W203, Test 1

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Q2.1 Solve for the constant
Given that

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_{X,Y}(x,y)\,dx\,dy = 1 \tag{1}$$

we can write for this case:

$$\int_{x=0}^{x=2}\int_{y=0}^{y=2} c * x^2\,y\,dx\,dy = 1 \tag{2}$$

consequently:

$$c * \Big|_{x=0}^{x=2}\frac{x^3}{3}\Big|_{y=0}^{y=2}\frac{y^2}{2} = \frac{c}{6} * 2^3 * 2^2 = 1 \tag{3}$$

Therefore

$$c = \frac{3}{16}$$

Q3.2 Expected value of $A = 2X_1X_2^2 + 3X_2X_3^3$
Using LOTUS:

$$\begin{aligned} E[A] = \iiint_V A(x_1,x_2,x_3) * f(x_1,x_2,x_3)\,dx_1\,dx_2\,dx_3 &= \\ \iiint_V 8(2x_1\,x_2^2 + 3x_2\,x_3^3)x_1\,x_2\,x_3\,dx_1\,dx_2\,dx_3 &= \\ \iiint_V 16x_1^2x_2^3x_3\,dx_1\,dx_2\,dx_3 + \iiint_V 24x_1x_2^2x_3^4\,dx_1\,dx_2\,dx_3 &= \\ \Big|_0^1\frac{16x_1^3x_2^4x_3^2}{3*4*2} + \Big|_0^1\frac{24x_1^2x_2^3x_3^5}{2*3*5} &= \\ = \frac{2}{3} + \frac{3}{5} = 1\frac{4}{15} \end{aligned}$$

Q3.3 Maximum Value
$Y = max[X_1, X_2, X_3] < y$

$$\begin{aligned} P(Y) = P(X_1 < y) \cap P(X_2 < y) \cap P(X_3 < y) &= \\ \text{because of independence} \\ \prod_{i=1}^{3}\int_0^y f_{X_i}(x_i)\,dx_i = \prod_{i=1}^{3}\int_0^y 2x_i\,dx_i = \prod_{i=1}^{3}\Big|_0^y x_i^2\,dx_i &= (y^2)^3 \\ P(Y|y = 0.5) &= \frac{1}{64} \end{aligned}$$

Q3.4 New maximum
$P(Y|X_1 = 0)$

$$\begin{aligned} P(Y|X_1) &= \frac{P(X_1 < y) \cap P(X_2 < y) \cap P(X_3 < y)}{P(X_1 < y)} \\ P(Y = 0.5|X_1 = 0) &= \frac{P(0 < 0.5) \cap P(X_2 < 0.5) \cap P(X_3 < 0.5)}{P(0 < 0.5)} = \\ P(Y = 0.5|X_1 = 0) = P(X_2 < 0.5) \cap P(X_3 < 0.5) &= \\ (y^2)^2 &= \frac{1}{16} \end{aligned}$$