Übungsblatt 8

(41)

$[6,2]_{17}$-RSC bzgl. $B = \{1,3,4,6,7,8\}$ mit $d(C) = 5$, $t = 2$

Notiz: Vielfache von 17: 17, 34, 51, 68, 85, 102, 119, 136, 153, 170, 187, 204, 221, 238, 255, 272, 289

und

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 & 7 & 8 \\ 1 & 9 & 16 & 2 & 15 & 13 \\ 1 & 10 & 13 & 12 & 3 & 2 \end{pmatrix}$$

$a_1 = (7, 10, 11, 7, 1, 13)$

$\leadsto$ Syndrome: $[a, x^0] = 7 + 10 + 11 + 7 + 1 + 13 = \underline{\underline{15}}$

$[a, x^1] = 7 + 3 \cdot 10 + 4 \cdot 11 + 6 \cdot 7 + 7 \cdot 1 + 8 \cdot 13 = 3 + 10 + 8 + 7 + 2 = \underline{\underline{13}}$

$[a, x^2] = 7 + 9 \cdot 10 + 16 \cdot 11 + 2 \cdot 7 + 15 \cdot 1 + 13 \cdot 13 = 7 + 5 + 6 + 14 + 15 + 16 = \underline{\underline{12}}$

$[a, x^3] = 7 + 10 \cdot 10 + 13 \cdot 11 + 12 \cdot 7 + 3 \cdot 1 + 13 \cdot 2 = 7 + 15 + 7 + 16 + 3 + 9 = \underline{\underline{6}}$

$\Rightarrow$ a_1 wurde nicht korrekt übertragen:

$\leadsto$ LGS um $L(x)$ zu finden:

$$\left(\begin{array}{ccc|c} 15 & 13 & 12 & 0 \\ 13 & 12 & 6 & 0 \end{array}\right) \begin{array}{l} \cdot 8 \\ \cdot 4 \end{array} \longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & 11 & 0 \\ 1 & 14 & 7 & 0 \end{array}\right) \begin{array}{l} \\ -\mathrm{I} \end{array} \longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & 11 & 0 \\ 0 & 12 & 13 & 0 \end{array}\right) \begin{array}{l} -2 \cdot 10 \cdot \mathrm{II} \\ \cdot 10 \end{array} \longrightarrow$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 6 & 0 \\ 0 & 1 & 11 & 0 \end{array}\right) \Rightarrow \text{Lösung } l = (11, 6, 1)$$

$\Rightarrow L(x) = 11 + 6x + x^2$

$\leadsto$ Potenzielle Fehlerstellen $N(L)$

$L(1) = 11 + 6 + 1 = 1$, $L(3) = 4$, $L(4) = 0$, $L(6) = 16$, $L(7) = 0$,
$L(8) = 4$

$\Rightarrow N(L) = \{3, 5\}$

$[a, x^0] = 9+1+15+11+8+9+7 = \underline{9}$

$[a, x^1] = 9+3\cdot 1+5\cdot 15+7\cdot 11+9\cdot 8+11\cdot 9+13\cdot 7 = 9+3+7+9+4+14+6 = \underline{1}$

$[a, x^2] = 9+9\cdot 1+8\cdot 15+15\cdot 11+13\cdot 8+2\cdot 9+16\cdot 7 = 9+9+1+12+2+1+10 = \underline{10}$

$[a, x^3] = 9+10\cdot 1+6\cdot 15+3\cdot 11+15\cdot 8+5\cdot 9+4\cdot 7 = 9+10+5+16+1+11+11 = \underline{12}$

$\Rightarrow$ fehlerhafte Übertragung

$\rightsquigarrow$ suche $L(x)$ per LGS:

$$\left(\begin{array}{ccc|c} 9 & 1 & 10 & 0 \\ 1 & 10 & 12 & 0 \end{array}\right) \xrightarrow[-2\mathrm{I}]{\cdot 2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 8 & 9 & 0 \end{array}\right) \xrightarrow[\cdot 15]{-2\cdot 15\,\mathrm{II}} \left(\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & 16 & 0 \end{array}\right)$$

$\Rightarrow L(x) = 12 + x + x^2$

$\rightsquigarrow$ bestimme pot. Fehlerstellen:

$L(1) = 14,\ L(3) = 7,\ L(5) = 8,\ L(7) = 0,\ L(9) = 0,$

$L(11) = 8,\ L(13) = 7$

$\Rightarrow N(L) = \{4, 5\}$

$\rightsquigarrow$ bestimme e_4, e_5 per LGS:

$$\left(\begin{array}{cc|c} 1 & 1 & 9 \\ 7 & 9 & 1 \\ 15 & 13 & 10 \\ 3 & 15 & 12 \end{array}\right) \begin{array}{l} \\ -7\mathrm{I} \\ -15\mathrm{I} \\ -3\mathrm{I} \end{array} \longrightarrow \left(\begin{array}{cc|c} 1 & 1 & 9 \\ 0 & 2 & 6 \\ 0 & 15 & 11 \\ 0 & 12 & 2 \end{array}\right) \begin{array}{l} -9\mathrm{II} \\ \cdot 9 \\ -15\cdot 9\mathrm{II} \\ -12\cdot 9\mathrm{II} \end{array} \longrightarrow \left(\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$\Rightarrow e_4 = 6,\ e_5 = 3$

$\rightsquigarrow$ korrigierte Codewort c:

$$c = a - e = (9,1,15,11,8,9,7) - (0,0,0,6,3,0,0)$$

$$c = (9,1,15,5,5,9,7)$$

$\Rightarrow$ Die originale Nachricht m:

$$\underline{m = (5,9,7)}$$

A3) $\mathbb{F}_8$ mit $\alpha^3=\alpha+1$; $[6,4]_{\mathbb{F}_8}$ RSC bzgl.: $\mathcal{B}=\{1,\alpha^2,\alpha^4,\alpha^6,\alpha,\alpha^3\}$

mit $d(C)=3$, $t=1$ und PPM

$$H=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha^4 & \alpha^6 & \alpha & \alpha^3 \end{pmatrix}$$

Empfangenes Wort $a_1=(1,\alpha,\alpha,\alpha^4,\alpha^4,\alpha^3)$

$\rightsquigarrow$ Syndrome:

$[a_1,x^0]=1+\alpha+\alpha+\alpha^4+\alpha^4+\alpha^3=\underline{\alpha}$

$[a_1,x^1]=1+\alpha\cdot\alpha^2+\alpha\cdot\alpha^4+\alpha^4\cdot\alpha^6+\alpha^4\cdot\alpha+\alpha^3\cdot\alpha^3$

$=1+\alpha^3+\alpha^5+\alpha^3+\alpha^5+\alpha^6=\underline{\alpha^2}$

$\rightsquigarrow L(x)$ per LGS:

$(\alpha\ \alpha^2\,|\,0)\longrightarrow(1\ \alpha\,|\,0)\Rightarrow l=(\alpha,1)$

$\Rightarrow L(x)=\alpha+x$

$\rightsquigarrow$ bestimme pot. Fehlerstellen:

$L(1)=\alpha^3$, $L(\alpha^2)=\alpha^4$, $L(\alpha^4)=\alpha^2$, $L(\alpha^6)=\alpha^5$, $L(\alpha)=0$

$L(\alpha^3)=1$

$\Rightarrow N(L)=\{5\}$ $\rightsquigarrow$ bestimme e_5 per LGS:

$$\left(\begin{array}{c|c} 1 & \alpha \\ \alpha & \alpha^2 \end{array}\right)\rightarrow\left(\begin{array}{c|c} 1 & \alpha \\ 0 & 0 \end{array}\right)\Rightarrow e_5=\alpha$$

$\rightsquigarrow$ Codewort korrigieren:

$c=a-e=(1,\alpha,\alpha,\alpha^4,\alpha^4,\alpha^3)-(0,0,0,0,\alpha,0)$

$c=(1,\alpha,\alpha,\alpha^4,\alpha^2,\alpha^3)$

$\Rightarrow$ originale Nachricht $\underline{m=(\alpha,\alpha^2+\alpha,\alpha^2,\alpha+1)}$

Empfangene Nachricht $a_2=(\alpha^2,\alpha^2,1,\alpha^3,\alpha^4,1)$

$\rightsquigarrow$ Syndrome:

$[a_2,x^0]=\alpha^2+\alpha^2+1+\alpha^3+\alpha^4+1=\underline{\underline{\alpha^6}}$

$[a_2,x^1]=\alpha^2+\alpha^4+\alpha^4+\alpha^2+\alpha^5+\alpha^3=\underline{\underline{\alpha^2}}$

$\rightsquigarrow$ bestimme $L(x)$ per LGS:

$(\alpha^6\ \alpha^2\,|\,0)\longrightarrow(1\ \alpha^3\,|\,0)\Rightarrow L(x)=\alpha^3+x$

⇝ bestimme pot. Fehlerstellen:

$L(1) = \alpha,\ L(\alpha^2) = \alpha^5,\ L(\alpha^4) = \alpha^6,\ L(\alpha^6) = \alpha^4,$

$L(\alpha) = 1,\ L(\alpha^3) = 0 \quad \Rightarrow \quad N(L) = \{6\}$

⇝ e_6 per LGS bestimmen:

$$\left(\begin{array}{c|c} 1 & \alpha^6 \\ \alpha^3 & \alpha^2 \end{array}\right) \xrightarrow{-\alpha^3 I} \left(\begin{array}{c|c} 1 & \alpha^6 \\ 0 & 0 \end{array}\right) \Rightarrow e_6 = \alpha^6$$

⇝ Codewort korrigieren:

$c = a - e = (\alpha^2, \alpha^2, 1, \alpha^3, \alpha^4, 1) - (0,0,0,0,0,\alpha^6)$

$c = (\alpha^2, \alpha^2, 1, \alpha^3, \alpha^4, \alpha^2)$

⇒ originale Nachricht $\underline{\underline{m = (1, \alpha+1, \alpha^2+\alpha, \alpha^2)}}$

A4

$[6,2]_8$-RSC bzgl.: $\mathcal{B} = \{1, \alpha^6, \alpha^5, \alpha^4, \alpha^3, \alpha^2\}$

mit $d(C) = 5$, $t = 2$ und PPM

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \alpha^6 & \alpha^5 & \alpha^4 & \alpha^3 & \alpha^2 \\ 1 & \alpha^5 & \alpha^3 & \alpha & \alpha^6 & \alpha^4 \\ 1 & \alpha^4 & \alpha & \alpha^5 & \alpha^2 & \alpha^6 \end{pmatrix},$$

empfanges Wort

$a = (\alpha, \alpha^2, \alpha, \alpha^5, \alpha^5, \alpha^2)$

⇝ Syndrome:

$[a, x^0] = \alpha + \alpha^2 + \alpha + \alpha^5 + \alpha^5 + \alpha^2 = \underline{\underline{0}}$

$[a, x^1] = \alpha + \alpha + \alpha^6 + \alpha^2 + \alpha + \alpha^4 = \underline{\underline{\alpha^6}}$

$[a, x^2] = \alpha + 1 + \alpha^4 + \alpha^6 + \alpha^4 + \alpha^6 = \underline{\underline{\alpha^3}}$

$[a, x^3] = \alpha + \alpha^6 + \alpha^2 + \alpha^3 + 1 + \alpha = \underline{\underline{\alpha^3}}$

⇝ $L(x)$ per LGS bestimmen:

$$\left(\begin{array}{ccc|c} 0 & \alpha^6 & \alpha^3 & 0 \\ \alpha^6 & \alpha^3 & \alpha^3 & 0 \end{array}\right) \begin{array}{c} \cdot\alpha \\ \cdot\alpha \end{array} \rightarrow \left(\begin{array}{ccc|c} 0 & 1 & \alpha^4 & 0 \\ 1 & \alpha^4 & \alpha^4 & 0 \end{array}\right) \xrightarrow{-\alpha^4 I} \left(\begin{array}{ccc|c} 0 & 1 & \alpha^4 & 0 \\ 1 & 0 & \alpha^2 & 0 \end{array}\right)$$

⇒ Lösung $L = (\alpha^2, \alpha^4, 1)$ bzw. $L(x) = \alpha^2 + \alpha^4 x + x^2$

⇝ pot. Fehlerstellen bestimmen:

$L(1) = \alpha^3,\ L(\alpha^6) = 0,\ L(\alpha^5) = \alpha^3,\ L(\alpha^4) = \alpha^2,$

$L(\alpha^3) = 0,\ L(\alpha^2) = \alpha^3$

⇒ $N(L) = \{2, 5\}$

$\rightsquigarrow$ e_2, e_5 bestimmen per LGS:

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ \alpha^6 & \alpha^3 & \alpha^6 \\ \alpha^5 & \alpha^6 & \alpha^3 \\ \alpha^4 & \alpha^2 & \alpha^3 \end{array}\right) \begin{array}{c} \\ -\alpha^6 \mathrm{I} \\ -\alpha^5 \mathrm{I} \\ -\alpha^4 \mathrm{I} \end{array} \longrightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & \alpha^4 & \alpha^6 \\ 0 & \alpha & \alpha^3 \\ 0 & \alpha & \alpha^3 \end{array}\right) \begin{array}{c} -\alpha^3 \mathrm{II} \\ \cdot \alpha^3 \\ -\alpha^4 \mathrm{II} \\ -\alpha^4 \mathrm{II} \end{array} \longrightarrow \left(\begin{array}{cc|c} 1 & 0 & \alpha^2 \\ 0 & 1 & \alpha^2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{array}{l} e_2 = \alpha^2, \\ e_5 = \alpha^2 \end{array}$$

$\rightsquigarrow$ korrigiertes Codewort:

$$c = a - e = (\alpha, \alpha^2, \alpha, \alpha^5, \alpha^5, \alpha^2) - (0, \alpha^2, 0, 0, \alpha^2, 0)$$

$$c = (\alpha, 0, \alpha, \alpha^5, \alpha^3, \alpha^2)$$

$\rightsquigarrow$ originale Nachricht $\underline{\underline{m = (\alpha^3, \alpha^2)}}$