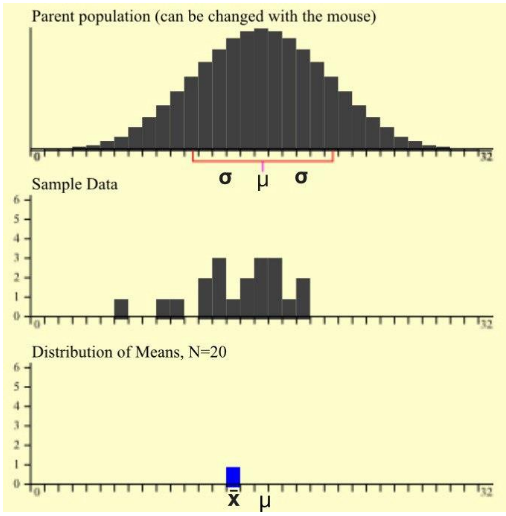
Population parameters:

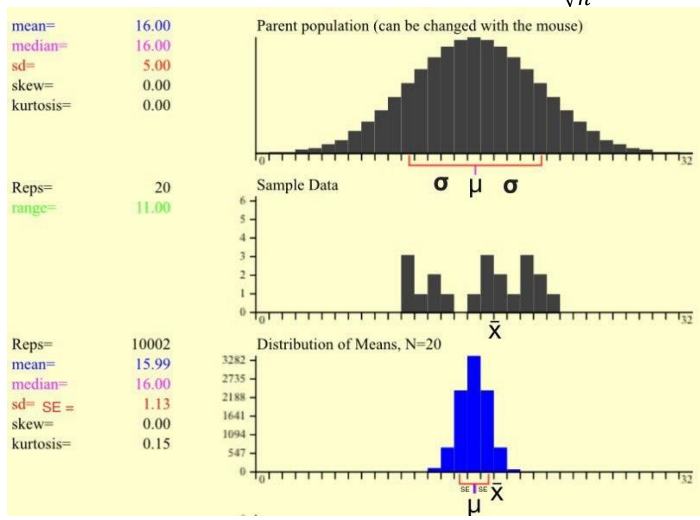
- μ is the population mean
- σ is the population standard deviation

According to **CLT for normal distributions** for any sample size n:

(1) The distribution of the sample mean will be normal.
(2) The mean of the distribution of the sample mean will be equal to μ.
(3) The standard deviation (standard error) of the distribution of the sample mean SE = $\frac{\sigma}{\sqrt{n}}$.



1 sample of size 20 is run



10002 samples of size 20 are run and the results follow (1), (2), (3)

In hypothesis testing:

We first assume(H_0 - null hypothesis) population mean to be equal to some value **μ**.
Then we get some sample of size **n**, calculate its mean(**x̄**) and variance(**s²**) and try to reject it with confidence level c or with significance level a = 1-c.

z_{score} calculates distance between x̄(sample mean) and μ(the mean of the distribution of the sample mean, not population mean) taking SE as a unit distance.

$z_{score} = \frac{\bar{x}-\mu}{SE} = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$

But why do we need z_{score}, why do we need distance between x̄ and μ?

According to **CLT for normal distributions** the distribution of the sample mean follows **(1), (2), (3)** rules.

(2) The mean of the distribution of the sample mean is equal to μ.
(3) SE(Standard deviation of the distribution of the sample mean) = $\frac{\sigma}{\sqrt{n}}$.
(1) The distribution of the sample mean is normal. **This lets us to know what percentage of the data(sample means) is covered by [μ-n*SE, μ+n*SE] interval.**

z_{score}	Percentage of data covered within z_{score}*SE of the μ(mean)	Maximum confidence level with which H_0 can be rejected	P value(Minimum significance level with which H_0 can be rejected)
1	68%	68%, 0.68	0.32
2	95%	95%, 0.95	0.05
3	99.7%	99.7%, 0.997	0.003

It means that if H_0 is true and we take a sample, then the probability of that sample mean(x̄) to be in [μ-SE, μ+SE] interval is 68%, to be in [μ-2*SE, μ+2*SE] interval is 95% and so on.

And if x̄ is out of some interval, then we can reject the H_0 with the confidence level that is equal to the probability of x̄ to be in that interval which is equal to the percentage of data(sample means) covered within that interval.

For example, if x̄ is out of [μ-2*SE, μ+2*SE] interval then we can reject the H_0 with the confidence level 95%, because the probability of x̄ to be in that interval is 95%, because 95% of the data(sample means) is covered by that interval.

And to reject the H_0 with the confidence level c%, x̄ has to be out of such interval that covers more than c% of data.

For example, to reject H_0 with confidence level 95%, x̄ has to be out of [μ-2*SE, μ+2*SE], in other words z_{score} has to be greater than 2, because [μ-2*SE, μ+2*SE] covers 95% of the data.

The bigger the confidence level, the bigger the z_{score}(distance between x̄ and μ) has to be for us to be able to reject H_0.

When the population variance(σ^2) is unknown, sample variance(s^2) is used instead and z_{score} "becomes" t_{score}.

SE = $\frac{s}{\sqrt{n}}$

t_{score} = $\frac{\bar{x}-\mu}{SE}$ = $\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$

There are tables where for each confidence level their corresponding minimum z and t scores are shown to reject the H_0.

An example problem

At a water-bottling factory, a machine is supposed to put 2 liters of water into the bottles. After an overhaul, management thinks the machine is no longer putting the correct amount of water in. They sample 20 bottles and find an avg of 2.10 L of water with standard deviation of 0.33 L. Test the claim at 0.01 level of significance.

H_0: μ = 2 n(sample size) = 20

H_a: μ ≠ 2 x̄(sample mean) = 2.1

c = 0.99 s(sample standard deviation) = 0.9

t = $\frac{x-\mu}{SE}$ = $\frac{x-\mu}{\frac{s}{\sqrt{n}}}$ = 0.4969

It's obvious that with t<1 we can't reject H_0 with 99% confidence level. Because we know that only 68% of the data is covered within 1 SE of the μ in normal distributions. So if x̄ isn't even out of that interval(x̄ is t*SE distant from μ) then we can't even reject H_0 with 68 % confidence level.

If it's not that obvious, then we can look at t-test table to see how much t at least has to be for us to be able to reject the H_0. T-test table shows that t has to be not less than 2.845 for us to be able to reject H_0 with 99% confidence level.

17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%
	Confidence Level									