

1. $O(nm \log(nm))$

2. The matching is stable if there is no any pair (s, c) , such that

- student s is not enrolled to college c
- student s prefers college c than s 's current admission state (we could consider unmatched to be of 0 evaluation value)
- either c is capable to enroll more student and s can increase c 's satisfaction value, or c prefers s than some other student s' that has been enrolled to c

3. Pf. (by contradiction) (We can consider unenrollment to be a college evaluated by every student with value 0, as well as well students evaluated by every college with value 0)

- Suppose (s, c) is a unstable pair: s is enrolled in c' , s prefers college c (i.e. $eval(s, c') < eval(s, c)$), and c prefers not to accept s or prefers another student s' (i.e. $eval(c, s') \leq eval(c, s)$).
 - Case 1: s never applied for c . Because students applied in decreasing order of preference, so s prefers c' to c , then (s, c) is stable.
 - Case 2: s applied for c . c unenrolled s (sooner or later), i.e. c prefer other students than s , so (s, c) is stable.

4. Yes, GS produces a student-optimal matching.