

Klausur: Grundlagen der Informatik

(Lehrveranstaltung)

Kurs: ...TINF16AIA.....

Dozent: Prof. Dr. Bernd Schwinn

Studierende(r)

(Matr.Nr.):

Semester: 1.

Hilfsmittel: keine

Dauer: 100 Min.

Bewertung: Erreichte Punktzahl:

Maximale
Punktzahl: 86

Note:

Signum:

Anmerkungen:

Aufgabennr.:		maximale Punkte	erreichte Punkte	Bemerkungen
1		6		
2		5		
3		6		
4		10		
5		8		
6		8		
7		6		
8		8		
9		8		
10		6		
11		9		
12		6		
Summe		86		

Task 1:

(6 Points)

Express the following phrases by logical formulae:

- The half of an even natural number is a natural number as well
- An odd natural number has an even predecessor and an even successor
- The successor of a natural number is odd or even
- A square number is a natural number that is the product of a natural number with itself

Task 2:

(5 Points)

Presume a signature with the sort symbols 's1' and 's2',

a 3-ary function $f(s1, s2, s1) \rightarrow s2$

a 2-ary function $g(s2, s1) \rightarrow s1$

a 2-ary predicate $P(s1, s2)$

a 3-ary predicate $Q(s2, s1, s2)$

and the variables x, y, z of sort $s1$ and a, b, c of sort $s2$.

Which of the following expressions are formulae in predicate logic? If not, give the reason why!

a) $Q(f(z, f(g(a, g(a, x))), b, g(b, g(c, x))), g(a, z), c) \vee P(g(f(g(c, z), a, y), x), f(g(a, y), b, z))$

b) $Q(f(g(a, x), c, z), g(g(f(z, b, x), z), y), c) \vee P(g(f(y, f(z, c, z), x), f(f(g(b, y), c, z), f(g(a, y), a, y), z)))$

c) $Q(f(g(a, g(a, x)), a, x), g(f(g(c, g(c, z))), f(z, c, f(x, f(z, a, z), z)), x), y), b) \rightarrow P(g(f(y, b, x), y), b)$

d) $P(z, f(g(f(z, f(x, c, z), g(f(y, c, x), z))), g(f(x, b, y), z)), f(g(b, y), a, z), g(f(g(a, z), f(x, f(y, a, g(b, x)), y), z), x)))$

e) $Q(f(y, a, g(f(x, b, z), y)), g(f(y, f(y, a, z), x), y), f(z, c, g(b, y))) \leftrightarrow P(g(f(y, b, x), z), f(z, f(z, a, z), y)))$

Task 3:

(6 Points)

Presume the following sets of formulae:

$$X = \{R \vee (S \rightarrow Q), \neg R \leftrightarrow P \vee \neg S, (\neg Q \vee S) \wedge \neg R\}$$

$$Y = \{(\neg R \wedge S) \vee Q \rightarrow P, \neg\neg Q \rightarrow \neg S, (R \rightarrow \neg P) \vee Q\}$$

$$Z = \{Q \wedge \neg S \rightarrow P, \neg R \vee (\neg S \wedge \neg P \wedge Q), \neg P \vee S \rightarrow Q\}$$

For each set determine all the replacements that makes it true. State all logical conclusions among the sets of formulae that are possible?

Task 4:

(3 + 3 + 4 Points)

Transform the following formulae into conjunctive normal form and notate **all** resulting Gentzen-formulae.

a) $R \leftrightarrow S \vee \neg Q \wedge P$

b) $S \vee \neg(P \leftrightarrow R \wedge \neg Q)$

c) $S \rightarrow \neg Q \vee (R \leftrightarrow \neg P) \wedge Q$

Task 5:

(3 + 5 Points)

Presume the following formulae. First transform them into Prenex normal form and scolemise them finally.

a) $\exists y P(x,y) \vee \forall x (\neg R(x,y,x) \rightarrow \neg \forall z Q(x,y,z)) \wedge \exists x \forall y R(x,y,z)$

b) $\neg \exists x (\forall y P(x,z,y) \leftrightarrow \exists y Q(x,y,z)) \wedge \forall z R(z,y,x)$

Task 6:

(8 Points)

Prove the correctness of the following specification

```
{ X>Y ; Y>=0; X, Y of type integer }  
  I := 0;  
  Z:=X;  
  while I<Y do  
    begin I := I + 1;  
          Z := Z + 1  
    end  
{ Z=X+Y ; X>Y ; Y>=0 ; X, Y of type integer }
```

Task 7:

(6 Points)

Presume the following statements with postcondition

```
    if  $X < Y$  then  $X := Y - X$ ;  
    if  $X < 0$  then  $Y := X - Y$   
        else  $Y := X + Y$   
{  $Y > X$ ;  $X, Y$  of type integer }
```

Determine the weakest precondition.

Task 8:

(8 Points)

Are the following pairs of literals unifiable? If yes, give the bindings resulting from unification. If not, give the reason why.

a) $p1([X|Y][X,Y],[Z|X])$ $p1([A|[B|B]],C)$ b) $p2([X|R][Y],[Y|X])$ $p2([Y|A],[Y])$ c) $p3([X|[Y][]],[Y|X])$ $p3([A|R],[R|A])$ d) $p4([X,[Y][[X]]],[X])$ $p4([A|[B,[A]]],A)$ e) $p5(X,[Y|[Y|X]])$ $p5([A],[B|[A|B]])$

Task 9:

(5 + 3 Points)

a) Define a Prolog predicate that is analogous to the Lisp function `mystery` given below.

```
(defun mystery (L)
  (cond ((listp L) (cond ((null L) 0)
                        ((< (car L) 0) (- (mystery (cdr L)) 1))
                        (T (+ (mystery (cdr L)) 1))))
    (T nil)))
```

b) Determine for the Lisp function given in a) the number of executed test functions if the function is activated with n ($n > 0$) positive numbers. Start the cost calculation with a recursion equation and solve it finally.

Task 10:

(6 Points)

Define Prolog clauses for a predicate `modify` that deletes all '0's within a list of numbers and that replaces each negative number by -1.

Task 11: (2 + 2 + 2 + 3 Points)
Assume L to be a list containing at least two (non-empty) sublists (with numbers). Give a Lisp expression that

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Assume L to be a list containing at least two (non-empty) sublists (with numbers). Give a Lisp expression that

- a) exchanges the first two sublists if the first number of the second sublist is negative
- b) calculates the sum of the first number of the first and the first number of the second sublist and inserts this into a new sublist located behind the first two sublists
- c) sorts the first as well as the second sublist and joins them zipperwise in a newly created first sublist (followed by the 3rd, 4th, ... sublist)
- d) calculates the sum of the last number of the first sublist and the last number of the second sublist and inserts this sum to the beginning of the second sublist

Task 12:

(6 Points)

Define a Lisp-function `modify` (analogous to Task 10), that deletes all '0's within a list of numbers and replaces each negative number by -1.

Example: `(modify '(1 -8 0 -3 3))` returns `(1 -1 -1 3)`

