Message integrity

Message Auth. Codes

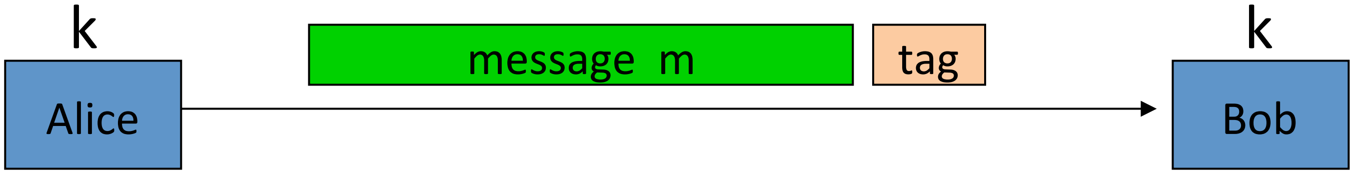
Message Integrity

Goal: **integrity**, no confidentiality.

Examples:

- Protecting public binaries on disk.
- Protecting banner ads on web pages.

Message integrity: MACs



Generate tag:

tag ← S(k, m)

Verify tag:

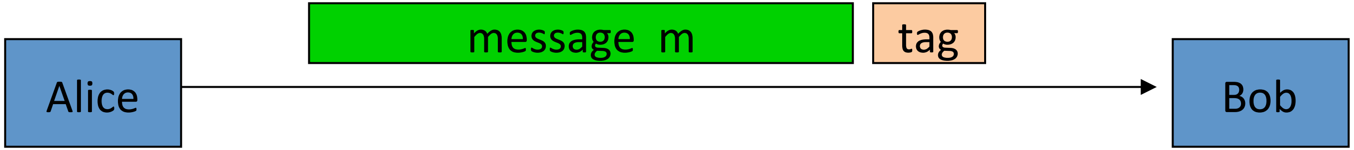
V(k, m, tag) $\stackrel{?}{=}$ \`yes'

Def: **MAC** I = (S,V) defined over (K,M,T) is a pair of algs:

– S(k,m) outputs t in T

– V(k,m,t) outputs \`yes' or \`no'

Integrity requires a secret key



Generate tag:

tag ← CRC(m)

Verify tag:

V(m, tag) $\stackrel{?}{=}$ \`yes'

- Attacker can easily modify message m and re-compute CRC.
- CRC designed to detect **random**, not malicious errors.

Secure MACs

Attacker's power: **chosen message attack**

- for $m_1, m_2, \ldots, m_q$ attacker is given $t_i \leftarrow S(k, m_i)$

Attacker's goal: **existential forgery**

- produce some **new** valid message/tag pair (m,t).

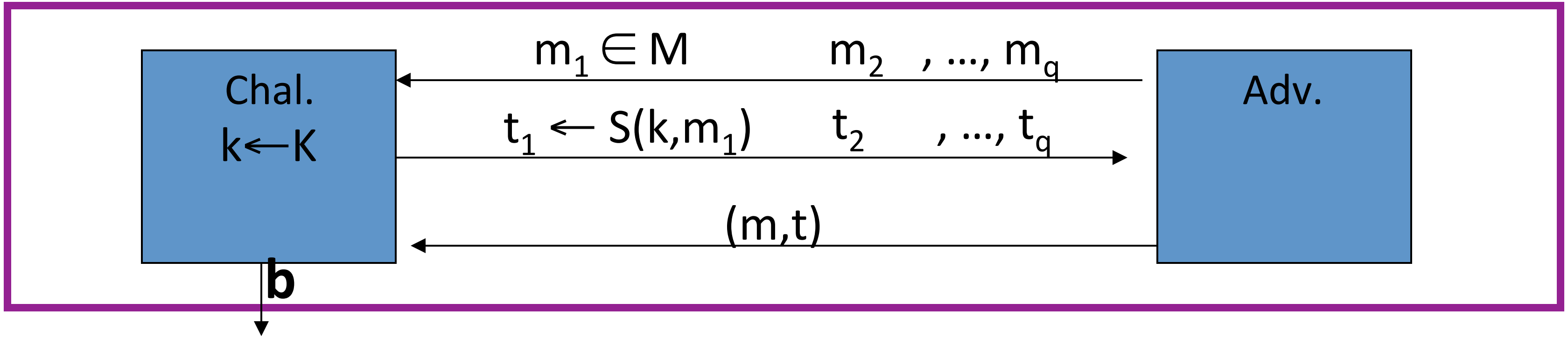
$$(m,t) \notin \{ (m_1,t_1), \ldots, (m_q,t_q) \}$$

⇒ attacker cannot produce a valid tag for a new message

⇒ given (m,t) attacker cannot even produce (m,t') for $t' \neq t$

Secure MACs

- For a MAC I=(S,V) and adv. A define a MAC game as:



$$\begin{cases} \mathbf{b}=1 & \text{if } V(k,m,t) = \text{`yes'} \text{ and } (m,t) \notin \{ (m_1,t_1), \ldots, (m_q,t_q) \} \\ \mathbf{b}=0 & \text{otherwise} \end{cases}$$

Def: I=(S,V) is a <u>**secure MAC**</u> if for all "efficient" A:

$\mathrm{Adv}_{\mathrm{MAC}}[A,I]$ = Pr[Chal. outputs 1]   is "negligible."

Let I = (S,V) be a MAC.

Suppose an attacker is able to find $m_0 \neq m_1$ such that

$$S(k, m_0) = S(k, m_1) \quad \text{for } ½ \text{ of the keys } k \text{ in } K$$

Can this MAC be secure?

- Yes, the attacker cannot generate a valid tag for m_0 or m_1
- No, this MAC can be broken using a chosen msg attack
- It depends on the details of the MAC
-

$Adv[A, I] = 1/2$

Let I = (S,V) be a MAC.

Suppose S(k,m) is always 5 bits long

Can this MAC be secure?

- No, an attacker can simply guess the tag for messages
- It depends on the details of the MAC
- Yes, the attacker cannot generate a valid tag for any message
-

$Adv[A,I] = 1/32$

Example: protecting system files

Suppose at install time the system computes:

filename F_1	filename F_2	…	filename F_n
$t_1 = S(k,F_1)$	$t_2 = S(k,F_2)$		$t_n = S(k,F_n)$

k derived from user's password

Later a virus infects system and modifies system files

User reboots into clean OS and supplies his password

– Then: secure MAC $\Rightarrow$ all modified files will be detected

End of Segment

Message Integrity

MACs based on PRFs

Review: Secure MACs

MAC: signing alg. **$S(k,m) \longrightarrow t$** and verification alg. **$V(k,m,t) \longrightarrow 0,1$**

Attacker's power: **chosen message attack**

- for $m_1, m_2, \ldots, m_q$ attacker is given $t_i \leftarrow S(k,m_i)$

Attacker's goal: **existential forgery**

- produce some **new** valid message/tag pair (m,t).

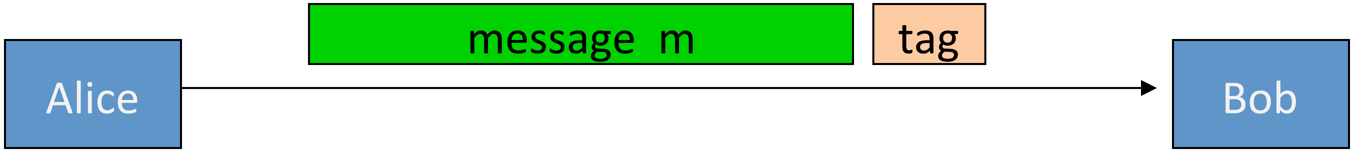
$$(m,t) \notin \{ (m_1,t_1), \ldots, (m_q,t_q) \}$$

$\Rightarrow$ attacker cannot produce a valid tag for a new message

Secure PRF $\Rightarrow$ Secure MAC

For a PRF $F: K \times X \longrightarrow Y$ define a MAC $I_F = (S,V)$ as:

- $S(k,m) := F(k,m)$
- $V(k,m,t)$: output \`yes' if $t = F(k,m)$ and \`no' otherwise.



$\textbf{tag} \leftarrow \textbf{F(k,m)}$

accept msg if

$\textbf{tag} = \textbf{F(k,m)}$

A bad example

Suppose $F: K \times X \longrightarrow Y$ is a secure PRF with $Y = \{0,1\}^{10}$

Is the derived MAC I_F a secure MAC system?

- Yes, the MAC is secure because the PRF is secure
- No tags are too short: anyone can guess the tag for any msg
- It depends on the function F
-

$Adv[A, I_F] = 1/1024$

Security

Thm: If $\mathbf{F: K \times X \longrightarrow Y}$ is a secure PRF and $1/|Y|$ is negligible

(i.e. $|Y|$ is large) then I_F is a secure MAC.

In particular, for every eff. MAC adversary A attacking I_F

there exists an eff. PRF adversary B attacking F s.t.:

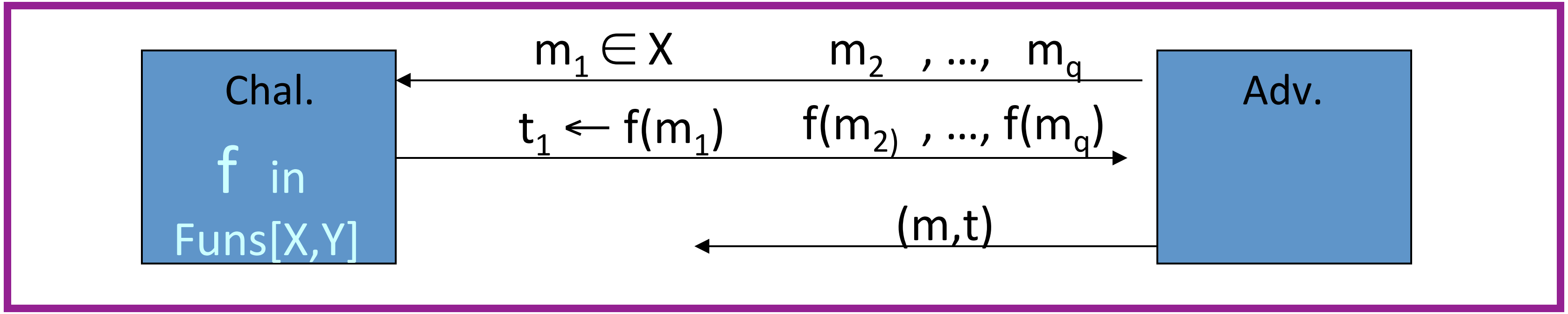
$$\mathrm{Adv}_{\mathrm{MAC}}[A, I_F] \leq \mathrm{Adv}_{\mathrm{PRF}}[B, F] + 1/|Y|$$

$\Rightarrow$ I_F is secure as long as $|Y|$ is large, say $|Y| = 2^{80}$.

Proof Sketch

Suppose $f: \mathbf{X} \longrightarrow \mathbf{Y}$ is a truly random function

Then MAC adversary A must win the following game:



A wins if $\mathbf{t = f(m)}$ and $\mathbf{m \notin \{ m_1 , \ldots , m_q \}}$

$\Rightarrow$ Pr[A wins] = 1/|Y|

same must hold for F(k,x)

Examples

- AES: a MAC for 16-byte messages.

- Main question: how to convert Small-MAC into a Big-MAC ?

- Two main constructions used in practice:
  - **CBC-MAC** (banking – ANSI X9.9, X9.19, FIPS 186-3)
  - **HMAC** (Internet protocols: SSL, IPsec, SSH, …)

- Both convert a small-PRF into a big-PRF.

Truncating MACs based on PRFs

Easy lemma: suppose $F: K \times X \longrightarrow \{0,1\}^n$ is a secure PRF.

Then so is $F_t(k,m) = \underbrace{F(k,m)[1\ldots t]}_{\text{first t-bit of output}}$ for all $1 \leq t \leq n$

⇒ if (S,V) is a MAC is based on a secure PRF outputting n-bit tags

the truncated MAC outputting w bits is secure

… as long as $1/2^w$ is still negligible (say $w \geq 64$)

End of Segment

Message Integrity

CBC-MAC and NMAC

MACs and PRFs

Recall: secure PRF **F** ⇒ secure MAC, as long as $|Y|$ is large

$S(k, m) = F(k, m)$

Our goal:

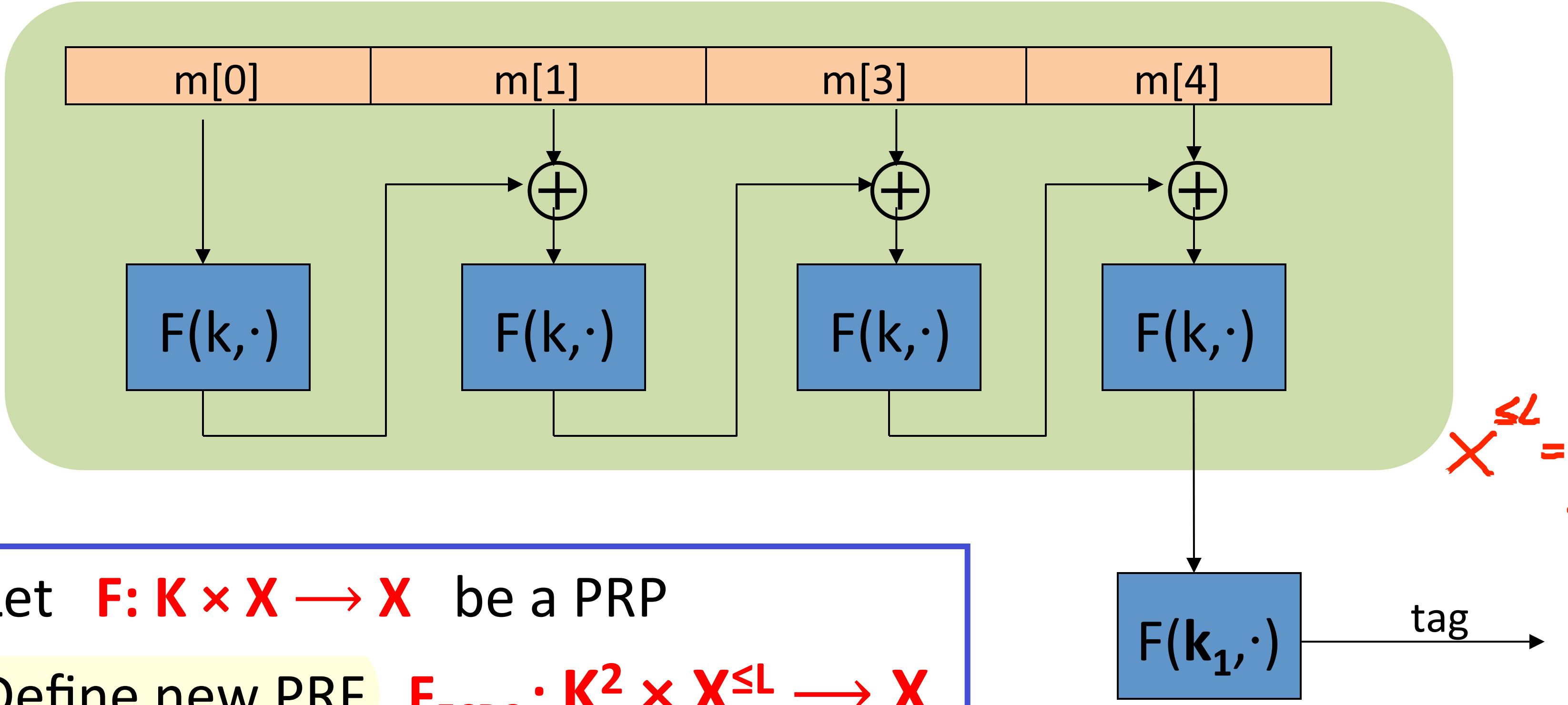
given a PRF for short messages (AES)

construct a PRF for long messages

From here on let $X = \{0,1\}^n$ (e.g. $n=128$)

Construction 1: encrypted CBC-MAC

raw CBC

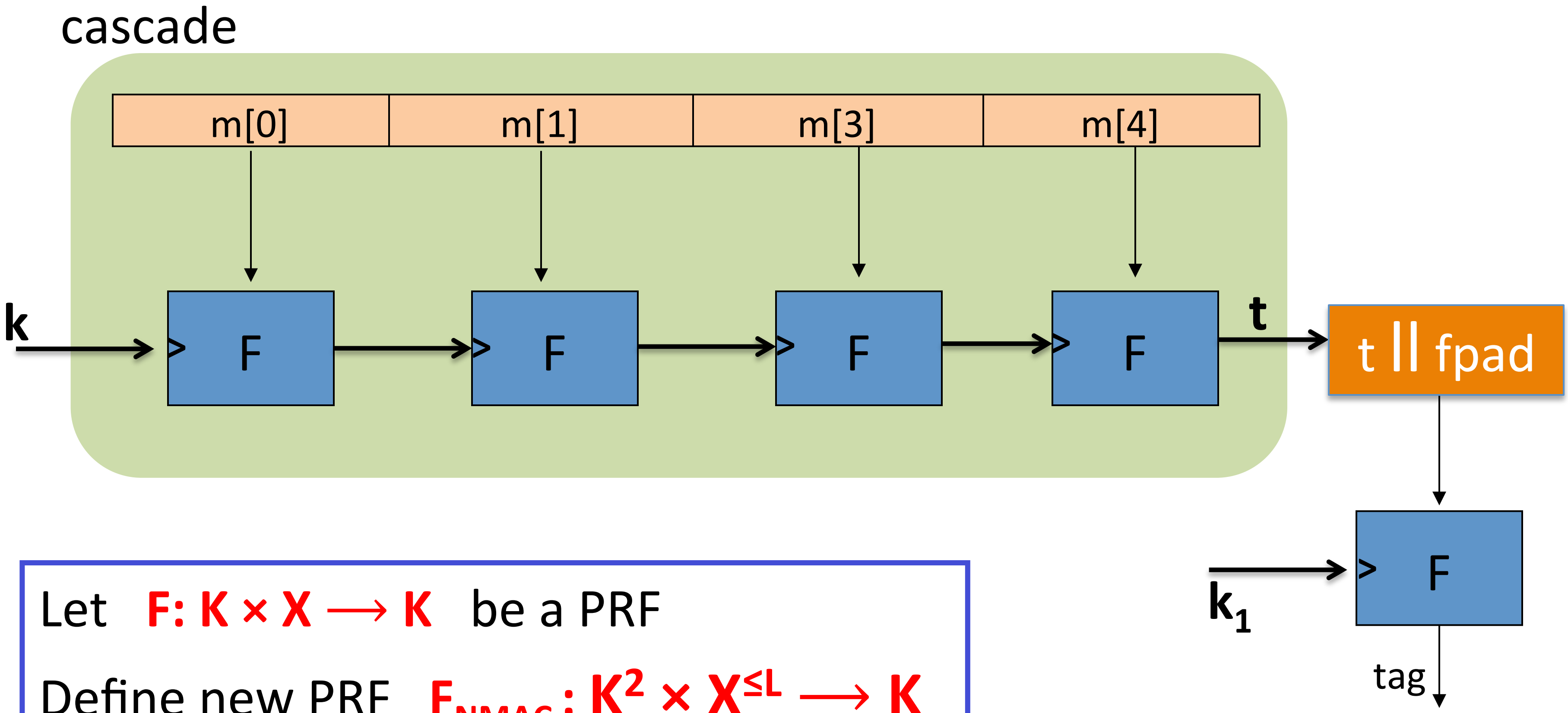


$X^{\leq L} = \bigcup_{i=1}^{L} X^i$

Let $\mathbf{F: K \times X \longrightarrow X}$ be a PRP

Define new PRF $\mathbf{F_{ECBC} : K^2 \times X^{\leq L} \longrightarrow X}$

Construction 2: NMAC (nested MAC)



Let $\mathbf{F: K \times X \longrightarrow K}$ be a PRF

Define new PRF $\mathbf{F_{NMAC} : K^2 \times X^{\leq L} \longrightarrow K}$

Why the last encryption step in ECBC-MAC and NMAC?

NMAC: suppose we define a MAC $I = (S,V)$ where

$$S(k,m) = \text{cascade}(k, m)$$

- This MAC is secure
- This MAC can be forged without any chosen msg queries
- This MAC can be forged with one chosen msg query
- This MAC can be forged, but only with two msg queries

$\text{Cascade}(k,m) \Rightarrow \text{cascade}(k, m\|w)$ for any w

Why the last encryption step in ECBC-MAC?

Suppose we define a MAC $I_{RAW} = (S,V)$ where

$$S(k,m) = \text{rawCBC}(k,m)$$

Then I_{RAW} is easily broken using a 1-chosen msg attack.

Adversary works as follows:

- Choose an arbitrary one-block message $m \in X$
- Request tag for m. Get $t = F(k,m)$
- Output t as MAC forgery for the 2-block message $(m,\ t \oplus m)$

Indeed: $\text{rawCBC}\big(k, (m,\ t\oplus m)\ \big) = F\big(k, F(k,m)\oplus(t\oplus m)\ \big) = F\big(k, t\oplus(t\oplus m)\ \big) = t$

ECBC-MAC and NMAC analysis

Theorem: For any $L>0$,

For every eff. q-query PRF adv. A attacking F_{ECBC} or F_{NMAC}

there exists an eff. adversary B s.t.:

$$\mathrm{Adv}_{PRF}[A, F_{ECBC}] \le \mathrm{Adv}_{PRP}[B, F] + 2\,q^2 / |X|$$

$$\mathrm{Adv}_{PRF}[A, F_{NMAC}] \le q \cdot L \cdot \mathrm{Adv}_{PRF}[B, F] + q^2 / 2|K|$$

CBC-MAC is secure as long as $q \ll |X|^{1/2}$

NMAC is secure as long as $q \ll |K|^{1/2}$ (2^{64} for AES-128)

An example

$$\mathrm{Adv}_{\mathrm{PRF}}[A, F_{\mathrm{ECBC}}] \le \mathrm{Adv}_{\mathrm{PRP}}[B, F] + \mathbf{2\,q^2 / |X|}$$

q = # messages MAC-ed with k

Suppose we want $\mathrm{Adv}_{\mathrm{PRF}}[A, F_{\mathrm{ECBC}}] \le 1/2^{32}$ $\Leftarrow$ $q^2 / |X| < 1/2^{32}$

- AES: $|X| = 2^{128}$ $\Rightarrow$ $q < 2^{48}$

  So, after 2^{48} messages must, must change key

- 3DES: $|X| = 2^{64}$ $\Rightarrow$ $q < 2^{16}$

The security bounds are tight: an attack

After signing $|X|^{1/2}$ messages with ECBC-MAC or
$|K|^{1/2}$ messages with NMAC
the MACs become insecure

Suppose the underlying PRF F is a PRP (e.g. AES)

- Then both PRFs (ECBC and NMAC) have the following extension property:

$$\forall\, x,y,w: \quad F_{BIG}(k, x) = F_{BIG}(k, y) \quad \Rightarrow \quad F_{BIG}(k, \mathbf{x \| w}) = F_{BIG}(k, \mathbf{y \| w})$$

The security bounds are tight: an attack

Let F_{BIG}: $\mathbf{K \times X \longrightarrow Y}$ be a PRF that has the extension property

$$F_{BIG}(k, x) = F_{BIG}(k, y) \quad \Rightarrow \quad F_{BIG}(k, \mathbf{x \| w}) = F_{BIG}(k, \mathbf{y \| w})$$

Generic attack on the derived MAC:

step 1: issue $|Y|^{1/2}$ message queries for rand. messages in X.

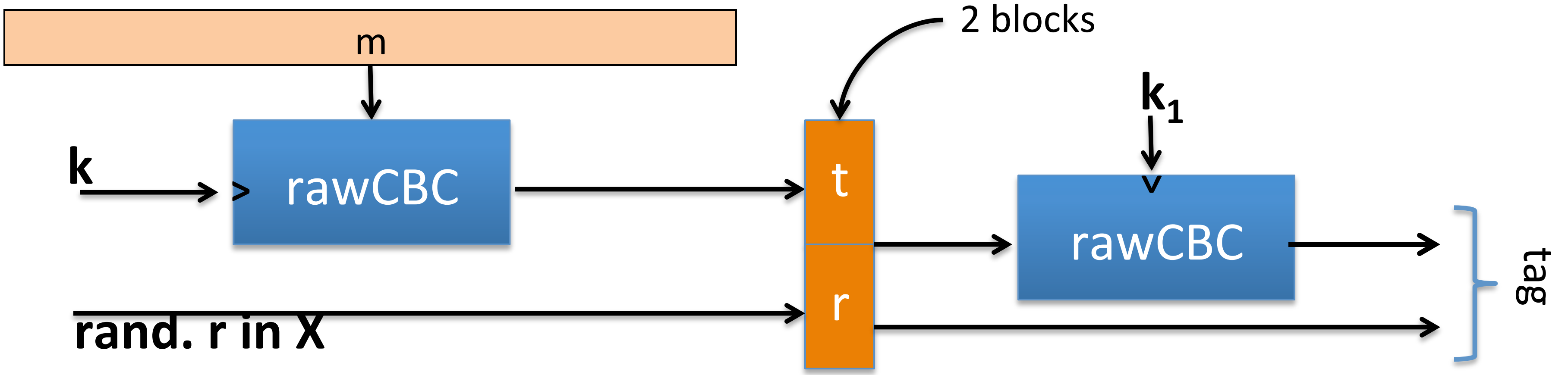
obtain (m_i, t_i) for $i = 1 ,\ldots, |Y|^{1/2}$

step 2: find a collision $t_u = t_v$ for $u \neq v$ (one exists w.h.p by b-day paradox)

step 3: choose some w and query for $t := F_{BIG}(k, \mathbf{m_u \| w})$

step 4: output forgery $(\mathbf{m_v \| w}, \ \mathbf{t})$. Indeed $t := F_{BIG}(k, \mathbf{m_v \| w})$

Better security: a rand. construction



Let **F: K × X ⟶ X** be a PRF. Result: MAC with tags in X^2.

Security: $\mathrm{Adv}_{MAC}[A, I_{RCBC}] \leq \mathrm{Adv}_{PRP}[B, F] \cdot (1 + 2\, q^2 / |X|\,)$

⇒ For 3DES: can sign $\mathbf{q=2^{32}}$ msgs with one key

Comparison

ECBC-MAC is commonly used as an AES-based MAC

- CCM encryption mode (used in 802.11i)
- NIST standard called CMAC

NMAC not usually used with AES or 3DES

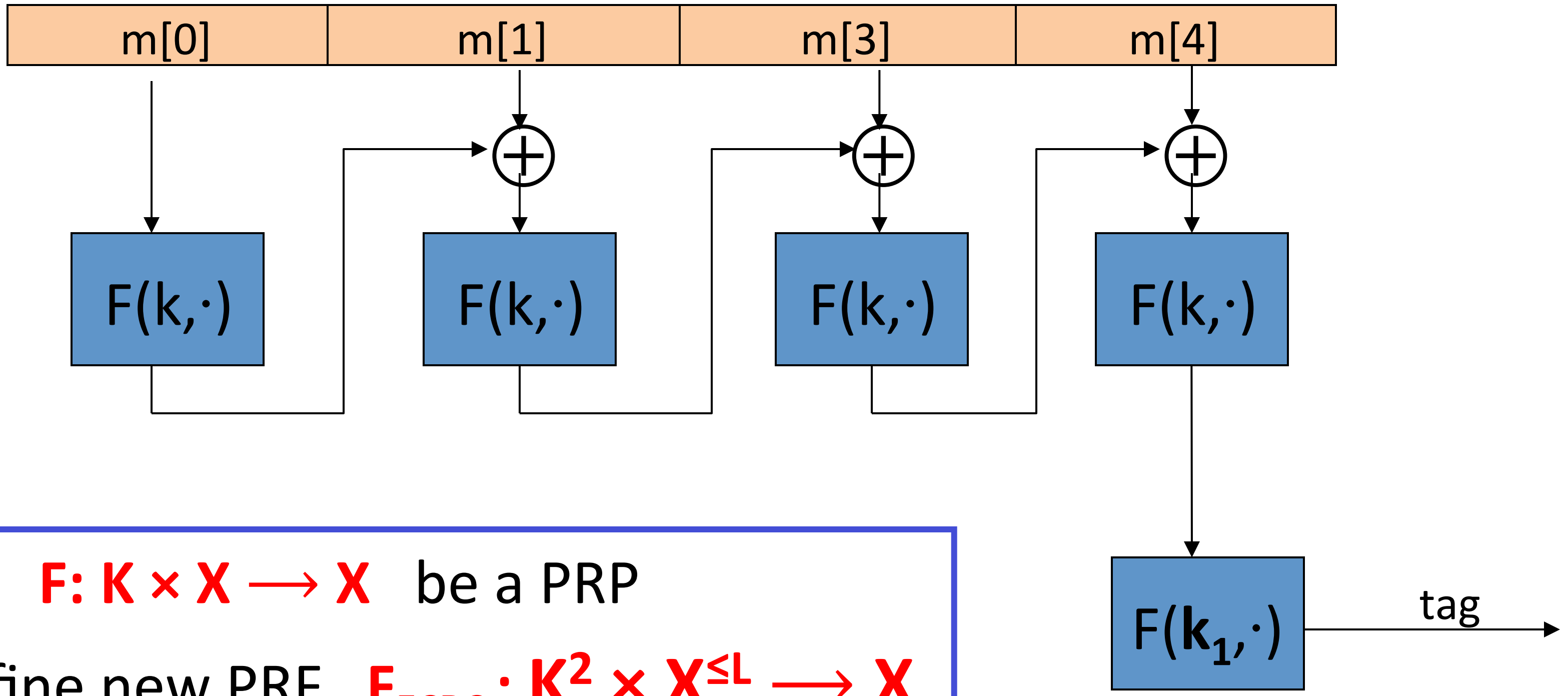
- Main reason: need to change AES key on every block
  requires re-computing AES key expansion
- But NMAC is the basis for a popular MAC called HMAC (next)

End of Segment

Message Integrity

MAC padding

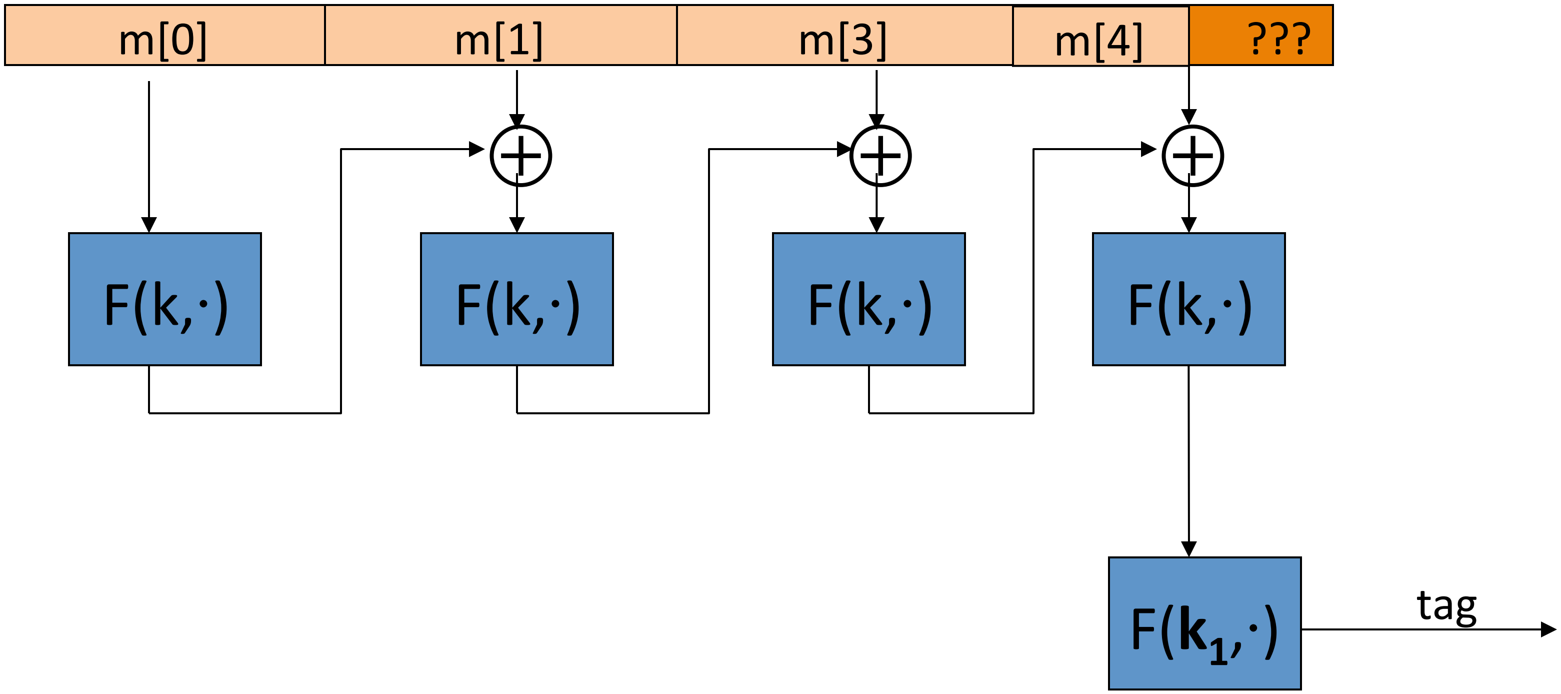
Recall: ECBC-MAC



Let $\mathbf{F: K \times X \longrightarrow X}$ be a PRP

Define new PRF $\mathbf{F_{ECBC} : K^2 \times X^{\leq L} \longrightarrow X}$

What if msg. len. is not multiple of block-size?



CBC MAC padding

Bad idea: pad m with 0's

m[0]	m[1]	→	m[0]	m[1]	0000

Is the resulting MAC secure?

- Yes, the MAC is secure
- It depends on the underlying MAC
- No, given tag on msg **m** attacker obtains tag on **m||0**
-

Problem: pad(m) = pad(m||0)

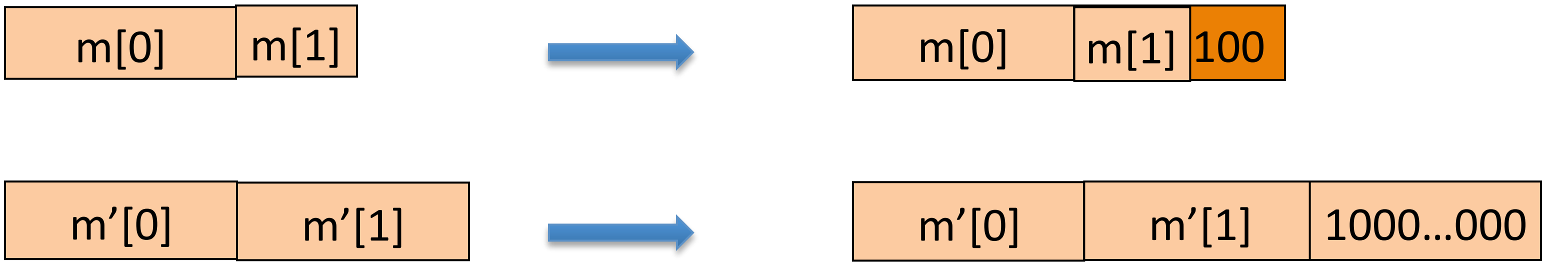
CBC MAC padding

For security, padding must be invertible !

$$m_0 \neq m_1 \quad \Rightarrow \quad pad(m_0) \neq pad(m_1)$$

ISO: pad with "1000…00". Add new dummy block if needed.

– The "1" indicates beginning of pad.

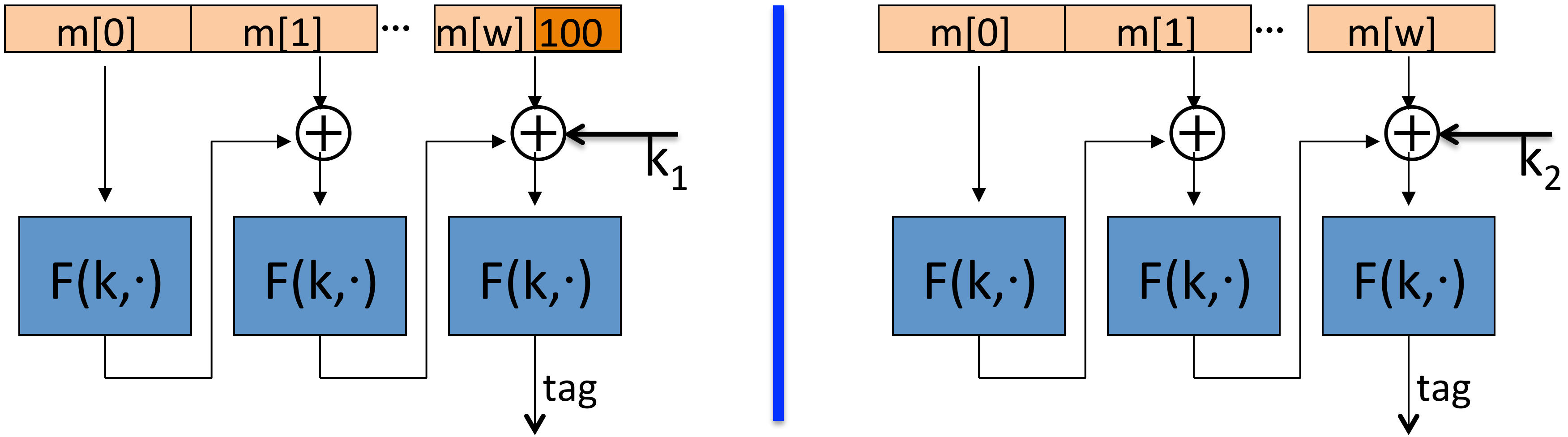


CMAC (NIST standard)

(k₁, k₂) derived from k

Variant of CBC-MAC where key = (k, k_1, k_2)

- No final encryption step (extension attack thwarted by last keyed xor)
- No dummy block (ambiguity resolved by use of k_1 or k_2)



End of Segment

Message Integrity

PMAC and Carter-Wegman MAC

- ECBC and NMAC are sequential.

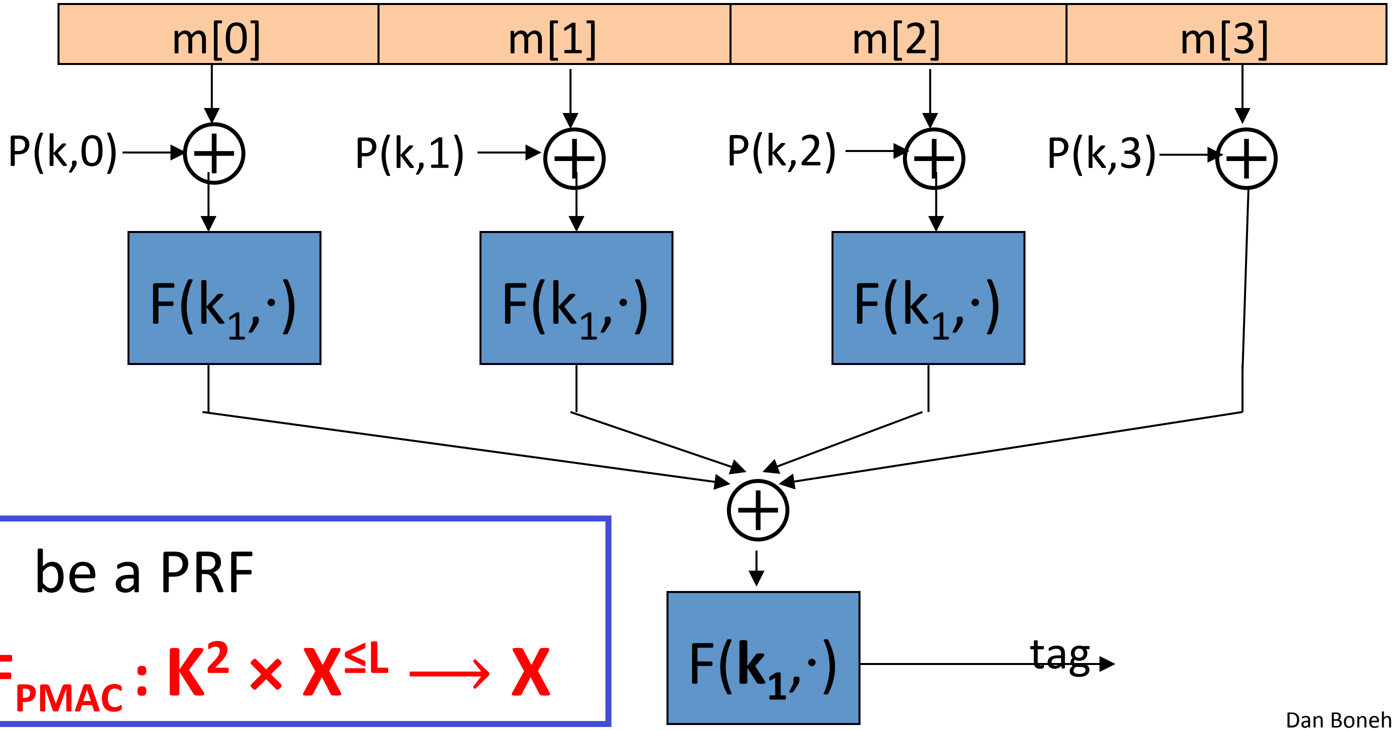
- Can we build a parallel MAC from a small PRF ??

Construction 3: PMAC – parallel MAC

P(k, i): an easy to compute function

key = (k, k_1)

Padding similar to CMAC



Let $\mathbf{F: K \times X \longrightarrow X}$ be a PRF

Define new PRF $\mathbf{F_{PMAC} : K^2 \times X^{\leq L} \longrightarrow X}$

PMAC: Analysis

PMAC Theorem: For any $L>0$,

If F is a secure PRF over (K,X,X) then

F_{PMAC} is a secure PRF over $(K, X^{\leq L}, X)$.

For every eff. q-query PRF adv. A attacking F_{PMAC}
there exists an eff. PRF adversary B s.t.:

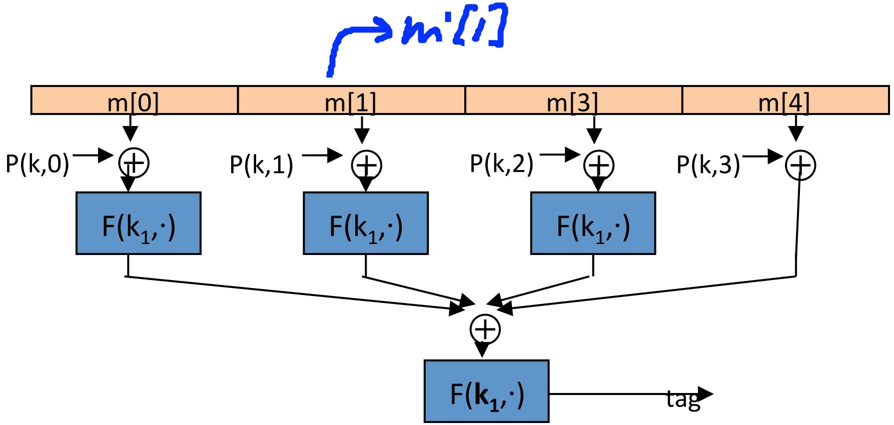
$$\mathrm{Adv}_{PRF}[A, F_{PMAC}] \leq \mathrm{Adv}_{PRF}[B, F] + \mathbf{2\, q^2\, L^2 / |X|}$$

PMAC is secure as long as $qL \ll |X|^{1/2}$

PMAC is incremental

Suppose F is a PRP.

When $m[1] \longrightarrow m'[1]$
can we quickly update tag?

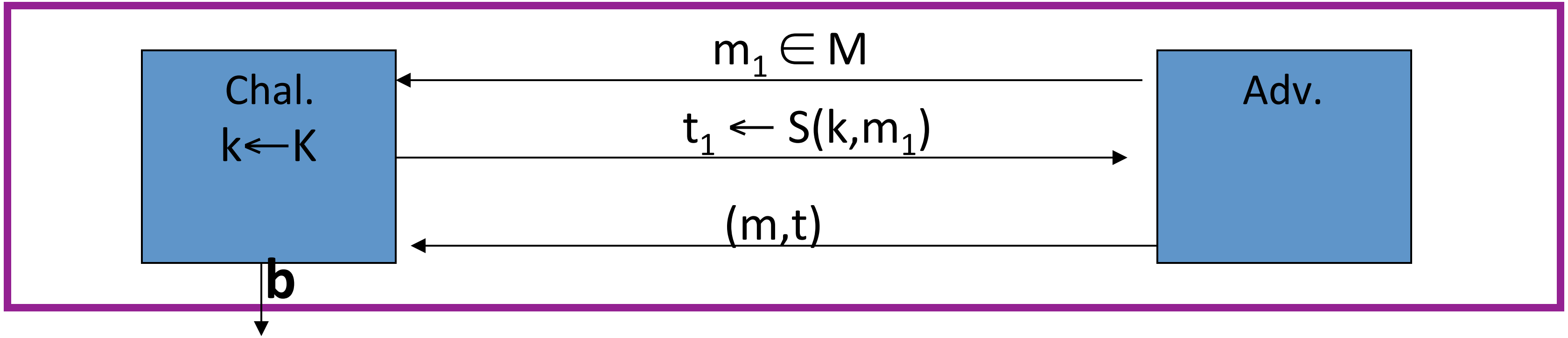


- no, it can't be done
- do $F^{-1}(k_1,tag) \oplus F\big(k_1, m'[1] \oplus P(k,1)\big)$
- do $F^{-1}(k_1,tag) \oplus F\big(k_1, m[1] \oplus P(k,1)\big) \oplus F\big(k_1, m'[1] \oplus P(k,1)\big)$
- do $tag \oplus F\big(k_1, m[1] \oplus P(k,1)\big) \oplus F\big(k_1, m'[1] \oplus P(k,1)\big)$

Then apply $F(k_1, \cdot)$

One time MAC (analog of one time pad)

- For a MAC $I=(S,V)$ and adv. A define a MAC game as:



$$\begin{cases} \mathbf{b}=1 & \text{if } V(k,m,t) = \text{`yes'} \text{ and } (m,t) \neq (m_1,t_1) \\ \mathbf{b}=0 & \text{otherwise} \end{cases}$$

Def: $I=(S,V)$ is a **secure MAC** if for all "efficient" A:

$\text{Adv}_{1\text{MAC}}[A,I] = \Pr[\text{Chal. outputs } 1]$ is "negligible."

One-time MAC: an example

Can be secure against **all** adversaries and faster than PRF-based MACs

Let q be a large prime (e.g. $\mathbf{q = 2^{128}+51}$)

key = (a, b) $\in \{1,\ldots,q\}^2$ (two random ints. in [1,q])

msg = (m[1], …, m[L]) where each block is 128 bit int.

$$\mathbf{S(\,key, msg\,) = P_{msg}(a) + b \pmod{q}}$$

where $P_{msg}(x) = x^{L+1} + m[L]\cdot x^L + \ldots + m[1]\cdot x$ is a poly. of deg L+1

We show: given **S(key, $\mathbf{msg_1}$)** adv. has no info about **S(key, $\mathbf{msg_2}$)**

One-time security (unconditional)

Thm: the one-time MAC on the previous slide satisfies (L=msg-len)

$$\forall m_1 \neq m_2, t_1, t_2: \quad \Pr_{a,b}\big[\, S((a,b), m_1) = t_1 \;\big|\; S((a,b), m_2) = t_2 \,\big] \leq L/q$$

Proof: $\forall m_1 \neq m_2,\ t_1,\ t_2$:

(1) $\Pr_{a,b}\big[\, S((a,b), m_2) = t_2 \,\big] = \Pr_{a,b}\big[P_{m_2}(a) + b = t_2\big] = 1/q$

(2) $\Pr_{a,b}\big[\, S((a,b), m_1) = t_1$ **and** $S((a,b), m_2) = t_2 \,\big] =$

$$\Pr_{a,b}\big[\, P_{m_1}(a) - P_{m_2}(a) = t_1 - t_2 \text{ and } P_{m_2}(a) + b = t_2 \,\big] \leq L/q^2 \quad \blacksquare$$

$\Rightarrow$ given valid (m_2,t_2), adv. outputs (m_1,t_1) and is right with prob. $\leq L/q$

One-time MAC ⇒ Many-time MAC

Let (S,V) be a secure one-time MAC over $(K_I, M, \{0,1\}^n)$.

Let $F: K_F \times \{0,1\}^n \longrightarrow \{0,1\}^n$ be a secure PRF.

Carter-Wegman MAC: $\quad CW\big((k_1,k_2), m \big) = \Big(r, \; \overbrace{F(k_1,r)}^{\text{slow but short inp}} \oplus \overbrace{S(k_2,m)}^{\text{fast long inp}} \Big)$

for random $r \longleftarrow \{0,1\}^n$.

<ins>**Thm**</ins>: If (S,V) is a secure **one-time** MAC and F a secure PRF then CW is a secure MAC outputting tags in $\{0,1\}^{2n}$.

$$CW(\ (k_1,k_2),\ m) = \ (r,\ \ F(k_1,r) \oplus S(k_2,m)\ )$$

How would you verify a CW tag **(r, t)** on message **m** ?

Recall that $V(k_2,m,.)$ is the verification alg. for the one time MAC.

- ○ Run $V(\ k_2,\ \ m,\ \ F(k_1,\ t) \oplus r)\ )$
- ○ Run $V(\ k_2,\ \ m,\ r\ )$
- ○ Run $V(\ k_2,\ \ m,\ t\ )$
- ○ Run $V(\ k_2,\ \ m,\ \ F(k_1,\ r) \oplus t)\ )$

Construction 4: HMAC (Hash-MAC)

Most widely used MAC on the Internet.

… but, we first we need to discuss hash function.

Further reading

- J. Black, P. Rogaway: CBC MACs for Arbitrary-Length Messages: The Three-Key Constructions. J. Cryptology 18(2): 111-131 (2005)
- K. Pietrzak: A Tight Bound for EMAC. ICALP (2) 2006: 168-179
- J. Black, P. Rogaway: A Block-Cipher Mode of Operation for Parallelizable Message Authentication. EUROCRYPT 2002: 384-397
- M. Bellare: New Proofs for NMAC and HMAC: Security Without Collision-Resistance. CRYPTO 2006: 602-619
- Y. Dodis, K. Pietrzak, P. Puniya: A New Mode of Operation for Block Ciphers and Length-Preserving MACs. EUROCRYPT 2008: 198-219

End of Segment