

Homework 1 | Bass Model

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Smartphone

For the innovation, I chose the smartphone. The smartphone is a widely recognized and prominent innovation in recent times, encompassing advanced mobile communication, internet browsing, apps, and various features beyond traditional cell phones.

Cellphone

The predecessor to the modern smartphone is the basic cell phone. Early cell phones were primarily used for voice calls and short messaging (SMS). The transition from basic cell phones to smartphones marked a significant shift, integrating advanced computing capabilities, touchscreens, internet access, GPS, high-quality cameras, and a vast array of applications.

I found Nokia net sales data in Statista. As we know Nokia was a leading company in cellphone market before 2008. I used their sales data to make a prediction.

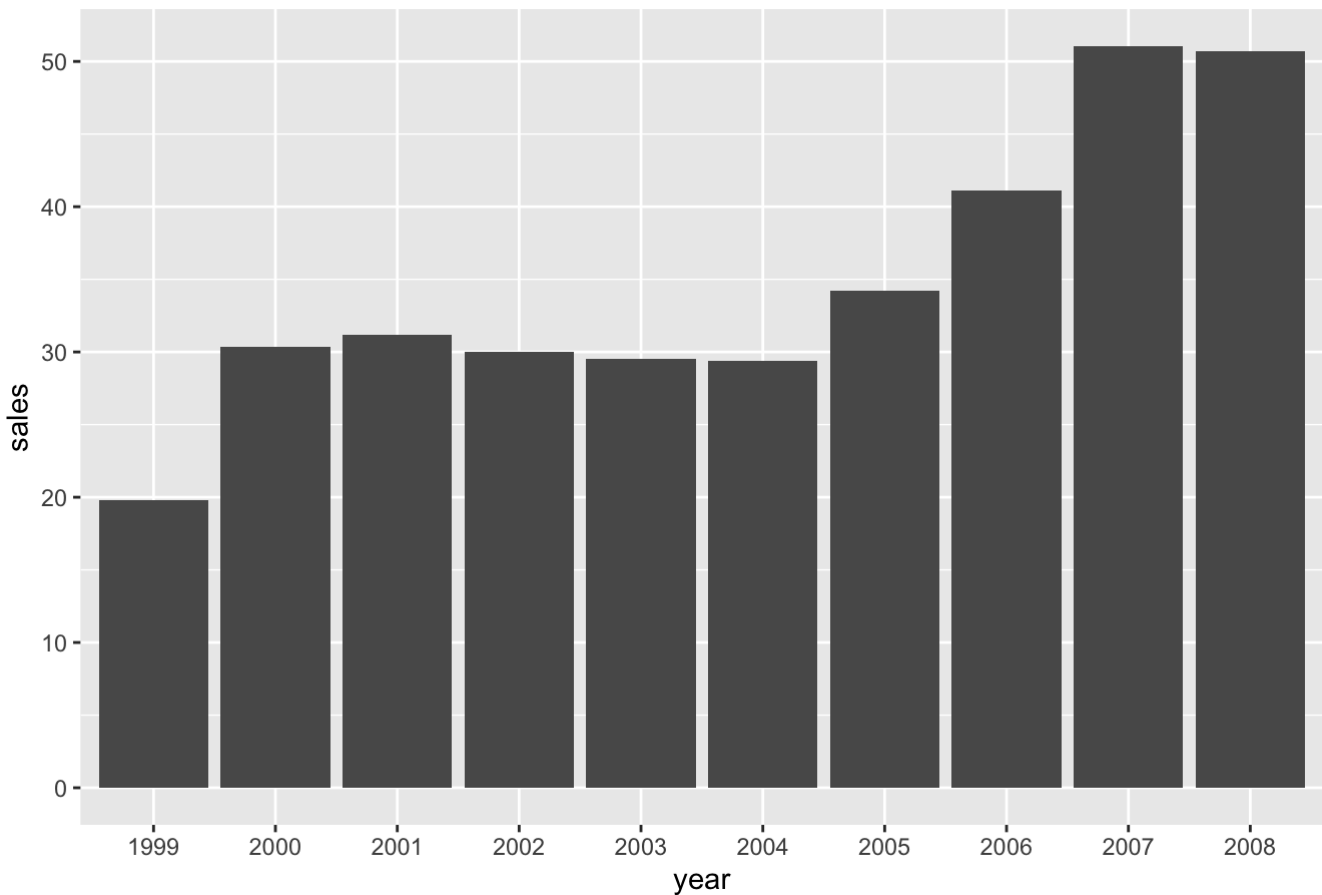
```
nokia <- read_excel("nokial.xlsx")
nokia
```

```
## # A tibble: 10 × 2
##   year  sales
##   <chr> <dbl>
## 1 1999   19.8
## 2 2000   30.4
## 3 2001   31.2
## 4 2002   30.0
## 5 2003   29.5
## 6 2004   29.4
## 7 2005   34.2
## 8 2006   41.1
## 9 2007   51.1
## 10 2008   50.7
```

```
sm_sales = ggplot(data = nokia, aes(x = year, y = sales)) +
  geom_bar(stat = 'identity') +
  ggtitle('Nokia sales')

sm_sales
```

Nokia sales



Define functions for $f(t)$ and $F(t)$:

```
bass.f <- function(t,p,q){
  ((p+q)^2/p)*exp(-(p+q)*t)/
  (1+(q/p)*exp(-(p+q)*t))^2
}
```

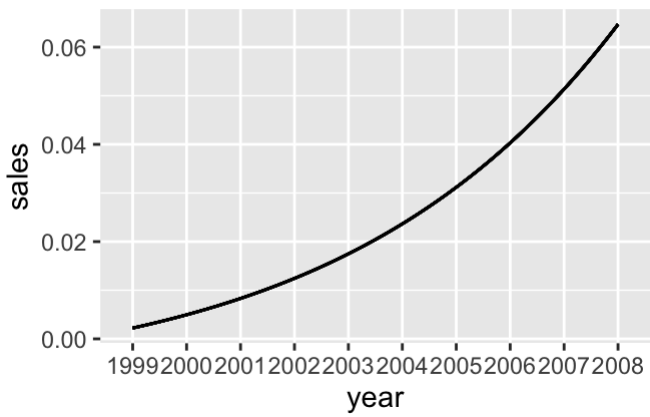
```
bass.F <- function(t,p,q){
  (1-exp(-(p+q)*t))/
  (1+(q/p)*exp(-(p+q)*t))
}
```

```
cum_ad = ggplot(data = nokia, aes(x = year, y = sales)) +
  stat_function(fun = bass.F, args = c(p=0.002, q=0.21)) +
  labs(title = 'Nokia net sales - cumulative adoptions')

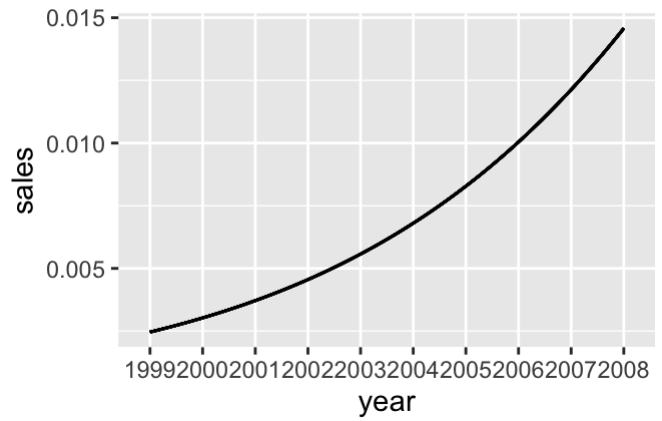
time_ad = ggplot(data = nokia, aes(x = year, y = sales)) +
  stat_function(fun = bass.f, args = c(p=0.002, q=0.21)) +
  labs(title = 'Nokia net sales - adoptions at time t')

suppressWarnings({ggarrange(cum_ad, time_ad, sm_sales)})
```

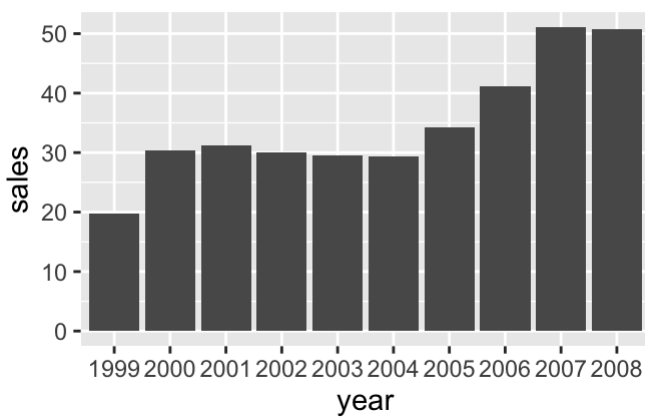
Nokia net sales - cumulative adoptio



Nokia net sales - adoptions at time



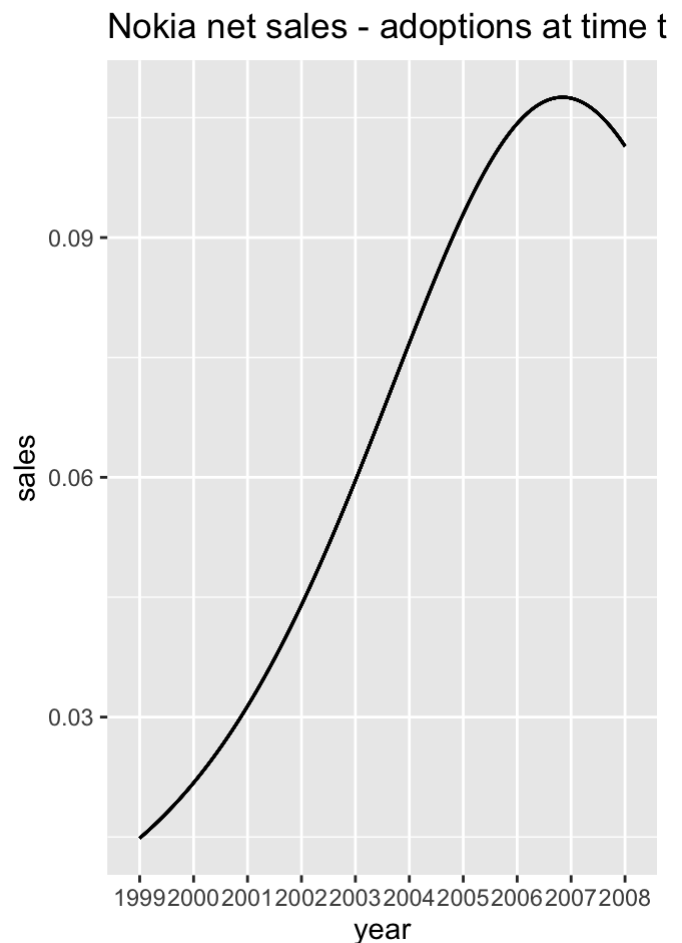
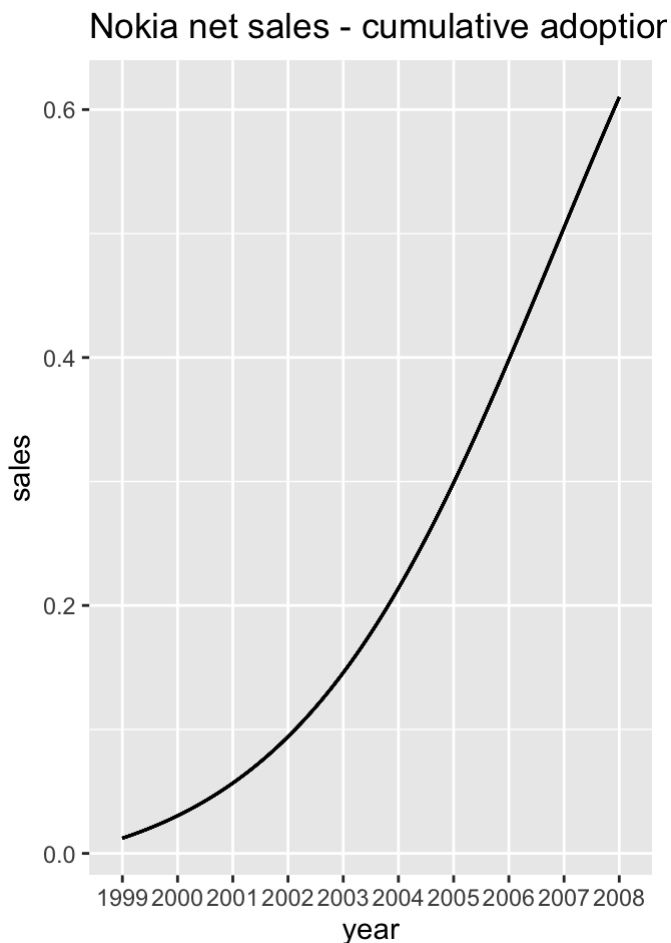
Nokia sales



```
cum_ad = ggplot(data = nokia, aes(x = year, y = sales)) +
  stat_function(fun = bass.F, args = c(p=0.01, q=0.41)) +
  labs(title = 'Nokia net sales - cumulative adoptions')

time_ad = ggplot(data = nokia, aes(x = year, y = sales)) +
  stat_function(fun = bass.f, args = c(p=0.01, q=0.41)) +
  labs(title = 'Nokia net sales - adoptions at time t')

suppressWarnings({ggarrange(cum_ad, time_ad)})
```



The “diffusion” library is an option for estimating the parameters of the bass model. The parameters are used in the Bass model equation to estimate the cumulative number of adopters (net sales in this case) over time:

```
diff_m = diffusion(nokia$sales)
p=round(diff_m$w,4)[1]
q=round(diff_m$w,4)[2]
m=round(diff_m$w,4)[3]
diff_m
```

```
## bass model
##
## Parameters:
##
##               Estimate p-value
## p - Coefficient of innovation  17.7275    NA
## q - Coefficient of imitation   0.7569     NA
## m - Market potential           39.4983     NA
##
## sigma: 35.9989
```

```
sales = nokia$sales
t = 1:length(sales)
bass_m = nls(sales ~ m*(((p+q)**2/p)*exp(-(p+q)*t))/
              (1+(q/p)*exp(-(p+q)*t))**2,
              start=c(list(m=sum(sales),p=0.02,q=0.4)),control=nls.control(maxiter = 1
00, minFactor = 1/1024, printEval = TRUE, warnOnly = TRUE))
```

```

## It. 1, fac= 1, eval (no.,total): ( 1, 1): new dev = 3980.84
## It. 1, fac= 0.5, eval (no.,total): ( 2, 2): new dev = 1224.77
## It. 2, fac= 1, eval (no.,total): ( 1, 3): new dev = 8191.04
## It. 2, fac= 0.5, eval (no.,total): ( 2, 4): new dev = 2486.24
## It. 2, fac= 0.25, eval (no.,total): ( 3, 5): new dev = 1249.71
## It. 2, fac= 0.125, eval (no.,total): ( 4, 6): new dev = 1096.34
## It. 3, fac= 0.25, eval (no.,total): ( 1, 7): new dev = 1388.93
## It. 3, fac= 0.125, eval (no.,total): ( 2, 8): new dev = 1044.92
## It. 4, fac= 0.25, eval (no.,total): ( 1, 9): new dev = 1931
## It. 4, fac= 0.125, eval (no.,total): ( 2, 10): new dev = 1119.72
## It. 4, fac= 0.0625, eval (no.,total): ( 3, 11): new dev = 1005.55
## It. 5, fac= 0.125, eval (no.,total): ( 1, 12): new dev = 1197.16
## It. 5, fac= 0.0625, eval (no.,total): ( 2, 13): new dev = 995.497
## It. 6, fac= 0.125, eval (no.,total): ( 1, 14): new dev = 1398.43
## It. 6, fac= 0.0625, eval (no.,total): ( 2, 15): new dev = 1032.04
## It. 6, fac= 0.03125, eval (no.,total): ( 3, 16): new dev = 977.097
## It. 7, fac= 0.0625, eval (no.,total): ( 1, 17): new dev = 1050.17
## It. 7, fac= 0.03125, eval (no.,total): ( 2, 18): new dev = 968.038
## It. 8, fac= 0.0625, eval (no.,total): ( 1, 19): new dev = 1093.62
## It. 8, fac= 0.03125, eval (no.,total): ( 2, 20): new dev = 971.623
## It. 8, fac= 0.015625, eval (no.,total): ( 3, 21): new dev = 955.651
## It. 9, fac= 0.03125, eval (no.,total): ( 1, 22): new dev = 967.632
## It. 9, fac= 0.015625, eval (no.,total): ( 2, 23): new dev = 945.532
## It. 10, fac= 0.03125, eval (no.,total): ( 1, 24): new dev = 967.691
## It. 10, fac= 0.015625, eval (no.,total): ( 2, 25): new dev = 938.074
## It. 11, fac= 0.03125, eval (no.,total): ( 1, 26): new dev = 972.878
## It. 11, fac= 0.015625, eval (no.,total): ( 2, 27): new dev = 933.824
## It. 12, fac= 0.03125, eval (no.,total): ( 1, 28): new dev = 984.776
## It. 12, fac= 0.015625, eval (no.,total): ( 2, 29): new dev = 933.561
## It. 13, fac= 0.03125, eval (no.,total): ( 1, 30): new dev = 1005.79
## It. 13, fac= 0.015625, eval (no.,total): ( 2, 31): new dev = 938.426
## It. 13, fac= 0.0078125, eval (no.,total): ( 3, 32): new dev = 928.394
## It. 14, fac= 0.015625, eval (no.,total): ( 1, 33): new dev = 936.434
## It. 14, fac= 0.0078125, eval (no.,total): ( 2, 34): new dev = 924.056
## It. 15, fac= 0.015625, eval (no.,total): ( 1, 35): new dev = 935.755
## It. 15, fac= 0.0078125, eval (no.,total): ( 2, 36): new dev = 920.658
## It. 16, fac= 0.015625, eval (no.,total): ( 1, 37): new dev = 936.617
## It. 16, fac= 0.0078125, eval (no.,total): ( 2, 38): new dev = 918.339
## It. 17, fac= 0.015625, eval (no.,total): ( 1, 39): new dev = 939.317
## It. 17, fac= 0.0078125, eval (no.,total): ( 2, 40): new dev = 917.277
## It. 18, fac= 0.015625, eval (no.,total): ( 1, 41): new dev = 944.241
## It. 18, fac= 0.0078125, eval (no.,total): ( 2, 42): new dev = 917.696
## It. 18, fac= 0.00390625, eval (no.,total): ( 3, 43): new dev = 914.255
## It. 19, fac= 0.0078125, eval (no.,total): ( 1, 44): new dev = 915.529
## It. 19, fac= 0.00390625, eval (no.,total): ( 2, 45): new dev = 911.458
## It. 20, fac= 0.0078125, eval (no.,total): ( 1, 46): new dev = 913.665
## It. 20, fac= 0.00390625, eval (no.,total): ( 2, 47): new dev = 908.904
## It. 21, fac= 0.0078125, eval (no.,total): ( 1, 48): new dev = 912.134
## It. 21, fac= 0.00390625, eval (no.,total): ( 2, 49): new dev = 906.616
## It. 22, fac= 0.0078125, eval (no.,total): ( 1, 50): new dev = 910.971
## It. 22, fac= 0.00390625, eval (no.,total): ( 2, 51): new dev = 904.616
## It. 23, fac= 0.0078125, eval (no.,total): ( 1, 52): new dev = 910.214
## It. 23, fac= 0.00390625, eval (no.,total): ( 2, 53): new dev = 902.934
## It. 24, fac= 0.0078125, eval (no.,total): ( 1, 54): new dev = 909.911
## It. 24, fac= 0.00390625, eval (no.,total): ( 2, 55): new dev = 901.601

```

```

## It. 25, fac= 0.0078125, eval (no.,total): ( 1, 56): new dev = 910.116
## It. 25, fac= 0.00390625, eval (no.,total): ( 2, 57): new dev = 900.656
## It. 26, fac= 0.0078125, eval (no.,total): ( 1, 58): new dev = 910.896
## It. 26, fac= 0.00390625, eval (no.,total): ( 2, 59): new dev = 900.142
## It. 27, fac= 0.0078125, eval (no.,total): ( 1, 60): new dev = 912.326
## It. 27, fac= 0.00390625, eval (no.,total): ( 2, 61): new dev = 900.113
## It. 28, fac= 0.0078125, eval (no.,total): ( 1, 62): new dev = 914.501
## It. 28, fac= 0.00390625, eval (no.,total): ( 2, 63): new dev = 900.63
## It. 28, fac= 0.00195312, eval (no.,total): ( 3, 64): new dev = 898.712
## It. 29, fac= 0.00390625, eval (no.,total): ( 1, 65): new dev = 899.532
## It. 29, fac= 0.00195312, eval (no.,total): ( 2, 66): new dev = 897.389
## It. 30, fac= 0.00390625, eval (no.,total): ( 1, 67): new dev = 898.531
## It. 30, fac= 0.00195312, eval (no.,total): ( 2, 68): new dev = 896.15
## It. 31, fac= 0.00390625, eval (no.,total): ( 1, 69): new dev = 897.635
## It. 31, fac= 0.00195312, eval (no.,total): ( 2, 70): new dev = 894.998
## It. 32, fac= 0.00390625, eval (no.,total): ( 1, 71): new dev = 896.85
## It. 32, fac= 0.00195312, eval (no.,total): ( 2, 72): new dev = 893.941
## It. 33, fac= 0.00390625, eval (no.,total): ( 1, 73): new dev = 896.184
## It. 33, fac= 0.00195312, eval (no.,total): ( 2, 74): new dev = 892.983
## It. 34, fac= 0.00390625, eval (no.,total): ( 1, 75): new dev = 895.647
## It. 34, fac= 0.00195312, eval (no.,total): ( 2, 76): new dev = 892.132
## It. 35, fac= 0.00390625, eval (no.,total): ( 1, 77): new dev = 895.247
## It. 35, fac= 0.00195312, eval (no.,total): ( 2, 78): new dev = 891.395
## It. 36, fac= 0.00390625, eval (no.,total): ( 1, 79): new dev = 894.996
## It. 36, fac= 0.00195312, eval (no.,total): ( 2, 80): new dev = 890.78
## It. 37, fac= 0.00390625, eval (no.,total): ( 1, 81): new dev = 894.906
## It. 37, fac= 0.00195312, eval (no.,total): ( 2, 82): new dev = 890.298
## It. 38, fac= 0.00390625, eval (no.,total): ( 1, 83): new dev = 894.99
## It. 38, fac= 0.00195312, eval (no.,total): ( 2, 84): new dev = 889.958
## It. 39, fac= 0.00390625, eval (no.,total): ( 1, 85): new dev = 895.264
## It. 39, fac= 0.00195312, eval (no.,total): ( 2, 86): new dev = 889.771
## It. 40, fac= 0.00390625, eval (no.,total): ( 1, 87): new dev = 895.744
## It. 40, fac= 0.00195312, eval (no.,total): ( 2, 88): new dev = 889.751
## It. 41, fac= 0.00390625, eval (no.,total): ( 1, 89): new dev = 896.451
## It. 41, fac= 0.00195312, eval (no.,total): ( 2, 90): new dev = 889.911
## It. 41, fac= 0.000976562, eval (no.,total): ( 3, 91): new dev = 889.036
## It. 42, fac= 0.00195312, eval (no.,total): ( 1, 92): new dev = 889.294
## It. 42, fac= 0.000976562, eval (no.,total): ( 2, 93): new dev = 888.346
## It. 43, fac= 0.00195312, eval (no.,total): ( 1, 94): new dev = 888.705
## It. 43, fac= 0.000976562, eval (no.,total): ( 2, 95): new dev = 887.682
## It. 44, fac= 0.00195312, eval (no.,total): ( 1, 96): new dev = 888.147
## It. 44, fac= 0.000976562, eval (no.,total): ( 2, 97): new dev = 887.046
## It. 45, fac= 0.00195312, eval (no.,total): ( 1, 98): new dev = 887.621
## It. 45, fac= 0.000976562, eval (no.,total): ( 2, 99): new dev = 886.437
## It. 46, fac= 0.00195312, eval (no.,total): ( 1,100): new dev = 887.128
## It. 46, fac= 0.000976562, eval (no.,total): ( 2,101): new dev = 885.858
## It. 47, fac= 0.00195312, eval (no.,total): ( 1,102): new dev = 886.67
## It. 47, fac= 0.000976562, eval (no.,total): ( 2,103): new dev = 885.31
## It. 48, fac= 0.00195312, eval (no.,total): ( 1,104): new dev = 886.249
## It. 48, fac= 0.000976562, eval (no.,total): ( 2,105): new dev = 884.794
## It. 49, fac= 0.00195312, eval (no.,total): ( 1,106): new dev = 885.865
## It. 49, fac= 0.000976562, eval (no.,total): ( 2,107): new dev = 884.311
## It. 50, fac= 0.00195312, eval (no.,total): ( 1,108): new dev = 885.522
## It. 50, fac= 0.000976562, eval (no.,total): ( 2,109): new dev = 883.864
## It. 51, fac= 0.00195312, eval (no.,total): ( 1,110): new dev = 885.221
## It. 51, fac= 0.000976562, eval (no.,total): ( 2,111): new dev = 883.454

```

```
## It. 52, fac= 0.00195312, eval (no.,total): ( 1,112): new dev = 884.964
## It. 52, fac= 0.000976562, eval (no.,total): ( 2,113): new dev = 883.082
## It. 53, fac= 0.00195312, eval (no.,total): ( 1,114): new dev = 884.754
## It. 53, fac= 0.000976562, eval (no.,total): ( 2,115): new dev = 882.752
## It. 54, fac= 0.00195312, eval (no.,total): ( 1,116): new dev = 884.594
## It. 54, fac= 0.000976562, eval (no.,total): ( 2,117): new dev = 882.464
## It. 55, fac= 0.00195312, eval (no.,total): ( 1,118): new dev = 884.485
## It. 55, fac= 0.000976562, eval (no.,total): ( 2,119): new dev = 882.221
## It. 56, fac= 0.00195312, eval (no.,total): ( 1,120): new dev = 884.431
## It. 56, fac= 0.000976562, eval (no.,total): ( 2,121): new dev = 882.025
## It. 57, fac= 0.00195312, eval (no.,total): ( 1,122): new dev = 884.435
## It. 57, fac= 0.000976562, eval (no.,total): ( 2,123): new dev = 881.88
## It. 58, fac= 0.00195312, eval (no.,total): ( 1,124): new dev = 884.501
## It. 58, fac= 0.000976562, eval (no.,total): ( 2,125): new dev = 881.787
## It. 59, fac= 0.00195312, eval (no.,total): ( 1,126): new dev = 884.632
## It. 59, fac= 0.000976562, eval (no.,total): ( 2,127): new dev = 881.75
## It. 60, fac= 0.00195312, eval (no.,total): ( 1,128): new dev = 884.832
## It. 60, fac= 0.000976562, eval (no.,total): ( 2,129): new dev = 881.772
```

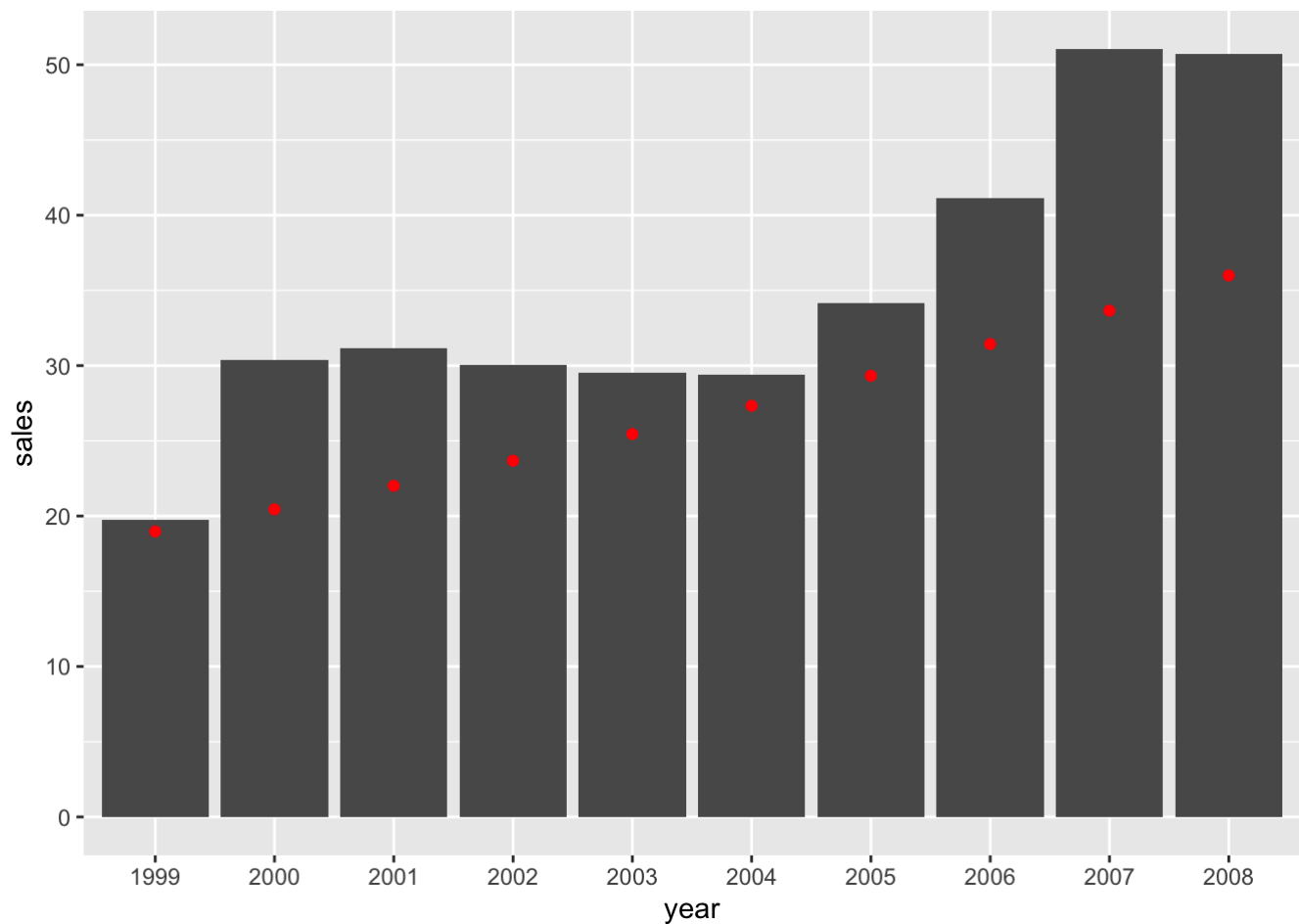
```
## Warning in nls(sales ~ m * (((p + q)^2/p) * exp(-(p + q) * t))/(1 + (q/p) * :
## step factor 0.000488281 reduced below 'minFactor' of 0.000976562
```

```
bass_m
```

```
## Nonlinear regression model
## model: sales ~ m * (((p + q)^2/p) * exp(-(p + q) * t))/(1 + (q/p) * exp(-(p
+ q) * t))^2
## data: parent.frame()
## m p q
## 4.602e+03 3.825e-03 7.929e-02
## residual sum-of-squares: 881.8
##
## Number of iterations till stop: 59
## Achieved convergence tolerance: 2.562
## Reason stopped: step factor 0.000488281 reduced below 'minFactor' of 0.000976562
```

A bass model was estimated using the given sales information. The gap between the product's estimated and actual sales is shown in the visualization below. Vizually, you can see that it is not very good, it is just generalizing.

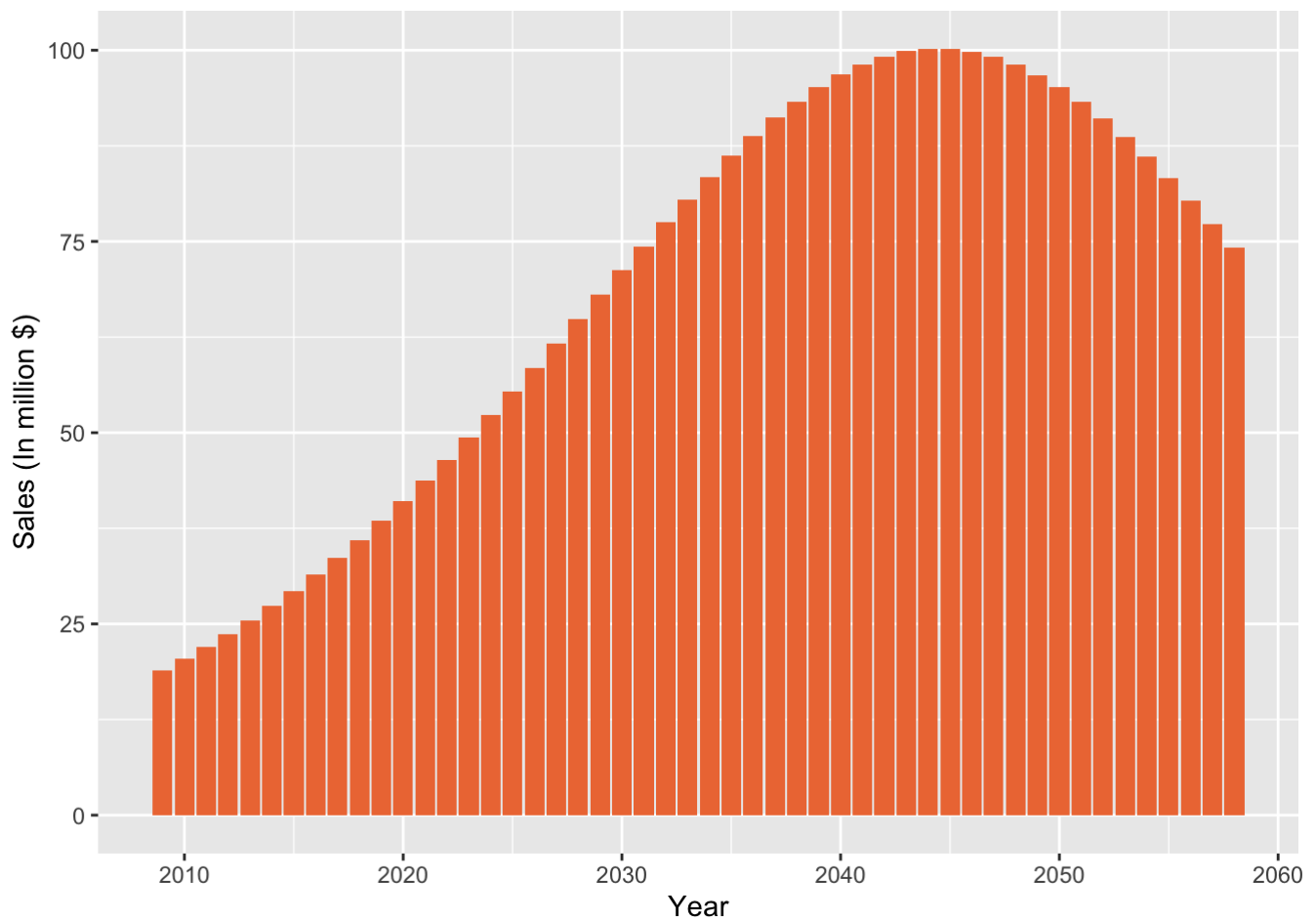
```
nokia$pred_sales = bass.f(1:10, p = 3.825e-03, q = 7.929e-02) * 4.602e+03
ggplot(data = nokia, aes(x = year, y = sales)) +
  geom_bar(stat = 'identity') +
  geom_point(mapping = aes(x=year, y=pred_sales), color = 'red')
```



Predicting based on our results of bass model.

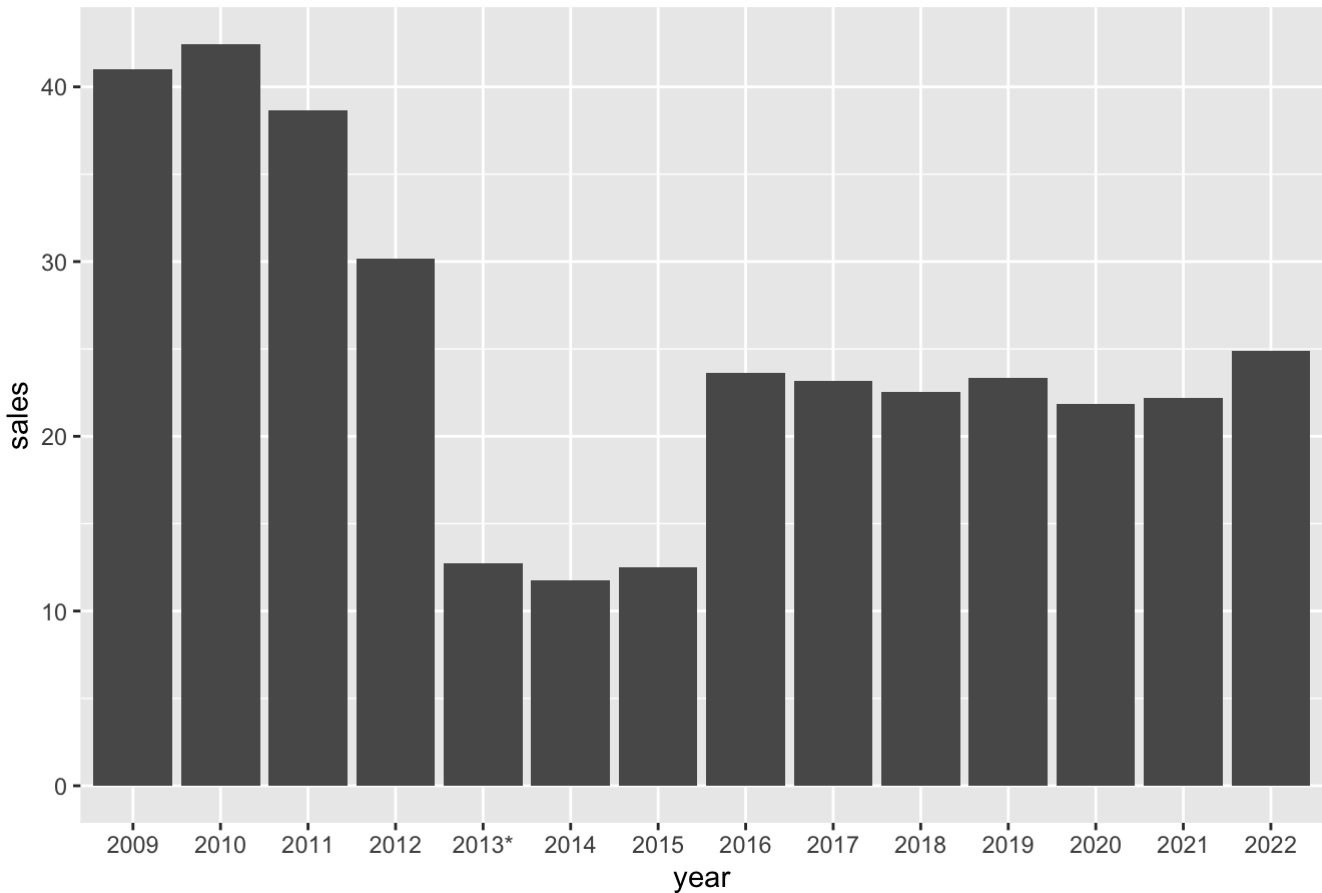
```
innovation_prediction <- bass.f(1:50, p = 3.825e-03, q = 7.929e-02) * 4.602e+03  
years <- seq(from = 2009, to = 2008 + 50, by = 1)  
innovation_data <- data.frame(Year = years, Sales = innovation_prediction)
```

```
ggplot(data = innovation_data, aes(x = Year, y = Sales)) +  
  geom_bar(stat='identity', fill = 'sienna2') + ylab("Sales (In million $)")
```

```
nokia_new <- read_excel("nokia2.xlsx")
ggplot(data = nokia_new, aes(x = year, y = sales)) +
  geom_bar(stat = 'identity') +
  ggtitle('Nokia sales')
```

Nokia sales



As you can see, I have the data of real Nokia sales which does not correspond with our prediction. The problem is that from 2007 new players with new ideas enter the market which made Nokia to lose its place in the market.

#Reference:

Nokia. (March 2, 2023). Nokia net sales worldwide from 1999 to 2022 (in billion euros) [Graph]. In Statista. Retrieved September 30, 2023, from <https://www.statista.com/statistics/267819/nokias-net-sales-since-1999/> (<https://www.statista.com/statistics/267819/nokias-net-sales-since-1999/>) Nokia sales data source: