Unit 1 proofs

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Intersection Minimum

Given: P(A) = 0.5 and P(B) = 0.75 What is the minimum possible value for P($A \cup B$)

We have already proved that $\exists P(A \cap B)$

Now we can prove that $P(A \cap B)_{min} = 0.25$

From the axioms:

$$P(\Omega) = P(A \cup B) + P(\overline{A \cup B})$$

From the probabilities of unions formula:

$$P(\Omega) = P(A) + P(B) - P(A \cap B) + P(\overline{A \cup B})$$

Rearrange:

$$P(A \cap B) = P(A) + P(B) - P(\Omega) + P(\overline{A \cup B})$$

$$P(A \cap B) = 0.5 + 0.75 - 1 + P(\overline{A \cup B})$$

Note that $P(\overline{A \cup B})_{min} = 0$, therefore:

$$P(A \cap B)_{min} = 0.25$$