Tutorial Quiz 3 - Group Part

MAT344 - Spring 2019

Instructions:

Please record **each Group member's Name and Student Number**.

Make sure to show your work, justifying where possible and annotating any interesting steps or features of your work. **Do not just give the final answer, and do not simplify your calculations (use notation from the course, like $\binom{n}{k}$ or $S(n, k)$ etc.)**

Recall that $\overline{G}$ is the **complement** of G: it has the same vertices as G, but two vertices are adjacent in $\overline{G}$ if and only if they are *not* adjacent in G.

We say that a (simple) graph G is **self-complimentary** if $\overline{G}$ is isomorphic to G.

1 **(2 points + 2 bonus points)** *(The first part of this question is identical to the individual version.)*

1.1 **(2 points)** Find a **self-complimentary** graph G with 5 vertices and a self-complimentary graph with 4 vertices. *You do not need to prove that $G \cong \overline{G}$ in either case, but make sure to draw both G and $\overline{G}$.*

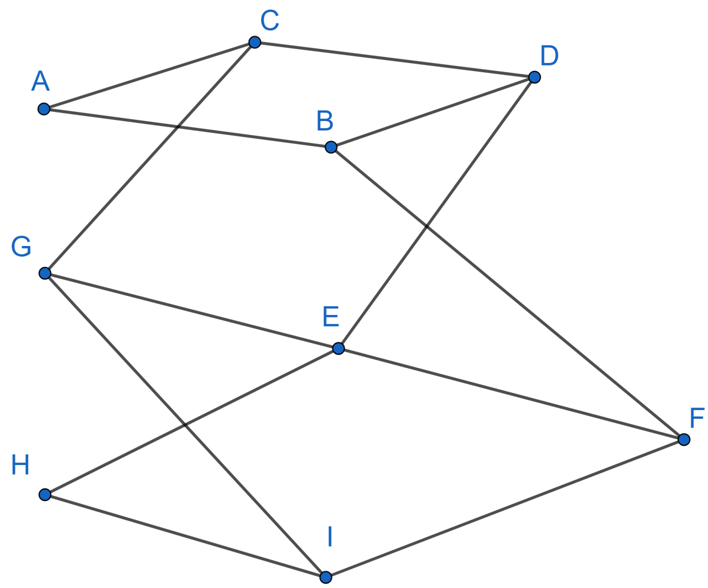
1.2 **(2 bonus points)** Prove that if G is **self-complimentary** and has n vertices, then $n = 4k$ or $n = 4k + 1$ for some non-negative integer k (i.e. n is 0 or 1 mod 4).

Hint: G and $\overline{G}$ have the same number of edges.

2 **(1+3 points & 2 + 1 bonus points)** *(The first two parts of this question are identical to the individual version.)*

2.1 **(1 point)** Write down the definition of a **Hamiltonian cycle**.

2.2 **(3 points)** *Use a proof by contradiction* to show that the following graph does **not** have a **Hamiltonian cycle**:



2.3 **(2 bonus points)** In class we discussed a theorem, called *Dirac's Theorem*, which has the form:

"If a graph G is/has ... then G has a Hamiltonian cycle."

(a) **(1 point)** Fill in the blank in the statement of *Dirac's Theorem*.

(b) **(1 point)** Find the fewest number of additional edges we could add to the graph in this question (previous page) so that the quoted theorem would imply that the graph had a Hamiltonian cycle. *Explain your work.*

2.4 **(1 bonus point)** Find the *actual* fewest number of additional edges we could add to the graph to create a Hamiltonian cycle. *Justify your answer (for instance, if you think the answer is 10, you should prove that adding 9 more edges is not sufficient).*

Recall that a **tournament** is a copy of K_n which has had each of it's edges *directed* in one direction or the other (but not both). Also, the textbook defines a directed graph G to be **balanced** if for each vertex $\mathbf{v}$ in G, in-deg($\mathbf{v}$) = out-deg($\mathbf{v}$).

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3 **(2 points)** *(This whole question is identical to the individual version.)*

For which values of $n \geqslant 1$ does there exist a **balanced tournament** with n vertices? *Justify your answer.*