Chapter 5 (5.1 & 5.2) - In-Class Worksheet

MAT344 - Spring 2019

A quick review... what do the following things represent *(in Chapter 5 terminology)*?

$$\binom{n+k-1}{k-1} \qquad \binom{n-1}{k-1} \qquad 2^{n-1} \qquad S(n,k) \qquad B(n)$$

Work earnestly! Work in groups!
Don't be afraid to ask questions, or check your work!

1 Recall that the **Multinomial Theorem** tells us that

$$(x_1 + ... + x_k)^n = \sum_{a_1+...+a_k=n} \binom{n}{a_1, ..., a_k} {x_1}^{a_1} \cdot ... \cdot {x_k}^{a_k}$$

where the sum is taken over all $a_1, ..., a_k$ that sum to n (where $a_i \geqslant 0$.)

For fixed n and k, how many terms appear in the sum? (i.e. how many things are being added together?)

Suggestion: count for low values, like $n = 3, k = 2$ *and* $n = k = 3$ *etc. What's the pattern?*

2.1 We are going to distribute n (identical) pieces of candy to k children (some children might receive none), but for some reason we are *only* going to give children *even* amounts of candy.
How many ways is this possible?

(i.e. for any given child, they can receive 0, or 2, or 4, or 6, or ... candies.)

2.2 What if no child can be given 0 candies?

2.3 What if we change the number of candies each child receives to *odd* amounts?

2.4 (*) What if the number of candies is 0 *or* odd?

3 What is the number of ways that we can ...

3.1 ... distribute 100 different books into 7 indistinguishable boxes (we are moving, and will pack the boxes, seal them, then label each box "books". We don't care about the order they are in in the box.)

3.2 ... distribute 50 identical loonies to 4 people, each getting at least one?

3.3 ... split up n different fish into any positive number *less than or equal to* n identical fish bowls?

3.4 .... split a class (of 725 students) into 4 rooms (with no empty rooms)?

You are on the sidewalk handing out 500 identical flyers (lucky you!). You hand them out to people who pass you in the street, *trying* to get one to each person. But **sometimes you miss people**, and **sometimes you accidentally hand out multiple copies at once**.

4.1 Suppose that by the end, 200 people walked by you on the street, and that you managed to hand out all of the flyers. Furthermore, suppose that the first ten people all got exactly one flyer each, and that the last ten people each got at least two. Then how many ways are there for you to have handed out the flyers?

4.2 Suppose that instead, you don't know how many people walked by you, but you do know that everyone who walked by got at least one flyer, and you know that the first ten people got exactly one flyer each again. How many ways are there for you to have handed out the flyers in this case?

4.3 Finally, suppose that 250 people walk by, and that you've handed out all of the flyers. But unbeknownst to you, this time *each flyer had a (single) QR-code number* on it, and that there were exactly 25 different QR-codes, each appearing the same number of times amongst the flyers. This means, of course, that the flyers weren't actually identical after all. Now how many ways are there for you to have handed out the flyers?

Warmup: what does k^n count?

Prove the following using a **combinatorial proof**:

$$S(n,k) = \sum_{j=k-1}^{n-1} k^{n-j-1} \cdot S(j, k-1)$$

Suggestion: to make this similar to "committee selection", think of n *as distinct books, and the* k *as indistinguishable boxes you will put them in: the LHS is then the number of ways to split* n *distinct books into* k *non-empty indistinguishable groups.*

Hint: The RHS only accounts for $n-1$ *of the objects, so something is happening implicitly to one of the objects for each value of* j*, just like earlier combinatorial proofs we've done!*

6 Prove the following using a **combinatorial proof**:

$$S(n, l+m) \cdot \binom{l+m}{l} = \sum_{k=l}^{n} \binom{n}{k} \cdot S(k, l) \cdot S(n-k, m)$$