

Idea

The Idea behind the ADT of a Queue is to provide a collection of data with two primary operations:

- *push*: Push a new item on top of the Queue.
- *pop*: Take the item on top of the Queue off.

Definition

We define the ADT as the following 5-Tuple:

$$\mathcal{D} = (N, P, Fs, Ts, Ax),$$

where the components are defined as follows:

1. $N := \text{Queue}$
2. $P := \{\text{Element}\}$
3. $Fs := \{\text{queue}, \text{enqueue}, \text{dequeue}, \text{peek}, \text{length}\}$
4. Ts is the set containing the following type specifications:
 - (a) $\text{queue} : \text{Queue}$
 - (b) $\text{length} : \text{Queue} \rightarrow \mathbb{N}_0$
 - (c) $\text{enqueue} : \text{Queue} \times \text{Element} \rightarrow \text{Queue}$
 - (d) $\text{dequeue} : \text{Queue} \rightarrow \text{Queue} \cup \{\Omega\}$
 - (e) $\text{peek} : \text{Queue} \rightarrow \text{Element} \cup \{\Omega\}$
5. Ax is the set containing the following axioms.
 $\forall Q \in \text{Queue} : x, y \in \text{Element} :$
 - (a) $\text{queue}().\text{peek}() = \Omega$
 - (b) $\text{queue}().\text{length}() = 0$
 - (c) $Q.\text{enqueue}(x).\text{length}() = Q.\text{length}() + 1$
 - (d) $Q.\text{length}() > 0 \rightarrow Q.\text{dequeue}().\text{length}() = Q.\text{length}() - 1$
 - (e) $Q.\text{length}() > 0 \rightarrow Q.\text{enqueue}(x).\text{dequeue}() = Q.\text{dequeue}().\text{enqueue}(x)$
 - (f) $\text{queue}().\text{enqueue}(x).\text{dequeue}() = \text{queue}()$
 - (g) $\text{queue}().\text{dequeue}() = \Omega$
 - (h) $Q.\text{length}() > 0 \rightarrow Q.\text{enqueue}(x).\text{peek}() = Q.\text{peek}()$
 - (i) $Q.\text{length}() = 0 \rightarrow Q.\text{enqueue}(x).\text{peek}() = x$

Implementation

TBD