3C05
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Assignment 6 Parameter Estimation –

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$x_1, x_2 \dots x_n$ 3 sample n size

$$L(x_1, x_2 \dots x_n) = f(x_1) \cdot f(x_2) \cdot f(x_3) \cdots f(x_n)$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}} \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right)$$

Taking ln both sides

$$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i-\mu)^2}{2\sigma^2}\right) \quad (1)$$

derivative μ

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{} -\frac{(2(x_i-\mu))}{2\sigma^2} = 0$$

$$\Rightarrow \sum_{i=1}^{n} (x_i - \mu) = 0$$

$$\Rightarrow n\bar{x} - n\mu = 0$$

$$\bar{x} - \mu$$

$Q_1 = \bar{x}$ i.e Sample mean

w.r.t σ^2

$$\frac{\partial \ln(L)}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{2(\sigma^2)^2} = 0$$

$$-n + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{\sigma^2} = 0$$

$$n = \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2} \qquad \sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\mu)^2$$

$$\therefore Q_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$$

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2) Binomial distribution

$$^nC_{x_i}\, Q^{x_i}(1-Q)^{n-x_i}$$

$$L = \prod_{i=1}^{n} {}^nC_{x_i}\, Q^{x_i}(1-Q)^{n-x_i}$$

log both side

$$\log L = \sum_{i=1}^{n} \log {}^nC_{x_i} + \log Q^{x_i} + \log (1-Q)^{n-x_i}$$

$$\log L = \sum_{i=1}^{n} \log({}^nC_{x_i}) + \log Q \sum_{i=1} x_i + \log(1-Q)\sum_{i=1}^{n} (n-x_i)$$

Q differenciate

$$\frac{d\log(L)}{dQ} = 0$$

$$\frac{1}{Q}\sum x_i - \frac{n^2}{1-Q} + \frac{1}{1-Q}\sum x_i = 0$$

$$\frac{1}{Q(1-Q)}\sum x_i = \frac{n^2}{1-Q}$$

$$\frac{\sum x_i}{Q} = n^2$$

$$\boxed{Q = \frac{\sum x_i}{n^2}}$$ ans.