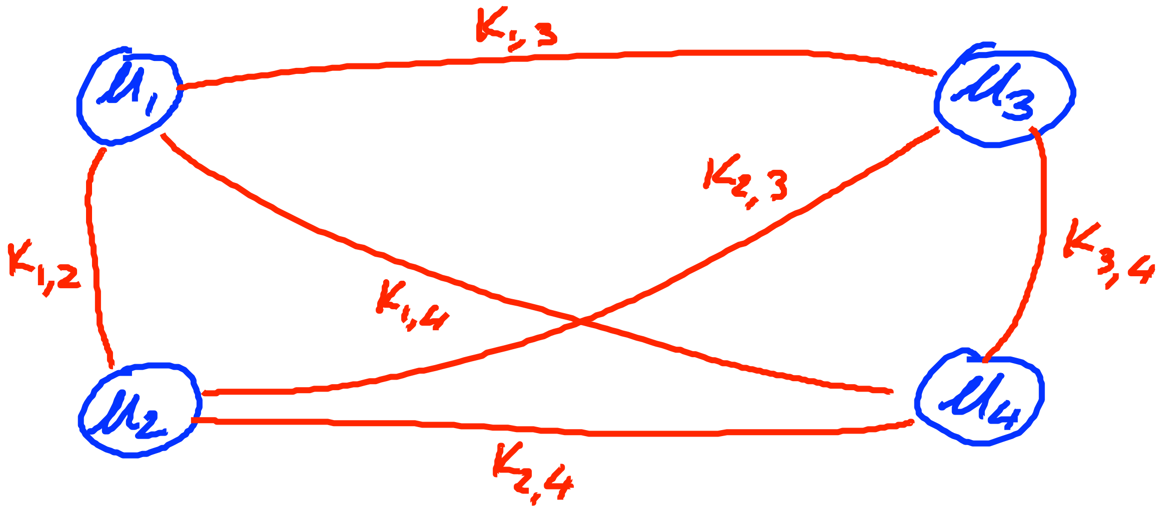
Basic key exchange

Trusted 3rd parties

Key management

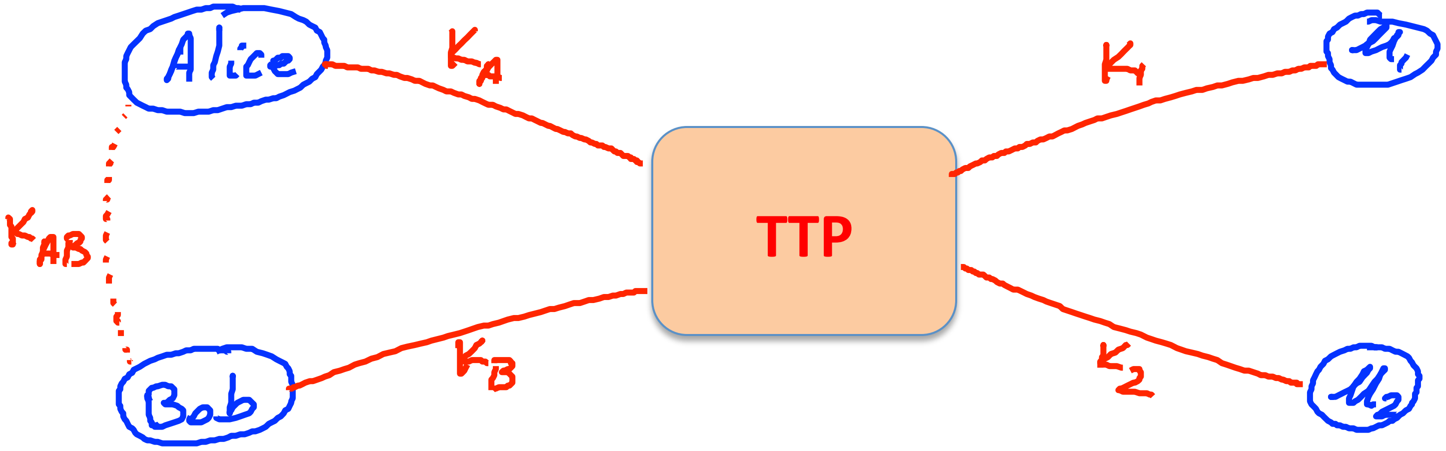
Problem: n users. Storing mutual secret keys is difficult



Total: O(n) keys per user

A better solution

Online Trusted 3rd Party (TTP)



Every user only remembers one key.

Generating keys: a toy protocol

Alice wants a shared key with Bob. Eavesdropping security only.

Bob (k_B) **Alice** (k_A) **TTP**

"Alice wants key with Bob"

choose random k_{AB}

$E(k_A, \text{"A,B"} \| k_{AB})$

$\text{ticket} \leftarrow E(k_B, \text{"A,B"} \| k_{AB})$

ticket

k_{AB} k_{AB}

$\boxed{(E,D) \text{ a CPA-secure cipher}}$

Generating keys: a toy protocol

Alice wants a shared key with Bob. Eavesdropping security only.

Eavesdropper sees: $E(k_A,$ "A, B" ll $k_{AB})$; $E(k_B,$ "A, B" ll $k_{AB})$

(E,D) is CPA-secure ⇒
eavesdropper learns nothing about k_{AB}

Note: TTP needed for every key exchange, knows all session keys.

(basis of Kerberos system)

Toy protocol: insecure against active attacks

Example: insecure against replay attacks

Attacker records session between Alice and merchant Bob

– For example a book order

Attacker replays session to Bob

– Bob thinks Alice is ordering another copy of book

Key question

Can we generate shared keys without an **online** trusted 3rd party?

Answer: yes!

Starting point of public-key cryptography:

- Merkle (1974), Diffie-Hellman (1976), RSA (1977)

- More recently: ID-based enc. (BF 2001), Functional enc. (BSW 2011)

End of Segment

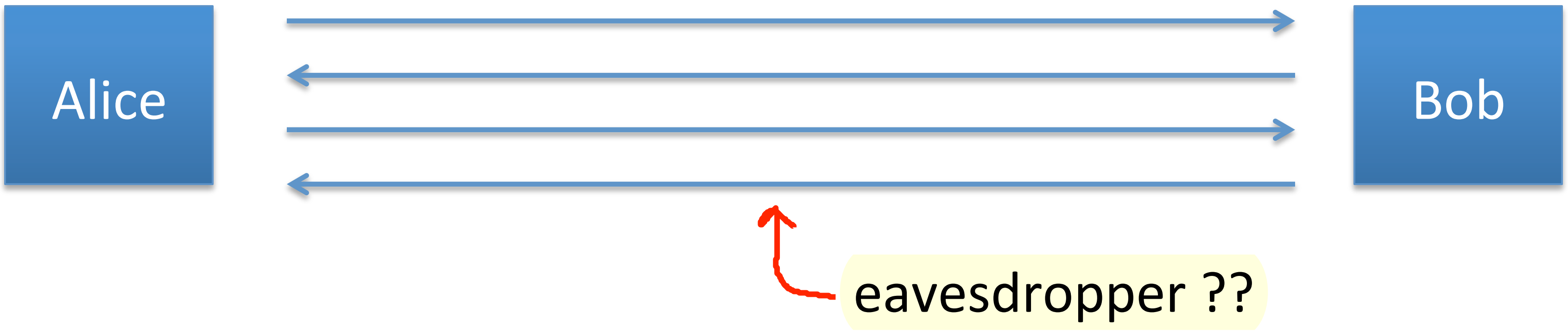
Basic key exchange

Merkle Puzzles

Key exchange without an online TTP?

Goal: Alice and Bob want shared key, unknown to eavesdropper

- For now: security against eavesdropping only (no tampering)



Can this be done using generic symmetric crypto?

Merkle Puzzles (1974)

Answer: yes, but very inefficient

Main tool: puzzles

- Problems that can be solved with some effort
- Example: $E(k,m)$ a symmetric cipher with $k \in \{0,1\}^{128}$
  - **puzzle(P) = E(P, "message")** where $P = 0^{96} \parallel b_1 \ldots b_{32}$
  - Goal: find P by trying all 2^{32} possibilities

Merkle puzzles

Alice: prepare 2^{32} puzzles

- For $i=1, \ldots, 2^{32}$ choose random $\mathbf{P_i} \in \{0,1\}^{32}$ and $\mathbf{x_i, k_i} \in \{0,1\}^{128}$

  set $\text{puzzle}_i \leftarrow E(\,0^{96} \,\|\, \mathbf{P_i}, \text{“Puzzle \# } x_i\text{”} \,\|\, k_i\,)$

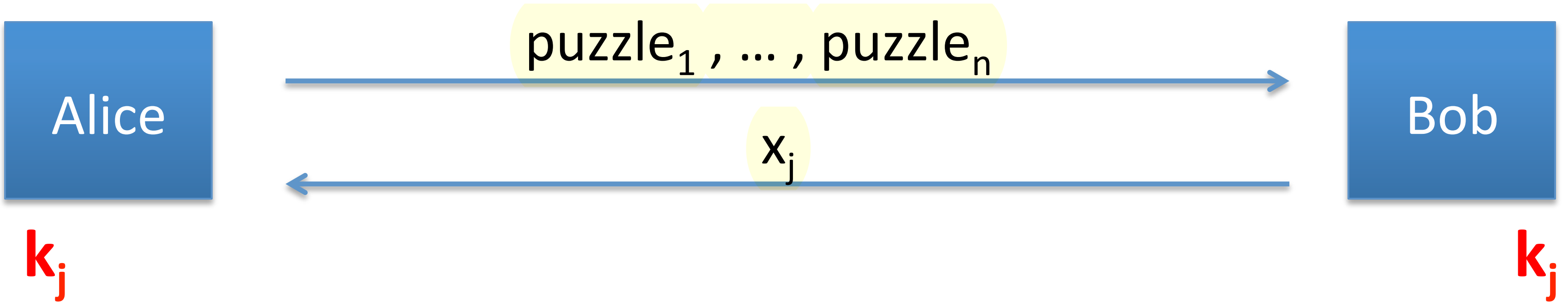
- Send $\text{puzzle}_1, \ldots, \text{puzzle}_{2^{32}}$ to Bob

Bob: choose a random puzzle_j and solve it. Obtain (x_j, k_j).

- Send x_j to Alice

Alice: lookup puzzle with number x_j. Use k_j as shared secret

In a figure



Alice's work: O(n) (prepare n puzzles)

Bob's work: O(n) (solve one puzzle)

Eavesdropper's work: $O(n^2)$ (e.g. 2^{64} time)

Impossibility Result

Can we achieve a better gap using a general symmetric cipher?

Answer: unknown

But: roughly speaking,

quadratic gap is best possible if we treat cipher as a black box oracle [IR'89, BM'09]

End of Segment

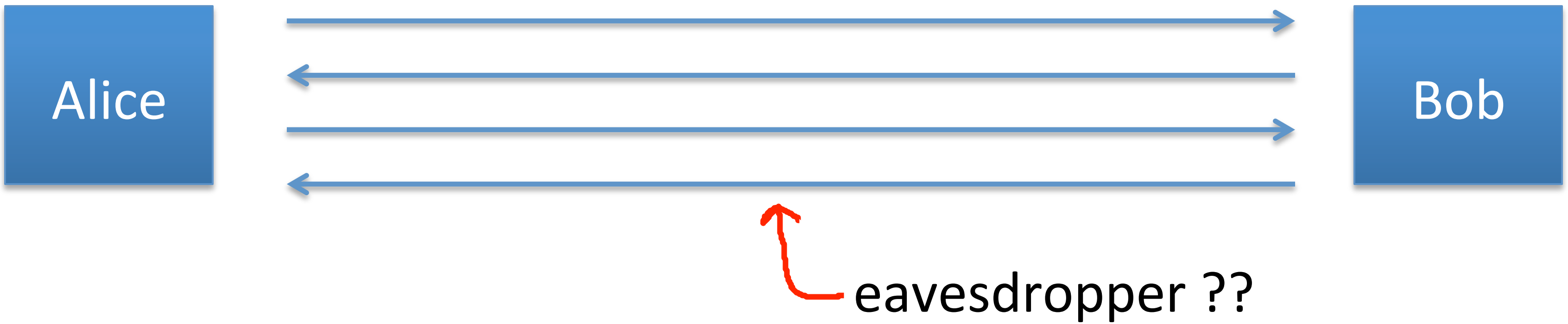
Basic key exchange

The Diffie-Hellman protocol

Key exchange without an online TTP?

Goal: Alice and Bob want shared secret, unknown to eavesdropper

- For now: security against eavesdropping only (no tampering)



Can this be done with an exponential gap?

The Diffie-Hellman protocol (informally)

Fix a large prime p (e.g. 600 digits)

Fix an integer g in $\{1, \ldots, p\}$

Alice

Bob

choose random **a** in $\{1,\ldots,p-1\}$

choose random **b** in $\{1,\ldots,p-1\}$

"Alice", $A \leftarrow g^a \pmod p$ →

← "Bob", $B \leftarrow g^b \pmod p$

$$B^a \pmod p = (g^b)^a = k_{AB} = g^{ab} \pmod p = (g^a)^b = A^b \pmod p$$

Security (much more on this later)

Eavesdropper sees: $p, g,$ $A=g^a \pmod p$, and $B=g^b \pmod p$

Can she compute $g^{ab} \pmod p$??

More generally: define $DH_g(g^a, g^b) = g^{ab} \pmod p$

How hard is the DH function mod p?

How hard is the DH function mod p?

Suppose prime p is n bits long.

Best known algorithm (GNFS): run time exp($\tilde{O}(\sqrt[3]{n})$)

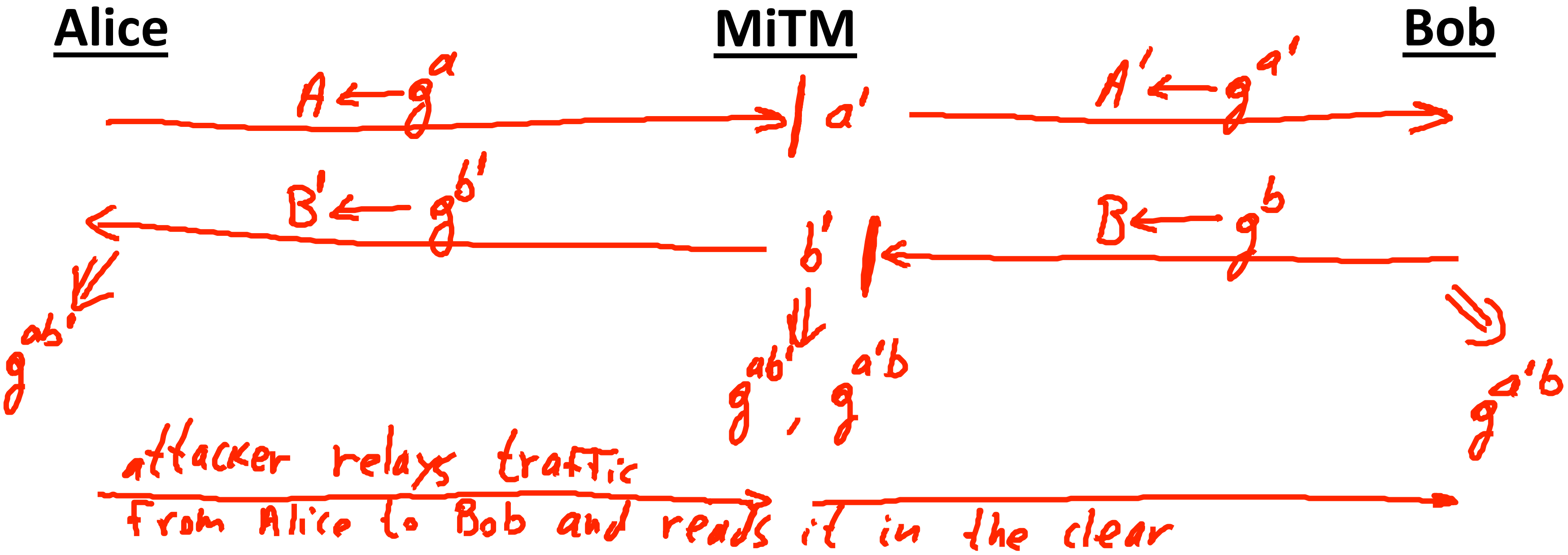
cipher key size	modulus size	Elliptic Curve size
80 bits	1024 bits	160 bits
128 bits	3072 bits	256 bits
256 bits (AES)	**15360** bits	512 bits

As a result: slow transition away from (mod p) to elliptic curves

Elliptic curve
Diffie-Hellman

Insecure against man-in-the-middle

As described, the protocol is insecure against **active** attacks



Another look at DH

Facebook

g^a g^b g^c g^d

Alice	**Bob**	**Charlie**	**David**	…
a	b	c	d	
$K_{AC}=g^{ac}$		$K_{AC}=g^{ac}$		

An open problem

$n=2$: DH
$n=3$: Known (Joux)
$n\geq 4$: open

Facebook

g^a g^b g^c g^d

Alice	**Bob**	**Charlie**	**David**	…
a	b	c	d	
K_{ABCD}	K_{ABCD}	K_{ABCD}	K_{ABCD}	

End of Segment

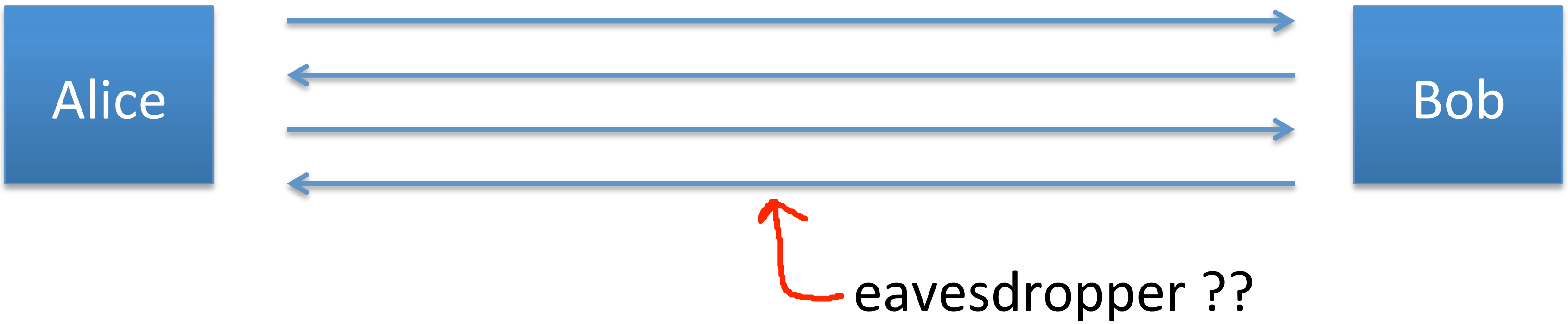
Basic key exchange

Public-key encryption

Establishing a shared secret

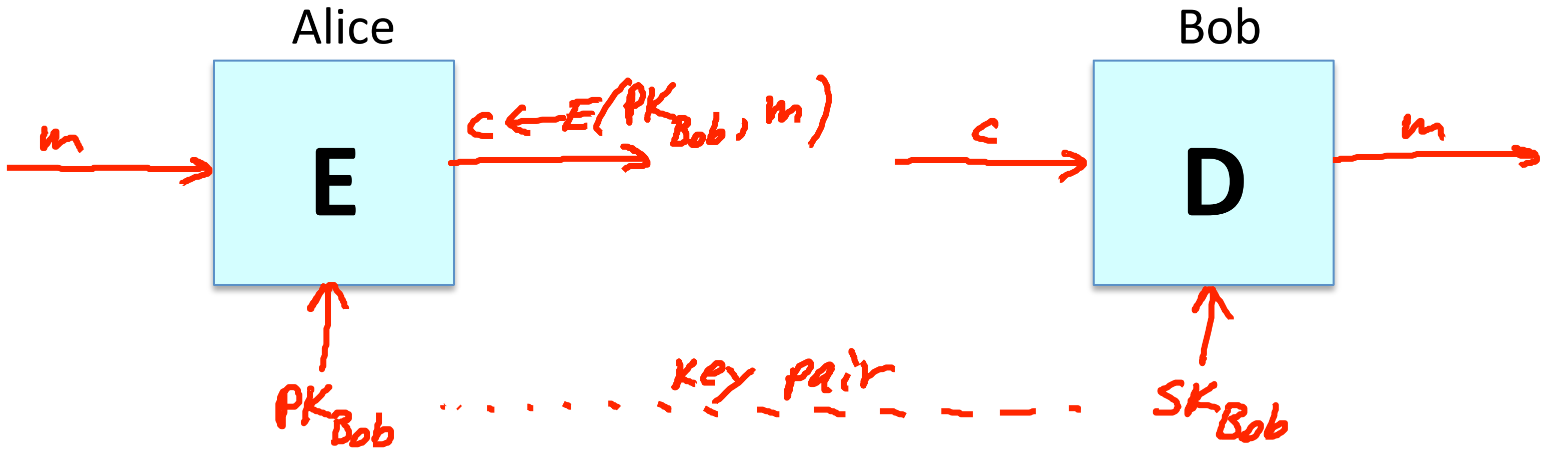
Goal: Alice and Bob want shared secret, unknown to eavesdropper

- For now: security against eavesdropping only (no tampering)



This segment: a different approach

Public key encryption



PK: public key, SK: secret key

Public key encryption

<u>Def</u>: a public-key encryption system is a triple of algs. (G, E, D)

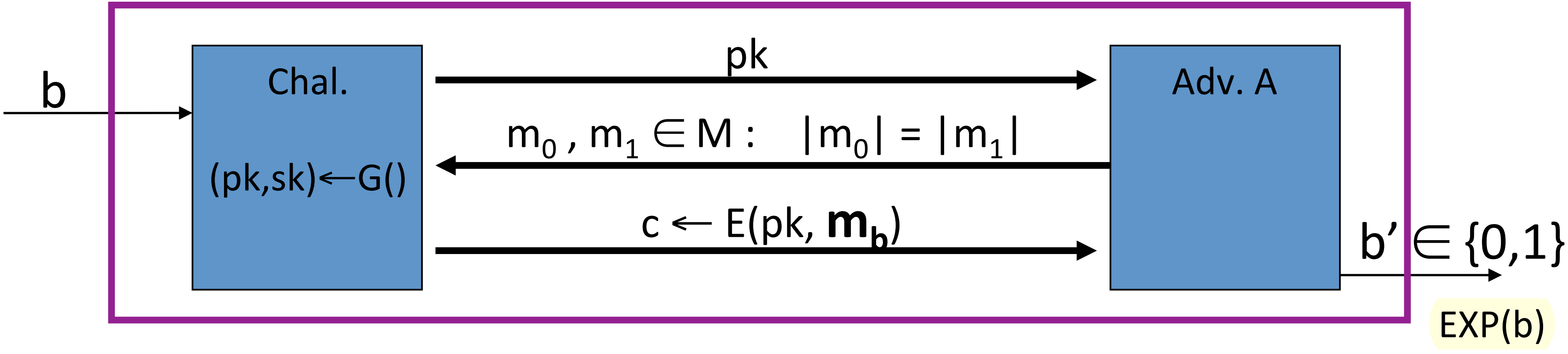
- G(): randomized alg. outputs a key pair (pk, sk)
- E(pk, m): randomized alg. that takes $m \in M$ and outputs $c \in C$
- D(sk,c): det. alg. that takes $c \in C$ and outputs $m \in M$ or $\perp$

Consistency: $\forall$ (pk, sk) output by G :

$$\forall m \in M: \quad D(sk,\ E(pk, m)\ ) = m$$

Semantic Security

For b=0,1 define experiments EXP(0) and EXP(1) as:



Def: E =(G,E,D) is sem. secure (a.k.a IND-CPA) if for all efficient A:

$$\text{Adv}_{\text{SS}}[A,E] = \Big|\Pr[\text{EXP}(0)=1] - \Pr[\text{EXP}(1)=1]\Big| \quad < \quad \text{negligible}$$

Establishing a shared secret

Alice **Bob**

$(pk, sk) \longleftarrow G()$

"Alice", pk

choose random

$x \in \{0,1\}^{128}$

"Bob", $c \leftarrow E(pk, x)$

$D(sk, c) \rightarrow x$

x: shared secret

Security (eavesdropping)

Adversary sees **pk, E(pk, x)** and wants $x \in M$

Semantic security $\Rightarrow$

adversary cannot distinguish

$\{ pk, E(pk, x), x \}$ from $\{ pk, E(pk, x), rand \in M \}$

$\Rightarrow$ can derive session key from x.

Note: protocol is vulnerable to man-in-the-middle

Insecure against man in the middle

As described, the protocol is insecure against **active** attacks

Alice	**MiTM**	**Bob**
$(pk, sk) \longleftarrow G()$	$(pk', sk') \longleftarrow G()$	

"Alice", pk $\longrightarrow$ | "Alice", pk' $\longrightarrow$

choose random
$x \in \{0,1\}^{128}$

× $\longleftarrow$ "Bob", E(pk, x) | "Bob", E(pk', x) $\longleftarrow$

×

Public key encryption: constructions

Constructions generally rely on hard problems from number theory and algebra

Next module:

- Brief detour to catch up on the relevant background

Further readings

- Merkle Puzzles are Optimal,
  B. Barak, M. Mahmoody-Ghidary, Crypto ’09

- On formal models of key exchange (sections 7-9)
  V. Shoup, 1999

End of Segment