Private Reinforcement Learning with PAC and Regret Guarantees

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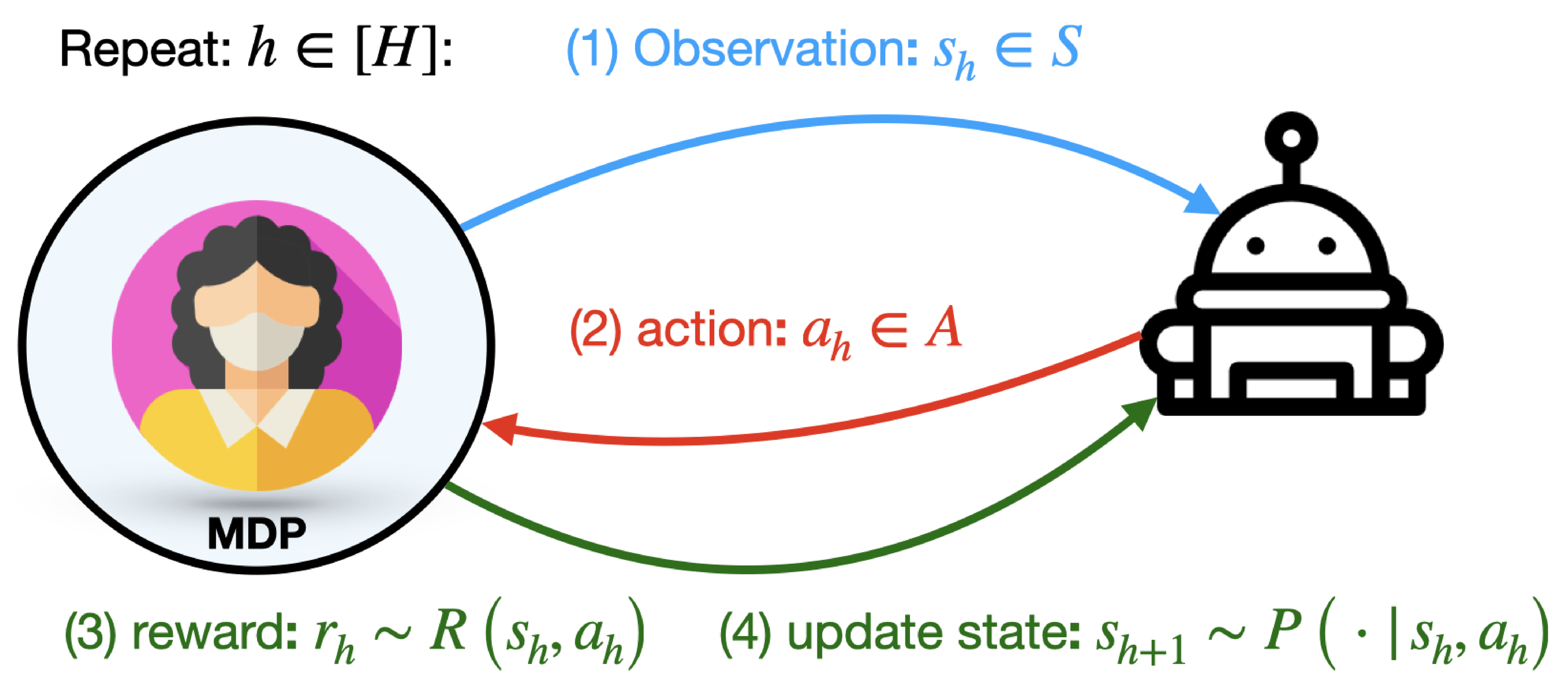
Online Learning MDP

1. An agent interacts with a fix horizon MDP over a sequence of episodes:
2. $M = (S, A, R, P, p_0, H)$ // Environment
3. **Privacy in RL**: we assume that sequence of states and rewards is sensitive data belonging to users.



Contributions

1. Definition of Joint Differential Privacy (**JDP**) in Reinforcement Learning (**RL**).
2. Almost optimal ε-**JDP** **RL** algorithm with bounded *sample complexity* and bounded *regret*.
3. Lower bound for any **RL** algorithm that satisfies **JDP**.

Results

1. Let α be a target accuracy and ε the privacy parameter.
   - *How many episodes does it take to learn an α-optimal policy?*
   - A smaller ε means the algorithm is more private.
2. Upper bound sample complexity:

$$\widetilde{O}\left(\frac{SAH^4}{\alpha^2} + \frac{S^2AH^4}{\epsilon\alpha}\right) \tag{1}$$

3. Lower bound sample complexity:

$$\widetilde{\Omega}\left(\frac{SAH^2}{\alpha^2} + \frac{SAH}{\epsilon\alpha}\right) \tag{2}$$

Privacy Definition

- We represent a user as a tree of depth H encoding all possible state and reward paths.
- Let $\mathcal{M}$ be a **RL** algorithm:
  - The INPUT is a sequence of T users.
  - The OUTPUT is a sequence of T actions and a final policy π_T.

$$\mathcal{M}(\ldots) = \left(a^{(1)}, \ldots, a^{(T)}, \pi_T\right) \qquad a^{(t)} = \left(a_1^{(t)}, \ldots, a_h^{(t)}\right)$$

- U and U' are t-neighboring if they differ only on user at position t:

$$U = (\ldots, \ldots, \ldots),\ U' = (\ldots, \ldots, \ldots)$$

Definition (KPRU14)**.** A mechanism $\mathcal{M}$ is ε-**JDP** if for all t, all t-neighboring user sequences U, U' and all events $E \subseteq A^{H\times[T-1]} \times \Pi$ we have

$$\Pr\left[\mathcal{M}_{-t}(U) \in E\right] \le e^{\varepsilon} \Pr\left[\mathcal{M}_{-t}(U') \in E\right] \tag{3}$$

PUCB

- **PUCB** is a **JDP** version of UBEV [DLB 2017].
- Non-private event counters: $\widehat{n}_t(s,a,h), \widehat{m}_t(s,a,h,s'), \widehat{r}_t(s,a,h)$.
- Private event counters: $\widetilde{n}_t(s,a,h), \widetilde{m}_t(s,a,h,s'), \widetilde{r}_t(s,a,h)$.
- Use Binary mechanism (**BM**) from [Dwork et at., 2010] and [Chan et al., 2011]. For any $t \in [T]$:

$$|\widehat{n}_t(s,a,h) - \widetilde{n}_t(s,a,h)| \le \tfrac{H}{\varepsilon}\log(T)^{5/2}\log(SAH/\beta) := E_\varepsilon \tag{4}$$

- Balance exploration/exploitation with optimism:
  - Compute $\widetilde{Q}_t^+$ with **DP**:

$$\widetilde{Q}_t^+(s,a,h) = (\text{Reward}) + (\text{Future reward}) + (\text{Bonus}) \tag{5}$$

  - Greedy policy: $\pi_t(s,h) = \arg\max_{a^*} \widetilde{Q}_t^+(s,a^*,h)$.

Private Optimistic Q-function

$$\widetilde{Q}^+(s,a,h) = \frac{\underbrace{\widetilde{r}(s,a,h)}_{\text{Immediate reward}} + \underbrace{\sum_{s'} \widetilde{V}_{h+1}(s')\widetilde{m}(s,a,h,s')}_{\text{Expected future reward}}}{\widetilde{n}(s,a,h)} + \underbrace{\widetilde{\mathrm{conf}}(s,a,h)}_{\text{Bonus term}}$$

$$\widetilde{\mathrm{conf}}(s,a,h) = \widetilde{\phi}(s,a,h) + \widetilde{\psi}(s,a,h)$$

$$\widetilde{\phi}(s,a,h) = (1+H)\sqrt{\frac{T/\beta}{\widetilde{n}(s,a,h)}}$$ sampling error [Dann et al. 2017]

$$\widetilde{\psi}(s,a,h) = (1+SH)\left(\frac{3E_\varepsilon}{\widetilde{n}(s,a,h)} + \frac{2E_\varepsilon^2}{\widetilde{n}(s,a,h)^2}\right)$$ privacy error [This work].

PUCB Sample Complexity Analysis

- Construct $\widetilde{Q}_t^+$ with DP counters and show that:

$$\widetilde{Q}_t^+(s,a,h) \ge Q^*(s,a,h) \tag{6}$$

- Optimality gap decomposition:

$$V^* - V^{\pi_t} := \Delta_t \le \sum_{(s,a,h)\in(S,A,[H])} w_t(s,a,h)\widetilde{\mathrm{conf}}_t(s,a,h) \tag{7}$$

- Bounding number of episodes where $\Delta_t > \alpha$.

Lower Bound Analysis

- Consider hard-MDP construction.
- Lower bound for **DP** best-arm-identification:
- We show that finding the the α-optimal arm takes $\widetilde{\Omega}\left(\frac{A}{\varepsilon\alpha}\ln\frac{1}{4\beta}\right)$ tries.
- We consider a Public Initial State (**PIS**) Setting:
  - And do a reduction to **DP** best-arm-identification.
  - We show that the learner makes at least $\frac{SAH}{24\varepsilon\alpha}\ln\frac{1}{4\beta}$ mistakes in the PIS setting.
  - If algorithm satisfies ε-**JDP** $\implies$ satisfies ε-**JDP** in the PIS setting.

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