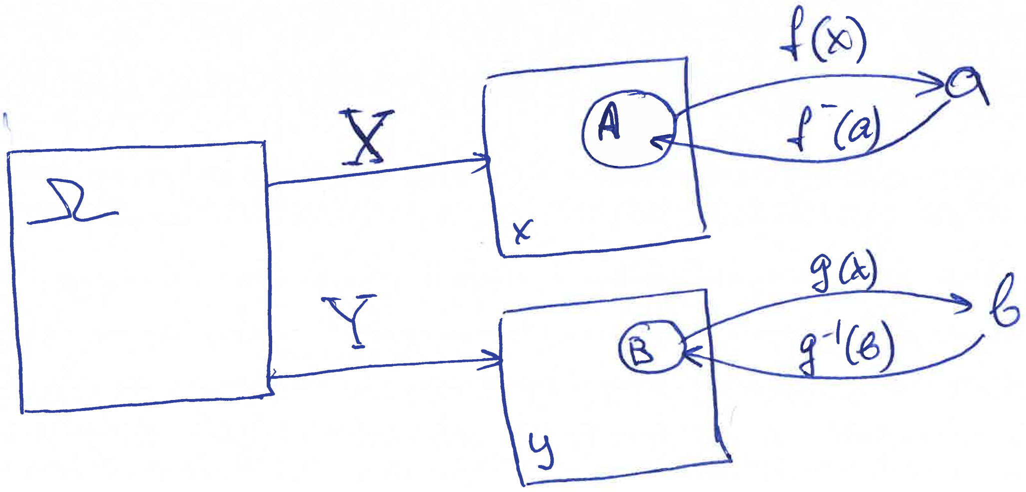

f, g - functions of random variables.

f^{-1}, g^{-1} - inverse transforms, not necessarily func

X, Y - independent random variables

what we need to prove:

$Pr(a, b) = Pr(a) \cdot Pr(b)$, given that $Pr(x, y) = Pr(x) \cdot Pr(y)$

Proof:

$Pr(f(x) = a, g(y) = b) = Pr(x \in A, y \in B) =$

$= \sum_{x \in A, y \in B} Pr(x, y) = \sum_{x \in A} Pr(x) \cdot \sum_{y \in B} Pr(y)$ because X and Y are independent.

Note, that $A = f^{-1}(a)$ and $B = g^{-1}(b) \Rightarrow$

$\sum_{x \in A} Pr(x) \cdot \sum_{y \in B} Pr(y) = Pr(x \in f^{-1}(a)) \cdot Pr(y \in g^{-1}(b)) =$

$= Pr(a) \cdot Pr(b)$ $\square$

This is a proof for any descrete random variable, including Bernoulli trials.

Here I assumed, that inverse transfoms exist, I'm not sure how to prove it.

The proof is inspired by the proof by Fang-Yi Y... posted on math.stackexchange.com.