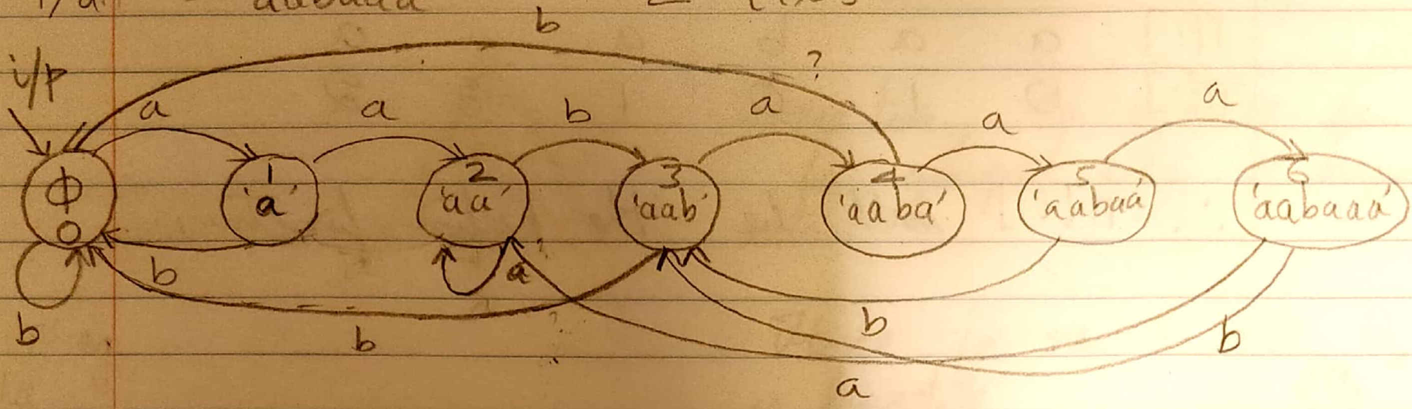
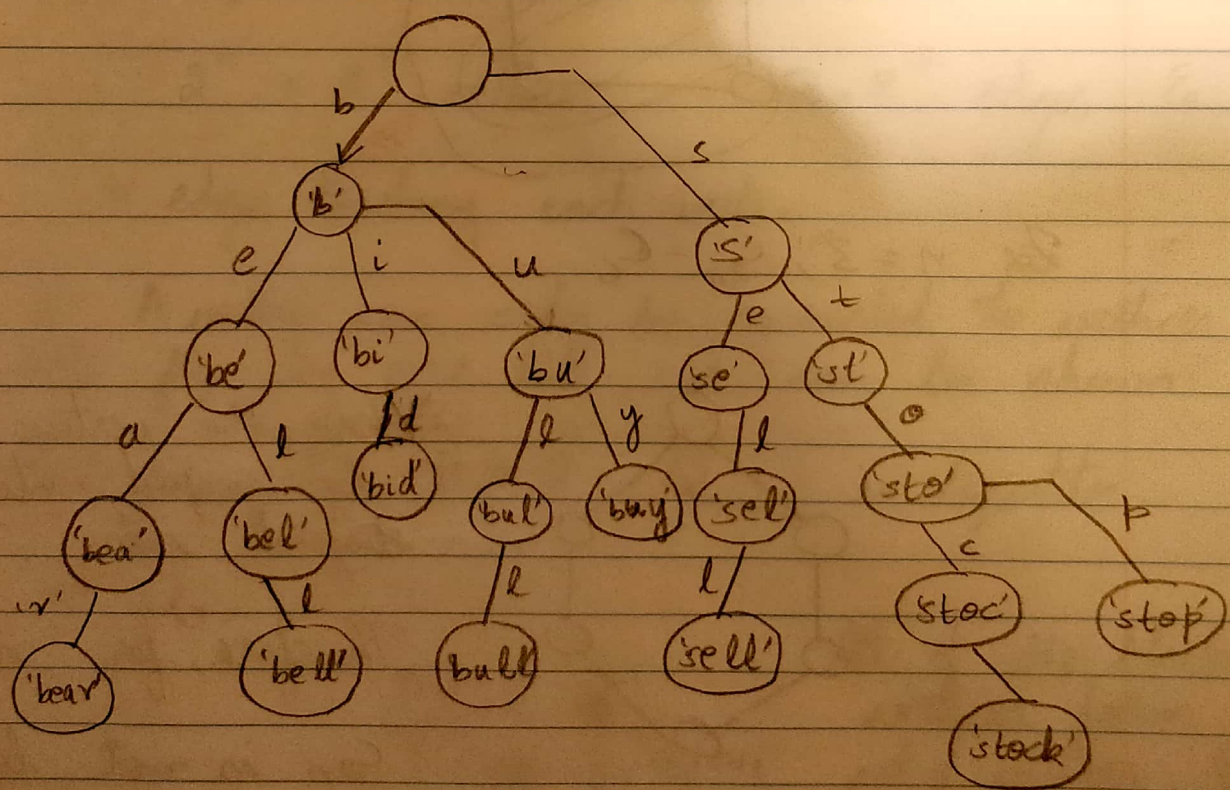
HOMEWORK - 6 ⟹ ARSHITHA BASAVARAJ

1) a. = "aabaaa" $\Sigma = \{a, b\}$



Anything other than a, b goes to the initial state.

b.



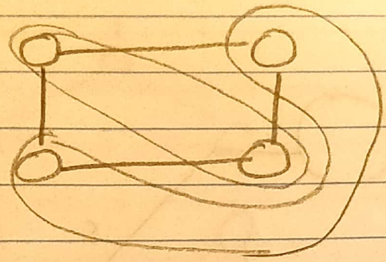
c.

i	1	2	3	4	5	6
P[i]	a	a	b	a	a	a
π[i]	0	1	0	1	2	2

∴ π[i] will be the prefix function.

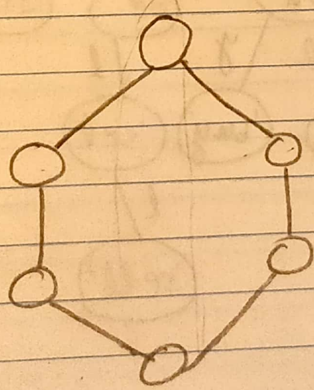
2)

a) For $n = 2$, $C_{2\times 2} = C_4$



C_4 is bipartite

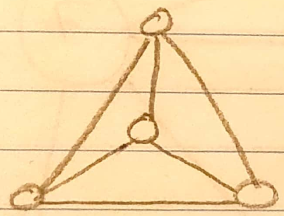
For $n = 3$, $C_{2n} = C_6$



Here the vertices can't be grouped into 2 disjoint sets.

Therefore, for $n \geq 3$, C_{2n} is not bipartite.

b) W_{2n} For $n = 2$, $W_{2n} = W_4$



Here too, the vertices can't be grouped into 2 disjoint sets.

Hence, for no $n > 0$, W_{2n} is bipartite

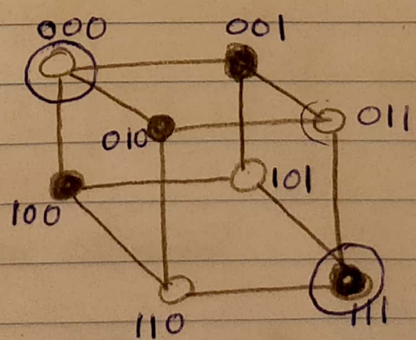
c) $Q_{n+7} = Q_8$

$2^n = 2^8$ vertices $8 \cdot 2^7$ edges &

n edges touching each vertex.

$\because$ A vertex can only be connected to vertices whose bits flip by one bit, it can be shown that for all $n > 0$, Q_{n+7} is bipartite.

For Q_3,



All of the blackened vertices form one set of vertices &

the other set of vertices are the white ones.

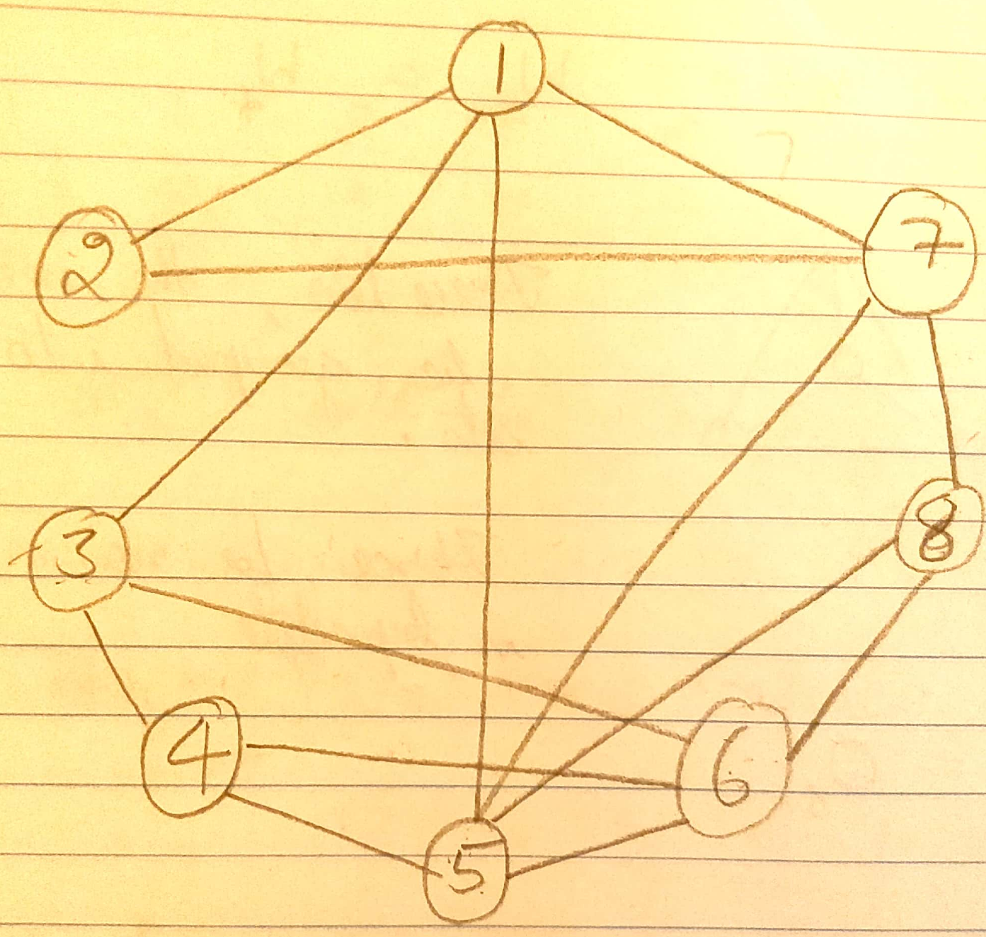
These sets are disjoint.

$\Rightarrow Q_3$ is bipartite

This theory can be extended to all $n > c$ of Q_{n+7} hyper cube graphs.

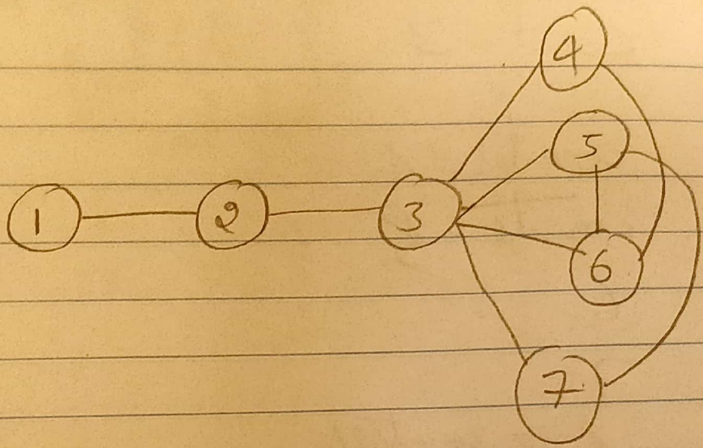
3.

a.



This would be the resulting graph G.

b.



Adjacency list

Vertex	Edges
1	2
2	1, 3
3	2, 4, 5, 6, 7
4	3, 6
5	3, 6, 7
6	3, 4, 5
7	3, 5

Algorithm

Step 1: Go to every vertex & check if there exists a one and only vertex which has a single edge. – $\Theta(V)$

Step 2: Vertex found in Step 1 would be the tail. Go to the vertex pointed by the tail.

$\Theta(1)$

Step 3: Check if this vertex has edges as the tail and another vertex. This would be the center vertex.

$\Theta(1)$

Step 4: Go to the vertex other than tail of the center vertex. ~~If~~ $\Theta(1)$

Step 5: If the vertex found in step 4 has $|V|-2$ edges, then it's the body vertex. $\Theta(1)$

Step 6: If the above conditions are satisfied then it's a kite graph.

Asymptotic runtime of the algorithm is $\Theta(V)$

Adjacency list is the most efficient graph representation of all the others.