

# TEST 1

## MAT344 - SPRING 2019

PROF. ALEX RENNET

NAME: \_\_\_\_\_

STUDENT ID: \_\_\_\_\_

SIGNATURE: \_\_\_\_\_

### Instructions

There are **6 questions** on this test, some with multiple parts.

There are **20 points** available.

This test has **8 pages**, including this one.

**No aids allowed.** (i.e. no calculators, cheat sheets, devices etc.)

### TUTORIAL SECTION (Leave blank if you can't remember)

WEDNESDAY	
3pm	<input type="checkbox"/> TUT101 - Arash
5pm	<input type="checkbox"/> TUT102 - Osaid
6pm	<input type="checkbox"/> TUT103 - Osaid

### FOR MARKING (Leave This Blank)

/2	/3	/3	/3	/3	/6	/20
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(2 points each  $\Rightarrow$  4 points total)

Put your answers in this question in terms of terminology or notation that we've used in this course, like  $\binom{n}{k}$  or  $S(n, k)$ , etc.

**You do not need to justify your answers in this question.**

In a science experiment gone wrong, a portal is created which connects our world to an unlimited number of *alternate realities*.

- 1.1 The scientists (in our world) that created the portal have already identified 5 alternate realities of special interest to investigate. You and nineteen others have been chosen to go on a mission to explore these realities.  
**How many ways can these 20 people be divided up amongst the 5 realities so that each world has at least one person going?**

- 1.2 Suppose you end up on a team with 5 other people on it. In the alternate reality you visit, you each have a **duplicate**: someone who for all purposes is *identical* to you. After you and your teammates find your duplicates, all of you end up at a ceremony together to meet the Queen; all of you, along with 50 other people from their reality, will wait in line to shake hands with the Queen.  
**How many ways are there to form this line?**

**For the remainder of the test, you must justify your answers.**

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2

**(3 points)**

For integers  $1 \leq n \leq m$ , how many functions  $f : [n] \rightarrow [m]$  are there for which  $f(1) < f(2) < \dots < f(n)$ ?

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3

**(3 points)** What is the coefficient on  $x^{2019}$  in the power series form of  $f(x) = \sqrt[5]{2-x}$ ?  
(Simplify your answer here as much as you can.)

**(3 points)**

How many arrangements are there of the letters in the word

A R I T H M O P H E L I A

so that the arrangements have the  $M$  *immediately beside* at least one  $A$  (i.e. either as ...MA... or as ...AM... or as ...AMA...)?

**(3 points)**

Prove the following identity using the **Binomial Theorem**:

$$\sum_{0 \leq k \text{ even}}^n \binom{n}{k} 2^k = \frac{3^n + (-1)^n}{2}$$

**(3 points each  $\Rightarrow$  6 points total)**

Prove each of the equations below using a **combinatorial proof**.

*(Recall that in a **combinatorial proof** you must explain in words why the two sides of the equation count the same thing in two different ways.)*

**You will receive 0 points for** an algebraic argument (for instance, don't rewrite  $\binom{n}{k}$  in terms of  $n!$  and  $k!$  etc), or for attempting to use induction or a proof by contradiction, applying a theorem, etc.

6.1 **(3 points)**

$$\binom{2n}{2} = 2\binom{n}{2} + n^2$$

*Hint: rewrite  $n$  as  $\binom{n}{1}$ .*

6.2 **(4 points)**

$$S(n+1, k+1) = \sum_{i=k}^n \binom{n}{i} S(i, k)$$