W203, Test 1 Practice

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Q3.1 Joint PDF

$$f(x_1, x_2, x_3) = \begin{cases} 8 * x_1 * x_2 * x_3, \ 0 < x_1, x_2, x_3 < 0 \\ 0, \ elsewhere \end{cases}$$

Q3.2 Expected value of $A = 2X_1X_2^2 + 3X_2X_3^3$
Using LOTUS:

$$E[A] = \iiint_V A(x_1, x_2, x_3) * f(x_1, x_2, x_3)\, dx_1\, dx_2\, dx_3 =$$
$$\iiint_V 8(2x_1\, x_2^2 + 3x_2\, x_3^3) x_1\, x_2\, x_3\, dx_1\, dx_2\, dx_3 =$$
$$\iiint_V 16 x_1^2 x_2^3 x_3\, dx_1\, dx_2\, dx_3 + \iiint_V 24 x_1 x_2^2 x_3^4\, dx_1\, dx_2\, dx_3 =$$
$$\Big|_0^1 \frac{16 x_1^3 x_2^4 x_3^2}{3 * 4 * 2} + \Big|_0^1 \frac{24 x_1^2 x_2^3 x_3^5}{2 * 3 * 5} =$$
$$= \frac{2}{3} + \frac{3}{5} = 1\frac{4}{15}$$

Q3.3 Maximum Value
$Y = max[X_1, X_2, X_3] < y$

$$P(Y) = P(X_1 < y) \cap P(X_2 < y) \cap P(X_3 < y) =$$
because of independence
$$\prod_{i=1}^{3} \int_0^y f_{X_i}(x_i)\, dx_i = \prod_{i=1}^{3} \int_0^y 2x_i\, dx_i = \prod_{i=1}^{3} \Big|_0^y x_i^2\, dx_i = (y^2)^3$$
$$P(Y|y = 0.5) = \frac{1}{64}$$

Q3.4 New maximum
$P(Y|X_1 = 0)$

$$P(Y|X_1) = \frac{P(X_1 < y) \cap P(X_2 < y) \cap P(X_3 < y)}{P(X_1 < y)}$$
$$P(Y = 0.5|X_1 = 0) = \frac{P(0 < 0.5) \cap P(X_2 < 0.5) \cap P(X_3 < 0.5)}{P(0 < 0.5)} =$$
$$P(Y = 0.5|X_1 = 0) = P(X_2 < 0.5) \cap P(X_3 < 0.5) =$$
$$(y^2)^2 = \frac{1}{16}$$