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MAT 344 ASSIGNMENT 01

COLLABORATORS: Speed Vuong, math Stack Exchange

1) $14 \times 11 \times \left(\begin{vmatrix} 13 \\ 2 \end{vmatrix} + \begin{vmatrix} 11 \\ 2 \end{vmatrix} \right)$

14 x 11 means Sam choses 1 dog and 1 cat out of 14 dog and 11 cats. Then Janet chooses 2 dogs of left over dogs which are 13. OR she picks 2 cats out of 11 cats. So there is one possibility out of 2 that's why OR "+" sign.

Another answer could be when Janet picks first. But final answer would be same.

2)

d First we choose 4 people out of 17, 11 choose 4 when we don't have any superheroes and 10 choose 4 when there are no super villians. 15 choose 2 when you choose 2 other people from left over 15 people. And $\begin{vmatrix} 4 \\ 4 \end{vmatrix}$ when neithe superhero nor villians.

$\begin{vmatrix} 17 \\ 4 \end{vmatrix} - \begin{vmatrix} 11 \\ 4 \end{vmatrix} - \begin{vmatrix} 15 \\ 2 \end{vmatrix} - \begin{vmatrix} 10 \\ 4 \end{vmatrix} + \begin{vmatrix} 4 \\ 4 \end{vmatrix}$

e) $\begin{vmatrix} 17 \\ 4 \end{vmatrix} - \begin{vmatrix} 4 \\ 2 \end{vmatrix} + \begin{vmatrix} 13 \\ 2 \end{vmatrix}$

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MAT344 ASSIGN 02 COLLABORATORS: YESAR

1) Prove, using a combinatorial argument, $n \geq r$ the following eq. holds: $\binom{n+1}{r+1} = \sum_{k=0}^{n-r} \binom{r+k}{r}$

First we work on L.H.S. $\binom{n+1}{r+1}$. Now, consider a or/b size of three with two "a" in each. Then possible combination are aab, aba, baa. So total number of such a or/and b are $3 = \binom{3}{2}$. Therefore $\binom{n+1}{r+1}$ is the number of a or/and b of length $(n+1)$ with $(r+1)$ "a".

let's move on to R.H.S. we know $\binom{r+k}{r}$ is summation from $k=0$ to $n-r$. with length $(n+1)$ with $(r+1)$ "a".
Consider the a or/and b with 2 "b" where the right most one is in 2nd position. The only possibility of such string is bba. Similarly, the strings of three a a or/and b where the right most one is in the 3rd position. The possibilities are abb, bab.
Thus, total no of a or/and b $= \binom{1}{1} + \binom{2}{1} = 3$

But also keep in mind, if the a or/and b of $(n+1)$ and there are total $(r+1)$ "b". The right most one must be in one of the position $(r+1), (r+2), \ldots, (n+1)$. if it is in position t, then n of the $(t-1)$ positions to its left will be a b. So, total possibilities of such a or/and b $= \binom{t-1}{r}$ letting $t-1 = r+k$, the result follows from Rule of Sum.

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Ans 2. Combinatorial argument to prove the following equation for $n \geq 2$

$$S(n,2) = 2^{n-1} - 1$$

$S(n,k)$

$S(1,2) = 2^{1-1} - 1$
$= 2^0 - 1$
$= 0$

$S(2,2) = 2^1 - 1$
$= 1$

$$S(n,k) = \left\{ {n \atop k} \right\} = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} (k c_j) j^n$$

$$S(n,2) = \left\{ {n \atop 2} \right\}$$

$$= \frac{1}{2!} \sum_{j=0}^{0} (-1)^{2-j} \cdot 2c_j \, j^n$$

$$= \frac{1}{2!} \left[(-1)^2 \cdot 2c_0 \, 0^n + (-1)^1 \, 2c_1 \, (-1)^n + (-1)^0 \, 2c_2 \, 2^n \right)$$

$$= \frac{1}{2!} \left[0 - 2 + 2^n \right]$$

$$= 2^{n-1} - 1$$ Proved

Ans 1

if $n-a_1 \geq 0$ then the number of ways for selecting a_1 items

$$\binom{n}{a_1}$$

if $n-a_1-2a_2 \geq 0$ then

$$\binom{n-a_1}{a_2}$$

likewise if $n-a_1-2a_1-\cdots-ka_k \geq 0$ then the number of ways for selecting a_k items from the set of $(n-a_1-a_2-\cdots-a_{k-1})$ elements

$$\binom{n-a_1-a_2-\cdots-a_{k-1}}{a_k} = \binom{n-\sum_{i=1}^{k-1} a_i}{a_i}$$

The number of partitions of the set $[n]=$

$$= \binom{n}{a_1}\binom{n-a_1}{a_2}\cdots\binom{n-\sum_{i=1}^{k-1} a_i}{a_1} = \prod_{j=1}^{k} \binom{n-\sum_{i=0}^{j-1} a_i}{a_j} \text{ with } a_0=0$$

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Ans) sub in some values

$F(n-2) = F(n) - F(n-1)$

$F(n-3) = F(n-1) - F(n-2)$

$\vdots$

$F(1) = F(3) - F(2)$

Adding all eq we get

$F(n-2) + F(n-3) + \cdots + F(1) = F(n) - 1$

Thus $F(n) = \sum_{k=2}^{n-1} F(n-k) + 1$

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Ans 2 It is $\sum_{k=1}^{n} \binom{n-1}{k-1} = 2^{n-1}$

$$B_n + B_{n+1} = \sum_{k=1}^{n} \binom{n-1}{k-1} + \sum_{k=0}^{n} \binom{n-1}{k}$$

$$= \sum_{k=1}^{n} \left[\binom{n-1}{k-1} + \binom{n-1}{k} \right] + \binom{n-1}{0}$$

$$= \sum_{k=1}^{n} \binom{n}{k} + \binom{n-1}{0} = \sum_{k=1}^{n} \binom{n}{k} + \binom{n}{0}$$

$$= \sum_{k=0}^{n} \binom{n}{k} = 2^n$$

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Solution to Q2 from Midterm

$[n] \longrightarrow [m]$

1	1
2	2
3	3
4	4
5	5
	6
	7

$\binom{m}{m-n} = \binom{7}{2}$ We choose 2 values in m to remove and since $f(1) < f(2) < \cdots < f(n)$, there is only one way to assign the $[n] \to [m]$.

$\therefore \binom{m}{m-n}$ function. or it can be rewritten as m choose n technically same thing.

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A7A7~~3~~4 ASSIGNMENT

COLLABORATORS: SPEED, HAMDY

Chap 5 #32

Ans① Prove that for all +ve $\mathbb{Z}$ n, the inequality
$p(n)^2 < p(n^2 + 2n)$ holds.

By explanation:
We know n can only be +ve $\mathbb{Z}$ so value of n will be minimum 1 which means the R.H.S will always be greater than L.H.S by atleast 2 blocks or two more orientation of the ferrers shape. So RHS is always greater than LHS by atleast 2.

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Ans 2 Prove that for all +ve $\mathbb{Z}$ n and d
we have $P_{dur}(n,d) = D(n,d)$

$P_{dur}(n,d)$ → number of partitions of n with durfee length d

$D(n,d)$ → removing the diagonal of size d and make the matrix that shows the ferrer shapes above and below, the particular diagonal you removed

So, using the hint the length D's diagonal is equal to d. So which means its counting the same thing
So thus proved by combinatorial proof.

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1) FACT $\binom{-m}{n} = (-1)^n \binom{n+m-1}{m-1}$ $\quad (1+x)^m = \sum_{n\geq 0} \binom{m}{n} x^n$

$$\binom{-m}{n} = \frac{-m(-m-1)\cdots(-m-n+1)}{n!}$$

$$= (-1)^n \frac{m(m+1)\cdots(m+n-1)}{n!}$$

$$= (-1)^n \frac{(m+n-1)!}{(m-1)!\,n!}$$

$$= (-1)^n \binom{m+n-1}{m-1}$$

$$f(x) = \frac{1+x}{(1-x)^3} = (1-x)^{-3} + x(1-x)^{-3}$$

$$= \sum_{n\geq 0} \binom{-3}{n}(-x)^n + x\sum_{n\geq 0}\binom{-3}{n}(-x)^n$$

$$= \left[1 + \sum_{n\geq 1}\binom{-3}{n}(-x)^n\right] - \left[\sum_{n\geq 0}\binom{-3}{n}(-x)^{n+1}\right]$$

$$= 1 + \sum_{n\geq 1}\binom{-3}{n}(-x)^n - \sum_{n\geq 1}\binom{-3}{n-1}(-x)^n$$

$$= 1 + \sum_{n\geq 1}\left[\binom{-3}{n} - \binom{-3}{n-1}\right](-1)^n x^n$$

$$a_n = (-1)^n\left[\binom{-3}{n} - \binom{-3}{n-1}\right]$$

$$= (-1)^n\left[(-1)^n\binom{n+2}{2} - (-1)^{n-1}\binom{n+1}{2}\right]$$

$$= \binom{n+2}{2} - \binom{n+1}{2}$$

$$= \frac{n+2!}{2!\,n!} - \frac{n+1!}{2!\,n-1!}$$

$$= (n+1)^2$$

$a_0 = 1$
$a_n = (n+1)^2$

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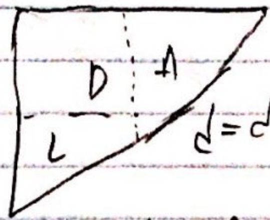
2)

a)



d = durfee length d.

A: partition of $n-d^2$, most d parts.

L: partition of $n-d^2$, at most size d

The arm of a partition is itself a partition of $n-d^2$, with the restriction that it has at most d parts.

The leg of a partition is itself a partition of $n-d^2$, with the restriction that its parts are at most size d.

d = durfee length d.

b) legs $= L(x) = \prod_{i=1}^{d} \frac{1}{(1-x)^i}$ Arms $= A(x) = \prod_{i=1}^{d} \frac{1}{(1-x)^i}$

Durfee $= D(x) = x^{d^2}$

$$A(x)\,L(x)\,C(x) = \prod_{i=1}^{d} \frac{1}{(1-x)^i} \cdot \prod_{i=1}^{d} \frac{1}{(1-x)^i} \cdot x^{d^2}$$

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MATH 344 ASSIGN 07 COLLABORATORS: SPEED

1) $A(x) = \sum a_n x^n$
closed form of it
the number of parts of size i is strictly less than i.
and a_n is number of partitions of n such that for
$\forall i \in \mathbb{Z}^+$

$A_1(x) : \sum_{n \geq 0} p_i(n) x^i = \prod_{k=1}^{i} \frac{x^i}{1-x^k}$ (where size i)

$A_2(x) : \sum_{n \geq 0} p_i(n) x^n = \prod_{k=1}^{i-1} \frac{1}{1-x^k}$ (where size is less i parts)

So we combine them;

$$A(x) = A_1(x) A_2(x) = \prod_{k=1}^{i} \frac{x^i}{1-x^k} \prod_{k=1}^{i-1} \frac{1}{1-x^k}$$

$$= \prod_{k=1}^{i} \frac{x^i}{1-x^k} \prod_{k=1}^{i} \frac{1-x^i}{1-x^k}$$

$$= \prod_{k=1}^{i} \frac{x^i(1-x^i)}{1-x^k} \text{ Ans}$$

2) $Q(x) = \sum q_n x^n$ where q_n is # of partitions of n into exactly k parts (distinct)

$$Q(x) = \sum_{n \geq 0} P_{distinct(k)}(n)\, x^k$$
$$= \prod_{i=1}^{k} (1 + x^i)$$
$$= \prod_{i=1}^{k} \left(\frac{1 - x^{2i}}{1 - x^i}\right)$$
$$= \prod_{i=1}^{k} \frac{1}{1 - x^{2i-1}} = \prod_{i=1}^{k} \frac{x^{2k-1}}{1 - x^{2i-1}}$$

$i - 1$

$(1 + x^1 + x^{1\cdot 2})\ (1 + x^{2\cdot 1} + x^{2\cdot 2})\ (1 + x^{3\cdot 1} + x^{3\cdot 2})\ (\quad)$

$\frac{1}{1-x}$ $\quad \frac{1}{1-x^2}$

$\sum_{k=0}^{2} \prod_{k=1}^{i} \frac{x^i}{1 - x^k}$ $\quad \prod_{i=1} \quad \prod$

$(x^i + x^{k+i} + x^{2k+i} + \ldots)(1 + x$

$\prod_{k=1}^{i-1} \frac{1}{1 - x^k}$ = limit $< i$

Test correction

Ans 3). $C(x) = \sum_{n \geq 0} c_n x^n$ $\quad c_n = \sum_{k=0}^{n} 3^k (n-k+1)$

$C(x) = A(x)\,B(x)$

$C(n) = \sum_{k=0}^{n} a_k\, b_{n-k}$

$a_k = 3^k \quad\quad b_{n-k} = n-k+1$

$\downarrow \quad\quad\quad\quad \downarrow$

$a_n = 3^n \quad\quad b_n = n+1$

$A(x) = \sum_{n \geq 0} 3^n x^n \quad\quad B(x) = \sum_{n \geq 0} (n+1) x^n$

$= \frac{1}{(1-3x)} \quad\quad = \frac{1}{(1-x)^2}$

$C(x) = \frac{1}{(1-3x)(1-x)^2}$

So from we set $c_n = \left(\sum_{n \geq 0} 3^n x^n\right)\left(\sum_{n \geq 0} (n+1) x^n\right)$

$= (3^n)\,(n+1)$ Ans.

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COLLABORATION: SPEED

2) By Contradiction, ~~[illegible]~~
Assume G has atleast 3 vertices, 2-connected and had path u to v vertex that must include w vertex, then we know atleast one vertex out of u, v and w has degree 1 and let's say it is u. Then it means it has only one adjacent vertex and so removing a vertex lets call it w which will be in the path to u, then we must disconnect G which contradicts that G is 2-connected since there exists a vertex w that such that G - w disconnects.

1c) $\chi(G) = k$ but $\chi(G - v) < k$ for any vertex v in G

Now Assume G is k-critical and degree less than $k-1$ or $\deg(v) < k-1$. then if $G - v$ is $k-1$ colored by definition of k-critical but only at most $k-2$ colors can be used for $G - v$ by assumption then adding back that to v to get G and assigning any unused color up to $k-1$, we get G is $k-1$ critical and which is a contradiction

d) lets say that is not isomorphic to an odd C_n and it would create two cases.

case 01: it has C_n in it as a sub graph but then we can remove the extra vertices and ~~the ... edges~~ you will get $\chi(G3)$ but thats ok.

Case 02: There is no C_n in it as a sub graph which means there are no odd cycles in it that further tell us that is isomorphic to a bipartite but we know bipartite's are 2 colorable only so thus contradiction.

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