Klausur: Grundlagen der Informatik

(Lehrveranstaltung)

Kurs: …TINF17AIA……………………….

Dozent: Prof. Dr. Bernd Schwinn

Studierende(r)
(Matr.Nr.): ....

Semester: 1.

Hilfsmittel: keine **Dauer: 100 Min.**

Bewertung: Erreichte Punktzahl: Maximale Punktzahl: 85

Note: .... Signum: ....

Anmerkungen:

Aufgabennr.:		maximale Punkte	erreichte Punkte	Bemerkungen
1		6		
2		5		
3		6		
4		10		
5		8		
6		8		
7		6		
8		8		
9		8		
10		6		
11		8		
12		6		
Summe		**85**		

Task 1: (6 Points)

Express the following phrases by logical formulae:

- The successor of each natural square number is a natural number
- The result of dividing an even natural number by 2 is a natural number
- Each natural square number is not negative
- The sum of the predecessor and the successor of a prime number is even

Task 2: (5 Points)

Presume a signature with the sort symbols ‘s1’ and ‘s2’,
a 2-ary function f(s1,s2) -> s2
a 3-ary function g(s2,s1,s2) -> s1
a 2-ary predicate P(s2,s1)
a 3-ary predicate Q(s1,s2,s1)
variables x, y and z of sort s1 as well as a, b and c of sort s2.
Which of the following expressions are formulae in predicate logic. If not, give the reason why!

a) $Q(x, f(g(f(y, a), z, b), f(g(b, g(c, x, f(y, a))), a)), y) \vee \neg P(f(g(c, z, f(g(f(z, c), y, b), a)), b), g(a, g(b, y, b), c))$

b) $Q(g(a, f(g(a, x, b), f(x, c)), c), f(y, a), y) \wedge P(f(g(f(x, b), g(a, x, a), f(z, f(x, b))), f(g(b, z, f(y, b)), a)), y)$

c) $Q(g(f(x, a), g(a, x, f(y, b)), f(g(c, z, a), b)), f(x, f(y, f(x, a))), g(a, y, b)) \leftrightarrow P(f(g(a, g(b, z, c), f(x, a)), b), x)$

d) $Q(g(f(x, f(y, c)), g(b, g(f(y, a), x, f(x, f(y, b))), f(g(a, x, b), c)), b), a, g(f(z, c), g(f(z, a), y, b), f(g(c, x, b), b)))$

e) $Q(g(a, z, f(y, b)), g(f(z, f(y, b)), g(a, y, f(x, b)), f(g(f(g(c, x, a), b), x, a), f(z, c))), z) \rightarrow P(f(y, a), g(b, y, c))$

Task 3: (6 Points)

Presume the following sets of formulae:

$X = \{R \vee S \rightarrow \neg Q, \neg R \rightarrow P \wedge \neg S, (\neg Q \leftrightarrow S) \wedge \neg R\}$

$Y = \{(R \rightarrow S) \vee \neg Q \rightarrow P, Q \leftrightarrow \neg S, R \rightarrow \neg P \wedge Q\}$

$Z = \{Q \wedge \neg R \rightarrow P, R \vee (\neg S \wedge P \wedge Q), P \vee \neg S \rightarrow Q\}$

For each set determine all the replacements that makes it true. State all logical conclusions among the sets of formulae that are possible?

Task 4: (3 + 3 + 4 Points)

Transform the following formulae into conjunctive normal form and notate **all** resulting Gentzen-formulae.

a) $R \vee \neg(S \rightarrow \neg Q \wedge P)$

b) $\neg(S \vee (P \leftrightarrow R) \wedge \neg Q)$

c) $S \vee \neg Q \rightarrow (R \leftrightarrow \neg P) \vee Q$

Task 5: (3 + 5 Points)

Presume the following formulae. First transform them into Prenex normal form and scolemise them finally.

a) $\exists y\ P(x,y) \vee \neg\exists x(R(z,y,x) \ \rightarrow \ \forall y\ Q(z,y,x)) \wedge \neg\exists x\forall y\ R(z,y,x)$

b) $\exists x\ (\forall y\ P(x,y,z) \ \leftrightarrow \exists z\ Q(z,y,x)) \wedge \neg\forall x\ R(z,y,x)$

Task 6: (8 Points)

Prove the correctness of the following specification

```
{ X>Y ; Y>=0; X, Y of type integer }
        I := Y;
        Z:=0;
        repeat
                I := I + 1;
                Z := Z + 1
        until I = X
{ Z=X-Y ; X>Y ; Y>=0 ; X, Y of type integer }
```

Task 7: (6 Points)

Presume the following statements with postcondition

```
if X<Y then begin if X<0 then Y := X – Y end
        else Y := Y – X;
Y := X – Y;
```

{ Y>X; X, Y of type integer }

Determine the weakest precondition.

Task 8: (8 Points)

Are the following pairs of literals unifiable? If yes, give the bindings resulting from unification. If not, give the reason why.

a) p1([[X,Y]|[X|Y]],[Y|Y]) p1([A|B],A)

b) p2([[X|R]|Y],R) p2([A|R],[A])

c) p3([X|[[]|Y]],[Y]) p3([A|[]],[B|A])

d) p4([[X],[Y,Y]],Y) p4([A|[B,C]],B)

e) p5(X,[Y,X,X|[Y]]) p5([A],[B,[A]|B])

Task 9: (5 + 3 Points)

Define a Lisp function that is analogous to the Prolog predicate mystery given below.

```
mystery([],0).
mystery([X],X).
mystery([A,B|R],N) :- mystery(R,M), N is M+A.
```

b) Determine for the Prolog predicate given in a) the number of **all** executed unifications of literals if the predicate is activated with a list containing n positive numbers whereby n is odd . Start the cost calculation with a recursion equation and solve it finally.

Task 10: (6 Points)

Define Prolog clauses for a predicate possum that sums up all positive numbers out of a list of integer-numbers.

Task 11: (2 + 3 + 1 + 2 Points)

Presume L to be a list containing at least two (non-empty) sublists of (at least two) numbers. Give a Lisp expression that

a) replaces within the first sublist the first two numbers by the sum of these two numbers

b) calculates the sum of the first number of the first sublist and the second number of the second sublist and inserts the result to the beginning of the second sublist

c) deletes the first sublist and inserts the reverse of this first sublist to the end of L

d) determines the length of the second sublist and inserts this to the end of the first sublist

Task 12: (6 Points)

Define a Lisp function possum (analogous to Task 10) that calculates for a list of integer numbers the sum of all positive numbers contained in it.

Example: (possum ‘(1 -8 0 -3 3)) returns 4