Tutorial Quiz 2 - Group Part

MAT344 - Spring 2019

Instructions:

Please record **each Group member's Name and Student Number**.

Make sure to show your work, justifying where possible and annotating any interesting steps or features of your work. **Do not just give the final answer, and do not simplify your calculations (use notation from the course, like $\binom{n}{k}$ or $S(n,k)$ etc.)**

Definition

First, recall that for $n \geqslant 1$, $\mathbf{p(n)}$ is the number of all partitions of n.

For $n \geqslant k \geqslant 1$, **we define:**

- $\mathbf{f}_k(n)$ to be the number of partitions of n whose *first* k *parts are equal*.
- $\mathbf{g}_k(n)$ to be the number of partitions of n whose parts *all* have size k *or more.*
- $\mathbf{s}_k(n)$ to be the number of partitions of n whose *last* part has size k.
  ($s_k(n)$ is needed only for Question 2.)

1 **(2+3 points ⇒ 5 points)** *(This whole question is identical to the individual version.)*

1.1 **(1 point each)** Draw the *Ferrers shapes* of all of the partitions enumerated by

(a) $f_3(9)$

(b) $g_3(9)$

1.2 **(3 points)** Fix arbitrary $n \geqslant k \geqslant 1$. Prove using a bijection that $f_k(n) = g_k(n)$.

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2 **(3 points)** *(This whole question is identical to the individual version.)*

Fix an arbitrary $k \geqslant 2$. Prove that $f_k(n) = p(n) - \sum_{i=1}^{k-1} s_i(n)$.

3 (2+2 = 4 bonus points)

3.1 **(1 point each)** Prove the following (for an arbitrary $n \geqslant 4$):

(a) $s_1(n) = p(n-1)$

(b) $s_2(n) = p(n-2) - p(n-3)$.

3.2 **(2 points)**
Use our observations so far to prove (for an arbitrary $n \geqslant 4$) that

$$f_3(n) = p(n) - p(n-1) - p(n-2) + p(n-3).$$

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4 **(1 bonus point)**
Conjecture and prove a formula for $f_4(n)$ which only involves various $p(n)$'s, as we did for $f_3(n)$.