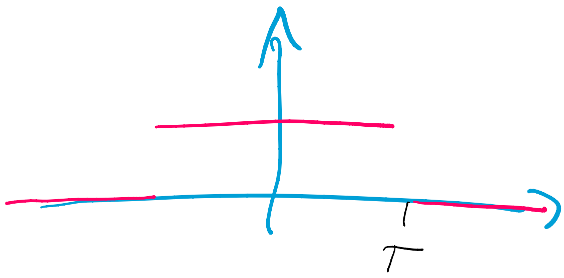
Codierungstheorie

Mittwoch, 5. Oktober 2022 12:45

Bsp.: $r_T(t) = \begin{cases} 1 & \text{für } t \in [-T, T] \\ 0 & \text{sonst} \end{cases}$



gerade

$$\hat{r}_T(\omega) \overset{\downarrow}{=} \frac{2}{\sqrt{2\pi}} \int_0^\infty r_T(t) \cos(\omega t)\, dt$$

$$= \frac{2}{\sqrt{2\pi}} \int_0^T \cos(\omega t)\, dt$$

$$= \frac{2}{\sqrt{2\pi}} \left[\frac{\sin(\omega t)}{\omega} \right]_0^T$$

$$= \frac{2}{\sqrt{2\pi}} \cdot \frac{\sin(\omega T)}{\omega}$$

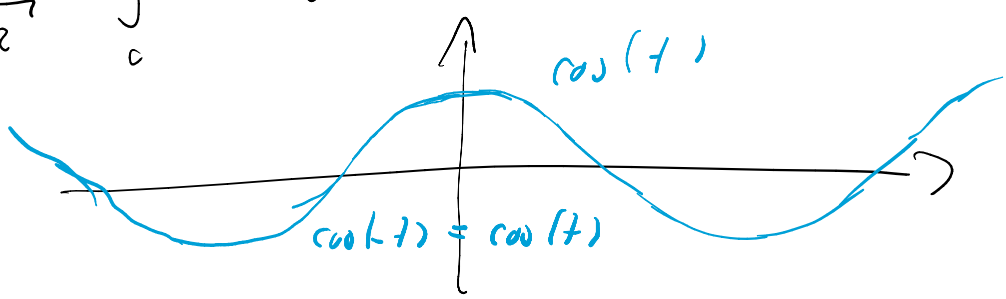
$$= \frac{2T}{\sqrt{2\pi}} \frac{\sin(\omega T)}{\omega T}$$

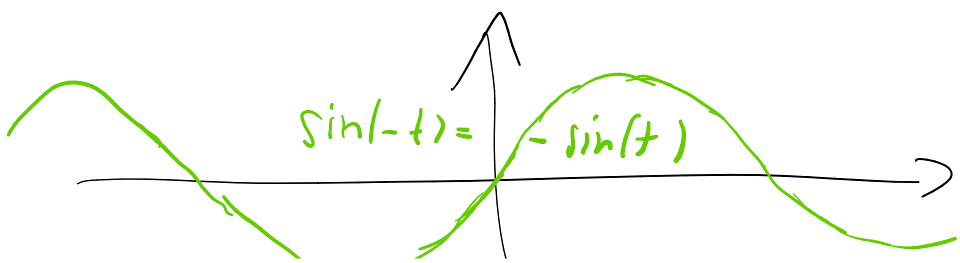
Bsp.: $f(t) = \begin{cases} \cos(\omega_0 t) & (-T \leq t \leq T) \\ 0 & \text{sonst} \end{cases}$

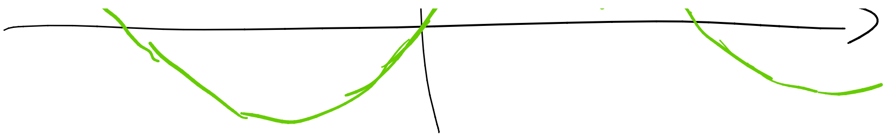
gerade

$$\hat{f}(\omega) = \frac{2}{\sqrt{2\pi}} \int_0^\infty f(t) \cos(\omega t)\, dt$$

$$= \frac{2}{\sqrt{2\pi}} \int_0^T \cos(\omega_0 t) \cdot \cos(\omega t)\, dt + \int 0\, dt =$$







$$\cos(a)\cdot\cos(b) = \frac{\cos(a+b)+\cos(a-b)}{2}$$

$$a = \omega t, \quad b = \omega_0 t$$

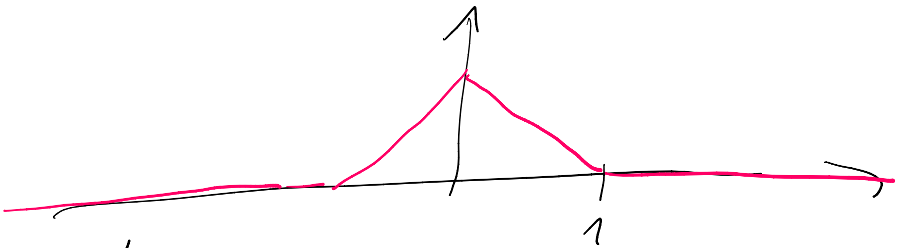
$$\cos(\omega t)\cos(\omega_0 t) = \frac{1}{2}\left(\cos((\omega+\omega_0)t) + \cos((\omega-\omega_0)t)\right)$$

$$= \frac{\cancel{2}}{\sqrt{2\pi}} \int_0^T \frac{1}{\cancel{2}} \left(\cos((\omega+\omega_0)t) + \cos((\omega-\omega_0)t)\right) dt$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{\sin((\omega+\omega_0)t)}{\omega+\omega_0} + \frac{\sin((\omega-\omega_0)t)}{\omega-\omega_0} \right]_0^T$$

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{\sin((\omega+\omega_0)T)}{\omega+\omega_0} + \frac{\sin((\omega-\omega_0)T)}{\omega-\omega_0} \right) - 0$$

Bsp:



$d(t)$ gerade

$$\hat{d}(\omega) = \frac{2}{\sqrt{2\pi}} \int_0^\infty d(t)\cos(\omega t)\,dt$$

$$= \frac{2}{\sqrt{2\pi}} \int_0^1 d(t)\cos(\omega t)\,dt + \frac{2}{\sqrt{2\pi}} \int 0\,dt$$

$$= \frac{2}{\sqrt{2\pi}} \int_0^1 (1-t)\cos(\omega t)\,dt$$

$$= \frac{2}{\sqrt{2\pi}} \underbrace{\int_0^1 \cos(\omega t)\,dt}_{A} - \frac{2}{\sqrt{2\pi}} \underbrace{\int_0^1 \overset{u(t)}{t}\, \overset{w'(t)}{\cos(\omega t)}\,dt}_{A}$$

$$v(t) = \frac{\sin(\omega t)}{\omega}$$

$\sqrt{2\pi}$ $\qquad$ $\sqrt{2\pi}$ $\qquad$ $v(t) = \frac{\sin(\omega t)}{\omega}$

$$= \frac{2}{\sqrt{2\pi}} \left[\frac{\sin(\omega t)}{\omega} \right]_0^1 - \frac{2}{\sqrt{2\pi}} \left(\left[\frac{t \sin(\omega t)}{\omega} \right]_0^1 - \int_0^1 \frac{\sin(\omega t)}{\omega} dt \right)$$

part. Integration: $\int_a^b u(t) v'(t) dt = [u(t) \cdot v(t)]_a^b - \int_a^b u'(t) v(t) dt$

$$= \frac{2}{\sqrt{2\pi}} \cancel{\frac{\sin(\omega)}{\omega}} - \frac{2}{\sqrt{2\pi}} \left(\cancel{\frac{\sin(\omega)}{\omega}} + \left[\frac{\cos(\omega t)}{\omega^2} \right]_0^1 \right)$$

$$= -\frac{2}{\sqrt{2\pi}} \left(\frac{\cos(\omega)}{\omega^2} - \frac{1}{\omega^2} \right)$$

$$= \frac{2}{\omega^2 \sqrt{2\pi}} \left(1 - \cos(\omega) \right) \quad \leftarrow$$

$$= \frac{4}{\omega^2 \sqrt{2\pi}} \sin^2\left(\frac{\omega}{2}\right)$$

f ist $2T_0$ periodisch

$$g(x) = f\left(\frac{T_0 x}{\pi}\right)$$

$$g(x + 2\pi) = f\left(\frac{T_0 (x + 2\pi)}{\pi}\right)$$

$$= f\left(\frac{T_0 x}{\pi} + 2T_0\right) = f\left(\frac{T_0 x}{\pi}\right)$$

$$= g(x)$$

Bsp.: $\quad \{ \; 1 \quad$ für $x \in [0, \pi[$

Bsp.:

$$f(x) = \begin{cases} 1 & \text{für } x \in [0, \pi[\\ -1 & \text{für } x \in [\pi, 2\pi[\end{cases}$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(t)\,dt = \frac{1}{\pi} \int_0^{\pi} 1\,dt + \frac{1}{\pi}\int_{\pi}^{2\pi} -1\,dt$$

$$= \frac{1}{\pi} \cdot \pi + \frac{1}{\pi} \cdot (-\pi) = 0$$

$n \geq 1$:

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(nt)\,dt$$

$$= \frac{1}{\pi} \int_0^{\pi} \cos(nt)\,dt + \frac{1}{\pi} \int_{\pi}^{2\pi} -\cos(nt)\,dt$$

$$= \frac{1}{\pi} \left[\frac{\sin(nt)}{n} \right]_0^{\pi} - \frac{1}{\pi} \left[\frac{\sin(nt)}{n} \right]_{\pi}^{2\pi}$$

$$= 0$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(nt)\,dt$$

$$= \frac{1}{\pi} \int_0^{\pi} \sin(nt)\,dt - \frac{1}{\pi} \int_{\pi}^{2\pi} \sin(nt)\,dt$$

$$= \frac{1}{\pi} \left[-\frac{\cos(nt)}{n} \right]_0^{\pi} - \frac{1}{\pi} \left[-\frac{\cos(nt)}{n} \right]_{\pi}^{2\pi}$$

$$= \frac{1}{\pi} \left(-\frac{\cos(n\pi)}{n} + \frac{1}{n} \right) - \frac{1}{\pi} \left(-\frac{1}{n} + \frac{\cos(n\pi)}{n} \right)$$

$$= \frac{1}{\pi}\left(\frac{2}{n} - \frac{2\cos(n\pi)}{n}\right)$$

$$= \frac{1}{\pi}\left(\frac{2}{n} - \frac{2(-1)^n}{n}\right) = \begin{cases} 0 & n \text{ gerade} \\ \frac{4}{n\pi} & n \text{ ungerade} \end{cases}$$

Fourierreihe

$$\mathcal{F}_f(x) = \sum_{n=0}^{\infty} \frac{4}{\pi(2n+1)} \sin((2n+1)x)$$

Bsp: $f(t) = t$ für $-\pi \leq t < \pi$

ungerade $\Rightarrow$ $a_n = 0$ für alle $n \geq 0$

$n \geq 1$:

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(nt)\,dt$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} t\sin(nt)\,dt$$

$$= \frac{1}{\pi}\cdot\left[-t\frac{\cos(nt)}{n}\right]_{-\pi}^{\pi} + \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{\cos(nt)}{n}\,dt$$

$$= \frac{1}{\pi}\left(\frac{-\pi\cos(n\pi)}{n} - \frac{\pi\cos(n(-\pi))}{n}\right) + \frac{1}{\pi}\left[\frac{\sin(nt)}{n^2}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\cancel{\pi}}\left(\frac{-\cancel{\pi}\cdot(-1)^n}{n} - \frac{\cancel{\pi}(-1)^n}{n}\right) + 0$$

$$= \frac{2\cdot(-1)^{n+1}}{n}$$

Bsp.:
$$f(t) = \begin{cases} 2 & \text{bis } -\pi \leq t < 0 \\ 1 & \text{bis } 0 \leq t < \pi \end{cases}$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t)\,dt = \frac{1}{\pi} \int_{-\pi}^{0} 2\,dt + \frac{1}{\pi} \int_{0}^{\pi} 1\,dt$$

$$= \frac{1}{\pi} \cdot 2 \cdot \pi + \frac{1}{\pi} \cdot \pi = 3$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(nt)\,dt = \frac{1}{\pi} \int_{-\pi}^{0} 2\cos(nt)\,dt + \frac{1}{\pi} \int_{0}^{\pi} \cos(nt)\,dt$$

$$= \frac{1}{\pi} \left[\frac{2\sin(nt)}{n} \right]_{-\pi}^{0} + \frac{1}{\pi} \left[\frac{\sin(nt)}{n} \right]_{0}^{\pi} = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(nt)\,dt = \frac{1}{\pi} \int_{-\pi}^{0} 2\sin(nt)\,dt + \frac{1}{\pi} \int_{0}^{\pi} \sin(nt)\,dt$$

$$= \frac{1}{\pi} \left[-\frac{2\cos(nt)}{n} \right]_{-\pi}^{0} + \frac{1}{\pi} \left[-\frac{\cos(nt)}{n} \right]_{0}^{\pi}$$

$$= \frac{1}{\pi} \left(-\frac{2}{n} + \frac{2\cos(n(-\pi))}{n} - \frac{\cos(n\pi)}{n} + \frac{1}{n} \right)$$

$$= \frac{1}{\pi} \left(-\frac{1}{n} + \frac{\cos(n\pi)}{n} \right) = \begin{cases} 0 & n \text{ gerade} \\ -\frac{2}{n\pi} & n \text{ ungerade} \end{cases}$$

$$F_f(x) = \frac{3}{2} + \sum_{n=0}^{\infty} -\frac{2}{(2n+1)\pi} \sin((2n+1)\cancel{x})$$