MAT334 - Week 1 Problems

Textbook Problems 1.1: 1, 2, 4, 5, 6, 7, 9, 10, 11, 12, 18 (Hint: induction.)

Additional Problems

1. For each pair of complex numbers z, w given below

   i) $z = 3$, $w = 1 + i$
   ii) $z = 2 - i$, $w = 4 + 2i$
   iii) $z = (2 + 2i)(1 - 4i)$, $w = 5 - 6i$
   iv) $z = \frac{1+i}{2-i}$, $w = 2 - i$

   verify the following properties of complex algebra:

   a) $z + w = w + z$ and $zw = wz$
   b) $\text{Re}(z + w) = \text{Re}(z) + \text{Re}(w)$ and $\text{Im}(z + w(= \text{Im}(z) + \text{Im}(w)$
   c) $\overline{zw} = \overline{z}\ \overline{w}$
   d) $|zw| = |z||w|$
   e) $|z| = |\overline{z}|$
   f) $z\overline{z} = |z|^2$
   g) $|\text{Re}(z)| \leq |z|$ and $|\text{Im}(z)| \leq |z|$

   Yes, some of these are in the properties I gave you in lecture, but seeing a proof is never as convincing as working out some examples. Also, it will probably help you greatly to simplify the more complicated expressions before working with them.

2. For each of the following z, find the polar form for z, z^3 and $z^1 7$.

   a) $z = -1 - i$
   b) $z = -1 + \sqrt{3}i$
   c) $z = 3 + 4i$
   d) $z = \frac{12-5i}{4}$
   e) $z = 6 - i$

   Some of these angles will not be nice. Do not give an approximate answer. Your answer should only involve multiples of π or inverse trig functions.

3. For each of the following complex numbers, find their principal argument:

   a) $z = -1 + i$
   b) $z = 2 - i$
   c) $z = -11 - 15i$
   d) $z = -11 + 15i$
   e) $z = \frac{1}{3} + \frac{1}{\sqrt{3}}i$

4. The goal of this exercise is to verify a very important claim that we use all the time: that for any $z \neq 0$, any two arguments for z differ by a multiple of 2π.

    a) Suppose $\cos\theta = \cos\Psi$ and $\sin\theta = \sin\Psi$. Prove that $\cos(\theta - \Psi) = 1$.

    b) For which $\theta \in \mathbb{R}$ is $\cos(\theta) = 1$? (This does not require proof.)

    c) Conclude that if $\cos\theta = \cos\Psi$ and $\sin\theta = \sin\Psi$, then $\Psi = \theta + 2k\pi$ for some integer k.

    d) Conclude that if $z = r(\cos(\theta) + i\sin(\theta) = s(\cos(\Psi) + i\sin(\Psi)$ is written in polar form in two different ways, then $\Psi = \theta + 2k\pi$. (Hint: remember that $r = |z|$ in the definition of polar form. What should s be?)

5. Prove the following:

    a) For any $z \in \mathbb{C}$, $\overline{(\overline{z})} = z$.

    b) For any $z, w \in \mathbb{C}$, $z\overline{w} - \overline{z}w$ is purely imaginary. (A complex number z is called *purely imaginary* if $z = 0 + yi$.)