Thesis Statement: Fractional Differential Equations with Caputo and Liouville Derivatives

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1. Overview of the Mathematical Topic

Fractional differential equations (FDEs) are equations that involve fractional derivatives of order α, where α can be a real number or even a complex number. These equations generalize classical integer-order differential equations and have applications in various fields such as physics, engineering, and finance. The Caputo and Liouville derivatives are fundamental operators in the theory of fractional calculus, providing tools to describe systems with memory effects and non-local behaviors.

2. Motivation for Choosing This Topic

The study of fractional differential equations appeals to me due to its interdisciplinary nature and its ability to model complex systems more accurately than integer-order models. As a researcher interested in advanced mathematical modeling, exploring the properties and solutions of FDEs with Caputo and Liouville derivatives offers both academic challenge and practical relevance in understanding real-world phenomena with fractional dynamics.

3. Relevant Mathematical Areas

The mathematical areas central to FDEs with Caputo and Liouville derivatives include fractional calculus, functional analysis, numerical methods for solving fractional equations, and the theory of special functions such as the Mittag-Leffler function. These areas provide the theoretical foundation necessary for analyzing and solving fractional differential equations in various applications.

4. Specific Mathematical Problem

The specific problem addressed in this thesis is to investigate the existence, uniqueness, and stability of solutions to fractional differential equations involving Caputo and Liouville derivatives. Particularly, we focus on establishing conditions under which solutions exist and are unique, and exploring the long-term behavior of solutions using analytical and numerical methods.

5. Historical Background

The study of fractional calculus dates back to the works of mathematicians such as Liouville, Riemann, and Caputo in the 19th century. These pioneers laid the groundwork for understanding fractional derivatives and their applications in physics and engineering. Recent advances have further expanded the applications of fractional calculus in various scientific disciplines, emphasizing its importance in modern mathematical modeling.

6. Thesis Statement

Through this research, we aim to contribute to the theoretical understanding and practical applications of fractional differential equations with Caputo and Liouville derivatives. By exploring the mathematical properties and solutions of these equations, we seek to provide insights into complex systems with memory effects and non-local behaviors, advancing the field of fractional calculus and its applications.