Tutorial Worksheet 3
MAT344 - Spring 2019

1

1.1 Draw the *Ferrer's shape* for the seven partitions of 5.

1.2 For each partition, identify its *conjugate*. Which of the partitions is self-conjugate?

1.3 What is the general fact (proved in the book!) about partitions of n that the previous part illustrates?

2

2.1 Which of the partitions of 5 have the property that all parts in the partition are of size *2 or more*?

2.2 What is $p(4)$? *(Recall that $p(n)$ is the number of partitions of n.)*

2.3 Prove that the number of partitions of n where each part has size at least 2 is equal to $p(n) - p(n-1)$. *(Our work so far in this worksheet has, in part, highlighted the case $n = 5$ of this statement.)*

3

In this question, we will consider the number of partitions of n in which the difference between the largest and the second largest part is *exactly 2*. Call the number of such partitions $l(n)$.

3.1 Draw the *Ferrer's shape* of the *four* partitions of this type for $n = 9$, and the conjugates of each of them. *(Notice in particular that partitions satisfying the restriction must have at least two parts!)*

3.2 Find a formula for $l(n)$ in terms of $p(n)$. Prove your answer is correct.

4

Let $n \geqslant 4$. Find the number of partitions of n in which the difference of the first two parts is ...

4.1 ... *at least* three. *(A partition of this type is allowed to have exactly one part.)*

4.2 ... *exactly* three. *(A partition of this type must have at least two parts.)*

Hint: consider partitions of some integer smaller than n.