Tutorial Worksheet 6
MAT344 - Spring 2019

*Please refer to the list of **Graph Theory Definitions** on Quercus.*

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1 Is there a bipartite graph G with vertex set $\{\mathbf{v}_1, ..., \mathbf{v}_9\}$ and **ordered degree sequence** 6, 6, 6, 5, 3, 3, 3, 3, 3?

*(Recall that the **ordered degree sequence** of a graph is just a non-increasing list of the degrees of its vertices.)*

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2 Let G be a connected bipartite graph, let $\mathbf{v}, \mathbf{w}$ be vertices in G and let $\mathcal{P}_1$ and $\mathcal{P}_2$ be distinct paths from $\mathbf{v}$ to $\mathbf{w}$.

Prove that the lengths of $\mathcal{P}_1$ and $\mathcal{P}_2$ are either both even or both odd.

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3 Prove, by induction on $n \geqslant 1$, that if a (simple) graph G has n vertices and (strictly) more than $\binom{n-1}{2}$ edges, then G is connected.

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4 Prove that in any tree T, any two paths of maximum length in T intersect in at least one vertex.