Übungsblatt 7

① a) $[6,2]_{17}$ - RSC bzgl. $\mathcal{B} = \{1,3,4,6,7,8\}$ mit $d(C)=5$, $t=2$

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 & 7 & 8 \\ 1 & 9 & 16 & 2 & 15 & 13 \\ 1 & 10 & 13 & 12 & 3 & 2 \end{pmatrix} \begin{matrix} \\ \text{(II)}-\text{(I)}\cdot 9 \\ \text{(III)}-\text{(I)}\cdot 15 \\ \text{(IV)}-\text{(I)}\cdot 2 \end{matrix} \rightarrow$$

Vielfache von 17:
17, 34, 51, 68, 85, 102, 119, 136, 153, 170, 187, 204, 221

$$\rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 10 & 11 & 3 & 12 \\ 0 & 1 & 4 & 15 & 6 & 10 \\ 0 & 1 & 7 & 5 & 4 & 2 \end{pmatrix} \begin{matrix} \text{I}-\text{II} \\ \\ \text{(III)}-\text{(II)}\cdot 14 \\ \text{(IV)}-\text{(II)}\cdot 11 \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & 8 & 7 & 15 & 6 \\ 0 & 1 & 10 & 11 & 3 & 12 \\ 0 & 0 & 1 & 5 & 8 & 6 \\ 0 & 0 & 1 & 2 & 11 & 9 \end{pmatrix}$$

$$\begin{matrix} \text{I}-8\,\text{III} \\ \text{II}-10\,\text{III} \\ \text{IV}-\text{III} \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 1 & 2 & 9 \\ 0 & 1 & 0 & 12 & 8 & 3 \\ 0 & 0 & 1 & 5 & 8 & 6 \\ 0 & 0 & 0 & 14 & 3 & 3 \end{pmatrix} \begin{matrix} \text{I}-11\,\text{IV} \\ \text{II}-12\cdot 11\,\text{IV} \\ \text{III}-5\cdot 11\,\text{IV} \\ 11\,\text{IV} \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 3 & 10 \\ 0 & 1 & 0 & 0 & 3 & 15 \\ 0 & 0 & 1 & 0 & 13 & 11 \\ 0 & 0 & 0 & 1 & 16 & 16 \end{pmatrix}$$

$$G = \begin{pmatrix} 14 & 4 & 1 & 1 & 0 \\ 2 & 6 & 1 & 0 & 1 \end{pmatrix}$$

b) $m = (9,7) \Rightarrow c = (4, 10, 16, 9, 7)$

$\tilde{m} = (15,2) \Rightarrow \tilde{c} = (10, 4, 0, 15, 2)$

② a) $[7,3]_{17}$-RSC bzgl. $\mathcal{B} = \{1, 3, 5, 7, 9, 11, 13\}$

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 5 & 7 & 9 & 11 & 13 \\ 1 & 9 & 8 & 15 & 13 & 2 & 16 \\ 1 & 10 & 6 & 3 & 15 & 5 & 4 \end{pmatrix} \begin{matrix} \text{I} - ((\text{II}-\text{I})\cdot 9) \\ (\text{II}-\text{I})\cdot 9 \\ (\text{III}-\text{I})\cdot 15 \\ (\text{IV}-\text{I})\cdot 2 \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & 16 & 15 & 14 & 13 & 12 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 3 & 6 & 10 & 15 & 9 \\ 0 & 1 & 10 & 4 & 11 & 8 & 6 \end{pmatrix} \begin{matrix} \text{I} - (\text{III}-\text{II})\cdot 16 \\ \text{II} - (\text{III}-\text{II})\cdot 2 \\ \text{III} - \text{II} \\ (\text{IV}-\text{II})\cdot 15 \end{matrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & 1 & 3 & 6 & 10 \\ 0 & 1 & 0 & 14 & 9 & 2 & 10 \\ 0 & 0 & 1 & 3 & 6 & 10 & 15 \\ 0 & 0 & 1 & 15 & 3 & 11 & 0 \end{pmatrix} \begin{matrix} \text{I} - 10\cdot(\text{IV}-\text{III}) \\ \text{II} - 14\cdot 10\cdot(\text{IV}-\text{III}) \\ \text{III} - 3\cdot((\text{IV}-\text{III})\cdot 10) \\ (\text{IV}-\text{III})\cdot 10 \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 16 & 13 & 7 \\ 0 & 1 & 0 & 0 & 4 & 15 & 2 \\ 0 & 0 & 1 & 0 & 11 & 14 & 6 \\ 0 & 0 & 0 & 1 & 4 & 10 & 3 \end{pmatrix}$$

$$\Rightarrow G = \begin{pmatrix} 1 & 13 & 6 & 13 & 7 & 0 & 0 \\ 4 & 2 & 3 & 7 & 0 & 1 & 0 \\ 10 & 15 & 11 & 14 & 0 & 0 & 1 \end{pmatrix}$$

b) $m = (6, 7, 8) \rightarrow c = (12, 10+14+1, 2+4+3, 10+15+10, 6, 7, 8)$

$$= (12, 8, 9, 7, 6, 7, 8)$$

$\tilde{m} = (11, 8, 4) \rightarrow \tilde{c} = (11+15+6, 7+16+9, 15+7+10, 7+5+5, 11, 8, 4)$

$$= (15, 15, 15, 0, 11, 8, 4)$$

③ $\mathbb{F}_8$ mit $\alpha^3 = \alpha + 1$, $[6,4]_8$-RSC bzgl. $\mathcal{B} = \{1, \alpha^2, \alpha^2+\alpha, \alpha^2+1, \alpha, \alpha+1\}$

$\rightsquigarrow \mathcal{B} = \{1, \alpha^2, \alpha^4, \alpha^6, \alpha, \alpha^3\}$

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha^2+\alpha & \alpha^2+1 & \alpha & \alpha+1 \end{pmatrix}$$

$$\begin{matrix} \text{I} - (\text{II}-\text{I})\alpha \\ (\text{II}-\text{I})\cdot\alpha \end{matrix} \begin{pmatrix} 1 & 0 & \alpha^2 & \alpha & \alpha^5 & \alpha^6 \\ 0 & 1 & \alpha^6 & \alpha^3 & \alpha^4 & \alpha^2 \end{pmatrix}$$

$$\Rightarrow G = \begin{pmatrix} \alpha^2 & \alpha^3+1 & 1 & 0 & 0 & 0 \\ \alpha & \alpha+1 & 0 & 1 & 0 & 0 \\ \alpha^3+\alpha+1 & \alpha^2+\alpha & 0 & 0 & 1 & 0 \\ \alpha^2+1 & \alpha^2 & 0 & 0 & 0 & 1 \end{pmatrix}$$

b)

$m = (\alpha, \alpha^2, \alpha+1, 1) \rightarrow c = (\alpha^3+\alpha^3+\alpha+\alpha^6, 1+\alpha^5+1+\alpha^2, \alpha, \alpha^2, \alpha^3, 1)$

$c = (\alpha^2+\alpha+1, \alpha+1, \alpha, \alpha^2, \alpha+1, 1)$

$\tilde{m} = (\alpha^2+1, \alpha^2+\alpha, 1, \alpha) \rightarrow \tilde{c} = (\alpha+\alpha^5+\alpha^5+1, \alpha^5+1+\alpha^3+\alpha^3, \alpha^6, \alpha^5, 1, \alpha)$

$\tilde{c} = (\alpha+1, \alpha+1, \alpha^2+1, \alpha^2+\alpha, 1, \alpha)$

a)

$[6,2]_8$ -RSC bzgl. $\mathcal{B} = \{1, \alpha^2+1, \alpha^2+\alpha+1, \alpha^2+\alpha, \alpha+1, \alpha^2\}$

$$+/= \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & (\alpha^3+1)(\alpha^2+\alpha+1) & (\alpha^2+\alpha) & (\alpha+1) & \alpha^2 & \\ 1 & (\alpha^2+\alpha+1)(\alpha+1) & \alpha & (\alpha^2+1) & (\alpha^2+\alpha) & \\ 1 & (\alpha^2+\alpha)(\alpha) & (\alpha^2+\alpha+1) & \alpha^2 & (\alpha^2+1) & \end{pmatrix}$$

$$\begin{matrix} \text{III}-\text{I})\,\alpha^5 \\ (\text{III}-\text{I})\cdot\alpha^3 \\ (\text{IV}-\text{I})\cdot\alpha^2 \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & \alpha^2 & \alpha^3 & \alpha^6 & \alpha^4 \\ 0 & 1 & \alpha^4 & \alpha^6 & \alpha^5 & \alpha \\ 0 & 1 & \alpha^5 & \alpha^6 & \alpha & \alpha^4 \end{pmatrix}$$

$$\begin{matrix} \text{I}-\text{II} \\ \text{II}-\alpha^2\cdot\alpha^6(\text{III}-\text{II}) \\ (\text{III}-\text{II})\cdot\alpha^6 \\ (\text{IV}-\text{II})\cdot\alpha^4 \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & \alpha^6 & \alpha & \alpha^2 & \alpha^5 \\ 0 & 1 & 0 & \alpha^2 & 1 & \alpha^6 \\ 0 & 0 & 1 & \alpha^3 & 1 & \alpha \\ 0 & 0 & 1 & \alpha & \alpha^2 & 0 \end{pmatrix}$$

$$\begin{matrix} \text{I}-\alpha^6\,\text{III} \\ \text{II}-(\text{IV}-\text{III})\cdot\alpha^2 \\ \text{III}-(\text{IV}-\text{III})\cdot\alpha^3 \\ \text{IV}-\text{III} \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & \alpha^3 & \alpha^4 & \alpha^5 \\ 0 & 1 & 0 & 0 & \alpha^3 & \alpha^4 \\ 0 & 0 & 1 & 0 & \alpha^6 & \alpha^2 \\ 0 & 0 & 0 & 1 & \alpha^6 & \alpha \end{pmatrix} \xrightarrow{\text{I}-\alpha^3\text{IV}} \begin{pmatrix} 1 & 0 & 0 & 0 & \alpha & 1 \\ 0 & 1 & 0 & 0 & \alpha^3 & \alpha^4 \\ 0 & 0 & 1 & 0 & \alpha^6 & \alpha^2 \\ 0 & 0 & 0 & 1 & \alpha^6 & \alpha \end{pmatrix}$$

$$\Rightarrow G = \begin{pmatrix} \alpha & \alpha+1 & \alpha^2+1 & \alpha^2+1 & 1 & 0 \\ 1 & \alpha^2+\alpha & \alpha^2 & \alpha & 0 & 1 \end{pmatrix}$$

b)

$m = (\alpha, \alpha+1) \rightarrow c = (\alpha^2+\alpha+1, \alpha^2+\alpha+1, \alpha^2+\alpha, \alpha^2+\alpha+1, \alpha, \alpha+1)$

$\tilde{m} = (\alpha^2+1, \alpha^2+\alpha) \rightarrow \tilde{c} = (\alpha^2+\alpha+1, \alpha^2+\alpha, \alpha, 0, \alpha^2+1, \alpha^2+\alpha)$