Tutorial Worksheet 4
MAT344 - Spring 2019

1 Find an explicit formula for a_n if for $n \geqslant 1$, $a_{n+1} = 3a_n + 2^n$ and $a_0 = 1$.

2 *This is taken from the In-Class Activities, Chapter 8, Part 1 (it was skipped in class.)*

Fix an arbitrary generating function $A(x) = \sum a_n x^n$.

2.1 What is b_n if $B(x) = \sum b_n x^n = \dfrac{A(\sqrt{x}) + A(-\sqrt{x})}{2}$?

2.2 What is c_n if $C(x) = \sum c_n x^n = \dfrac{A(\sqrt{x}) - A(-\sqrt{x})}{2\sqrt{x}}$?

2.3 Find the **closed form** of $D(x) = \sum d_n x^n$ where $(d_n) = (1, 0, 1, 0, 1, 0, ...)$.

2.4 Find the **closed form** of $E(x) = \sum e_n x^n$ where $(e_n) = (0, 1, 0, 1, 0, 1, ...)$.

3 The *Skrulls*, a race of alien shapeshifters is planning an invasion of Earth. Naturally their first target is Toronto, and in particular the University of Toronto mathematics department. Kl'rt, their leader, commands an army of n Skrulls *(we'll name them with elements of* $[n]$*.)*

Kl'rt's attack proceeds as follows:

- First Kl'rt splits the n Skrulls into two *(consecutive, possibly empty)* groups (i.e. $1, 2, 3, ..., k$ and $k+1$,...,n for some $0 \leqslant k \leqslant n$).
- Then, from the first group, Kl'rt chooses an even number of them to attack the city of Toronto and cause as much mayhem and chaos as possible (perhaps by nefariously increasing the winter snowfall *beyond seasonal norms*).
- From the second part, Kl'rt chooses three Skrulls to disguise themselves as UofT mathematics professors; they will blend in while the others cause their distraction, with the goal of stealing some universe-altering, top secret mathematics research.

Find the closed form of the generating function $F(x) = \sum f_n x^n$ for the number of ways, f_n, for Kl'rt to do this, then derive a formula for f_n by converting it to a series.