Tutorial Worksheet 5
MAT344 - Spring 2019

Please refer to the list of ***Graph Theory Definitions*** *on Quercus.*

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1 Prove that in any (simple) graph G with $n \geqslant 2$ vertices, there are at least two vertices with the same degree. *Hint: use a proof by contradiction.*

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2 Ten players participate in a (round-robin) chess tournament. It is a round-robin tournament, so each player will play nine games. At noon, a total of thirteen games have been played by the players. *(A new game is started as soon as two players are available.)* Prove that at noon, at least one player has played at least three games.

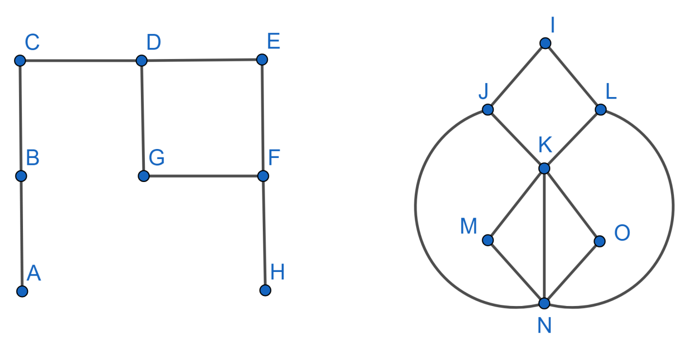
3

Definition

- The **distance** between vertices $\mathbf{x}$ and $\mathbf{y}$ in a (connected) graph G, written $\mathtt{dist}(\mathbf{x}, \mathbf{y})$, is the length of the *shortest* path between $\mathbf{x}$ and $\mathbf{y}$.
- The **eccentricity** of a vertex $\mathbf{x}$ in G (connected), written $\mathrm{ecc}(\mathbf{x})$, is the *maximum* distance between $\mathbf{x}$ and any other vertex $\mathbf{y}$ in G.
- A vertex $\mathbf{x}$ in G is a **centre vertex** if it has the lowest eccentricity of all vertices in G.

Find all centre vertice(s) in the following two graphs:

Suggestion: compute the eccentricity of the vertices in each graph - from this you can read of the centre vertices.



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4 Is it possible to construct a (connected) graph with vertices **c** and **d** which are both centre vertices, but so that **c** and **d** are not adjacent?