W271, Unit 1 Question 1

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1. **Expectation of binomial distribution**

   We can think of binomially distributed random variable X as a sum of outcomes of n independent Bernoulli trials x_i.

   $X = \sum_{i=1}^{n} x_i$

   $E[X] = E[\sum_{i=1}^{n} x_i] = \sum_{i=1}^{n} E[x_i]$, due to linearity of expectation

   note that $E[x] = \pi$ for all i, by definition and independence

   $E[X] = \sum_{i=1}^{n} \pi = n\pi$

2. **Variance of binomially distributed random variable**

   $V[x] = E[x^2] - E^2[x]$, almost definition

   $E^2[x] = \pi^2$

   $E[x^2] = 1^2 * \pi + 0^2 * (1 - \pi) = \pi$, therefore:

   $V[x] = \pi - \pi^2 = \pi(1 - \pi)$

   similarly to expectation of X, variance of X is also a sum of variances of x :

   $V[X] = V\sum_{i=1}^{n} x_i] = \sum_{i=1}^{n} V[x_i]$, therefore:

   $V[X] = nV[x] = n\pi(1 - \pi)$