# Unit 1 proofs

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*1. Let's prove that $P(A \cap B)$ even exists:* $\exists A \cap B \equiv \exists x : x \in A, x \in B$

Let's assume the opposite, i.e.

$$\not\exists x : x \in A, x \in B \equiv A \cap B = \varnothing$$

with this assumption

$$P(\Omega) = P(A) + P(B) + P(\overline{A \cup B})$$

$P(\Omega) = 1$ because of axiom, calculating the expression above:

$$P(\overline{A \cup B}) = -0.25$$

which is against the axioms, therefore original assumption is false

*2. Now we can prove that given a set of events $A = \{A_0, A_1 \ldots A_n\}$ the highest probability of all events occurring at the same time $P\left(\cap_{i=0}^{n} A_i\right)_{max} = P(A_{min})$*

i.e. probability of the intersection of a series of events can not exceed the probability of the smallest event but can be equal to it.

Note that $\exists x \in \cap_{i=0}^{n} A_i : x \notin A_i$ is impossible by definition of union, therefore:

$$\forall A_i \in A, \bigcap_{i=0}^{n} A_i \leq A_i$$

The statement above holds for $A_{min}$ just as well as for any other $A_i$, therefore:

$$P\left(\bigcap_{i=0}^{n} A_i\right) \leq P(A_{min})$$

At the same time $x \in \cap_{i=0}^{n} A_i$ can include at least $\forall x \in A_{min}$, therefore:

$$P\left(\bigcap_{i=0}^{n} A_i\right)_{max} \geq P(A_{min})$$

Combining these two statements

$$P\left(\bigcap_{i=0}^{n} A_i\right)_{max} = P(A_{min})$$

In case of the events in question, $P(A \cap B)_{max} = P(A) = 0.5$