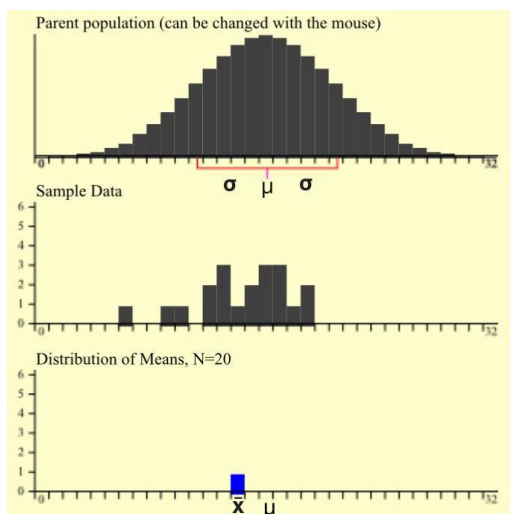


Population parameters:

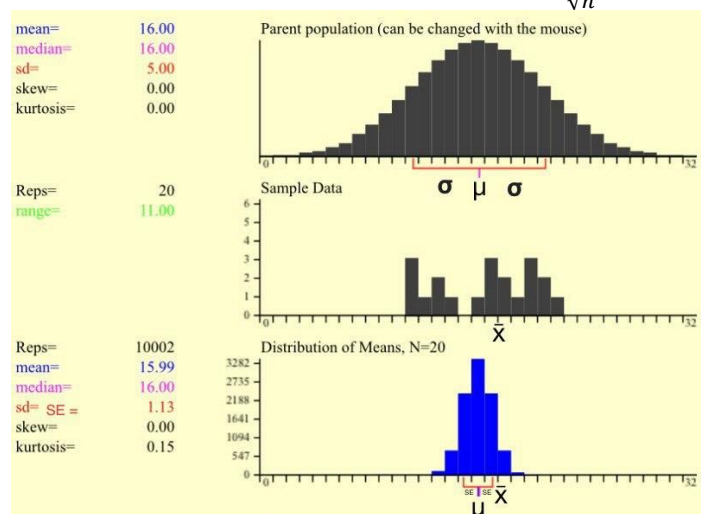
- μ is the population mean
- σ is the population standard deviation

According to **CLT for normal distributions** for any sample size n :

- (1) The distribution of the sample mean will be normal.
- (2) The mean of the distribution of the sample mean will be equal to μ .
- (3) The standard deviation (standard error) of the distribution of the sample mean $SE = \frac{\sigma}{\sqrt{n}}$.



1 sample of size 20 is run



10002 samples of size 20 are run and the results follow (1), (2), (3)

In hypothesis testing:

We first assume (H_0 - null hypothesis) population mean to be equal to some value μ .

Then we get some sample of size n , calculate its mean (\bar{x}) and variance (s^2) and try to reject it with confidence interval $c\%$.

z_{score} calculates distance between \bar{x} (sample mean) and μ (the mean of the distribution of the sample mean, not population mean) taking SE as a unit distance.

$$Z_{score} = \frac{\bar{x} - \mu}{SE} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

But why do we need z_{score} , why do we need distance between \bar{x} and μ ?

According to **CLT for normal distributions** the distribution of the sample mean follows (1), (2), (3) rules.

- (2) The mean of the distribution of the sample mean is equal to μ .
- (3) SE (Standard deviation of the distribution of the sample mean) = $\frac{\sigma}{\sqrt{n}}$.
- (1) The distribution of the sample mean is normal. This lets us to know what percentage of the data (sample means) is covered by $[\mu - n \cdot SE, \mu + n \cdot SE]$ interval.

$n \cdot SE$	Percentage of data covered within $n \cdot SE$ of the mean
1 SE	68%
2 SE	95%
3 SE	99.7%

It means that if H_0 is true and we take a sample, then the probability of that sample mean (\bar{x}) to be in $[\mu - SE, \mu + SE]$ interval is 68%, to be in $[\mu - 2 \cdot SE, \mu + 2 \cdot SE]$ interval is 95% and so on.

17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098
Z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%