Chapter 3 - In-Class Worksheet

MAT344 - Spring 2019

Introduce yourself to your neighbours:

- For instance, tell them your name and program;
- Tell them what you've heard about this course;
- Ask them whether they've done any of the readings, etc... *(Move closer to other people if you need to; don't be shy!)*

1 *Work earnestly! Work in groups!*
Don't be afraid to ask questions, or check your work!

1.1 How many different arrangements are there of letters from the word

C O M B I N A T O R I C S ?

1.2 What if we insist that the two C's are never next to each other?

1.3 (*) What if instead, we insist that the T comes before at least one *vowel*?

Write answers using notation from the book, like $\binom{n}{k}$, $n!$, $(n)_k$, *etc. (i.e. Don't compute an exact answer, say by using a calculator.)*

2

2.1 How many functions $f : [5] \to [5]$ are there with *exactly two* values of $x \in [5]$ so that $f(x) = x$? *(These are called **fixed points** of f.)*

2.2 What if we insist that the functions must be *injective*?

2.3 (*) Explore other versions of the problem to see which of them seem *hard*.
For instance, what if we ask about *surjective* functions? *Bijective*?
What if we change 5 to some other number like 6 or 7? What if the domain and codomain can be $[n]$ and $[m]$ (for n, m positive integers)? etc.

3 *If you solve a problem, try to help make sure everyone in your group understands the solution!*

3.1 How many different four-digit strings of numbers are there that have *one or more* repeated digits?

3.2 What if the strings have to be **numbers** themselves?

For each of the first two problems, there is at least one "direct" way to count, and one "indirect" way to count. Try to find the other if you've found one.

3.3 (*) Find solutions to the version of these problems where the strings/numbers are of *length* n.

And note that something interesting happens when n *is around* 10.

4 Let n be a positive integer, and X an n-element set.

4.1 How many subsets of X are there? *(Prove your answer.)*

4.2 How many *odd-sized* subsets of X are there, for $n = 1, 2, 3, 4$? *(Compute by hand.)*

4.3 Conjecture an answer to the number of *odd-sized subsets* of X for a general n. *(Check it with me if you're unsure!)*

4.4 Prove your answer correct.

4.5 (*) What if we ask about subsets of size $k \pmod 3$, for $k = 0, 1, 2$?