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| Федеральное государственное бюджетное  образовательное учреждение высшего образования «Новосибирский государственный технический университет» | | |
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| Кафедра прикладной математики | | |
| Практическое задание № 2 | | |
| по дисциплине «Численные методы» | | |
| **ИТЕРАЦИОННЫЕ МЕТОДЫ РЕШЕНИЯ СЛАУ** | | |
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| Группа ПМ-12 | Лойченко данила, Овчинников иван |
| Вариант 8 |  |
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| Преподаватели | задорожный александр генадьевич |
|  | Леонович дарьяна александровна |
| Новосибирск, 2024 | | |

1. **Задание**

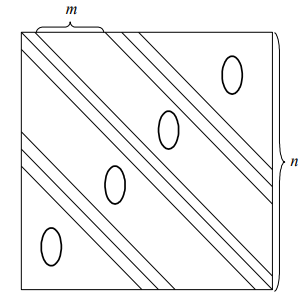
Разработать программы решения СЛАУ методами Якоби, Гаусса-Зейделя, блочной релаксации с хранением матрицы в диагональном формате. Исследовать сходимость методов для различных тестовых матриц и её зависимость от параметра релаксации. Изучить возможность оценки порядка числа обусловленности матрицы путем вычислительного эксперимента.

1. **Математическая модель**

Пусть дана система линейных алгебраических уравнений следующего вида:



С матрицей следующего вида: 9-диагональная матрица m – количество нулевых диагоналей, n – размерность матрицы. Размер блока в реализации блочной релаксации фиксированный.



Выбирается начальное приближение (при отсутствии априорных данных для выбора приближения в качестве начального приближения можно выбрать нулевой вектор).

Метод Якоби. Каждое следующее приближение в методе Якоби рассчитывается по формуле:



где k – номер текущей итерации.

Метод Гаусса-Зейделя. Каждое последующее приближение рассчитывается по формуле:



Для ускорения сходимости итерационного процесса можно использовать параметр релаксации.

Итерационный процесс в методе Якоби с параметром релаксации выглядит следующим образом:





Подставляя (1.4) и (1.5) в (1.2), получаем:



В методе Гаусса-Зейделя с параметром релаксации (другое название – метод релаксации) итерационный процесс описывается следующим образом:



Условия выхода из итерационного процесса для рассмотренных методов:

* выход по относительной невязке:



* защита от зацикливания: k > maxiter, maxiter – максимальное количество итераций.

1. **Исследование на матрице с диагональным числом обусловленности**

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|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Метод Якоби | | | | | |
| w | x | x\* - x | Число итераций | Отн. невязка | Число обусловленности |
| 0.1 | 0,99999999990257 | 9,743E-11 | 64325 | 1,00E-13 | 148,353799 |
| 1,99999999989193 | 1,081E-10 |
| 2,99999999988952 | 1,105E-10 |
| 3,99999999988747 | 1,125E-10 |
| 4,99999999988692 | 1,131E-10 |
| 5,99999999988768 | 1,123E-10 |
| 6,99999999989309 | 1,069E-10 |
| 7,99999999989436 | 1,056E-10 |
| 8,99999999989302 | 1,070E-10 |
| 9,99999999989006 | 1,099E-10 |
| 10,99999999988780 | 1,122E-10 |
| 11,99999999988680 | 1,132E-10 |
| 0.2 | 0,99999999990268 | 9,732E-11 | 32155 | 9,99E-14 | 148,346350 |
| 1,99999999989205 | 1,080E-10 |
| 2,99999999988964 | 1,104E-10 |
| 3,99999999988759 | 1,124E-10 |
| 4,99999999988705 | 1,129E-10 |
| 5,99999999988781 | 1,122E-10 |
| 6,99999999989321 | 1,068E-10 |
| 7,99999999989448 | 1,055E-10 |
| 8,99999999989314 | 1,069E-10 |
| 9,99999999989018 | 1,098E-10 |
| 10,99999999988790 | 1,121E-10 |
| 11,99999999988690 | 1,131E-10 |
| 0.3 | 0,99999999990278 | 9,722E-11 | 22172 | 9,99E-14 | 148,128556 |
| 1,99999999989217 | 1,078E-10 |
| 2,99999999988976 | 1,102E-10 |
| 3,99999999988771 | 1,123E-10 |
| 4,99999999988717 | 1,128E-10 |
| 5,99999999988793 | 1,121E-10 |
| 6,99999999989332 | 1,067E-10 |
| 7,99999999989459 | 1,054E-10 |
| 8,99999999989326 | 1,067E-10 |
| 9,99999999989030 | 1,097E-10 |
| 10,99999999988800 | 1,120E-10 |
| 11,99999999988700 | 1,130E-10 |
| 0.4 | 0,99999999990269 | 9,731E-11 | 16071 | 1,00E-13 | 148,101251 |
| 1,99999999989207 | 1,079E-10 |
| 2,99999999988966 | 1,103E-10 |
| 3,99999999988762 | 1,124E-10 |
| 4,99999999988707 | 1,129E-10 |
| 5,99999999988783 | 1,122E-10 |
| 6,99999999989323 | 1,068E-10 |
| 7,99999999989450 | 1,055E-10 |
| 8,99999999989317 | 1,068E-10 |
| 9,99999999989021 | 1,098E-10 |
| 10,99999999988790 | 1,121E-10 |
| 11,99999999988690 | 1,131E-10 |
| 0.5 | 0,99999999990285 | 9,715E-11 | 12855 | 9,98E-14 | 148,126195 |
| 1,99999999989224 | 1,078E-10 |
| 2,99999999988984 | 1,102E-10 |
| 3,99999999988779 | 1,122E-10 |
| 4,99999999988725 | 1,127E-10 |
| 5,99999999988801 | 1,120E-10 |
| 6,99999999989340 | 1,066E-10 |
| 7,99999999989466 | 1,053E-10 |
| 8,99999999989333 | 1,067E-10 |
| 9,99999999989038 | 1,096E-10 |
| 10,99999999988810 | 1,119E-10 |
| 11,99999999988710 | 1,129E-10 |
| 0.6 | 0,99999999990296 | 9,704E-11 | 10711 | 9,99E-14 | 147,911944 |
| 1,99999999989236 | 1,076E-10 |
| 2,99999999988996 | 1,100E-10 |
| 3,99999999988792 | 1,121E-10 |
| 4,99999999988738 | 1,126E-10 |
| 5,99999999988814 | 1,119E-10 |
| 6,99999999989352 | 1,065E-10 |
| 7,99999999989478 | 1,052E-10 |
| 8,99999999989345 | 1,065E-10 |
| 9,99999999989050 | 1,095E-10 |
| 10,99999999988820 | 1,118E-10 |
| 11,99999999988720 | 1,128E-10 |
| 0.7 | 0,99999999990294 | 9,706E-11 | 9179 | 9,99E-14 | 147,880105 |
| 1,99999999989234 | 1,077E-10 |
| 2,99999999988994 | 1,101E-10 |
| 3,99999999988789 | 1,121E-10 |
| 4,99999999988735 | 1,127E-10 |
| 5,99999999988811 | 1,119E-10 |
| 6,99999999989349 | 1,065E-10 |
| 7,99999999989476 | 1,052E-10 |
| 8,99999999989343 | 1,066E-10 |
| 9,99999999989048 | 1,095E-10 |
| 10,99999999988820 | 1,118E-10 |
| 11,99999999988720 | 1,128E-10 |
| 0.8 | 0,99999999990321 | 9,679E-11 | 8031 | 9,95E-14 | 147,979073 |
| 1,99999999989265 | 1,073E-10 |
| 2,99999999989025 | 1,097E-10 |
| 3,99999999988822 | 1,118E-10 |
| 4,99999999988767 | 1,123E-10 |
| 5,99999999988843 | 1,116E-10 |
| 6,99999999989380 | 1,062E-10 |
| 7,99999999989506 | 1,049E-10 |
| 8,99999999989374 | 1,063E-10 |
| 9,99999999989079 | 1,092E-10 |
| 10,99999999988850 | 1,115E-10 |
| 11,99999999988750 | 1,125E-10 |
| 0.9 | 0,99999999990312 | 9,688E-11 | 7137 | 9,98E-14 | 147,790628 |
| 1,99999999989255 | 1,075E-10 |
| 2,99999999989015 | 1,098E-10 |
| 3,99999999988811 | 1,119E-10 |
| 4,99999999988756 | 1,124E-10 |
| 5,99999999988832 | 1,117E-10 |
| 6,99999999989370 | 1,063E-10 |
| 7,99999999989496 | 1,050E-10 |
| 8,99999999989363 | 1,064E-10 |
| 9,99999999989069 | 1,093E-10 |
| 10,99999999988840 | 1,116E-10 |
| 11,99999999988740 | 1,126E-10 |
| 1,0 | 0,99999999990311 | 9,690E-11 | 6487 | 9,98E-14 | 147,658346 |
| 1,99999999989253 | 1,075E-10 |
| 2,99999999989013 | 1,099E-10 |
| 3,99999999988809 | 1,119E-10 |
| 4,99999999988755 | 1,124E-10 |
| 5,99999999988831 | 1,117E-10 |
| 6,99999999989368 | 1,063E-10 |
| 7,99999999989495 | 1,051E-10 |
| 8,99999999989362 | 1,064E-10 |
| 9,99999999989067 | 1,093E-10 |
| 10,99999999988840 | 1,116E-10 |
| 11,99999999988740 | 1,126E-10 |
| 1,1 | 0,99999999990308 | 9,692E-11 | 5837 | 9,99E-14 | 147,715696 |
| 1,99999999989250 | 1,075E-10 |
| 2,99999999989010 | 1,099E-10 |
| 3,99999999988806 | 1,119E-10 |
| 4,99999999988752 | 1,125E-10 |
| 5,99999999988827 | 1,117E-10 |
| 6,99999999989365 | 1,063E-10 |
| 7,99999999989491 | 1,051E-10 |
| 8,99999999989359 | 1,064E-10 |
| 9,99999999989064 | 1,094E-10 |
| 10,99999999988840 | 1,116E-10 |
| 11,99999999988740 | 1,126E-10 |
| 1,2 | 0,99999999990328 | 9,672E-11 | 5350 | 9,96E-14 | 147,731811 |
| 1,99999999989272 | 1,073E-10 |
| 2,99999999989033 | 1,097E-10 |
| 3,99999999988829 | 1,117E-10 |
| 4,99999999988775 | 1,122E-10 |
| 5,99999999988851 | 1,115E-10 |
| 6,99999999989388 | 1,061E-10 |
| 7,99999999989514 | 1,049E-10 |
| 8,99999999989381 | 1,062E-10 |
| 9,99999999989087 | 1,091E-10 |
| 10,99999999988860 | 1,114E-10 |
| 11,99999999988760 | 1,124E-10 |
| 1,3 | 0,99999999990353 | 9,648E-11 | 4938 | 9,95E-14 | 147,540425 |
| 1,99999999989299 | 1,070E-10 |
| 2,99999999989061 | 1,094E-10 |
| 3,99999999988858 | 1,114E-10 |
| 4,99999999988804 | 1,120E-10 |
| 5,99999999988879 | 1,112E-10 |
| 6,99999999989414 | 1,059E-10 |
| 7,99999999989540 | 1,046E-10 |
| 8,99999999989408 | 1,059E-10 |
| 9,99999999989114 | 1,089E-10 |
| 10,99999999988890 | 1,111E-10 |
| 11,99999999988790 | 1,121E-10 |
| 1,31 | 0,99999999990342 | 9,658E-11 | 4900 | 9,96E-14 | 147,575272 |
| 1,99999999989288 | 1,071E-10 |
| 2,99999999989049 | 1,095E-10 |
| 3,99999999988846 | 1,115E-10 |
| 4,99999999988791 | 1,121E-10 |
| 5,99999999988867 | 1,113E-10 |
| 6,99999999989403 | 1,060E-10 |
| 7,99999999989529 | 1,047E-10 |
| 8,99999999989396 | 1,060E-10 |
| 9,99999999989103 | 1,090E-10 |
| 10,99999999988880 | 1,112E-10 |
| 11,99999999988780 | 1,122E-10 |
| 1,32 | 0,99999999990353 | 9,648E-11 | 4863 | 9,94E-14 | 147,743074 |
| 1,99999999989300 | 1,070E-10 |
| 2,99999999989061 | 1,094E-10 |
| 3,99999999988858 | 1,114E-10 |
| 4,99999999988804 | 1,120E-10 |
| 5,99999999988879 | 1,112E-10 |
| 6,99999999989415 | 1,059E-10 |
| 7,99999999989540 | 1,046E-10 |
| 8,99999999989408 | 1,059E-10 |
| 9,99999999989114 | 1,089E-10 |
| 10,99999999988890 | 1,111E-10 |
| 11,99999999988790 | 1,121E-10 |

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| --- | --- | --- | --- | --- | --- |
| Метод Зейделя | | | | | |
| w | x | x\* - x | Число итераций | Относительная невязка | Число обусловленности |
| 0.1 | 0,999999999907378 | 9,262E-11 | 61305 | 1,00E-13 | 140,970811 |
| 1,999999999897270 | 1,027E-10 |
| 2,999999999894990 | 1,050E-10 |
| 3,999999999893040 | 1,070E-10 |
| 4,999999999892530 | 1,075E-10 |
| 5,999999999893260 | 1,067E-10 |
| 6,999999999898390 | 1,016E-10 |
| 7,999999999899600 | 1,004E-10 |
| 8,999999999898330 | 1,017E-10 |
| 9,999999999895530 | 1,045E-10 |
| 10,999999999893400 | 1,066E-10 |
| 11,999999999892400 | 1,076E-10 |
| 0.2 | 0,999999999912225 | 8,778E-11 | 29130 | 1,00E-13 | 133,542887 |
| 1,999999999902650 | 9,735E-11 |
| 2,999999999900500 | 9,950E-11 |
| 3,999999999898660 | 1,013E-10 |
| 4,999999999898190 | 1,018E-10 |
| 5,999999999898880 | 1,011E-10 |
| 6,999999999903730 | 9,627E-11 |
| 7,999999999904880 | 9,512E-11 |
| 8,999999999903680 | 9,632E-11 |
| 9,999999999901030 | 9,897E-11 |
| 10,999999999899000 | 1,010E-10 |
| 11,999999999898100 | 1,019E-10 |
| 0.3 | 0,999999999917139 | 8,286E-11 | 18404 | 1,00E-13 | 126,069269 |
| 1,999999999908110 | 9,189E-11 |
| 2,999999999906080 | 9,392E-11 |
| 3,999999999904360 | 9,564E-11 |
| 4,999999999903920 | 9,608E-11 |
| 5,999999999904580 | 9,542E-11 |
| 6,999999999909150 | 9,085E-11 |
| 7,999999999910230 | 8,977E-11 |
| 8,999999999909110 | 9,089E-11 |
| 9,999999999906620 | 9,338E-11 |
| 10,999999999904700 | 9,530E-11 |
| 11,999999999903900 | 9,610E-11 |
| 0.4 | 0,999999999922053 | 7,795E-11 | 13039 | 9,98E-14 | 118,705630 |
| 1,999999999913570 | 8,643E-11 |
| 2,999999999911670 | 8,833E-11 |
| 3,999999999910060 | 8,994E-11 |
| 4,999999999909650 | 9,035E-11 |
| 5,999999999910280 | 8,972E-11 |
| 6,999999999914560 | 8,544E-11 |
| 7,999999999915580 | 8,442E-11 |
| 8,999999999914530 | 8,547E-11 |
| 9,999999999912210 | 8,779E-11 |
| 10,999999999910400 | 8,960E-11 |
| 11,999999999909600 | 9,040E-11 |
| 0.5 | 0,999999999926799 | 7,320E-11 | 9817 | 9,99E-14 | 111,291458 |
| 1,999999999918840 | 8,116E-11 |
| 2,999999999917070 | 8,293E-11 |
| 3,999999999915560 | 8,444E-11 |
| 4,999999999915190 | 8,481E-11 |
| 5,999999999915780 | 8,422E-11 |
| 6,999999999919790 | 8,021E-11 |
| 7,999999999920750 | 7,925E-11 |
| 8,999999999919770 | 8,023E-11 |
| 9,999999999917600 | 8,240E-11 |
| 10,999999999915900 | 8,410E-11 |
| 11,999999999915200 | 8,480E-11 |
| 0.6 | 0,999999999931696 | 6,830E-11 | 7668 | 9,99E-14 | 103,846671 |
| 1,999999999924280 | 7,572E-11 |
| 2,999999999922630 | 7,737E-11 |
| 3,999999999921230 | 7,877E-11 |
| 4,999999999920900 | 7,910E-11 |
| 5,999999999921460 | 7,854E-11 |
| 6,999999999925190 | 7,481E-11 |
| 7,999999999926080 | 7,392E-11 |
| 8,999999999925170 | 7,483E-11 |
| 9,999999999923160 | 7,684E-11 |
| 10,999999999921600 | 7,840E-11 |
| 11,999999999920900 | 7,910E-11 |
| 0.7 | 0,999999999936538 | 6,346E-11 | 6131 | 9,99E-14 | 96,435102 |
| 1,999999999929660 | 7,034E-11 |
| 2,999999999928140 | 7,186E-11 |
| 3,999999999926840 | 7,316E-11 |
| 4,999999999926550 | 7,345E-11 |
| 5,999999999927070 | 7,293E-11 |
| 6,999999999930520 | 6,948E-11 |
| 7,999999999931360 | 6,864E-11 |
| 8,999999999930520 | 6,948E-11 |
| 9,999999999928670 | 7,133E-11 |
| 10,999999999927200 | 7,280E-11 |
| 11,999999999926600 | 7,340E-11 |
| 0.8 | 0,999999999941514 | 5,849E-11 | 4977 | 9,98E-14 | 88,942387 |
| 1,999999999935180 | 6,482E-11 |
| 2,999999999933790 | 6,621E-11 |
| 3,999999999932610 | 6,739E-11 |
| 4,999999999932350 | 6,765E-11 |
| 5,999999999932840 | 6,716E-11 |
| 6,999999999936010 | 6,399E-11 |
| 7,999999999936770 | 6,323E-11 |
| 8,999999999936010 | 6,399E-11 |
| 9,999999999934320 | 6,568E-11 |
| 10,999999999932900 | 6,710E-11 |
| 11,999999999932400 | 6,760E-11 |
| 0.9 | 0,999999999946227 | 5,377E-11 | 4077 | 9,99E-14 | 81,573649 |
| 1,999999999940420 | 5,958E-11 |
| 2,999999999939150 | 6,085E-11 |
| 3,999999999938080 | 6,192E-11 |
| 4,999999999937850 | 6,215E-11 |
| 5,999999999938310 | 6,169E-11 |
| 6,999999999941200 | 5,880E-11 |
| 7,999999999941910 | 5,809E-11 |
| 8,999999999941210 | 5,879E-11 |
| 9,999999999939680 | 6,032E-11 |
| 10,999999999938400 | 6,160E-11 |
| 11,999999999938000 | 6,200E-11 |
| 1,0 | 0,999999999951218 | 4,878E-11 | 3356 | 9,97E-14 | 74,089364 |
| 1,999999999945960 | 5,404E-11 |
| 2,999999999944830 | 5,517E-11 |
| 3,999999999943860 | 5,614E-11 |
| 4,999999999943670 | 5,633E-11 |
| 5,999999999944090 | 5,591E-11 |
| 6,999999999946700 | 5,330E-11 |
| 7,999999999947340 | 5,266E-11 |
| 8,999999999946710 | 5,329E-11 |
| 9,999999999945350 | 5,465E-11 |
| 10,999999999944200 | 5,580E-11 |
| 11,999999999943800 | 5,620E-11 |
| 1,1 | 0,999999999956078 | 4,392E-11 | 2764 | 9,97E-14 | 66,676810 |
| 1,999999999951360 | 4,864E-11 |
| 2,999999999950350 | 4,965E-11 |
| 3,999999999949490 | 5,051E-11 |
| 4,999999999949340 | 5,066E-11 |
| 5,999999999949730 | 5,027E-11 |
| 6,999999999952060 | 4,794E-11 |
| 7,999999999952640 | 4,736E-11 |
| 8,999999999952070 | 4,793E-11 |
| 9,999999999950870 | 4,913E-11 |
| 10,999999999949800 | 5,020E-11 |
| 11,999999999949500 | 5,050E-11 |
| 1,2 | 0,999999999961076 | 3,892E-11 | 2269 | 9,94E-14 | 59,154587 |
| 1,999999999956910 | 4,309E-11 |
| 2,999999999956030 | 4,397E-11 |
| 3,999999999955280 | 4,472E-11 |
| 4,999999999955170 | 4,483E-11 |
| 5,999999999955520 | 4,448E-11 |
| 6,999999999957560 | 4,244E-11 |
| 7,999999999958080 | 4,192E-11 |
| 8,999999999957590 | 4,241E-11 |
| 9,999999999956550 | 4,345E-11 |
| 10,999999999955600 | 4,440E-11 |
| 11,999999999955300 | 4,470E-11 |
| 1,3 | 0,999999999966099 | 3,390E-11 | 1848 | 9,90E-14 | 51,650098 |
| 1,999999999962490 | 3,751E-11 |
| 2,999999999961740 | 3,826E-11 |
| 3,999999999961100 | 3,890E-11 |
| 4,999999999961030 | 3,897E-11 |
| 5,999999999961340 | 3,866E-11 |
| 6,999999999963090 | 3,691E-11 |
| 7,999999999963540 | 3,646E-11 |
| 8,999999999963130 | 3,687E-11 |
| 9,999999999962250 | 3,775E-11 |
| 10,999999999961400 | 3,860E-11 |
| 11,999999999961200 | 3,880E-11 |
| 1,31 | 0,999999999966462 | 3,354E-11 | 1809 | 9,92E-14 | 50,994647 |
| 1,999999999962890 | 3,711E-11 |
| 2,999999999962150 | 3,785E-11 |
| 3,999999999961530 | 3,847E-11 |
| 4,999999999961450 | 3,855E-11 |
| 5,999999999961760 | 3,824E-11 |
| 6,999999999963490 | 3,651E-11 |
| 7,999999999963940 | 3,606E-11 |
| 8,999999999963530 | 3,647E-11 |
| 9,999999999962670 | 3,733E-11 |
| 10,999999999961900 | 3,810E-11 |
| 11,999999999961700 | 3,830E-11 |
| 1,32 | 0,999999999967033 | 3,297E-11 | 1771 | 9,90E-14 | 50,217245 |
| 1,999999999963530 | 3,647E-11 |
| 2,999999999962800 | 3,720E-11 |
| 3,999999999962190 | 3,781E-11 |
| 4,999999999962120 | 3,788E-11 |
| 5,999999999962420 | 3,758E-11 |
| 6,999999999964120 | 3,588E-11 |
| 7,999999999964560 | 3,544E-11 |
| 8,999999999964160 | 3,584E-11 |
| 9,999999999963310 | 3,669E-11 |
| 10,999999999962500 | 3,750E-11 |
| 11,999999999962300 | 3,770E-11 |
| 1,40 | 0,999999999970827 | 2,917E-11 | 1484 | 9,94E-14 | 44,199045 |
| 1,999999999967740 | 3,226E-11 |
| 2,999999999967120 | 3,288E-11 |
| 3,999999999966590 | 3,341E-11 |
| 4,999999999966540 | 3,346E-11 |
| 5,999999999966820 | 3,318E-11 |
| 6,999999999968300 | 3,170E-11 |
| 7,999999999968690 | 3,131E-11 |
| 8,999999999968350 | 3,165E-11 |
| 9,999999999967620 | 3,238E-11 |
| 10,999999999966900 | 3,310E-11 |
| 11,999999999966800 | 3,320E-11 |
| 1,50 | 0,999999999975968 | 2,403E-11 | 1166 | 9,84E-14 | 36,648244 |
| 1,999999999973450 | 2,655E-11 |
| 2,999999999972960 | 2,704E-11 |
| 3,999999999972540 | 2,746E-11 |
| 4,999999999972530 | 2,747E-11 |
| 5,999999999972770 | 2,723E-11 |
| 6,999999999973970 | 2,603E-11 |
| 7,999999999974290 | 2,571E-11 |
| 8,999999999974020 | 2,598E-11 |
| 9,999999999973460 | 2,654E-11 |
| 10,999999999972900 | 2,710E-11 |
| 11,999999999972800 | 2,720E-11 |
| 1,60 | 0,999999999980942 | 1,906E-11 | 883 | 9,80E-14 | 29,032770 |
| 1,999999999978970 | 2,103E-11 |
| 2,999999999978610 | 2,139E-11 |
| 3,999999999978300 | 2,170E-11 |
| 4,999999999978330 | 2,167E-11 |
| 5,999999999978530 | 2,147E-11 |
| 6,999999999979440 | 2,056E-11 |
| 7,999999999979700 | 2,030E-11 |
| 8,999999999979500 | 2,050E-11 |
| 9,999999999979100 | 2,090E-11 |
| 10,999999999978600 | 2,140E-11 |
| 11,999999999978600 | 2,140E-11 |
| 1,70 | 0,999999999985868 | 1,413E-11 | 626 | 9,83E-14 | 21,291060 |
| 1,999999999984440 | 1,556E-11 |
| 2,999999999984210 | 1,579E-11 |
| 3,999999999984010 | 1,599E-11 |
| 4,999999999984080 | 1,592E-11 |
| 5,999999999984240 | 1,576E-11 |
| 6,999999999984870 | 1,513E-11 |
| 7,999999999985060 | 1,494E-11 |
| 8,999999999984930 | 1,507E-11 |
| 9,999999999984700 | 1,530E-11 |
| 10,999999999984300 | 1,570E-11 |
| 11,999999999984400 | 1,560E-11 |
| 1,74 | 0,999999999987887 | 1,211E-11 | 528 | 9,85E-14 | 18,104667 |
| 1,999999999986700 | 1,330E-11 |
| 2,999999999986520 | 1,348E-11 |
| 3,999999999986370 | 1,363E-11 |
| 4,999999999986440 | 1,356E-11 |
| 5,999999999986590 | 1,341E-11 |
| 6,999999999987100 | 1,290E-11 |
| 7,999999999987270 | 1,273E-11 |
| 8,999999999987170 | 1,283E-11 |
| 9,999999999987000 | 1,300E-11 |
| 10,999999999986700 | 1,330E-11 |
| 11,999999999986700 | 1,330E-11 |
| 1,75 | 0,999999999988747 | 1,125E-11 | 504 | 9,95E-14 | 16,838373 |
| 1,999999999987410 | 1,259E-11 |
| 2,999999999987320 | 1,268E-11 |
| 3,999999999987190 | 1,281E-11 |
| 4,999999999987280 | 1,272E-11 |
| 5,999999999987370 | 1,263E-11 |
| 6,999999999987830 | 1,217E-11 |
| 7,999999999988070 | 1,193E-11 |
| 8,999999999987940 | 1,206E-11 |
| 9,999999999987820 | 1,218E-11 |
| 10,999999999987400 | 1,260E-11 |
| 11,999999999987600 | 1,240E-11 |
| 1,76 | 0,999999999999035 | 9,650E-13 | 589 | 9,90E-14 | 0,500111 |
| 2,000000000000410 | -4,099E-13 |
| 2,999999999999940 | 5,995E-14 |
| 3,999999999999880 | 1,199E-13 |
| 4,999999999999800 | 1,998E-13 |
| 6,000000000000180 | -1,803E-13 |
| 7,000000000000210 | -2,096E-13 |
| 7,999999999999750 | 2,496E-13 |
| 8,999999999999970 | 3,020E-14 |
| 9,999999999999780 | 2,203E-13 |
| 11,000000000000300 | -3,002E-13 |
| 11,999999999999600 | 3,997E-13 |
| 1,8 | 1,000000000000980 | -9,801E-13 | 17724 | 9,97E-14 | 0,518447 |
| 1,999999999999540 | 4,601E-13 |
| 3,000000000000030 | -3,020E-14 |
| 4,000000000000100 | -1,004E-13 |
| 5,000000000000190 | -1,901E-13 |
| 5,999999999999780 | 2,203E-13 |
| 6,999999999999750 | 2,496E-13 |
| 8,000000000000240 | -2,398E-13 |
| 9,000000000000010 | 0,000E+00 |
| 10,000000000000200 | -2,007E-13 |
| 10,999999999999600 | 3,997E-13 |
| 12,000000000000300 | -3,002E-13 |

В силу того, что у данной матрицы СЛАУ имеет строгое диагональное преобладание, то этого свойства достаточно для сходимости методов Якоби и Зейделе.

К тому же стоит заметить, что из-за того, что матрица обладает данным свойством метод Зейделя сходится быстрее чем, метод Якоби



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| w | Метод Якоби | | | | |
| x | x\* - x | Число итераций | Отн. невязка | Число обусловленности |
| 0.1 | 1,00000000000438 | -4,380E-12 | 514 | 9,83E-14 | 4,050466 |
| 1,99999999999701 | 2,990E-12 |
| 3,00000000000172 | -1,720E-12 |
| 4,00000000000003 | -3,020E-14 |
| 5,00000000000012 | -1,199E-13 |
| 6,00000000000319 | -3,190E-12 |
| 6,99999999999312 | 6,880E-12 |
| 8,00000000000338 | -3,380E-12 |
| 8,99999999999884 | 1,160E-12 |
| 10,00000000000050 | -4,992E-13 |
| 10,99999999999920 | 7,994E-13 |
| 11,99999999999920 | 7,994E-13 |
| 0.2 | 1,00000000000409 | -4,090E-12 | 251 | 9,25E-14 | 3,841535 |
| 1,99999999999721 | 2,790E-12 |
| 3,00000000000161 | -1,610E-12 |
| 4,00000000000003 | -3,020E-14 |
| 5,00000000000010 | -1,004E-13 |
| 6,00000000000298 | -2,980E-12 |
| 6,99999999999357 | 6,430E-12 |
| 8,00000000000315 | -3,149E-12 |
| 8,99999999999892 | 1,080E-12 |
| 10,00000000000050 | -4,992E-13 |
| 10,99999999999920 | 7,994E-13 |
| 11,99999999999930 | 6,999E-13 |
| 0.3 | 1,00000000000396 | -3,960E-12 | 163 | 9,91E-14 | 3,630505 |
| 1,99999999999730 | 2,700E-12 |
| 3,00000000000156 | -1,560E-12 |
| 4,00000000000004 | -3,997E-14 |
| 5,00000000000009 | -8,971E-14 |
| 6,00000000000289 | -2,890E-12 |
| 6,99999999999378 | 6,220E-12 |
| 8,00000000000305 | -3,050E-12 |
| 8,99999999999895 | 1,050E-12 |
| 10,00000000000050 | -4,992E-13 |
| 10,99999999999920 | 7,994E-13 |
| 11,99999999999930 | 6,999E-13 |
| 0.4 | 1,00000000000303 | -3,030E-12 | 120 | 8,04E-14 | 3,420731 |
| 1,99999999999794 | 2,060E-12 |
| 3,00000000000119 | -1,190E-12 |
| 4,00000000000003 | -3,020E-14 |
| 5,00000000000006 | -6,040E-14 |
| 6,00000000000221 | -2,210E-12 |
| 6,99999999999524 | 4,760E-12 |
| 8,00000000000233 | -2,331E-12 |
| 8,99999999999920 | 7,994E-13 |
| 10,00000000000030 | -3,002E-13 |
| 10,99999999999940 | 6,004E-13 |
| 11,99999999999950 | 4,992E-13 |
| 0.5 | 1,00000000000314 | -3,140E-12 | 93 | 8,89E-14 | 3,209332 |
| 1,99999999999786 | 2,140E-12 |
| 3,00000000000124 | -1,240E-12 |
| 4,00000000000004 | -3,997E-14 |
| 5,00000000000006 | -6,040E-14 |
| 6,00000000000229 | -2,290E-12 |
| 6,99999999999507 | 4,930E-12 |
| 8,00000000000242 | -2,419E-12 |
| 8,99999999999917 | 8,296E-13 |
| 10,00000000000030 | -3,002E-13 |
| 10,99999999999940 | 6,004E-13 |
| 11,99999999999940 | 6,004E-13 |
| 0.6 | 1,00000000000321 | -3,210E-12 | 75 | 9,70E-14 | 3,163526 |
| 1,99999999999782 | 2,180E-12 |
| 3,00000000000127 | -1,270E-12 |
| 4,00000000000005 | -4,974E-14 |
| 5,00000000000005 | -4,974E-14 |
| 6,00000000000234 | -2,340E-12 |
| 6,99999999999497 | 5,030E-12 |
| 8,00000000000247 | -2,469E-12 |
| 8,99999999999915 | 8,509E-13 |
| 10,00000000000040 | -3,997E-13 |
| 10,99999999999930 | 6,999E-13 |
| 11,99999999999940 | 6,004E-13 |
| 0.7 | 1,00000000000223 | -2,230E-12 | 63 | 7,26E-14 | 3,000586 |
| 1,99999999999848 | 1,520E-12 |
| 3,00000000000088 | -8,802E-13 |
| 4,00000000000003 | -3,020E-14 |
| 5,00000000000003 | -3,020E-14 |
| 6,00000000000162 | -1,620E-12 |
| 6,99999999999650 | 3,500E-12 |
| 8,00000000000171 | -1,711E-12 |
| 8,99999999999941 | 5,898E-13 |
| 10,00000000000020 | -2,007E-13 |
| 10,99999999999950 | 4,992E-13 |
| 11,99999999999960 | 3,997E-13 |
| 0,77 | 1,00000000000208 | -2,080E-12 | 56 | 7,19E-14 | 2,630843 |
| 1,99999999999858 | 1,420E-12 |
| 3,00000000000082 | -8,198E-13 |
| 4,00000000000003 | -3,020E-14 |
| 5,00000000000002 | -2,043E-14 |
| 6,00000000000151 | -1,510E-12 |
| 6,99999999999672 | 3,280E-12 |
| 8,00000000000160 | -1,600E-12 |
| 8,99999999999944 | 5,596E-13 |
| 10,00000000000020 | -2,007E-13 |
| 10,99999999999950 | 4,992E-13 |
| 11,99999999999960 | 3,997E-13 |
| 0,78 | 1,00000000000223 | -2,230E-12 | 55 | 7,38E-14 | 2,669332 |
| 1,99999999999860 | 1,400E-12 |
| 3,00000000000093 | -9,299E-13 |
| 4,00000000000011 | -1,101E-13 |
| 5,00000000000010 | -1,004E-13 |
| 6,00000000000165 | -1,650E-12 |
| 6,99999999999667 | 3,330E-12 |
| 8,00000000000173 | -1,730E-12 |
| 8,99999999999950 | 4,992E-13 |
| 10,00000000000030 | -3,002E-13 |
| 10,99999999999960 | 3,997E-13 |
| 11,99999999999970 | 3,002E-13 |
| 0,79 | 1,00000000000059 | -5,900E-13 | 56 | 8,47E-14 | 1,012569 |
| 1,99999999999914 | 8,600E-13 |
| 3,00000000000005 | -5,018E-14 |
| 3,99999999999972 | 2,802E-13 |
| 4,99999999999971 | 2,904E-13 |
| 6,00000000000032 | -3,197E-13 |
| 6,99999999999838 | 1,620E-12 |
| 8,00000000000037 | -3,695E-13 |
| 8,99999999999949 | 5,098E-13 |
| 9,99999999999981 | 1,901E-13 |
| 10,99999999999950 | 4,992E-13 |
| 11,99999999999950 | 4,992E-13 |
| 0,80 | 0,99999999999986 | 1,440E-13 | 60 | 6,14E-14 | 0,546903 |
| 1,99999999999971 | 2,900E-13 |
| 2,99999999999978 | 2,198E-13 |
| 3,99999999999975 | 2,500E-13 |
| 4,99999999999975 | 2,496E-13 |
| 5,99999999999980 | 1,998E-13 |
| 6,99999999999964 | 3,597E-13 |
| 7,99999999999982 | 1,803E-13 |
| 8,99999999999974 | 2,593E-13 |
| 9,99999999999976 | 2,398E-13 |
| 10,99999999999970 | 3,002E-13 |
| 11,99999999999970 | 3,002E-13 |
| 0,90 | 1,00000000000043 | -4,301E-13 | 133 | 9,21E-14 | 0,724932 |
| 2,00000000000048 | -4,801E-13 |
| 3,00000000000049 | -4,898E-13 |
| 4,00000000000050 | -5,000E-13 |
| 5,00000000000051 | -5,098E-13 |
| 6,00000000000050 | -5,000E-13 |
| 7,00000000000048 | -4,796E-13 |
| 8,00000000000047 | -4,707E-13 |
| 9,00000000000048 | -4,796E-13 |
| 10,00000000000040 | -3,997E-13 |
| 11,00000000000050 | -4,992E-13 |
| 12,00000000000050 | -4,992E-13 |
| 0,94 | 0,99999999999948 | 5,190E-13 | 228 | 9,90E-14 | 0,797250 |
| 1,99999999999942 | 5,800E-13 |
| 2,99999999999941 | 5,902E-13 |
| 3,99999999999940 | 6,000E-13 |
| 4,99999999999939 | 6,102E-13 |
| 5,99999999999940 | 6,004E-13 |
| 6,99999999999943 | 5,702E-13 |
| 7,99999999999943 | 5,702E-13 |
| 8,99999999999943 | 5,702E-13 |
| 9,99999999999941 | 5,898E-13 |
| 10,99999999999940 | 6,004E-13 |
| 11,99999999999930 | 6,999E-13 |
| 0,95 | 1,00000000000051 | -5,100E-13 | 275 | 9,60E-14 | 0,815697 |
| 2,00000000000057 | -5,702E-13 |
| 3,00000000000058 | -5,800E-13 |
| 4,00000000000059 | -5,898E-13 |
| 5,00000000000059 | -5,898E-13 |
| 6,00000000000059 | -5,898E-13 |
| 7,00000000000056 | -5,604E-13 |
| 8,00000000000055 | -5,507E-13 |
| 9,00000000000056 | -5,596E-13 |
| 10,00000000000050 | -4,992E-13 |
| 11,00000000000050 | -4,992E-13 |
| 12,00000000000050 | -4,992E-13 |
| 1,00 | 0,99999999999941 | 5,950E-13 | 7738 | 9,99E-14 | 0,906633 |
| 1,99999999999934 | 6,599E-13 |
| 2,99999999999932 | 6,799E-13 |
| 3,99999999999931 | 6,901E-13 |
| 4,99999999999931 | 6,901E-13 |
| 5,99999999999931 | 6,901E-13 |
| 6,99999999999934 | 6,599E-13 |
| 7,99999999999935 | 6,501E-13 |
| 8,99999999999934 | 6,608E-13 |
| 9,99999999999932 | 6,803E-13 |
| 10,99999999999930 | 6,999E-13 |
| 11,99999999999930 | 6,999E-13 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| w | Метод Зейделя | | | | |
| x | x\* - x | Число итераций | Отн. невязка |  |
| 0.1 | 1,000000000004350 | -4,350E-12 | 503 | 9,64E-14 | 3,953993 |
| 1,999999999997180 | 2,820E-12 |
| 3,000000000001600 | -1,600E-12 |
| 4,000000000000090 | -8,971E-14 |
| 4,999999999999990 | 9,770E-15 |
| 6,000000000003170 | -3,170E-12 |
| 6,999999999993440 | 6,560E-12 |
| 8,000000000003080 | -3,080E-12 |
| 8,999999999998900 | 1,100E-12 |
| 10,000000000000500 | -4,992E-13 |
| 10,999999999999200 | 7,994E-13 |
| 11,999999999999300 | 6,999E-13 |
| 0.2 | 1,000000000004280 | -4,280E-12 | 239 | 9,91E-14 | 3,637964 |
| 1,999999999997380 | 2,620E-12 |
| 3,000000000001460 | -1,460E-12 |
| 4,000000000000160 | -1,599E-13 |
| 4,999999999999850 | 1,501E-13 |
| 6,000000000003110 | -3,110E-12 |
| 6,999999999993840 | 6,160E-12 |
| 8,000000000002760 | -2,760E-12 |
| 8,999999999998960 | 1,039E-12 |
| 10,000000000000400 | -3,997E-13 |
| 10,999999999999300 | 6,999E-13 |
| 11,999999999999400 | 6,004E-13 |
| 0.3 | 1,000000000003900 | -3,900E-12 | 151 | 9,52E-14 | 3,315200 |
| 1,999999999997750 | 2,250E-12 |
| 3,000000000001230 | -1,230E-12 |
| 4,000000000000220 | -2,203E-13 |
| 4,999999999999740 | 2,602E-13 |
| 6,000000000002820 | -2,820E-12 |
| 6,999999999994670 | 5,330E-12 |
| 8,000000000002260 | -2,260E-12 |
| 8,999999999999090 | 9,095E-13 |
| 10,000000000000300 | -3,002E-13 |
| 10,999999999999400 | 6,004E-13 |
| 11,999999999999600 | 3,997E-13 |
| 0.4 | 1,000000000003190 | -3,190E-12 | 107 | 8,32E-14 | 2,979451 |
| 1,999999999998290 | 1,710E-12 |
| 3,000000000000920 | -9,202E-13 |
| 4,000000000000250 | -2,496E-13 |
| 4,999999999999670 | 3,304E-13 |
| 6,000000000002290 | -2,290E-12 |
| 6,999999999995880 | 4,120E-12 |
| 8,000000000001640 | -1,640E-12 |
| 8,999999999999290 | 7,105E-13 |
| 10,000000000000200 | -2,007E-13 |
| 10,999999999999500 | 4,992E-13 |
| 11,999999999999700 | 3,002E-13 |
| 0.5 | 1,000000000002720 | -2,720E-12 | 80 | 7,68E-14 | 2,633637 |
| 1,999999999998650 | 1,350E-12 |
| 3,000000000000710 | -7,101E-13 |
| 4,000000000000290 | -2,904E-13 |
| 4,999999999999620 | 3,801E-13 |
| 6,000000000001940 | -1,940E-12 |
| 6,999999999996700 | 3,300E-12 |
| 8,000000000001230 | -1,229E-12 |
| 8,999999999999420 | 5,809E-13 |
| 10,000000000000200 | -2,007E-13 |
| 10,999999999999600 | 3,997E-13 |
| 11,999999999999800 | 2,007E-13 |
| 0.6 | 1,000000000002870 | -2,870E-12 | 61 | 8,98E-14 | 2,277455 |
| 1,999999999998690 | 1,310E-12 |
| 3,000000000000670 | -6,701E-13 |
| 4,000000000000400 | -3,997E-13 |
| 4,999999999999470 | 5,302E-13 |
| 6,000000000002020 | -2,020E-12 |
| 6,999999999996760 | 3,240E-12 |
| 8,000000000001130 | -1,130E-12 |
| 8,999999999999400 | 6,004E-13 |
| 10,000000000000100 | -9,948E-14 |
| 10,999999999999600 | 3,997E-13 |
| 11,999999999999900 | 9,948E-14 |
| 0.7 | 1,000000000002650 | -2,650E-12 | 47 | 9,54E-14 | 1,917268 |
| 1,999999999998870 | 1,130E-12 |
| 3,000000000000590 | -5,902E-13 |
| 4,000000000000530 | -5,302E-13 |
| 4,999999999999390 | 6,102E-13 |
| 6,000000000001820 | -1,820E-12 |
| 6,999999999997210 | 2,790E-12 |
| 8,000000000000920 | -9,202E-13 |
| 8,999999999999480 | 5,205E-13 |
| 10,000000000000100 | -9,948E-14 |
| 10,999999999999600 | 3,997E-13 |
| 11,999999999999900 | 9,948E-14 |
| 0,77 | 1,000000000001640 | -1,640E-12 | 39 | 6,79E-14 | 1,659493 |
| 1,999999999999230 | 7,701E-13 |
| 3,000000000000410 | -4,099E-13 |
| 4,000000000000500 | -5,000E-13 |
| 4,999999999999640 | 3,597E-13 |
| 6,000000000001070 | -1,070E-12 |
| 6,999999999998320 | 1,680E-12 |
| 8,000000000000540 | -5,400E-13 |
| 8,999999999999760 | 2,398E-13 |
| 10,000000000000000 | 0,000E+00 |
| 10,999999999999600 | 3,997E-13 |
| 11,999999999999900 | 9,948E-14 |
| 0,78 | 1,000000000001290 | -1,290E-12 | 38 | 5,53E-14 | 1,608260 |
| 1,999999999999360 | 6,399E-13 |
| 3,000000000000340 | -3,402E-13 |
| 4,000000000000430 | -4,299E-13 |
| 4,999999999999760 | 2,398E-13 |
| 6,000000000000820 | -8,198E-13 |
| 6,999999999998680 | 1,320E-12 |
| 8,000000000000420 | -4,192E-13 |
| 8,999999999999850 | 1,492E-13 |
| 10,000000000000000 | 0,000E+00 |
| 10,999999999999700 | 3,002E-13 |
| 11,999999999999900 | 9,948E-14 |
| 0,79 | 1,000000000001670 | -1,670E-12 | 36 | 8,41E-14 | 1,511881 |
| 1,999999999998900 | 1,100E-12 |
| 3,000000000000680 | -6,799E-13 |
| 4,000000000000820 | -8,198E-13 |
| 4,999999999999760 | 2,398E-13 |
| 6,000000000000960 | -9,601E-13 |
| 6,999999999998110 | 1,890E-12 |
| 8,000000000000680 | -6,803E-13 |
| 8,999999999999890 | 1,101E-13 |
| 9,999999999999920 | 7,994E-14 |
| 10,999999999999400 | 6,004E-13 |
| 11,999999999999900 | 9,948E-14 |
| 0,80 | 1,000000000000750 | -7,501E-13 | 35 | 6,63E-14 | 1,134839 |
| 1,999999999999150 | 8,500E-13 |
| 3,000000000000550 | -5,498E-13 |
| 4,000000000000650 | -6,501E-13 |
| 5,000000000000130 | -1,297E-13 |
| 6,000000000000310 | -3,100E-13 |
| 6,999999999998970 | 1,030E-12 |
| 8,000000000000400 | -3,997E-13 |
| 9,000000000000140 | -1,403E-13 |
| 9,999999999999840 | 1,599E-13 |
| 10,999999999999500 | 4,992E-13 |
| 11,999999999999800 | 2,007E-13 |
| 0,90 | 0,999999999998990 | 1,010E-12 | 30 | 5,37E-14 | 1,094722 |
| 2,000000000000240 | -2,398E-13 |
| 3,000000000000010 | -1,021E-14 |
| 3,999999999999770 | 2,300E-13 |
| 5,000000000000180 | -1,803E-13 |
| 5,999999999999280 | 7,203E-13 |
| 7,000000000000720 | -7,203E-13 |
| 7,999999999999900 | 1,004E-13 |
| 9,000000000000110 | -1,101E-13 |
| 9,999999999999960 | 4,086E-14 |
| 10,999999999999900 | 9,948E-14 |
| 12,000000000000000 | 0,000E+00 |
| 0,94 | 0,999999999999923 | 7,705E-14 | 28 | 3,82E-14 | 0,481761 |
| 1,999999999999840 | 1,601E-13 |
| 3,000000000000000 | 0,000E+00 |
| 4,000000000000000 | 0,000E+00 |
| 5,000000000000330 | -3,304E-13 |
| 5,999999999999910 | 8,971E-14 |
| 7,000000000000050 | -4,974E-14 |
| 7,999999999999900 | 1,004E-13 |
| 9,000000000000150 | -1,492E-13 |
| 9,999999999999970 | 3,020E-14 |
| 11,000000000000000 | 0,000E+00 |
| 11,999999999999800 | 2,007E-13 |
| 0,95 | 1,000000000000280 | -2,800E-13 | 27 | 7,42E-14 | 0,532124 |
| 1,999999999999650 | 3,499E-13 |
| 2,999999999999880 | 1,199E-13 |
| 4,000000000000000 | 0,000E+00 |
| 5,000000000000640 | -6,404E-13 |
| 6,000000000000160 | -1,599E-13 |
| 6,999999999999900 | 1,004E-13 |
| 7,999999999999740 | 2,602E-13 |
| 9,000000000000240 | -2,398E-13 |
| 10,000000000000000 | 0,000E+00 |
| 11,000000000000200 | -2,007E-13 |
| 11,999999999999500 | 4,992E-13 |
| 1,00 | 0,999999999999835 | 1,650E-13 | 27 | 5,17E-14 | 0,544727 |
| 1,999999999999700 | 3,000E-13 |
| 3,000000000000220 | -2,198E-13 |
| 4,000000000000360 | -3,597E-13 |
| 5,000000000000170 | -1,696E-13 |
| 5,999999999999830 | 1,696E-13 |
| 6,999999999999760 | 2,398E-13 |
| 8,000000000000090 | -9,059E-14 |
| 9,000000000000170 | -1,705E-13 |
| 9,999999999999920 | 7,994E-14 |
| 10,999999999999700 | 3,002E-13 |
| 11,999999999999800 | 2,007E-13 |
| 1,03 | 1,000000000000360 | -3,599E-13 | 27 | 7,59E-14 | 0,595526 |
| 2,000000000000410 | -4,099E-13 |
| 2,999999999999620 | 3,801E-13 |
| 3,999999999999420 | 5,800E-13 |
| 4,999999999999770 | 2,300E-13 |
| 6,000000000000300 | -3,002E-13 |
| 7,000000000000350 | -3,499E-13 |
| 7,999999999999830 | 1,696E-13 |
| 8,999999999999720 | 2,807E-13 |
| 10,000000000000100 | -9,948E-14 |
| 11,000000000000300 | -3,002E-13 |
| 12,000000000000100 | -9,948E-14 |
| 1,04 | 0,999999999999661 | 3,390E-13 | 28 | 2,32E-14 | 0,935955 |
| 2,000000000000010 | -1,021E-14 |
| 3,000000000000180 | -1,799E-13 |
| 4,000000000000110 | -1,101E-13 |
| 4,999999999999900 | 1,004E-13 |
| 5,999999999999740 | 2,602E-13 |
| 7,000000000000040 | -3,997E-14 |
| 8,000000000000130 | -1,297E-13 |
| 9,000000000000030 | -3,020E-14 |
| 9,999999999999920 | 7,994E-14 |
| 10,999999999999800 | 2,007E-13 |
| 12,000000000000000 | 0,000E+00 |
| 1,10 | 0,999999999998715 | 1,285E-12 | 29 | 8,16E-14 | 1,043781 |
| 1,999999999999920 | 7,994E-14 |
| 3,000000000000780 | -7,798E-13 |
| 4,000000000000610 | -6,102E-13 |
| 4,999999999999760 | 2,398E-13 |
| 5,999999999999020 | 9,797E-13 |
| 6,999999999999990 | 9,770E-15 |
| 8,000000000000530 | -5,294E-13 |
| 9,000000000000280 | -2,807E-13 |
| 9,999999999999650 | 3,499E-13 |
| 10,999999999999200 | 7,994E-13 |
| 12,000000000000000 | 0,000E+00 |
| 1,20 | 1,000000000000270 | -2,700E-13 | 36 | 4,37E-14 | 0,827822 |
| 2,000000000000290 | -2,900E-13 |
| 2,999999999999740 | 2,598E-13 |
| 3,999999999999540 | 4,601E-13 |
| 4,999999999999800 | 1,998E-13 |
| 6,000000000000160 | -1,599E-13 |
| 7,000000000000400 | -3,997E-13 |
| 7,999999999999910 | 8,971E-14 |
| 8,999999999999750 | 2,505E-13 |
| 10,000000000000000 | 0,000E+00 |
| 11,000000000000100 | -9,948E-14 |
| 12,000000000000200 | -2,007E-13 |
| 1,30 | 1,000000000001260 | -1,260E-12 | 44 | 9,02E-14 | 1,169580 |
| 1,999999999999870 | 1,299E-13 |
| 2,999999999998990 | 1,010E-12 |
| 3,999999999999550 | 4,499E-13 |
| 5,000000000000750 | -7,496E-13 |
| 6,000000000001240 | -1,240E-12 |
| 7,000000000000050 | -4,974E-14 |
| 7,999999999999420 | 5,800E-13 |
| 8,999999999999720 | 2,807E-13 |
| 10,000000000000600 | -6,004E-13 |
| 11,000000000001200 | -1,201E-12 |
| 11,999999999999800 | 2,007E-13 |
| 1,40 | 1,000000000001170 | -1,170E-12 | 57 | 9,25E-14 | 1,358172 |
| 1,999999999999560 | 4,401E-13 |
| 2,999999999998680 | 1,320E-12 |
| 3,999999999998650 | 1,350E-12 |
| 5,000000000000070 | -7,017E-14 |
| 6,000000000001240 | -1,240E-12 |
| 6,999999999999040 | 9,601E-13 |
| 7,999999999999150 | 8,500E-13 |
| 8,999999999999530 | 4,707E-13 |
| 10,000000000000700 | -6,999E-13 |
| 11,000000000000800 | -7,994E-13 |
| 12,000000000000700 | -6,999E-13 |
| 1,50 | 0,999999999998019 | 1,981E-12 | 78 | 8,51E-14 | 1,576368 |
| 2,000000000000020 | -1,998E-14 |
| 3,000000000000540 | -5,400E-13 |
| 3,999999999999400 | 6,000E-13 |
| 4,999999999998840 | 1,160E-12 |
| 5,999999999998710 | 1,290E-12 |
| 6,999999999999590 | 4,103E-13 |
| 8,000000000001020 | -1,020E-12 |
| 9,000000000000410 | -4,103E-13 |
| 9,999999999999290 | 7,105E-13 |
| 10,999999999998800 | 1,201E-12 |
| 12,000000000000900 | -9,006E-13 |
| 1,60 | 0,999999999997026 | 2,974E-12 | 115 | 9,79E-14 | 2,088675 |
| 1,999999999999220 | 7,800E-13 |
| 3,000000000000240 | -2,398E-13 |
| 3,999999999998610 | 1,390E-12 |
| 4,999999999998800 | 1,200E-12 |
| 5,999999999998930 | 1,070E-12 |
| 6,999999999997650 | 2,350E-12 |
| 8,000000000001670 | -1,670E-12 |
| 9,000000000000960 | -9,592E-13 |
| 9,999999999999400 | 6,004E-13 |
| 10,999999999998900 | 1,100E-12 |
| 12,000000000001500 | -1,499E-12 |
| 1,70 | 1,000000000003260 | -3,260E-12 | 216 | 8,76E-14 | 2,223261 |
| 2,000000000000570 | -5,702E-13 |
| 2,999999999999990 | 1,021E-14 |
| 4,000000000001180 | -1,180E-12 |
| 5,000000000000050 | -4,974E-14 |
| 6,000000000000720 | -7,203E-13 |
| 7,000000000001880 | -1,880E-12 |
| 7,999999999997630 | 2,370E-12 |
| 8,999999999998870 | 1,130E-12 |
| 10,000000000000400 | -3,997E-13 |
| 11,000000000000000 | 0,000E+00 |
| 11,999999999998900 | 1,100E-12 |
| 1,80 | 0,999999999997734 | 2,266E-12 | 1102 | 8,82E-14 | 2,791861 |
| 1,999999999998320 | 1,680E-12 |
| 3,000000000000420 | -4,201E-13 |
| 3,999999999998880 | 1,120E-12 |
| 4,999999999999220 | 7,798E-13 |
| 6,000000000000080 | -7,994E-14 |
| 6,999999999995340 | 4,660E-12 |
| 8,000000000001810 | -1,810E-12 |
| 9,000000000001460 | -1,460E-12 |
| 9,999999999999510 | 4,903E-13 |
| 10,999999999999200 | 7,994E-13 |
| 12,000000000001200 | -1,201E-12 |
| 1,82 | 1,000000000002320 | -2,320E-12 | 5313 | 9,59E-14 | 2,843768 |
| 2,000000000001870 | -1,870E-12 |
| 2,999999999999470 | 5,298E-13 |
| 4,000000000001150 | -1,150E-12 |
| 5,000000000000910 | -9,104E-13 |
| 5,999999999999780 | 2,203E-13 |
| 7,000000000005260 | -5,260E-12 |
| 7,999999999998080 | 1,920E-12 |
| 8,999999999998380 | 1,620E-12 |
| 10,000000000000500 | -4,992E-13 |
| 11,000000000000900 | -9,006E-13 |
| 11,999999999998600 | 1,400E-12 |

1. Метод Якоби сходится к решению системы в том и только в том случае, когда все корни уравнения



По модулю меньше единицы

Удостоверимся в этом:



Отсюда:

Следовательно, для данной матрицы, метод Якоби будет сходиться.

1. Метод Зейделя сходится к решению системы в том и только в том случае, когда все корни уравнения



По модулю меньше единицы

Удостоверимся в этом:



Отсюда:

Следовательно, для данной матрицы, метод Якоби будет сходиться.

**Выводы по работе:**

Метод Зейделя получает решение за меньшее число итераций по сравнению с методом Якоби. Это можно объяснить тем, что в методе Зейделя используются ранее вычисленные значения компонент вектора на этом же шаге итерации, тем самым полученные значения сразу же начинают использоваться для текущих подсчетов.

Коэффициент релаксации позволяет в некоторых случаях ускорить сходимость итерационного метода в десятки раз, но сложность его использования заключается в поиске оптимального w. В случае, когда необходимо решить одну СЛАУ, процедура поиска оптимального w будет излишней, но если необходимо вычислить множество близких СЛАУ, то это будет обосновано.

В каждом из методов есть значения коэффициента релаксации, при которых итерационный метод начинает расходится, хотя еще находится в области допустимых параметров. Это можно объяснить тем, что изначальные границы определены для нормальных (положительно-определённых симметричных) матриц

При оценке чисел обусловленности матрицы в методе Якоби в методе Зейделя, получились значения <1, чего не может быть. Но так как проверка представляет из себя оценку снизу, она может быть меньше единицы.

**Приложение**

**PZ2.cpp**

#include <iostream>

#include "matrix.h"

#include <math.h>

int main()

{

setlocale(LC\_ALL, "Russian");

FILE\* inputMatrix, \* inputVector, \* inputParam, \* outFile;

fopen\_s(&inputMatrix, "matrixDA2.txt", "r");

fopen\_s(&inputVector, "vectorDA2.txt", "r");

fopen\_s(&inputParam, "param.txt", "r");

fopen\_s(&outFile, "out.txt", "w+");

SLAE slae;

slae.Input(inputMatrix, inputVector, inputParam);

slae.OutputDense();

if (slae.method == 0) //Якоби

{

for (int i = 1; i <= 200; i++)

{

slae.w += 0.01;

slae.IterativeMethod(i);

slae.OutputSolutionVector(outFile);

}

}

else // Зейдель

{

for (int i = 1; i <= 200; i++)

{

slae.w += 0.01;

slae.IterativeMethod(i);

slae.OutputSolutionVector(outFile);

}

}

slae.OutputResultParametrs();

return 0;

}

**matrix.h**

#pragma once

#include <cstdio>

#include <math.h>

#include <iostream>

#define REALOUT "%.15lf\n"

#define REALOUTD "%.0lf\t"

#define EPS 1e-13

using namespace std;

class SLAE {

public:

int n, m, MaxNumOfIterations, method;

int\* NumOfIterationsDependingOnW;

double AccuracyOfTheSolution, w = 0.00;

double\* x, \*x0, \*b, \*vectorForDiscrepancy, \*local\_x0, \*TableOfNumOfConditionality, \*xtrue;

double\*\* matrix;

int TableOfShifts[9] = { -4, -3, -2, -1, 0, 1, 2, 3, 4 };

void Input(FILE\* matrixFile, FILE\* vectorFile, FILE\* paramFile);

void OutputDense();

void IterativeMethod(int NumOfW);

void VectorOutput(double\*curX);

void OutputSolutionVector(FILE\* out);

void InitializeShiftsTable();

double VectorNorm(double\* first);

double CalculateRelativeDiscrepancy(double\* first);

double CalculateNumOfConditionality(double RelativeDiscrepancy);

void MatrixVectorMultiplicationForDiscrepancy(double\* vectorMult);

void VecotorCopy(double\* first, double\* second);

void VecotorSubtract(double\* first, double\* second);

void OutputResultParametrs();

protected:

void AllocateMemory();

};

Matrix.cpp

#include "matrix.h"

void SLAE::IterativeMethod(int NumOfW)

{

double\* curX, \*prevX;

VecotorCopy(x0, local\_x0);

if (method == 0) //Якоби

{

prevX = local\_x0;

curX = x;

}

else { //Зейдель

curX = local\_x0;

prevX = local\_x0;

}

double normB = VectorNorm(b);

double RelativeDiscrepancy = CalculateRelativeDiscrepancy(x0);

int curIteration = 0;

double DiscrepancyF\_Ax = 0;

for (; curIteration < MaxNumOfIterations and RelativeDiscrepancy > AccuracyOfTheSolution; curIteration++)

{

DiscrepancyF\_Ax = 0;

for (int i = 0; i < n; i++)

{

int indX = 0;

double sum = 0;

for (int j = 0; j < 4; j++)

{

indX = i + TableOfShifts[j];

if (indX >= 0)

{

sum += prevX[indX] \* matrix[i][j];

}

}

sum += prevX[i] \* matrix[i][4];

for (int j = 5; j < 9; j++)

{

indX = i + TableOfShifts[j];

if (indX < n)

{

sum += prevX[indX] \* matrix[i][j];

}

}

curX[i] = prevX[i] + w \* (b[i] - sum) / matrix[i][4];

DiscrepancyF\_Ax += (b[i] - sum) \* (b[i] - sum);

}

std::swap(curX, prevX);

RelativeDiscrepancy = sqrt(DiscrepancyF\_Ax) / normB;

//printf\_s("%.15lf\n", RelativeDiscrepancy);

if (isinf(RelativeDiscrepancy) or isnan(RelativeDiscrepancy))

break;

}

VecotorCopy(prevX, x);

TableOfNumOfConditionality[NumOfW-1] = CalculateNumOfConditionality(RelativeDiscrepancy);

if (curIteration >= MaxNumOfIterations)

{

//printf("Выход по причиние выхода за макс. число итераций\n");

NumOfIterationsDependingOnW[NumOfW - 1] = -1;

}

else if (RelativeDiscrepancy < AccuracyOfTheSolution) {

//printf("Выход по причиние достигнутой желаемой погрешности\n");

printf("%.2lf\n", w);

printf("%E\n", RelativeDiscrepancy);

NumOfIterationsDependingOnW[NumOfW - 1] = curIteration;

printf("%d\n", curIteration);

VectorOutput(x);

}

else if (isnan(RelativeDiscrepancy)) {

//printf("Невязка nan\n");

NumOfIterationsDependingOnW[NumOfW - 1] = -2;

}

else if (isinf(RelativeDiscrepancy)) {

//printf("Невязка inf\n");

NumOfIterationsDependingOnW[NumOfW - 1] = -2;

}

}

void SLAE::Input(FILE\* matrixFile, FILE\* vectorFile, FILE\* paramFile)

{

fscanf\_s(matrixFile, "%d", &n);

fscanf\_s(matrixFile, "%d", &m);

fscanf\_s(paramFile, "%d", &method);

fscanf\_s(paramFile, "%lf", &AccuracyOfTheSolution);

fscanf\_s(paramFile, "%d", &MaxNumOfIterations);

AllocateMemory();

InitializeShiftsTable();

for (int Icount = 0; Icount < 9; Icount++)

{

int curDiag = TableOfShifts[Icount];

if (curDiag <= 0)

{

for (int i = abs(TableOfShifts[Icount]); i < n; i++)

{

fscanf\_s(matrixFile, "%lf", &matrix[i][Icount]);

}

}

else

{

for (int i = 0; i < (n - abs(curDiag)); i++)

{

fscanf\_s(matrixFile, "%lf", &matrix[i][Icount]);

}

}

}

for (int i = 0; i < n; i++)

fscanf\_s(vectorFile, "%lf", &b[i]);

for (int i = 0; i < n; i++)

fscanf\_s(vectorFile, "%lf", &x0[i]);

for (int i = 0; i < n; i++)

{

xtrue[i] = (double)(i+1);

}

}

void SLAE::AllocateMemory()

{

matrix = new double\* [n];

for (int i = 0; i < n; ++i) {

matrix[i] = new double[n];

}

b = new double[n];

x = new double[n];

x0 = new double[n];

xtrue = new double[n];

local\_x0 = new double[n];

vectorForDiscrepancy = new double[n];

NumOfIterationsDependingOnW = new int[200];

TableOfNumOfConditionality = new double[200];

}

void SLAE::MatrixVectorMultiplicationForDiscrepancy(double \*first)

{

for (int i = 0; i < n; i++)

{

int indX = 0;

double sum = 0;

for (int j = 5; j < 9; j++)

{

indX = i + TableOfShifts[j];

if (indX < n)

{

sum += first[indX] \* matrix[i][j];

}

}

sum += first[i] \* matrix[i][4];

for (int j = 0; j < 4; j++)

{

indX = i + TableOfShifts[j];

if (indX >= 0)

{

sum += first[indX] \* matrix[i][j];

}

}

vectorForDiscrepancy[i] = b[i] - sum;

//cout << vectorOut[i] << endl;

}

}

double SLAE::CalculateRelativeDiscrepancy(double\* first)

{

MatrixVectorMultiplicationForDiscrepancy(first);

return VectorNorm(vectorForDiscrepancy)/VectorNorm(b);

}

double SLAE::CalculateNumOfConditionality(double RelativeDiscrepancy)

{

VecotorSubtract(x, xtrue);

double VectorXRelDiscrepancy = VectorNorm(vectorForDiscrepancy)/VectorNorm(xtrue);

return VectorXRelDiscrepancy / RelativeDiscrepancy;

}

void SLAE::OutputDense()

{

double\*\* matrixDense;

matrixDense = new double\* [n];

for (int i = 0; i < n; ++i) {

matrixDense[i] = new double[n];

}

for (int i = 0; i < n; i++)

{

for (int j = 0; j < n; j++)

{

matrixDense[i][j] = 0.0;

}

}

for (int Icount = 0; Icount < 9; Icount++)

{

int curDiagonal = TableOfShifts[Icount];

if (curDiagonal <= 0)

{

for (int i = abs(curDiagonal), j = 0; i < n and j < n; i++, j++)

{

matrixDense[i][j] = matrix[i][Icount];

}

}

else {

for (int i = 0, j = abs(curDiagonal); i < n and j < n; i++, j++)

{

matrixDense[i][j] = matrix[i][Icount];

}

}

}

for (int i = 0; i < n; i++)

{

for (int j = 0; j < n; j++)

{

printf(REALOUTD, matrixDense[i][j]);

}

printf("\n");

}

}

double SLAE::VectorNorm(double\* first)

{

double norm = 0;

for (int i = 0; i < n; i++)

{

norm += first[i] \* first[i];

}

return sqrt(norm);

}

void SLAE::VecotorSubtract(double\* first, double\* second)

{

for (int i = 0; i < n; i++)

{

vectorForDiscrepancy[i] = first[i] - second[i];

}

}

void SLAE::VecotorCopy(double\* first, double\* second)

{

for (int i = 0; i < n; i++)

{

second[i] = first[i];

}

}

void SLAE::InitializeShiftsTable()

{

for (int i = 0; i < 3; i++)

{

TableOfShifts[i] -= m;

TableOfShifts[8 - i] += m;

}

}

void SLAE::OutputSolutionVector(FILE\* out) {

for (int i = 0; i < n; i++)

fprintf\_s(out, REALOUT, x[i]);

fprintf\_s(out, "\n");

}

void SLAE::VectorOutput(double\* curX)

{

for (int i = 0; i < n; i++)

printf\_s(REALOUT, curX[i]);

printf\_s("\n");

}

void SLAE::OutputResultParametrs()

{

for (int i = 1; i <= (200); i++)

{

printf("%.2lf ", 0.01 \* i);

printf("%d ", NumOfIterationsDependingOnW[i - 1]);

printf("%lf\n", TableOfNumOfConditionality[i - 1]);

}

}