Homework 1

1.) Sums: Provide closed-form sol'n, along with brief explanation

a.) $\sum_{i=12}^{N} 5^i \Rightarrow$ exponential series $\Rightarrow \sum_{k=0}^{n} x^k = \frac{x^{n+1}-1}{x-1}$

$\sum_{i=12}^{N} 5^i = \sum_{i=0}^{N} 5^i - \sum_{i=0}^{11} 5^i$ ↳ only applies to sums 0:n!

$\frac{5^{N+1}-1}{5-1} - \frac{5^{11+1}-1}{5-1} = \frac{1}{4}\left(5^{N+1}-1-\left(5^{12}-1\right)\right)$

$\boxed{\frac{1}{4}\left(5^{N+1}-5^{12}\right)}$

b.) $\sum_{i=0}^{\infty} \frac{3}{11^i} = 3\sum_{i=0}^{\infty}\left(\frac{1}{11}\right)^i \Rightarrow$ Geometric series

$= 3\left(\frac{1}{1-\frac{1}{11}}\right) = 3\left(\frac{1}{\frac{10}{11}}\right)$ $\quad \sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$

$= 3\left(\frac{11}{10}\right) = \boxed{\frac{33}{10}}$

c.) $\sum_{i=1}^{N}\left(8i^2-21i+9\right) \Rightarrow$ linearity

$= 8\cdot\sum_{i=1}^{N}\left(i^2\right) - 21\cdot\sum_{i=1}^{N}(i) + 9\cdot\sum_{i=1}^{N} 1$ $\Rightarrow$ sum of squares; arithmetic series

$= 8\left(\frac{N(N+1)(2N+1)}{6}\right) - 21\left(\frac{N(N+1)}{2}\right) + 9(N)$

$\boxed{= \frac{4}{3}\left(N(N+1)(2N+1)\right) - \frac{21}{2}\left(N(N+1)\right) + 9(N)}$

d.) $\sum_{i=6}^{315}\left(\frac{1}{i}\right) = \sum_{i=1}^{315}\left(\frac{1}{i}\right) - \sum_{i=1}^{5}\left(\frac{1}{i}\right) \Rightarrow$ Harmonic Series $\sum_{k=1}^{n}\frac{1}{k} = \ln(n) + \text{constant}$

$\ln(315) + \text{constant} - (\ln(5) + \text{constant}) \Rightarrow \ln(a) - \ln(b) = \ln(a/b)$

$\ln(315/5) + \text{constant} = \boxed{\ln(63) + \text{constant}}$

e.) $\sum_{i=1}^{3^N}\left(\log_{18}(i)\right) \Rightarrow$ substitute $m = 3^N$

$\sum_{i=1}^{m}\left(\log_{18}(i)\right) \Rightarrow \log(x) + \log(y) = \log(xy)$

$= \log_{18}(m!) = \boxed{\log_{18}(3^N!)}$

2.) Exponents and logs

a.) $x^{11}\cdot x^{12}\cdot x^{13}\cdots x^{N} \Rightarrow x^a\cdot x^b = x^{a+b}$

$= x^{\sum_{i=1}^{N} i} = x^{\sum_{i=1}^{N} i - \sum_{i=1}^{10} i} = \boxed{x^{\frac{N(N+1)-10\cdot 11}{2}}}$

b.) $\log_{17}(47\cdot 47\cdot 47\cdot 47) \Rightarrow \log(x^y) = y\log(x)$

$\boxed{4\log_{17}(47)} \approx 5.436$

c.) $\log_x((2x)^x) \Rightarrow \log(x^y) = y\log(x)$

$x\log_x(2x) \Rightarrow \log(a \cdot b) = \log(a) + \log(b)$

$x(\log_x(x) + \log_x(2)) = \boxed{x(1 + \log_x(2))}$

d.) $72^{\log_{72}(152)} \Rightarrow b^{\log_b(k)} = k$

$\boxed{= 152}$

3.) Combinatorics

a.) How many 6-digit decimal numbers do not contain any digits < 3?

can choose digits $3:9 = 7$ digits

order matters, repeats allowed $\Rightarrow n^r$

$\boxed{= 7^6}$

b.) How many ways can you pick 9 <u>different</u> numbers between 17 and 68? Order doesn't matter.

$68 - 16 = 52$ (include 17); 52 choose 9, no replacement

$\boxed{52C_9 \; ; \; \binom{52}{9}}$