Q 1). Given a random sample $(X_1, X_2, -- X_n)$

$\mu = \theta_1$ (mean) & $\sigma^2 = \theta_2$ (variance)

Likelihood function $L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \exp\left(-\frac{(x_i - \theta_1)^2}{2\theta_2}\right)$

To maximize take log on both sides

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[-\frac{1}{2} \ln(2\pi\theta_2) - \frac{(x_i - \theta_1)^2}{2\theta_2} \right]$$

(i) differentiate wrt θ_1 [for θ_1]

$$\frac{d}{d\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$

$$x_i - n\theta_1 = 0$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{n} \quad [\text{mean}]$$

(ii) differentiate wrt θ_2 [for θ_2]

$$\frac{d}{d\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[\frac{-1}{2\theta_2} + \frac{(x_i - \theta_1)^2}{2\theta_2^2} \right] = 0$$

$$\frac{n}{\cancel{2}\theta_2} = \frac{1}{\cancel{2}\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2 \quad [\text{Variance}]$$

Q2 Binomial distribution $B(n, \theta)$

$p = \theta$

$q = 1-\theta$

PMF (Prob mass function)

$$f(x; n, \theta) = {}^{n}C_{x}\, \theta^{x} (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

$$\ln L(\theta) = \sum_{i=1}^{n} \left[\ln {}^{n}C_{x_i} + x_i \ln\theta + n-x_i \ln(1-\theta) \right]$$

differentiate w.r.t θ

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right] = 0$$

Find θ

$$\sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right] = 0$$

$$\sum_{i=1}^{n} \left(\frac{(1-\theta)x_i - (n-x_i)\theta}{\theta(1-\theta)} \right) = 0$$

$$\sum_{i=1}^{n} (1-\theta) x_i - (n-x_i)\theta = 0$$

$$\theta \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i\, n$$

$$\boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{n \cdot n}} \rightarrow \text{MLE}$$