightarrow \sum_{i=12}^{N} 5^i = \frac{a(r^n - 1)}{(r-1)} = \frac{5^{12}(5^{N-11} - 1)}{5-1}$$

$$\Rightarrow \sum_{i=12}^{N} 5^i = \frac{5^{12}(5^{N-11} - 1)}{5-1}$$

1b.

$$\sum_{i=0}^{\infty} 3/11^i = \frac{3}{11^0} + 3/11^1 + \cdots$$

$$= 3 + 3/11 + 3/11^2 + \cdots$$

Here, $r = 1/11$ & $a = 3$

$\because 0 \leq r < 1$ & $n \to \infty$

$$\sum_{i=0}^{\infty} 3/11^i = \frac{a}{1-r}$$

$$= \frac{3}{10/11} = \frac{3 \times 11}{10} = 33/10$$

$$\boxed{\sum_{i=0}^{\infty} 3/11^i = 33/10}$$

1c.

$$\sum_{i=1}^{N} (8i^2 - 21i + 9)$$

$$= 8\sum_{i=1}^{N} i^2 - 21\sum_{i=1}^{N} i + 9$$

$$= 8\left[\frac{N(N+1)(2N+1)}{6}\right] - 21\left[\frac{N(N+1)}{2}\right] + 9$$

1d.

$$\sum_{i=6}^{315} 1/i = \sum_{i=1}^{5} 1/i - \sum_{i=1}^{5} 1/i + \sum_{i=6}^{315} 1/i$$

$$= \sum_{i=1}^{315} 1/i - \sum_{i=1}^{5} 1/i$$

$$= \ln 315 + \underbrace{O(1) - \sum_{i=1}^{5} 1/i}_{\text{constant}}$$

$$= \ln 315 + c,$$

where c is a constant.

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1e.

$$\sum_{i=1}^{3^N} \log_{18} i$$

$$= \log_{18} 1 + \log_{18} 2 + \log_{18} 3 + \dots + \log_{18} 3^N$$

~~using~~

$$= \frac{1}{\log 18} \sum_{i=1}^{3^N} \log i$$

$$= \frac{1}{\log 18} \left[\log 1 + \log 2 + \dots + \log 3^N \right]$$

Now, using Stirling's approximation,

$$\approx \frac{1}{\log 18} \left[3^N \log 3^N - 3^N + O(\log 3^N) \right]$$

$$\approx \frac{1}{\log 18} \left[N . 3^N \log 3^N - 3^N + O(\log 3^N) \right]$$

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2a.

$$x^{11}.x^{12}.\ldots x^{N} = \frac{x^1.x^2\ldots x^{10}.x^{11}.x^{12}\ldots x^N}{x^1.x^2.x^3\ldots x^{10}}$$

$$= \frac{x^{N(N+1)/2}}{x^{\frac{10.11}{2}}} = \frac{x^{N(N+1)/2}}{x^{55}}$$

$$= \frac{x^{\frac{N(N+1)}{2}}}{x^{55}}$$

2b.

$$\log_{17}(47.47.47.47)$$

$$= \log_{17}(47^4)$$

$$= 4 \cdot \log_{17}(47)$$

2c.

$$\log_x\left((2x)^x\right)$$

$$= x \cdot \log_x(2x)$$

$$= x\left[\log_x 2 + \log_x x\right]$$

$$= x\left[\log_x 2 + 1\right]$$

2d. $72^{\log_{72} 152} \longrightarrow$ (1)

By definition of logarithms,

$72^x = 152$, where x here is an unknown.

$\Rightarrow \log_{72} 152 = x$

Substituting in (1)

$\Rightarrow 72^x = 152$

$\Rightarrow \boxed{72^{\log_{72} 152} = 152}$

3a.

MSD LSD

$7 \times 7 \times 7 \times 7 \times 7 \times 7$

MSD: The most significant digit can be filled with digits 3, 4, 5 .... 9.

Every other digit in a 6-digit decimal no. can be filled with any digit from 3, 4, 5 ... 9.

$\Rightarrow$ No of 6-digit decimal no.'s without a ~~xxxx~~ no. smaller than 3 are $\Big\} = 7 \times 7 \times 7 \times 7 \times 7 \times 7$

$= 7^6$

both included, i.e,

3b. ASSUMPTION: 17 & 68 are both included, ie,

[17, 68]

There are 52 numbers in the set [17, 68].

$$\Rightarrow \boxed{{}^{52}C_9 = \frac{52!}{(52-9)!\,9!}}$$

$$= \frac{52 \times 51 \times 50 \times \cancel{49}^{7} \times \ldots \times \cancel{42}^{7} \times 43}{9 \times 8 \times \cancel{7} \times \cancel{6} \times 5 \times 4 \times 3 \times 2}$$

$$\boxed{{}^{52}C_9 = \frac{52!}{43!\,9!}}$$

5a.

① Pick any integer in the range $0 \leq N \leq 7$.

② Create a blank grid of ROWS × COLUMNS.
In this case, 10×10.

③ If N is even, i.e., if $N \% 2 = 0$, then fill position $(N+2, N+2)$ with 'O'.

④ If N is odd, then
fill positions $(N+1, N+3)$ & $(N+3, N+3)$ with 'O's

⑤ For filling 'Xs', N+1 are used at the max..

⑥ For each columns 0 to N, positions (i, j) & $(i+1, j)$ are filled with X's.

j → column no.

i → row no.

⑦ Once one column is filled with X's, i is incremented by 1 before repeating ⑥ for the next column.