$$f(x,y) = \begin{cases} \frac{0.5}{x} & 0 < y < x < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

Unit 2, Q 1.2

Note, that

$$\int_{x=0}^{2} dx \int_{y=0}^{x} \frac{0.5}{x}\, dy = \int_{0}^{2} dx \cdot 0.5 = 1,$$ i.e. total prob. = 1, consistent w axioms.

this is equivalent to:

$$\int_{y=0}^{2} dy \int_{x=y}^{2} \frac{0.5}{x}\, dx = \int_{y=0}^{2} dy \left[0.5(\ln 2 - \ln y)\right] = 0.5\int_{0}^{2} \ln 2\, dy - 0.5\int_{0}^{2} \ln y\, dy$$

$$= \ln 2 - 0.5 \Big|_{0}^{2} (y \ln y - y) = \ln 2 - \ln 2 + 1 = 1$$ again, total prob = 1

Unit 2 Q 1.3

Marginal PDF for Y

$$f_Y(y) = \int_{x=y}^{2} \frac{0.5}{x}\, dx = \frac{\ln 2 - \ln y}{2}$$

Unit 2 Q 1.4

Conditional PDF of X, given Y =

$$f_{X|Y}(x,y) = \frac{f(x,y)}{f_Y(y)} = \frac{0.5}{x} \div \frac{\ln 2 - \ln y}{2} =$$

$$= \frac{1}{x(\ln 2 - \ln y)} \text{ for } 0 < y < x < 2$$