

## Naïve Bayesian classifier

class label is status attribute

$$P(c_i | X) = P(X | c_i) P(c_i)$$

| department | status | age     | salary    |
|------------|--------|---------|-----------|
| sales      | senior | 31...35 | 46K...50K |
| sales      | junior | 26...30 | 26K...30K |
| sales      | junior | 31...35 | 31K...35K |
| systems    | junior | 21...25 | 46K...50K |
| systems    | senior | 31...35 | 66K...70K |
| systems    | junior | 26...30 | 46K...50K |
| systems    | senior | 41...45 | 66K...70K |
| marketing  | senior | 36...40 | 46K...50K |
| marketing  | junior | 31...35 | 41K...45K |
| secretary  | senior | 46...50 | 36K...40K |
| secretary  | junior | 26...30 | 26K...30K |

a) Data sample :  $X = (\text{marketing}, 31...35, 46K...50K)$ 

$$P(c_i) : \begin{aligned} P(\text{status} = \text{"junior"}) &= 6/11 = 0.5455 \\ P(\text{status} = \text{"senior"}) &= 5/11 = 0.4545 \end{aligned}$$

$$\begin{aligned} \text{junior} &= 6 \\ \text{senior} &= 5 \end{aligned}$$

Compute  $P(X | c_i)$  for each class

$$\begin{aligned} P(\text{department} = \text{"marketing"} | \text{status} = \text{"junior"}) &= 1/6 = 0.1667 \\ P(\text{department} = \text{"marketing"} | \text{status} = \text{"senior"}) &= 1/5 = 0.2 \end{aligned}$$

$$\begin{aligned} P(\text{age} = \text{"31...35"} | \text{status} = \text{"junior"}) &= 2/6 = 0.3333 \\ P(\text{age} = \text{"31...35"} | \text{status} = \text{"senior"}) &= 2/5 = 0.4 \end{aligned}$$

$$\begin{aligned} P(\text{salary} = \text{"46k...50k"} | \text{status} = \text{"junior"}) &= 2/6 = 0.3333 \\ P(\text{salary} = \text{"46k...50k"} | \text{status} = \text{"senior"}) &= 2/5 = 0.4 \end{aligned}$$

$$\begin{aligned} P(X | c_i) : \quad P(X | \text{status} = \text{"junior"}) &= 0.1667 \times 0.3333 \times 0.3333 = 0.0185 \\ P(X | \text{status} = \text{"senior"}) &= 0.2 \times 0.4 \times 0.4 = 0.032 \end{aligned}$$

$$\begin{aligned} P(X | c_i) * P(c_i) : \quad P(X | \text{status} = \text{"junior"}) * P(\text{status} = \text{"junior"}) &= 0.0185 \times 0.5455 = 0.0101 \\ P(X | \text{status} = \text{"senior"}) * P(\text{status} = \text{"senior"}) &= 0.032 \times 0.4545 = 0.0145 \rightarrow \text{highest} \end{aligned}$$

Therefore,  $X$  belongs to class (status = "senior") #

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| sales      | senior | 31...35 | 46K...50K |
| sales      | junior | 26...30 | 26K...30K |
| sales      | junior | 31...35 | 31K...35K |
| systems    | junior | 21...25 | 46K...50K |
| systems    | senior | 31...35 | 66K...70K |
| systems    | junior | 26...30 | 46K...50K |
| systems    | senior | 41...45 | 66K...70K |
| marketing  | senior | 36...40 | 46K...50K |
| marketing  | junior | 31...35 | 41K...45K |
| secretary  | senior | 46...50 | 36K...40K |
| secretary  | junior | 26...30 | 26K...30K |

b) Data sample :  $X = (\text{sale}, 31...35, 66K...70K)$ 

$$P(c_i) : \begin{aligned} P(\text{status} = \text{"junior"}) &= 6/11 = 0.5455 \\ P(\text{status} = \text{"senior"}) &= 5/11 = 0.4545 \end{aligned}$$

$$\begin{aligned} \text{junior} &= 6 \\ \text{senior} &= 5 \end{aligned}$$

Compute  $P(X | c_i)$  for each class

$$P(\text{department} = \text{"sale"} | \text{status} = \text{"junior"}) = 2/6 = 0.3333$$

$$P(\text{department} = \text{"sale"} | \text{status} = \text{"senior"}) = 1/5 = 0.2$$

$$P(\text{age} = \text{"31...35"} | \text{status} = \text{"junior"}) = 2/6 = 0.3333$$

$$P(\text{age} = \text{"31...35"} | \text{status} = \text{"senior"}) = 2/5 = 0.4$$

$$P(\text{salary} = \text{"66K...70K"} | \text{status} = \text{"junior"}) = 0/6 = 0 \rightarrow \text{prob. min'v 0. In Laplace estimate}$$

Generalized Laplace estimate:

Salary has 6 values (26K...30K, 31K...35K, 36K...40K, 41K...45K, 46K...50K, 66K...70K).

So,  $s_i = 6$ 

$$P(A_i = v_j | C_k) = \frac{n_{ijk} + 1}{n_k + s_i}$$

Thus,

$$P(\text{salary} = \text{"66K...70K"} | \text{status} = \text{"junior"}) = 0+1 / 6+6 = 1/12 = 0.0833$$

$$P(\text{salary} = \text{"66K...70K"} | \text{status} = \text{"senior"}) = 2+1 / 5+6 = 3/11 = 0.2727$$

$$P(X | c_i) : P(X | \text{status} = \text{"junior"}) = 0.3333 \times 0.3333 \times 0.0833 = 0.0093$$

$$P(X | \text{status} = \text{"senior"}) = 0.2 \times 0.4 \times 0.2727 = 0.0218$$

$$P(X | c_i) * P(c_i) : P(X | \text{status} = \text{"junior"}) * P(\text{status} = \text{"junior"}) = 0.0093 \times 0.5455 = 0.0051$$

$$P(X | \text{status} = \text{"senior"}) * P(\text{status} = \text{"senior"}) = 0.0218 \times 0.4545 = 0.0099 \rightarrow \text{highest}$$

Therefore,  $X$  belongs to class (status = "senior") #