Tutorial Quiz 1 - Group Part

MAT344 - Spring 2019

Instructions:

Please record **each Group member's Name and Student Number**.

Make sure to show your work, justifying where possible and annotating any interesting steps or features of your work. **Do not just give the final answer, and do not simplify your calculations (use notation from the course, like $\binom{n}{k}$ or $S(n,k)$ etc.)**

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1 **(2 points each $\Rightarrow$ 4 points)** *(This whole question is identical to the individual version.)*

3333 aliens head towards planet Earth from a distant star. They are from a race of identical clones, each completely indistinguishable from the others.

1.1 When the spaceships reach Earth, they split up into groups to visit some of Earth's 100 largest cities. Some cities could have no spaceships visit.
How many ways are there for the spaceships to split up in this way?

1.2 Now suppose a spaceship with 123 of the aliens arrives above Toronto. This time, for the sake of the humans, the aliens decide to give themselves each unique names , like "Ziggy" and "Marvin" (and they put on name tags) to distinguish themselves from each other.
Then they split into groups and get into landing pods to take them from the spacecraft down to the ground. These landing pods are *identical* to each other and are unlimited in number, but each pod that gets launched from the ship will have at least one alien in it.
How many ways can the 123 aliens split up into landing pods?
(Ignore the other 3210 aliens and 677 spaceships.)

2 **(2 points + 1 bonus point available)**

2.1 **(2 points)** *(This part of the question is identical to #2 from the individual version.)*

How many ways are there to hand out 12 identical pieces of red licorice candy and 16 identical pieces of black licorice to four children, if each child must receive at least one piece of *black* licorice and must receive an even number of pieces of *red* licorice?

2.2 **(1 bonus point)** What if instead, there are the same starting amounts of candy, but only two children, and the restriction is that each child must receive an odd amount of licorice pieces *in total* (irrespective of colour)?

3 **(2 + 1 bonus point available)**

Suppose there are $n \geqslant 2$ students to be split into $k \geqslant 1$ groups. We will count ways to form the groups, where we will only be concerned with which students end up together (i.e. the groups are not numbered or otherwise distinguishable from each other).

3.1 **(2 points)** *(This part of the question is identical to #3 from the individual version.)*

Determine a formula for $T(n, k)$, the number of ways to split up the students with the following restriction: there are two particular students, *Legolas* and *Gimli*, who cannot be in a group together.

3.2 **(1 bonus point)** Repeat the previous part, but this time *Legolas* and *Gimili* can *either* be in a group together with no one else *or* they have to be put in different groups from each other.

4 **(2 bonus points)**

Let $N(n)$ be the number of all partitions of $[n]$ with *no* singleton blocks.

Let $A(n)$ be the number of all partitions of $[n]$ with *at least one* singleton block.

Prove that for all $n \geqslant 1$, $N(n+1) = A(n)$.