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$$a(X) = m(X) \cdot G(X) + \underline{\alpha^4 X + \alpha^6}$$

$$\underbrace{a(X) + \alpha^4 X + \alpha^6}_{\text{Code-Polynom}} = m(X) \cdot G(X)$$

Code-Polynom, gehört zu

$$c = a + (\alpha^6, \alpha^4, 0, 0, 0, 0)$$
$$= (\alpha, 1, \alpha, 1, \alpha+1, \alpha, \alpha)$$
$$+ (\alpha^2+1, \alpha^2+\alpha, 0, 0, 0, 0)$$
$$= (\alpha^2+\alpha+1, \alpha^2+\alpha+1, \alpha, 1, \alpha+1, \alpha, \alpha)$$

Beispiel:

$$a(X) = m(X) \cdot G(X) + s(X)$$
$$\alpha^5 \cdot X^6 = \tilde{m}(X) \cdot G(X) + s(X) \quad \Big] +$$

$$a(X) + \alpha^5 X^6 = (m(X) + \tilde{m}(X)) G(X) + \cancel{s(X) + s(X)} \; 0$$
$$= (m(X) + \tilde{m}(X)) G(X)$$

Codepolynom zu $c = a + (0, 0, 0, 0, 0, 0, \alpha^5)$

Beispiel: $\mathbb{F}_{13}$, $k = 3$

$$\mathcal{L}(k-1) = \mathcal{L}(2) = \{ \text{Polynome vom Grad} \le 2 \}$$
$$= \{ a + bX + cX^2 \mid a, b, c \in \mathbb{F}_{13} \}$$

$\vec{a}_1 = 1$

$\vec{a}_2 = X$

$$l(X) = a + bX + cX^2$$
$$= a \cdot \vec{a}_1 + b\,\vec{a}_2 + c\,\vec{a}_3$$

$\vec{a_2} = X$

$\vec{a_3} = X^2$

$= a \cdot \vec{a_1} + b\,\vec{a_2} + c\,\vec{a_3}$

$b_1 = 0,\ b_2 = 1,\ b_3 = 2,\ b_4 = 3,\ b_5 = 4$

$Ev: \mathcal{L}(2) \longrightarrow \mathbb{F}_{13}^5, \quad \ell \longmapsto (\ell(0), \ell(1), \ell(2), \ell(3), \ell(4))$

$\ell(X) = 1 \quad : \quad Ev(1) = (1, 1, 1, 1, 1)$

$\ell(X) = X \quad : \quad Ev(X) = (0, 1, 2, 3, 4)$

$\ell(X) = X^2 \quad : \quad Ev(X^2) = (0, 1, 4, 9, 3)$

Basis von $\mathrm{Im}(Ev)$

$Ev(3 + 2X + X^2) = 3 \cdot Ev(1) + 2 \cdot Ev(X) + Ev(X^2)$

$= 3 \cdot (1,1,1,1,1) + 2 \cdot (0,1,2,3,4) + 1 \cdot (0,1,4,9,3)$

$(3,3,3,3,3) + (0,2,4,6,8) + (0,1,4,9,3)$

$= (3, 6, 11, 5, 1)$

Beispiel: duale $[6,4]_{17}$-RS-Code zu $b_1 = 0, b_2 = 1, b_3 = 2, b_4 = 3, b_5 = 4, b_6 = 5$

hat Erzeugermatrix

$$G = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0^2 & 1^2 & 2^2 & 3^2 & 4^2 & 5^2 \\ 0^3 & 1^3 & 2^3 & 3^3 & 4^3 & 5^3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 4 & 9 & 16 & 8 \\ 0 & 1 & 8 & 10 & 13 & 6 \end{pmatrix}$$

$$\left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 1 & 4 & 9 & 16 & 8 & 0 \\ 0 & 1 & 8 & 10 & 13 & 6 & 0 \end{array}\right) \xrightarrow[\mathrm{IV}-\mathrm{II}]{\mathrm{III}-\mathrm{II}} \left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 2 & 6 & 12 & 3 & 0 \\ 0 & 0 & 6 & 7 & 9 & 1 & 0 \end{array}\right) \cdot 9$$

$$\longrightarrow \left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 1 & 3 & 6 & 10 & 0 \end{array}\right) \xrightarrow{\mathrm{IV}-6\,\mathrm{III}} \left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 1 & 3 & 6 & 10 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cccccc|c} 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 1 & 3 & 6 & 10 & 0 \\ 0 & 0 & 6 & 7 & 9 & 8 & 0 \end{array}\right) \xrightarrow{III - 6\,II} \left(\begin{array}{cccccc|c} 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 1 & 3 & 6 & 10 & 0 \\ 0 & 0 & 0 & 6 & 7 & 9 & 0 \end{array}\right) \cdot 3$$

$$\longrightarrow \left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 1 & 3 & 6 & 10 & 0 \\ 0 & 0 & 0 & 1 & 4 & 10 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 16 & 13 & 0 \\ 0 & 1 & 0 & 0 & 4 & 15 & 0 \\ 0 & 0 & 1 & 0 & 11 & 14 & 0 \\ 0 & 0 & 0 & 1 & 4 & 10 & 0 \end{array}\right)$$

x_5 x_6 frei

$$\begin{aligned} x_1 \qquad\qquad + 16 x_5 + 13 x_6 &= 0 \\ x_2 \qquad + 4 x_5 + 15 x_6 &= 0 \\ x_3 \quad + 11 x_5 + 14 x_6 &= 0 \\ x_4 + 4 x_5 + 10 x_6 &= 0 \end{aligned}$$

① h_1 : $x_5 = 1$, $x_6 = 0 \Rightarrow x_1 = 1$, $x_2 = 13$, $x_3 = 6$, $x_4 = 13$

$$h_1 = (1, 13, 6, 13, 1, 0)$$

② h_2 : $x_5 = 0$, $x_6 = 1$:

$$h_2 = (4, 2, 3, 7, 0, 1)$$

$$H = \begin{pmatrix} 1 & 13 & 6 & 13 & 1 & 0 \\ 4 & 2 & 3 & 7 & 0 & 1 \end{pmatrix}$$

Übung

$$G = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ \alpha & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 \\ \alpha^2 & \alpha^4 & \alpha^6 & \alpha & \alpha^3 & \alpha^5 \\ \alpha^3 & \alpha^6 & \alpha^2 & \alpha^5 & \alpha & \alpha^4 \end{pmatrix}$$

$$\left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ \alpha & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 & 0 \\ \alpha^2 & \alpha^4 & \alpha^6 & \alpha & \alpha^3 & \alpha^5 & 0 \\ \alpha^3 & \alpha^6 & \alpha^2 & \alpha^5 & \alpha & \alpha^4 & 0 \end{array}\right) \begin{array}{l} \\ \cdot \alpha^6 \\ \alpha^5 \\ \alpha^4 \end{array} \longrightarrow \left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & 0 \\ 1 & \alpha^2 & \alpha^4 & \alpha^6 & \alpha & \alpha^3 & 0 \\ 1 & \alpha^3 & \alpha^6 & \alpha^2 & \alpha^5 & \alpha & 0 \end{array}\right)$$

$II - I$ $\left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{array}\right)$ $\qquad$ $\left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{array}\right)$

$$\begin{array}{l} \\ \mathrm{II}-\mathrm{I} \\ \mathrm{III}-\mathrm{I} \\ \mathrm{IV}-\mathrm{I} \end{array} \longrightarrow \left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & \alpha^3 & \alpha^6 & \alpha & \alpha^5 & \alpha^4 & 0 \\ 0 & \alpha^5 & \alpha^5 & \alpha^2 & \alpha^3 & \alpha & 0 \\ 0 & \alpha & \alpha^2 & \alpha^6 & \alpha^4 & \alpha^3 & 0 \end{array}\right) \begin{array}{l} \\ \cdot\alpha^4 \\ \alpha \\ \alpha^6 \end{array} \to \dots \to \left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & \alpha^3 & \alpha^5 & \alpha^2 & \alpha & 0 \\ 0 & 0 & 1 & \alpha^5 & \alpha^4 & 1 & 0 \\ 0 & 0 & 0 & 1 & \alpha^2 & 1 & 0 \end{array}\right)$$

$$\to \dots \to \left(\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & \alpha^6 & \alpha & 0 \\ 0 & 1 & 0 & 0 & \alpha^5 & \alpha^2 & 0 \\ 0 & 0 & 1 & 0 & \alpha^5 & \alpha^4 & 0 \\ 0 & 0 & 0 & 1 & \alpha^2 & 1 & 0 \end{array}\right)$$

$$h_1 = (\alpha^6, \alpha^5, \alpha^5, \alpha^2, 1, 0)$$
$$h_2 = (\alpha, \alpha^2, \alpha^4, 1, 0, 1)$$

$$H = \begin{pmatrix} \alpha^6 & \alpha^5 & \alpha^5 & \alpha^2 & 1 & 0 \\ \alpha & \alpha^2 & \alpha^4 & 1 & 0 & 1 \end{pmatrix}$$

Bsp.: $[6,2]_{17}$ - RS - Code zu $0, 1, 2, 3, 4, 5$
hat Paritätsprüfmatrix

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 4 & 9 & 16 & 8 \\ 0 & 1 & 8 & 10 & 13 & 6 \end{pmatrix}$$

(Erzeuge dualen $[6,4]_{17}$ RS - Code zu $0,1,2,3,4,5$)

und Erzeugermatrix

$$G = \begin{pmatrix} 1 & 13 & 6 & 13 & 1 & 0 \\ 4 & 2 & 3 & 7 & 0 & 1 \end{pmatrix}$$

Übung: $[6,2]_{13}$ - RS - Code zu $0, 2, 4, 6, 8, 10$:

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 & 8 & 10 \\ 0 & 4 & 3 & 10 & 12 & 9 \\ 0 & 8 & 12 & 8 & 5 & 12 \end{pmatrix}$$

$$\left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & 4 & 6 & 8 & 10 & 0 \\ 0 & 4 & 3 & 10 & 12 & 9 & 0 \\ 0 & 8 & 12 & 8 & 5 & 12 & 0 \end{array}\right) \xrightarrow[\substack{III - 4 II' \\ IV - 8 II'}]{II : 2} \left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 8 & 11 & 9 & 2 & 0 \\ 0 & 0 & 9 & 10 & 12 & 11 & 0 \end{array}\right) \cdot 5$$

$$\xrightarrow[IV - 9 III']{} \left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 1 & 3 & 6 & 11 & 0 \\ 0 & 0 & 0 & 9 & 10 & 12 & 0 \end{array}\right) \cdot 3 \longrightarrow \left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 1 & 3 & 6 & 10 & 0 \\ 0 & 0 & 0 & 1 & 4 & 10 & 0 \end{array}\right)$$

$$\longrightarrow \dots \longrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 12 & 9 \\ 0 & 1 & 0 & 0 & 4 & 2 \\ 0 & 0 & 1 & 0 & 7 & 6 \\ 0 & 0 & 0 & 1 & 4 & 10 \end{pmatrix}$$

$g_1 = (1, 9, 6, 9, 1, 0)$

$g_2 = (4, 11, 7, 3, 0, 1)$

$$G = \begin{pmatrix} 1 & 9 & 6 & 9 & 1 & 0 \\ 4 & 11 & 7 & 3 & 0 & 1 \end{pmatrix}$$

Bsp.: $[6,2]_{13}$-RS-Code zu $0, 1, 2, 3, 4, 5$

$g_1 = (1, 13, 6, 13, 1, 0)$

$g_2 = (4, 2, 3, 7, 0, 1)$

$m = (m_1, m_2)$ wird codiert zu $m_1 g_1 + m_2 g_2$

$m = (3,4) \longrightarrow 3 g_1 + 4 g_2 = 3 \cdot (1, 13, 6, 13, 1, 0)$

$+ 4 \cdot (4, 2, 3, 7, 0, 1)$

$= (3, 5, 1, 5, 3, 0)$

$$= (3, 5, 1, 5, 3, 0)$$
$$+ (16, 8, 12, 11, 0, 4)$$
$$= (2, 13, 13, 16, \underline{3, 4})$$

Übung: $[6,4]_{13}$-Code zu $0, 1, 2, 3, 4, 5$

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

$$\left(\begin{array}{cccccc|c} 1 & 0 & 12 & 11 & 10 & 9 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 0 \end{array}\right)$$

$$g_1 = (1, 11, 1, 0, 0, 0)$$
$$g_2 = (2, 10, 0, 1, 0, 0)$$
$$g_3 = (3, 9, 0, 0, 1, 0)$$
$$g_4 = (4, 8, 0, 0, 0, 1)$$

$$(2, 6, 9, 7) \mapsto 2g_1 + 6g_2 + 9g_3 + 7g_4 = (2, 9, 2, 0, 0, 0)$$
$$+ (12, 8, 0, 6, 0, 0)$$
$$+ (1, 3, 0, 0, 9, 0)$$
$$+ (2, 4, 0, 0, 0, 7)$$
$$= (4, 11, 2, 6, 9, 7)$$

Übung:

$$m = (\alpha^3, \alpha^5) \longmapsto \alpha^3 \cdot (\alpha^6, \alpha^5, \alpha^5, \alpha^2, 1, 0)$$
$$+ \alpha^5 (\alpha, \alpha^2, \alpha^4, 1, 0, 1)$$

$$+ \alpha^5 (\alpha, \alpha^2, \alpha^4, 1, 0, 1)$$

$$= (\alpha^2, \alpha, \alpha, \alpha^5, \alpha^3, 0)$$

$$+ (\alpha^6, 1, \alpha^2, \alpha^5, 0, \alpha^5)$$

$$= (1, \alpha+1, \alpha^2+\alpha, 0, \alpha+1, \alpha^2+\alpha+1)$$