Duale Hochschule Baden-Württemberg Stuttgart

Klausur: Theoretische Informatik I

(Lehrveranstaltung)

Kurs: …TINF19AIA……………………….

Dozent: Prof. Dr. Bernd Schwinn

Studierende(r)
(Matr.Nr.): ……………………………………………………….

Semester: 1.

Hilfsmittel: keine

Dauer: 100 Min.

Bewertung: Erreichte Punktzahl:

Maximale Punktzahl: 85

Note: …………….

Signum: …………….

Anmerkungen:

Aufgabennr.:		maximale Punkte	erreichte Punkte	Bemerkungen
1		6		
2		5		
3		6		
4		10		
5		8		
6		8		
7		6		
8		8		
9		8		
10		6		
11		8		
12		6		
Summe		**85**		

Task 1: (6 Points)

Express the following phrases by logical formulae:

- The predecessor of an odd natural number is an even natural number
- The root of a natural square number is a natural number or it is negative
- The sum of the negative root and the positive root of a natural square number is zero
- The sum of the predecessor and the successor of a natural square numbers is even

Task 2: (5 Points)

Presume a signature with the sort symbols ‘s1’ and ‘s2’,
a 3-ary function f(s1,s2,s1) -> s2
a 3-ary function g(s2,s1) -> s1
a 2-ary predicate P(s1,s2)
a 3-ary predicate Q(s2,s1,s2)
, variables x, y and z of sort s1 as well as a, b and c of sort s2.
Which of the following expressions are formulae in predicate logic? If not, give the reason why!

a) Q(b, f(z, f(g(a, g(a, x)), b, g(b, g(c, x))), g(a, z))) $\vee$ P(g(b, f(g(c, z), a, y)), f(g(a, y), b, z))

b) ¬Q(f(g(a, x), c, z), g(g(f(z, b, x), z), y), c) $\vee$ P(g(f(y, f(z, c, z), x), f(f(g(b, y), c, z), f(g(a, y), a, y), z)))

c) Q(f(g(a, g(a, x)), a, x), g(f(g(c, g(c, z)), f(z, c, f(x, f(z, a, z), z)), x), a)) $\rightarrow$ P(g(f(y, b, x), y), b)

d) ¬Q(f(g(f(z, f(x, c, z), g(f(y, c, x), z)), g(f(x, b, y), z)), f(g(b, y), a, z), g(f(g(a, z), f(x, f(y, a, g(b, x)), y), z), x)), a)

e) Q(f(y, a, g(f(x, b, z), y)), g(f(y, f(y, a, z), x), y), f(z, c, g(b, y))) $\leftrightarrow$ P(g(f(y, b, x), z), f(z, f(z, a, z), y))

Task 3: (6 Points)

Presume the following sets of formulae:

$X = \{ R \wedge (S \rightarrow \neg Q), \neg Q \vee P, P \vee \neg S \leftrightarrow \neg R \}$

$Y = \{ \neg(\neg R \vee S) \vee P, \neg(Q \rightarrow \neg S), R \rightarrow \neg P \vee Q\}$

$Z = \{ (\neg Q \wedge S) \rightarrow P, (R \vee \neg S) \wedge (P \vee \neg R), \neg P \vee S \rightarrow Q\}$

For each set determine all the replacements that makes it true. State all possible logical conclusions among the sets of formulae!

Task 4:

(3 + 3 + 4 Points)

Transform the following formulae into conjunctive normal form and notate **all** resulting Gentzen-formulae.

a) $(R \rightarrow S) \vee \neg Q \rightarrow P$

b) $\neg(S \leftrightarrow \neg P) \vee (R \wedge \neg Q)$

c) $S \wedge (\neg Q \rightarrow R) \leftrightarrow \neg P \vee Q$

Task 5:

(3 + 5 Points)

Presume the following formulae. First transform them into Prenex normal form and scolemise them finally.

a) $\forall x\, (\neg P(x,y) \vee \exists y R(x,y,y)) \rightarrow \neg(\exists z\, \forall x\, Q(x,y,z) \wedge \forall y\, R(x,y,x))$

b) $\neg \exists y\, (\forall x\, P(x,z,y) \leftrightarrow \exists z\, Q(y,z,x) \wedge \forall x\, R(x,y,x))$

Task 6:

(8 Points)

Prove the correctness of the following specification

```
{ X>Y ; Y>=0; X, Y of type integer }
        I := Y;
        Z:=0;
        repeat
                Z := Z + 2;
                I := I + 1
        until I = X
{ Z=2*(X-Y) ; X>Y ; Y>=0 ; X, Y of type integer }
```

Task 7: (6 Points)

Presume the following statements with postcondition

```
Y := Y - X;
if X<0 then
    if Y<0 then Y := X * Y
        else Y := Y - X
{ Y>0; X, Y of type integer }
```

Determine the weakest precondition.

Task 8: (8 Points)

Are the following pairs of literals unifiable? If yes, give the bindings resulting from unification. If not, give the reason why.

a) p1([X|[Y|Y]],[]) p1([A,A],A)

b) p2([Y|Y],[Y]) p2([A,R],[R])

c) p3([[]|[X|Y]],Y) p3([A,Y],Y)

d) p4([[X]|[X,Y]],Y) p4([A,[B|C]],[])

e) p5(X,Y,[X|[Y]]) p5(A,[A],[A,A])

Task 9: (5 + 3 Points)

Define a Prolog program that is analogous to the Lisp function mystery given below.

```
(defun  mystery (L)
        (cond  ((listp L)       (cond  ((null L)        0)
                                       ((< (car L) 0)  (- (mystery (cdr L)) (car L)))
                                       ((> (car L) 0)  (+ (mystery (cdr L)) (car L)))
                                       (T (mystery (cdr L))))
               (T       nil)))
```

b) Determine for the Lisp-Funktion given in a) the number of executed test operations, if the function is activated with a list containing n (n>0) positive numbers. Start the cost calculation with a recursion equation and solve it finally.

Task 10: (6 Points)

Define Prolog clauses for a predicate **onlypos** that deletes within a list of integer numbers all 0`s and replaces each negative number by its doubled absolute value.

Task 11: (1 + 2 + 2 + 3 Points)

Presume L to be a list containing at least two (non-empty) sublists of (at least two) numbers. Give a Lisp expression that

a) joins zipperwise the first two sublists into the first sublist and inserts the result to the end of L

b) reverses and exchanges the first two sublists if the list contains more than two sublists

c) inserts the length of the two first sublists to the end of the according sublist

d) inserts the sum of the first element of the first sublist and the first element of the second sublist to the beginning of the third sublist (if no third sublist exists then a new 3rd sublist with the sum is generated)

Task 12: (6 Points)

Define a Lisp function **onlypos** (analogous to Task 10) that deletes within a list of integer numbers all 0`s and replaces each negative number by its doubled absolute value

Example: (onlypos ‘(1 -8 0 3 0 -3)) returns (1 16 3 6)